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Advanced Workshop on Anderson Localization, Nonlinearity and Turbulence: a Cross-Fertilization

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Inertial particle clustering and random walks in random environments

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Inertial particle clustering and random walks in random environments

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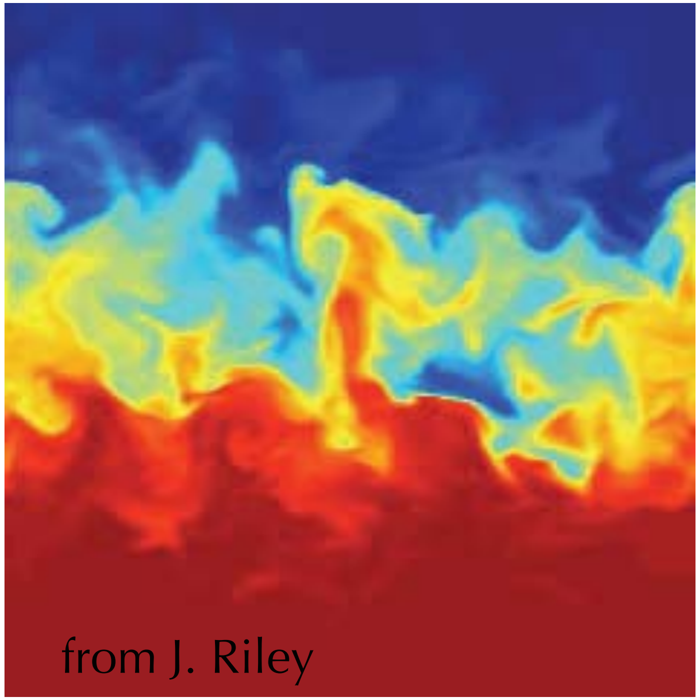
Anderson Localization, Nonlinearity and Turbulence, August 2010, ICTP, Trieste

Turbulent Transport/Mixing

- **Industrial/Natural problems:** passive or active transport of species by a turbulent flow

- **Effect of turbulence:**
  enhance mixing/dispersion
  (w.r.t. molecular diffusion)

  Eddy diffusivity
  (~mean-field effect)



- **Quantifying fluctuations?** What are the mechanisms leading to the presence of very high concentrations?

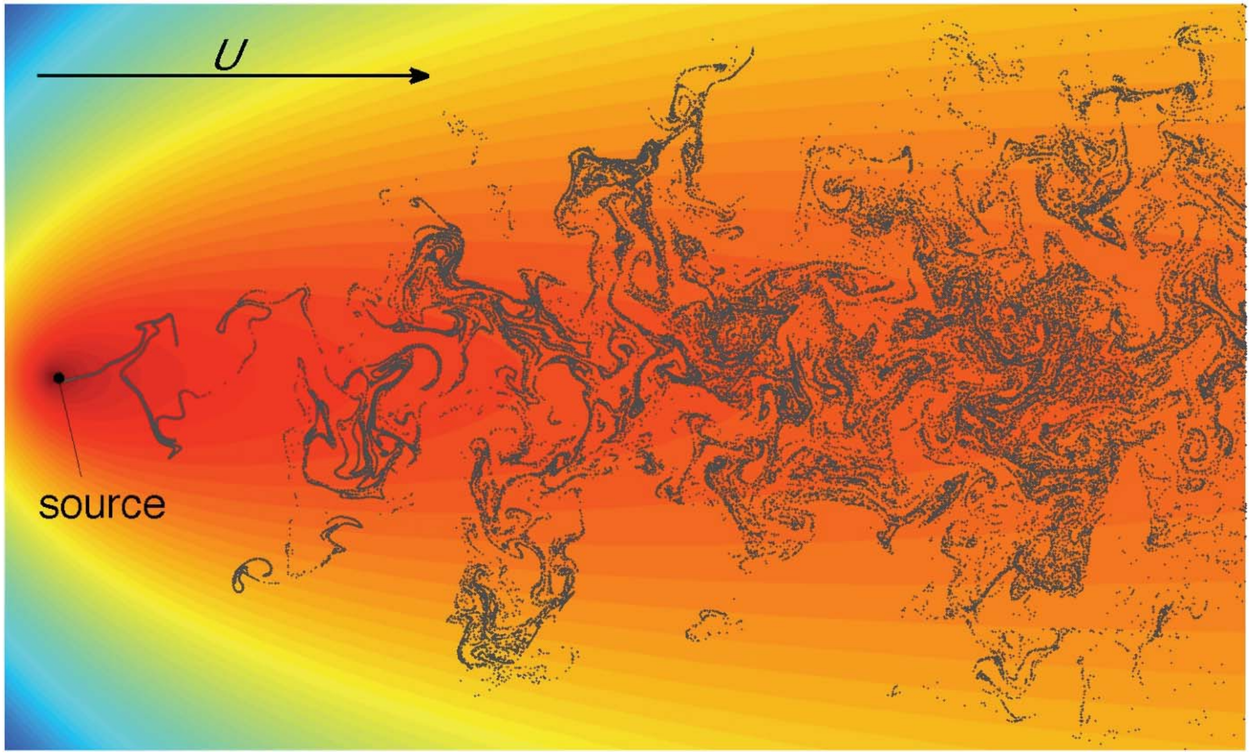
Fluctuations in turbulent transport

- Fluctuations are important for risk assessments

- **Models/Observations:** space and/or time averages

Mean vs. meandering plumes

- One source of fluctuations = the turbulent transport itself



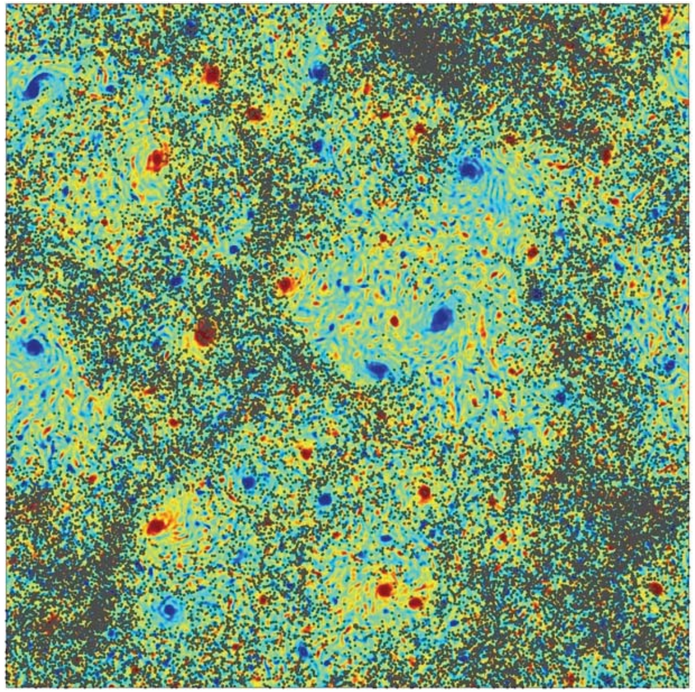
- Concentration PDFs have tails rather far from Gaussian (trapping events for the random walk of particles in the random environment of subjacent turbulent fluctuations?)

Universality??

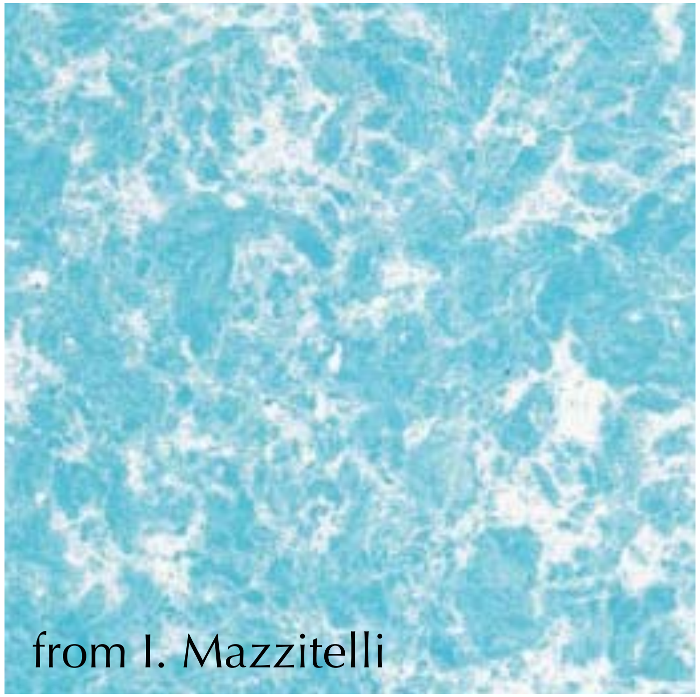
Particles finite mass

- **Most particles are not tracers but have inertia**

Heavy particles are ejected from eddies



Light particles cluster in their cores



Preferential concentration

Very heavy particles

- Spherical particles much much heavier than the fluid, feeling no gravity, evolving with moderate velocities: **one of the simplest model**

$$\ddot{\boldsymbol{X}} = -\frac{1}{\tau}\left(\dot{\boldsymbol{X}} - \boldsymbol{u}(\boldsymbol{X},t)\right)$$

↑ Prescribed velocity field (random or solution to NS)

Stokes time:

$$\tau = \frac{2\,\rho_p\,a^2}{9\,\rho_f\,\nu}$$

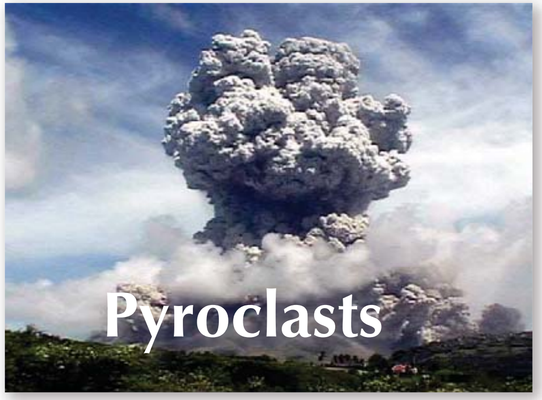
Inertia measured by the Stokes number

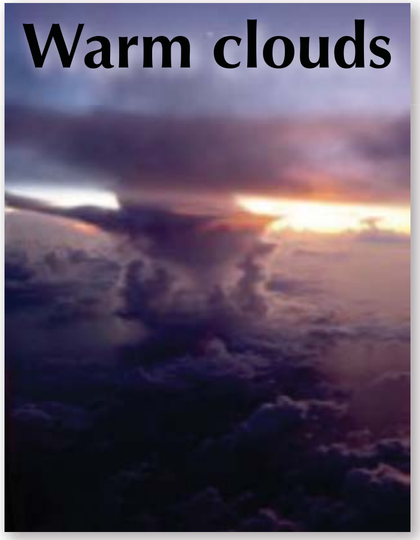
$$\mathrm{St} = \tau/\tau_{\mathrm{f}}$$

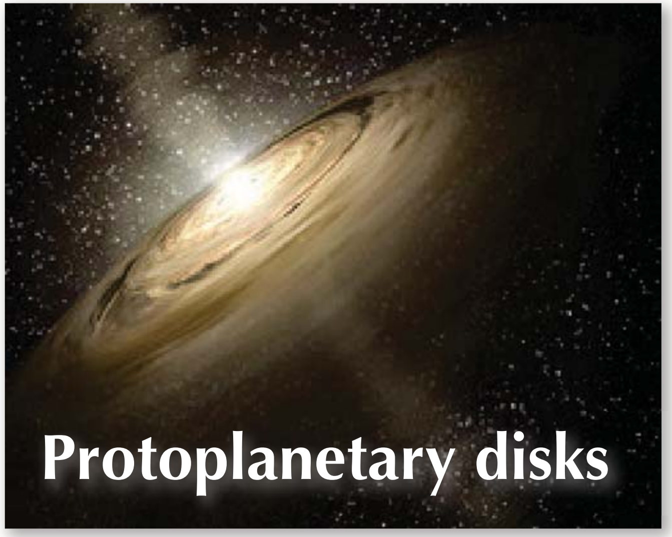
- **Dissipative dynamics** (even if $\boldsymbol{u}(\boldsymbol{x},t)$ is incompressible)

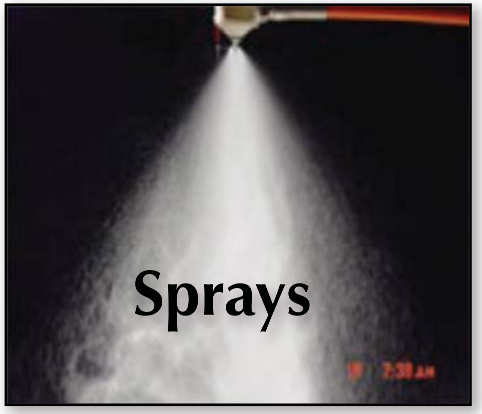
  Lagrangian averages correspond to an SRB measure (with support on the attractor) that depends on time and on the realization of the fluid velocity field.

Very heavy particles







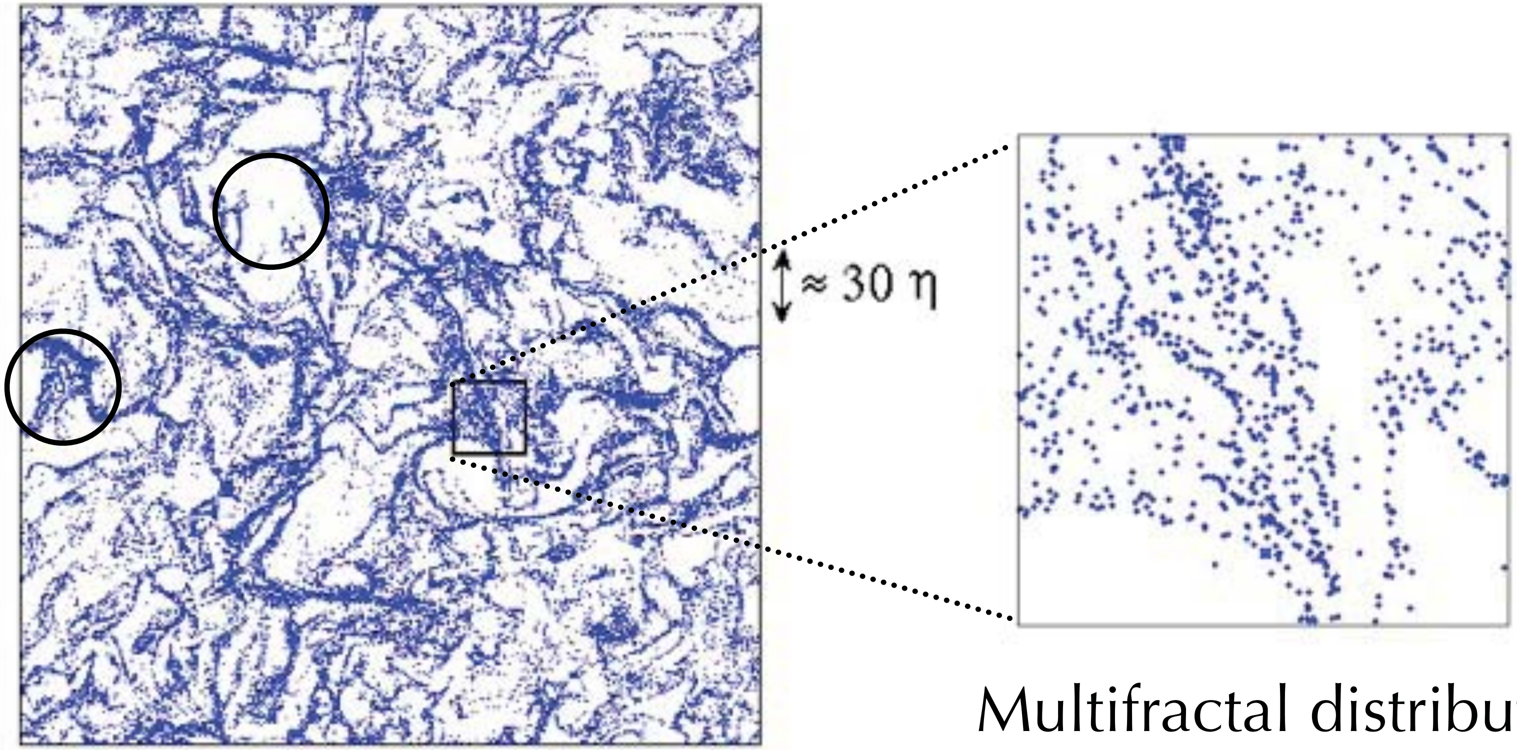


- Need for quantifying inertia effects on concentration fluctuations, which in turn affect global quantities

Preferential concentrations

- Observed for a long time in experiments
  Eaton & Fessler (1994); Douady, Couder, & Brachet (1991)

- **Quantifying them is important for**
  - the rates at which particles interact
  - the fluctuations in the concentration of a pollutant
  - the possible feedback of the particles on the fluid



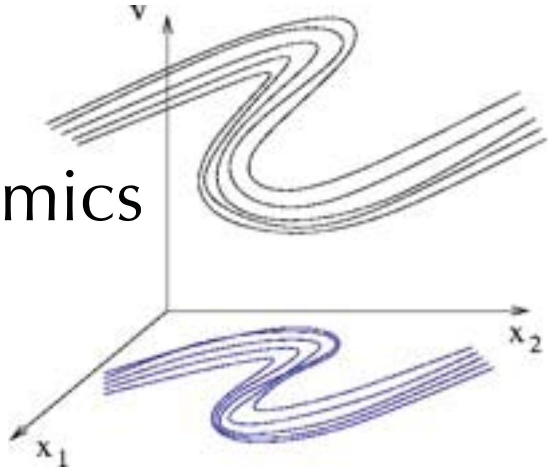
Inertial-range clusters and voids

Multifractal distribution at dissipative scales

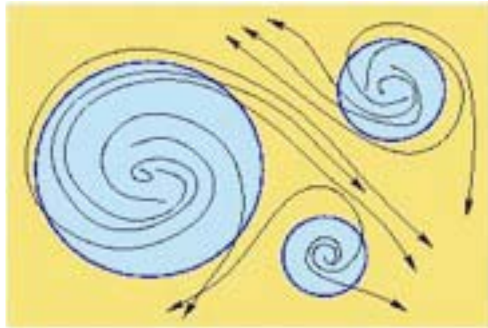
Phenomenology of clustering

- **Different mechanisms:**

Dissipative dynamics
$\Rightarrow$ **attractor**



Ejection from **eddies**
by centrifugal forces



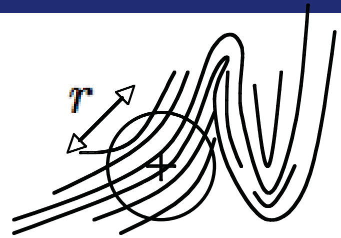
- **Theory:** requires elaborating models to disentangle these two effects. For instance:
  - flows with no structures (uncorrelated in time) to isolate the effects of a dissipative dynamics
  - coarse-grained closures to understand ejection from eddies

- Numerics show that these effects act at different scales

Small-scale clustering

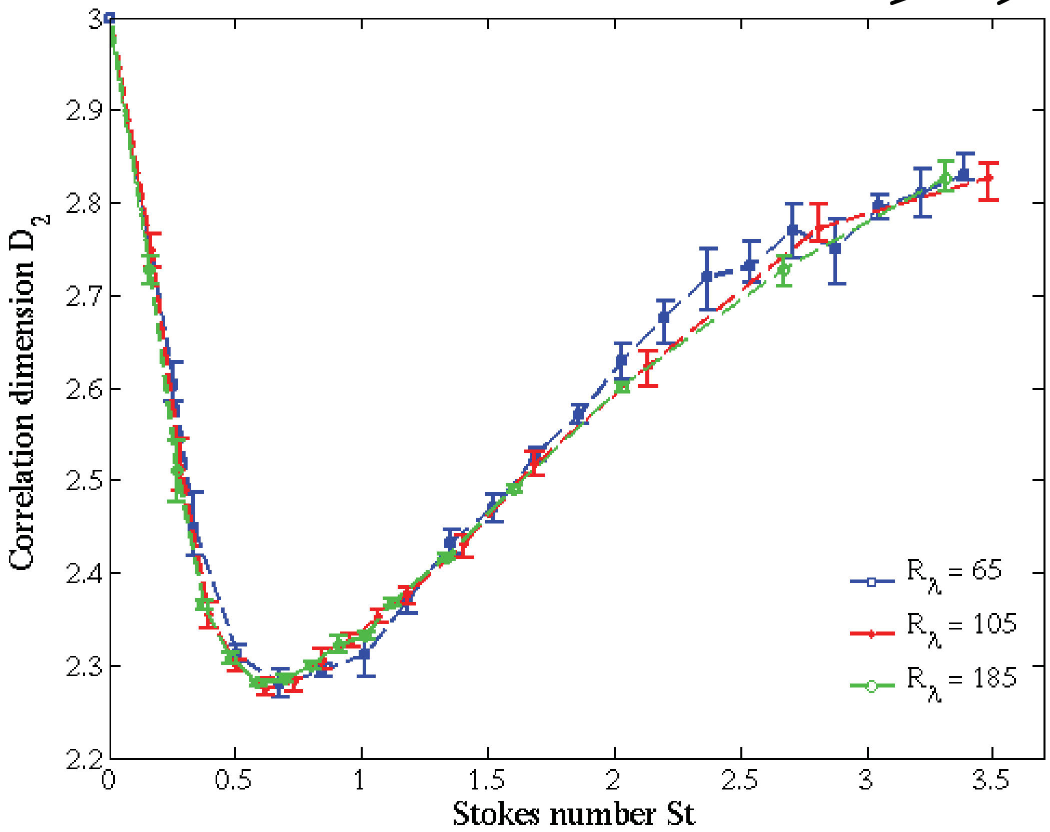
- **Fractal dimensions** $r \ll \eta$

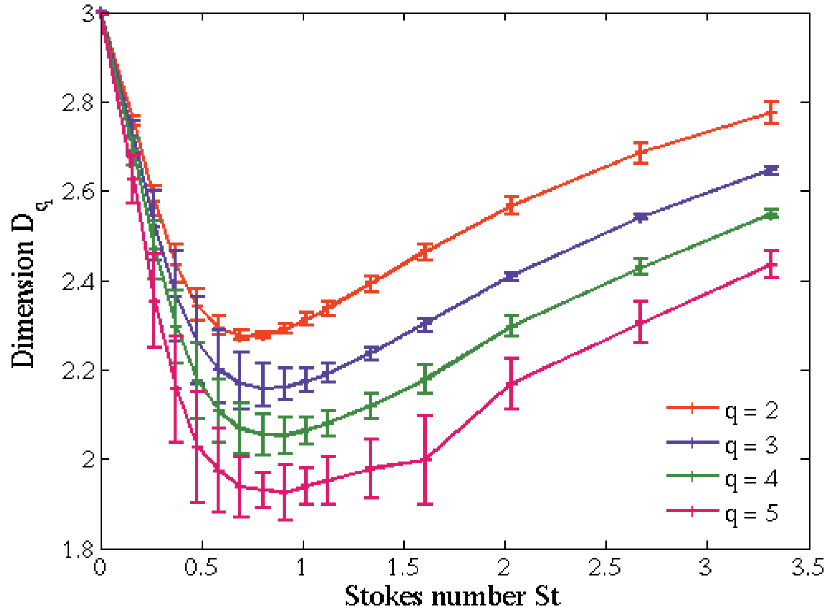
  Coarse-grained density $\bar{\rho}_r$



$$\langle \bar{\rho}_r^p \rangle \sim r^{p(\mathcal{D}_{p+1}-d)}$$

Hentschel–Procaccia (1983) spectrum for dimensions





- Spectrum $\mathcal{D}_p$ is a function of St but almost not depends on Re

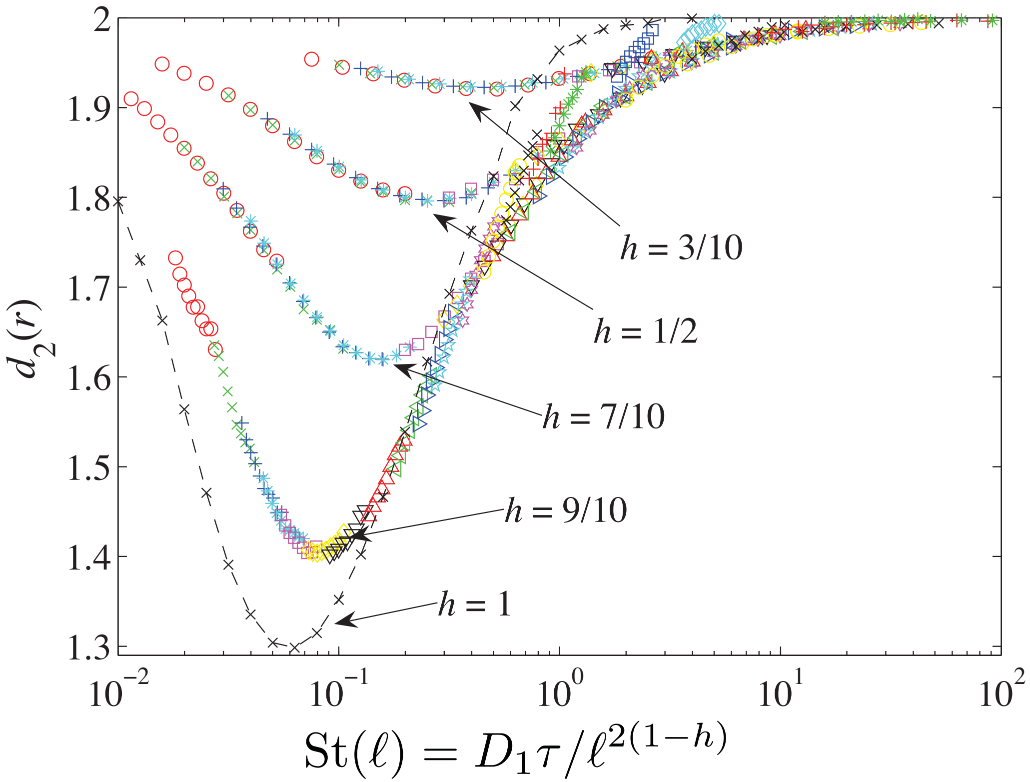
  PDF local dimension $\delta_r = \dfrac{\ln \bar{\rho}_r}{\ln r}$ $\qquad p_r(\delta) \propto r^{\mathcal{S}(\delta,\mathrm{St})}$

Inertial-range clustering ?

- Case of *non-differentiable* Kraichnan: particle dynamics at scale ℓ depends on a **local (scale-dependent) Stokes number**

$$\mathrm{St}(\ell) = \tau/\tau_\ell = \varepsilon^{1/3}\tau/\ell^{2/3}$$

Falkovich, Fouxon, Stepanov (2003)
JB, Cencini, Hillerbrand (2007)



Both the scale-invariance of the fluid flow and that of the particle distribution are broken

$\ell \to \infty \quad \mathrm{St}(\ell) \to 0$

inertia becomes negligible

$\ell \to 0 \quad \mathrm{St}(\ell) \to \infty$

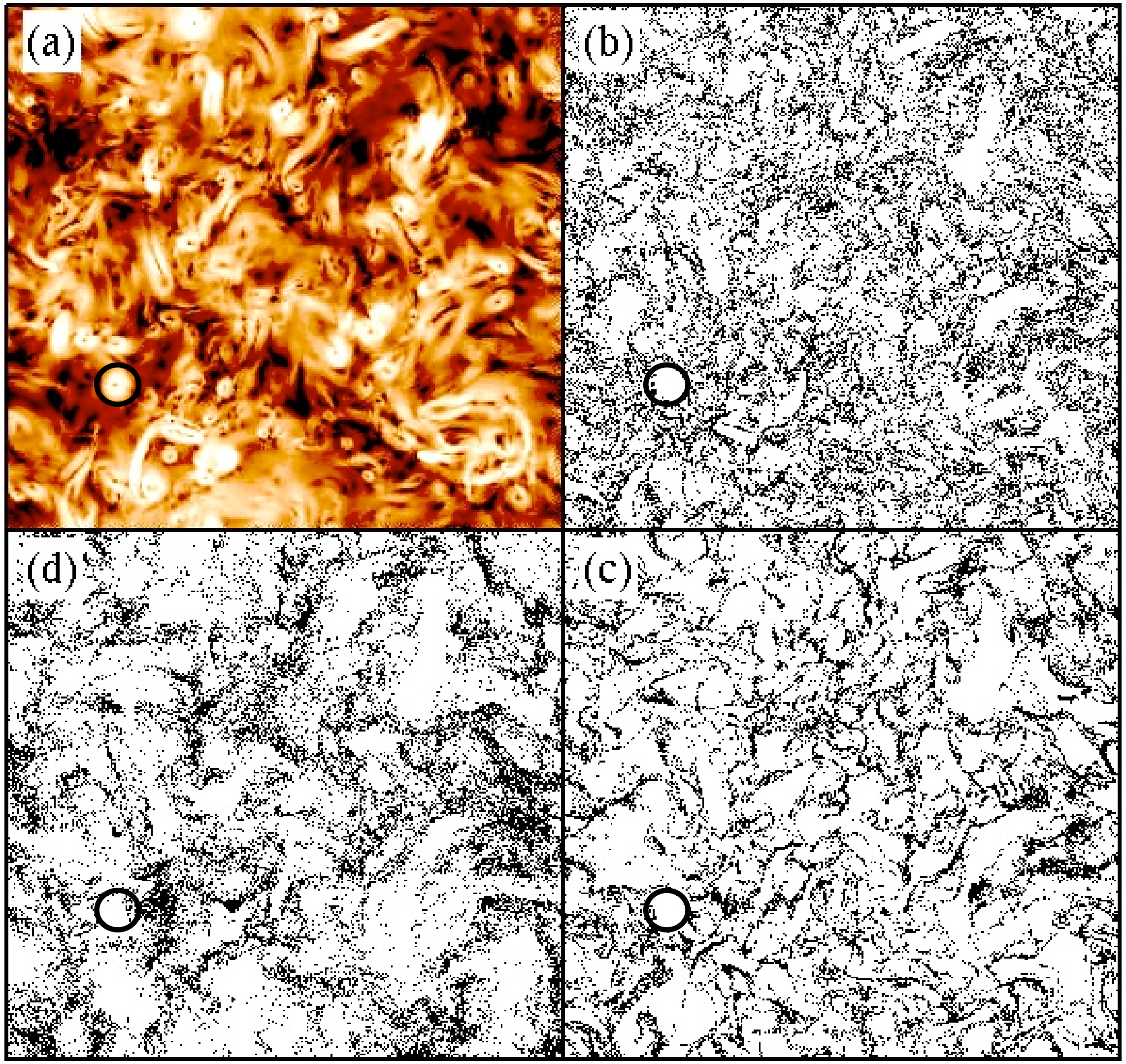
particles move almost ballistically

Particles in turbulent flow

Real flow have structure and particle distribution correlates with the acceleration field

Modulus of acceleration

$R_\lambda = 185$



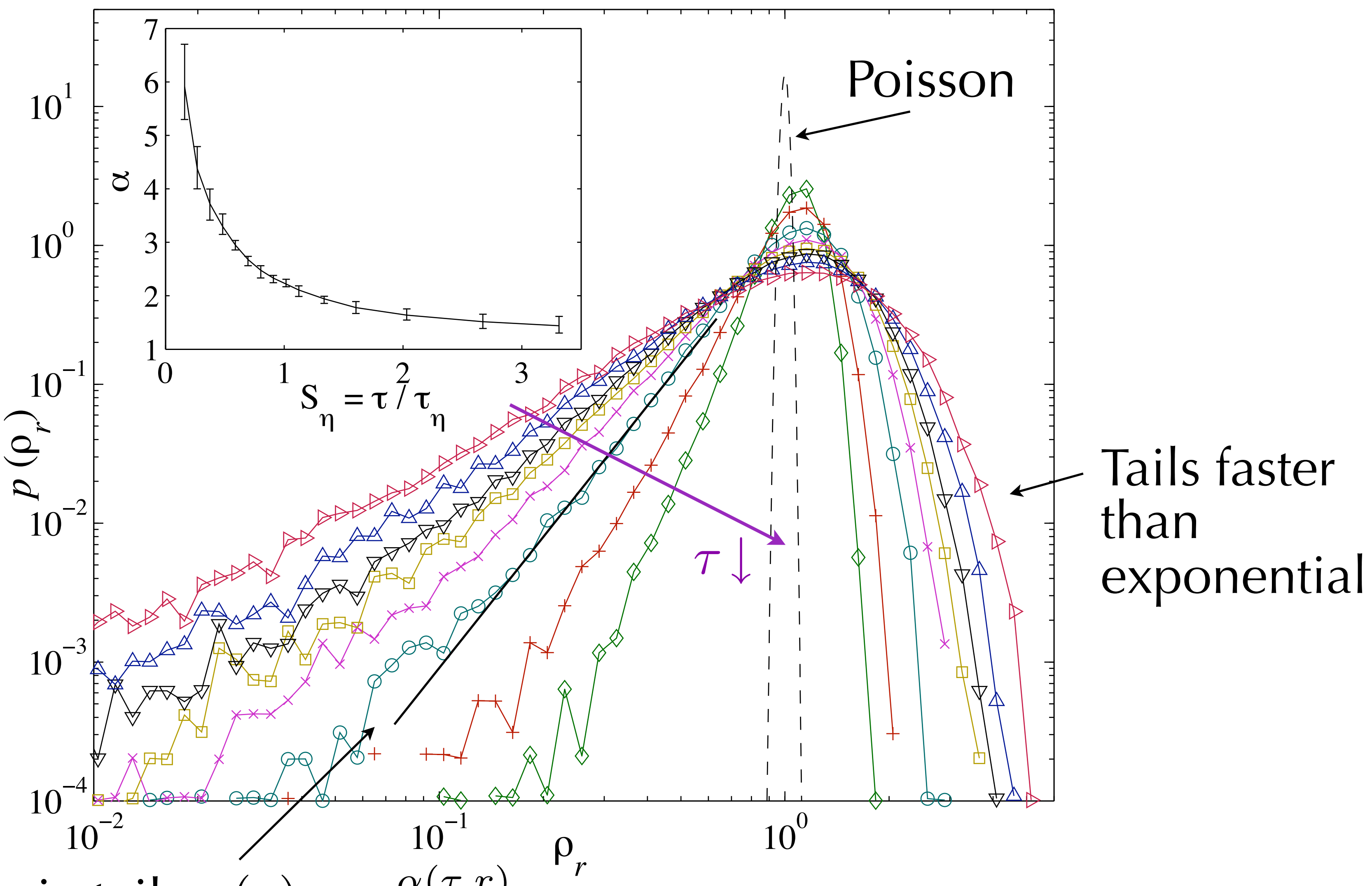
$\mathrm{St} = 0.16$

JB, Biferale *et al.* (2007)

$\mathrm{St} = 3.3$

$\mathrm{St} = 0.8$

Coarse-grained density



Algebraic tails $p(\rho) \propto \rho^{\alpha(\tau,r)}$
(signature of voids)

Time scales of clustering

- The local Stokes number $\mathrm{St}(\ell) = \varepsilon^{1/3}\tau/\ell^{2/3}$ is not relevant
- **Non dimensional contraction rate**

When inertia is very weak: $\tau \ll \tau_\mathrm{f}$

$$\ddot{\boldsymbol{X}} = -\frac{1}{\tau}\left(\dot{\boldsymbol{X}} - \boldsymbol{u}(\boldsymbol{X},t)\right) \Rightarrow \dot{\boldsymbol{X}} = \boldsymbol{u}(\boldsymbol{X},t) - \tau\ddot{\boldsymbol{X}}$$

$$\dot{\boldsymbol{X}} \approx \boldsymbol{v}(\boldsymbol{X},t) = \boldsymbol{u}(\boldsymbol{X},t) - \tau\left[\partial_t\boldsymbol{u} + \boldsymbol{u}\cdot\nabla\boldsymbol{u}\right]$$ Maxey's approximation

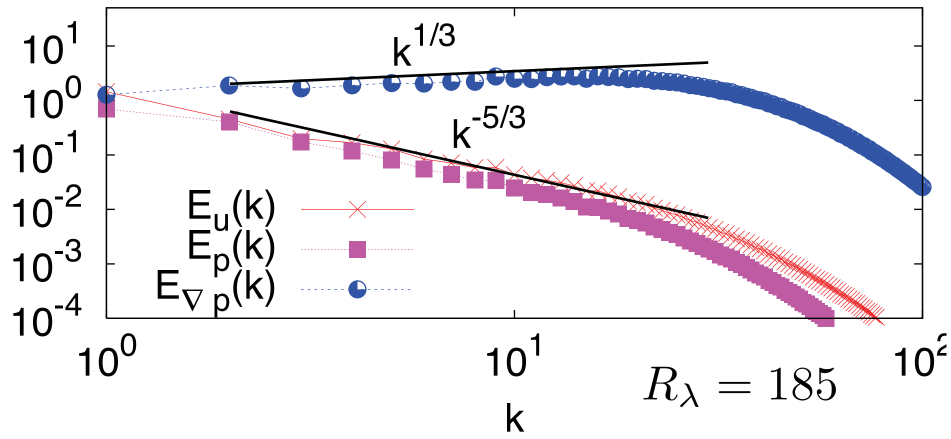
Rate at which a particle blob with size r is contracted

$$\Gamma_{r,\tau} = \frac{1}{r^3}\int_{|\boldsymbol{x}|<r} \nabla\cdot\boldsymbol{v}\ \mathrm{d}^3x \simeq \frac{\tau}{r^2}\delta_r p$$

Pressure scaling

Sweeping $\delta_r p \sim \mathrm{U}(\varepsilon r)^{1/3}$

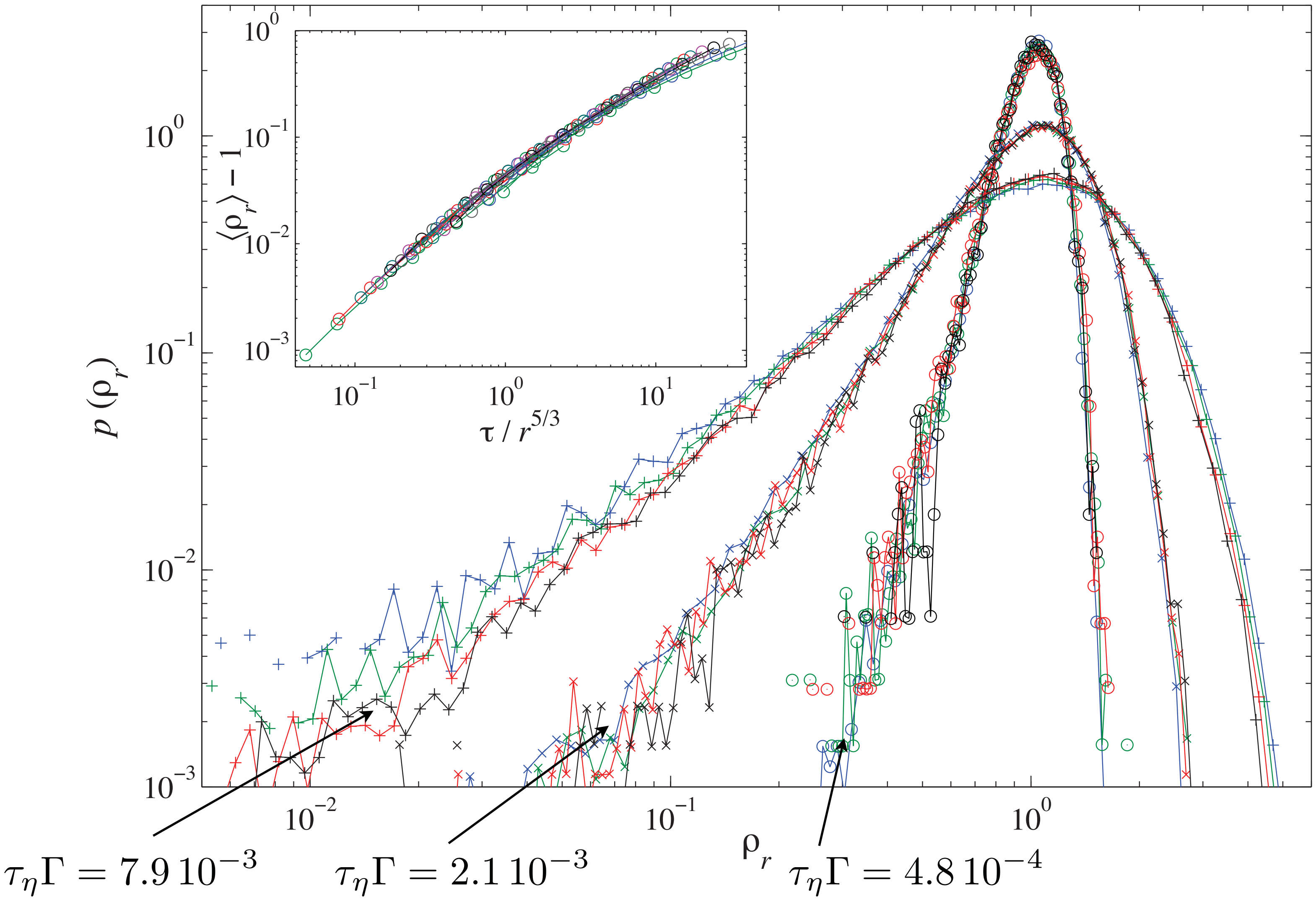
$$\Gamma_{r,\tau} \sim \tau/r^{5/3}$$



$R_\lambda = 185$

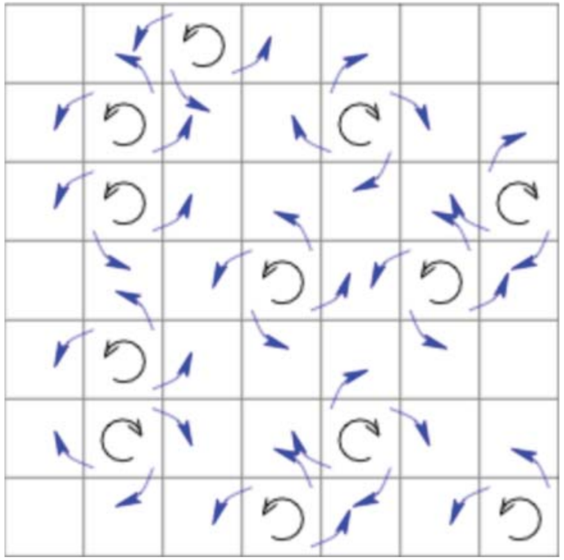
Scalable deviations from uniformity

- Mass distribution depends only on
  $$\tau_\eta \Gamma_{r,\tau} \sim \mathrm{Re}^{1/4}\,\mathrm{St}(r/\eta)^{5/3} \sim \mathrm{Re}^{-1}\,\mathrm{St}(r/L)^{5/3}$$

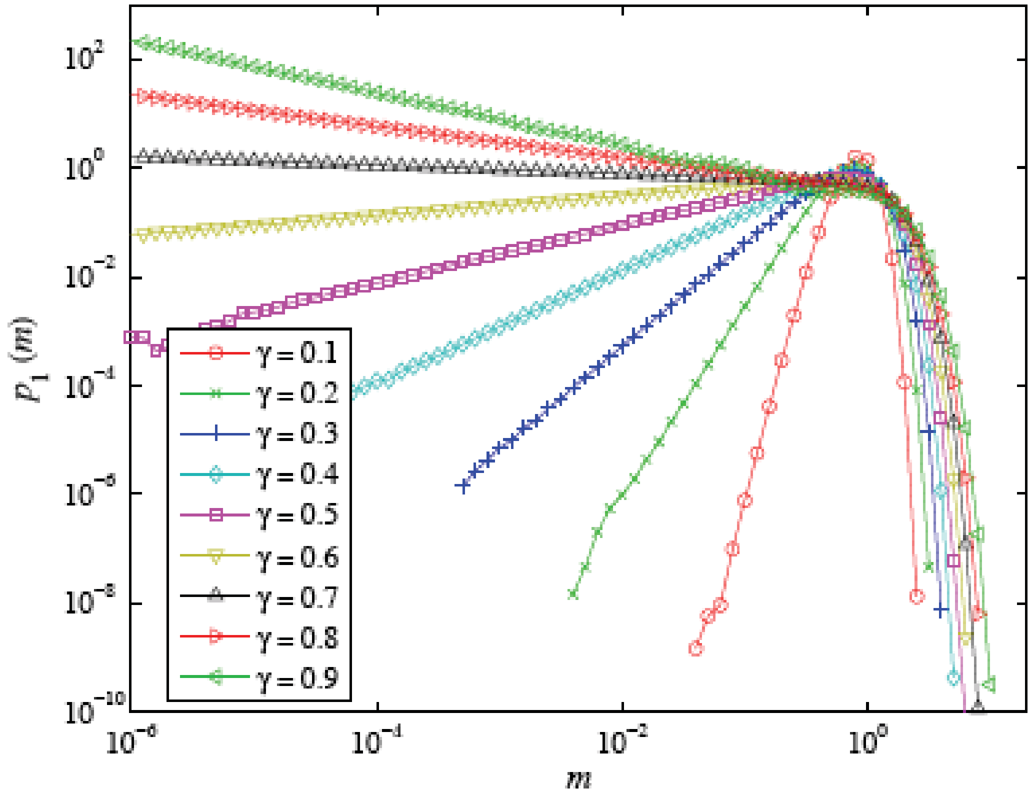


Mass transport model

- Find models belonging to the same universality class
- Discreteness in time and space
- At each time step some (randomly chosen with probability p) cells eject a fraction of their mass to their neighbors
- Parameter = γ ejection rate (related to particle inertia)

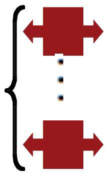


JB, R. Chétrite (2007)



Tails

- Right tail = algebraic $p(m) \propto m^{\alpha(\gamma)}$

N times 

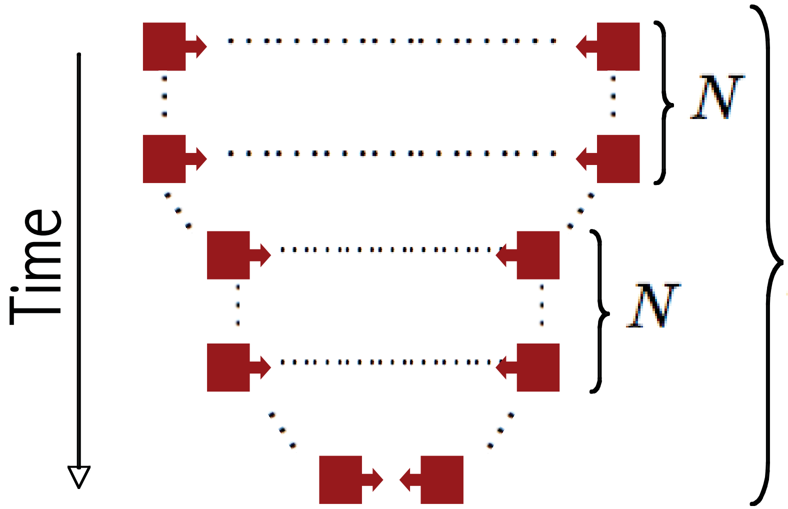
$m_0 \approx 1$

$m_N \approx (1-\gamma)^N$

$$\text{Prob} = p^N (1-p)^{2N}$$

$$\Rightarrow \alpha(\gamma) = \frac{\ln p(1-p)}{\ln(1-\gamma)}$$

- Left tail = super-exponential



$$\text{Prob} = [p^2(1-p)]^{N\,M}$$

$$m_{N\,M} = \frac{1-[1-(1-\gamma)]^N]^M}{(1-\gamma)^N}$$

$$\Rightarrow p(m) \propto \exp(-C\, m \ln m)$$

More general model

- **Continuous limit** $\mathrm{d}m_n = \gamma_{n+1} m_{n+1} + \gamma_{n-1} m_{n-1} - 2\gamma_n m_n$

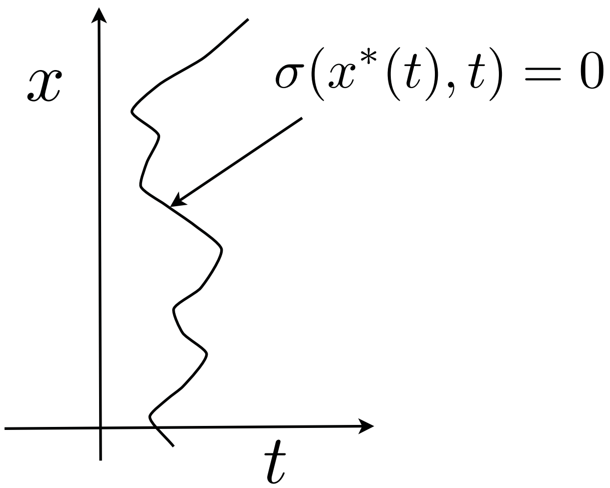
$$\left.\begin{array}{l}(\mathrm{d}x^2/\mathrm{d}t)\,\gamma_n \to \sigma^2(x,t)/2 \\ m_n/\mathrm{d}x \to \rho(x,t)\end{array}\right\} \Rightarrow \boxed{\partial_t \rho = \frac{1}{2}\partial_x^2\left(\sigma^2\,\rho\right)}$$

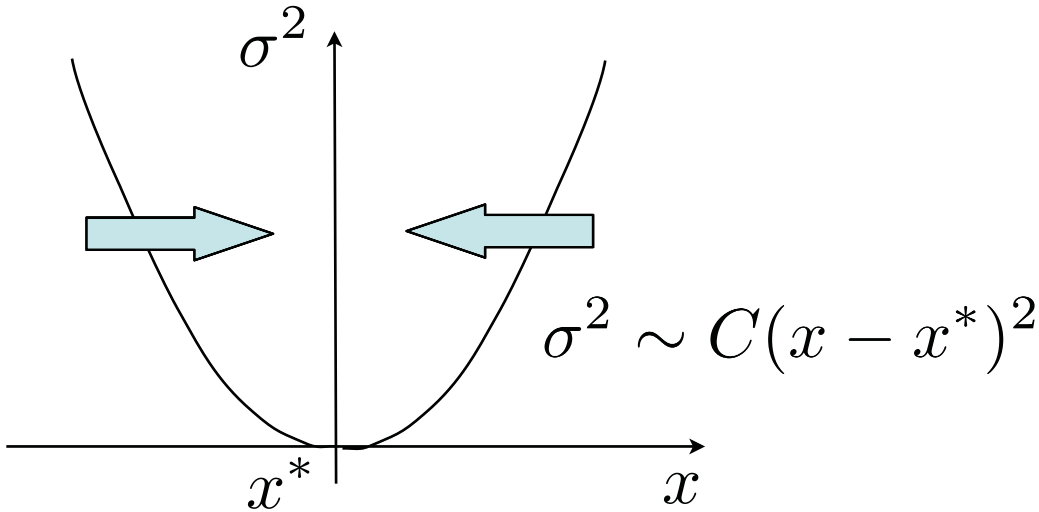
σ random $\Rightarrow$ diffusion in a random environment

Green function $\leftrightarrow$ transition probability for $\mathrm{d}X = \sigma(X,t)\,\mathrm{d}W_t$

- **High densities:** near the zeros of the diffusion coefficient σ

Example: $\sigma(x,t)$ smooth, generic, Gaussian

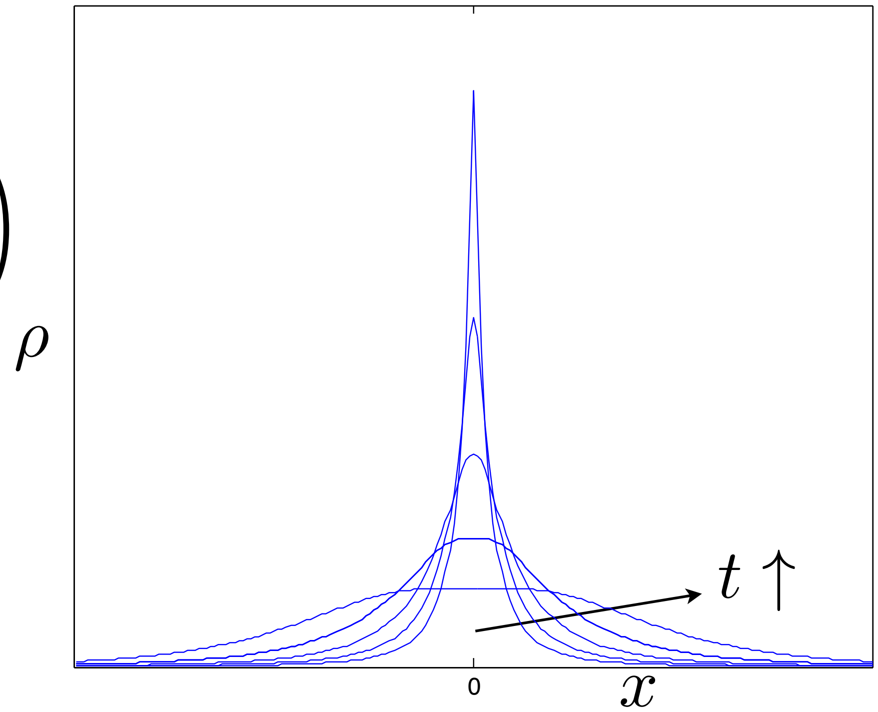




Large density tail

$$\partial_t \rho = \partial_x^2 \left(x^2 \rho\right)$$

$$\rho(x,t) = \frac{1}{2}\mathrm{e}^{2t}\mathrm{erfc}\left(\frac{\ln|x| + 3t}{2\sqrt{t}}\right)$$



Cumulative probability $P^{>}(\rho)$
= fraction of space-time where
the density is larger than ρ

$$\lambda = 2t/\ln\rho$$

$$P^{>}(\rho) \simeq \ln\rho \int_1^\infty \mathrm{e}^{-\mu(\lambda)\ln\rho}\mathrm{d}\lambda \qquad \mu(\lambda) = \frac{3}{2}\lambda - \sqrt{2\lambda(\lambda-1)}$$

Saddle-point: $P^{>}(\rho) \propto \rho^{-1}$

Universal intermediate asymptotics where $\boxed{p(\rho) = -\frac{\mathrm{d}P^{>}}{\mathrm{d}x} \propto \rho^{-2}}$

+ non-universal cut-off (distribution of zeros lifetime)

Conclusions

- **Summary**

  Clustering of heavy particles:
  of two kinds, depending on the observation scale:
  - multifractal in the dissipative range
  - dependent on a rescaled contraction rate in the inertial range

  Connection to random walks in random environments:
  ejection models reproduce most features (in particular tails)

- **Open questions**
  - **Universality** of mass probability distribution?
  - **Spatial/temporal correlations** (e.g. scale invariance) of the ejection/diffusion rate?