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# Many-body localization and dynamics

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# Many-body localization and dynamics

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ICTP workshop on
Anderson localization, nonlinearity and turbulence: X-fertilization

# outline

- Background:

  summary of Aleiner's talk;

  the "phase diagram" and some numerical evidence for quantum many-body localization

- Classical many-body localization?

- Physical properties of non-Anderson insulators

# Summary of Aleiner's talk (Basko etal 2006)

$$H = \sum_j U_j c_j^\dagger c_j + \sum_{j\eta} V_\eta c_j^\dagger c_j c_{j+\eta}^\dagger c_{j+\eta} + \sum_{j\eta} t_\eta c_j^\dagger c_{j+\eta}$$

- Start with strong disorder Uj and ZERO interactions, $V_\eta = 0$
- At low temperature there only a few localized particle-hole excitations – and these <u>remain</u> localized for WEAK interactions for T<Tc !
- Or, equivalently, there is a mobility edge "at"

$$E_c = L^d \epsilon(T_c)$$

- For a typical configuration with

$$D(\epsilon < \epsilon(T_c)) \sim e^{-\beta(E-E_c)} = e^{-\beta L^d(\epsilon - \epsilon(T_c))} = 0$$

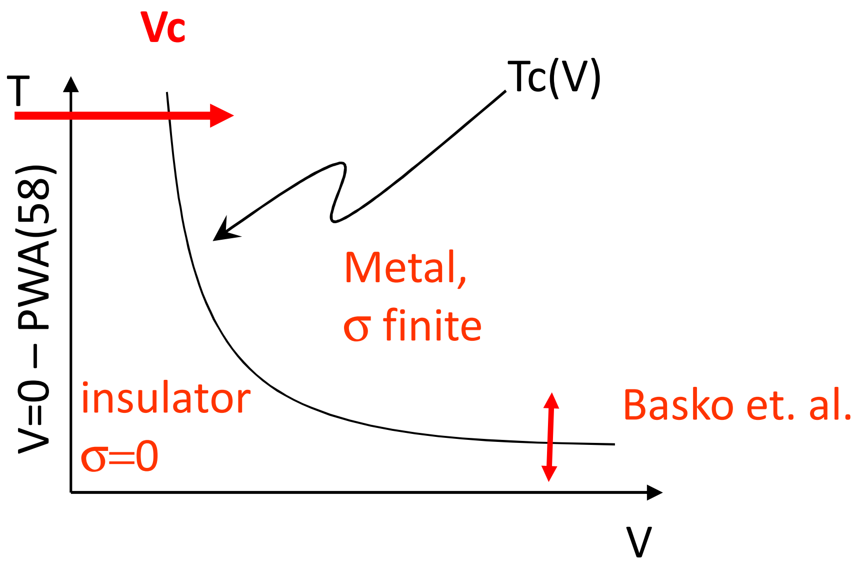
# MIT phase diagram

PRB 75, 155111 (2007)

E.g. $H = \sum_{j\eta}(t_\eta c_j^\dagger c_{j+\eta} + V_\eta \hat{n}_j \hat{n}_{j+\eta}) + \sum_j U_j \hat{n}_j \qquad |U_j| >> t, V$

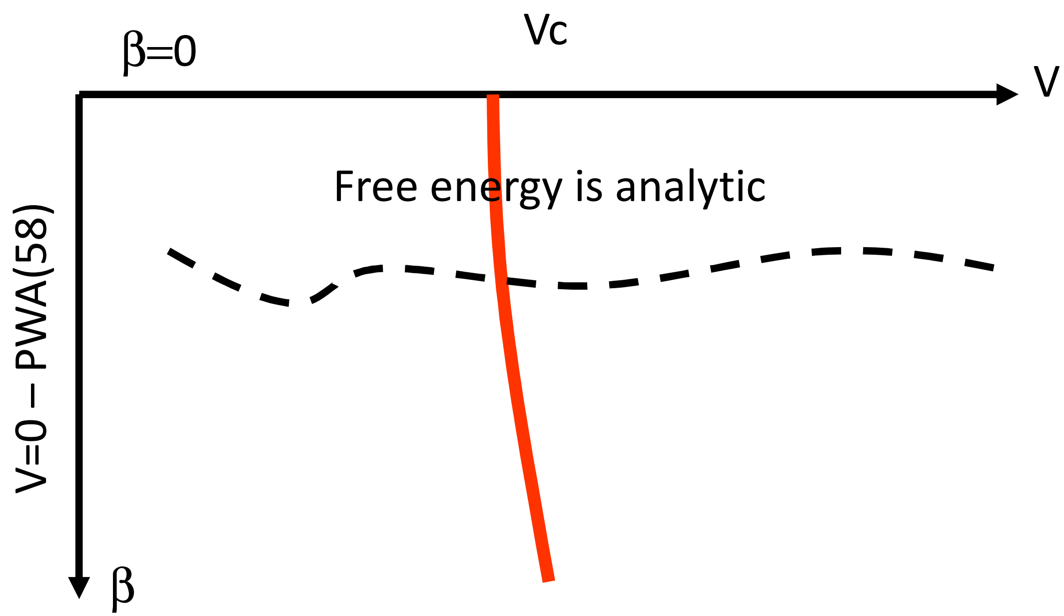
Can think of localization as a consequence of phase space restriction for delocalization of excitations by temperature OR bandwidth

**The infinite T trajectory (V.O., Huse)**




Many-body localization can survive at infinite (!) "temperature", there is a critical interaction strength

# No thermodynamic signatures near infinite temperature




The many-body localization transition is a purely dynamic phenomenon!!!

# A general comment re: lattice models and T>0

- Quite generally, asymptotic low frequency and small q behavior at ANY finite T (even small T) is better represented by INCREASING T rather than T $\rightarrow$ 0, e.g. hydrodynamic behavior is easiest to observe when all other scales have been made very short.
- Signal-to-noise ratio is maximized by sampling over all configurations and/or eigenstates, i.e. at infinite T
- Lattice models often afford easier and higher quality simulations (at the expense of not capturing some important physics of real experiments ☹)

# Numerical evidence of the many-body localization transition

By analogy with Anderson transition we can study:

a) single state properties – density of states, participation ratios and entanglement

or

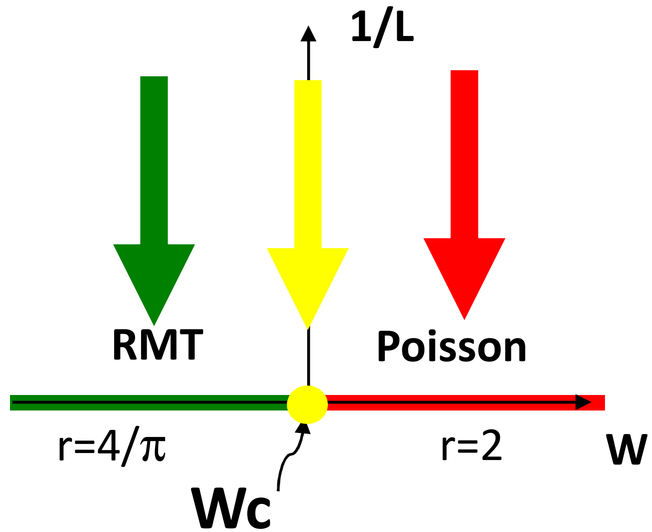
b) two level properties, e.g. conductivity and level statistics

# “One parameter” finite size scaling for level statistics

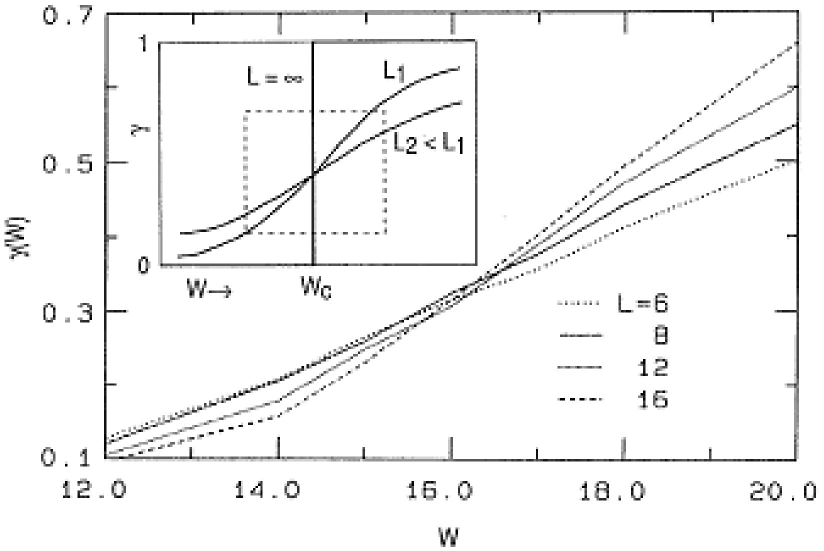
<u>Universality of gap statistics:</u>

consider $\delta_n = E_{n+1} - E_n$
and
$r = \langle\delta^2\rangle / \langle\delta\rangle^2$

Can be used
like a Binder
cumulant
e.g. in
Monte Carlo
of 3D Ising model




**“Data collapse” across 3D Anderson** (B. Shklovskii etal PRB ’93, I.Zharekheshev 80’s)



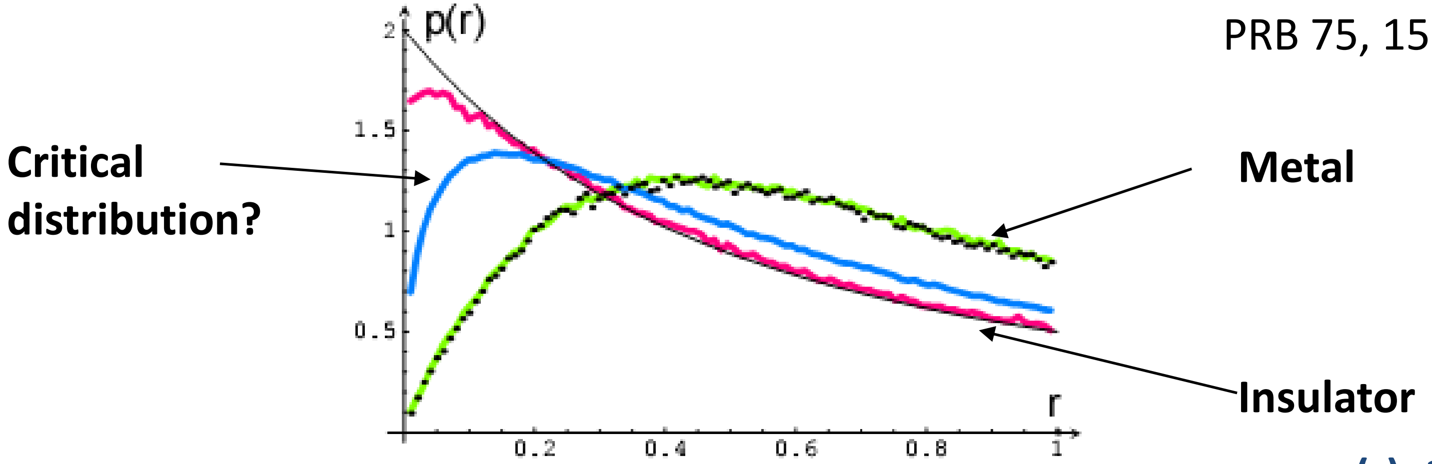

Upshot:
No phase transition in finite volume,
yet finite size corrections to
universal level statistics
can be used to find and study
the critical point

# We used two-gap distribution function

- Instead consider a dimensionless number constructed from two adjacent gaps:

$$r_n = \min(\delta_n, \delta_{n+1}) / \max(\delta_n, \delta_{n+1})$$

- Two universal distributions of r can be identified corresponding to Wigner-Dyson and Poisson stats

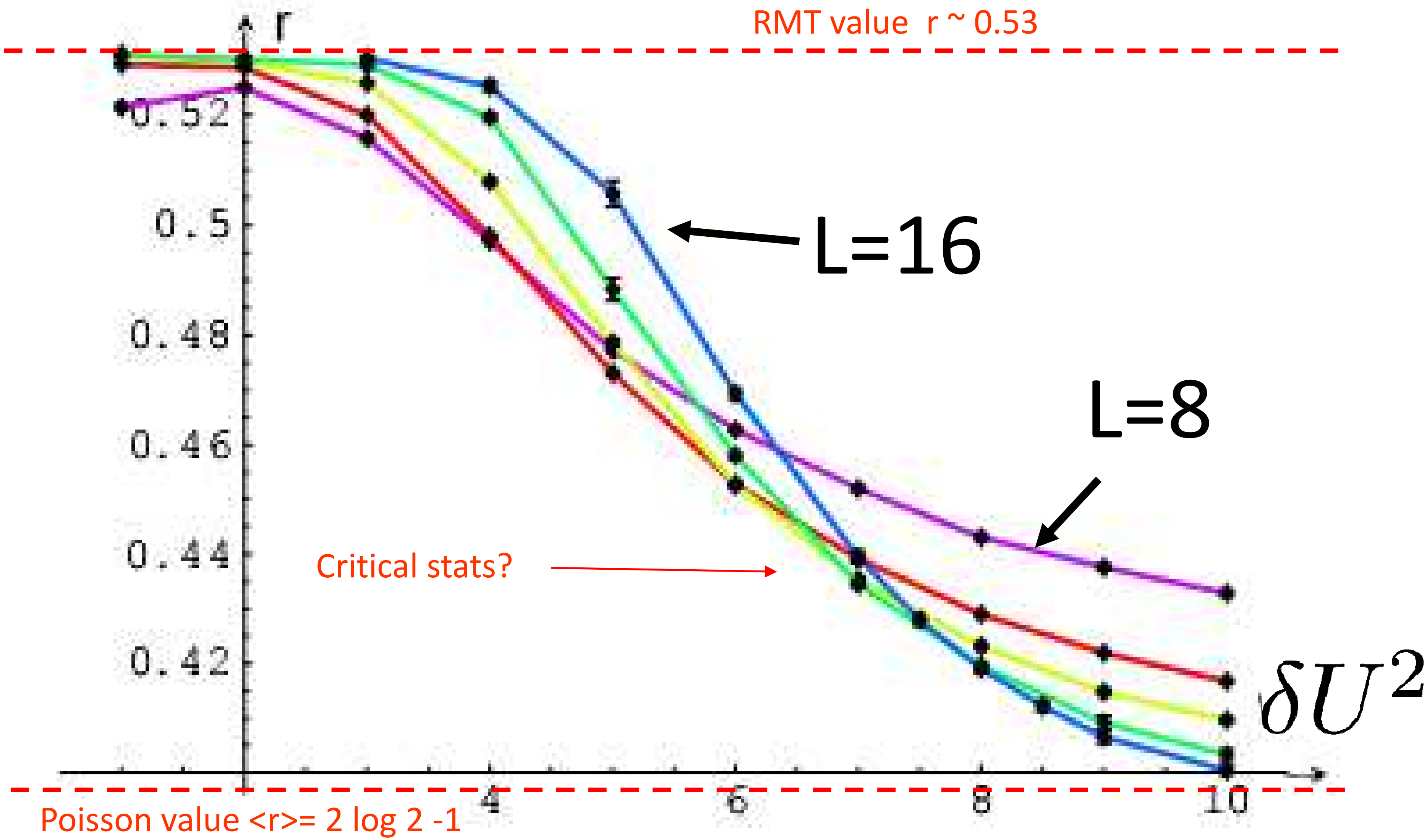



PRB 75, 155111 (2007)

**$p(r) = 2/(1+r)^2$**

- More simply $\langle r \rangle = 2 \log 2 - 1$ in the insulator, while $\langle r \rangle \sim 0.53$ in the diffusive phase, $\langle r \rangle_c = ?$

# Crossover sharpens (but drifts southeast)

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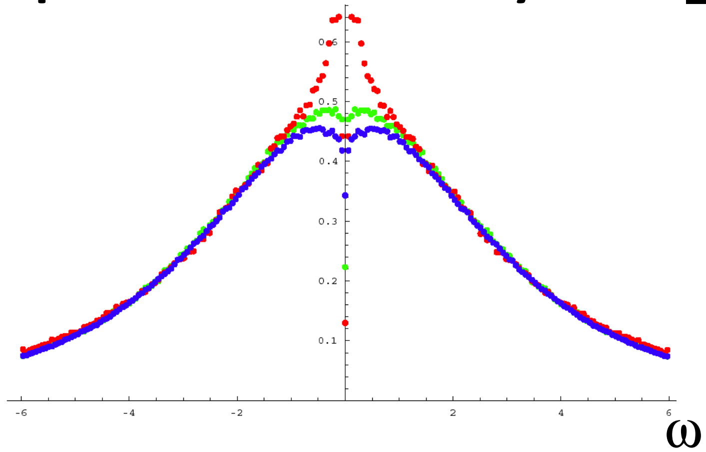
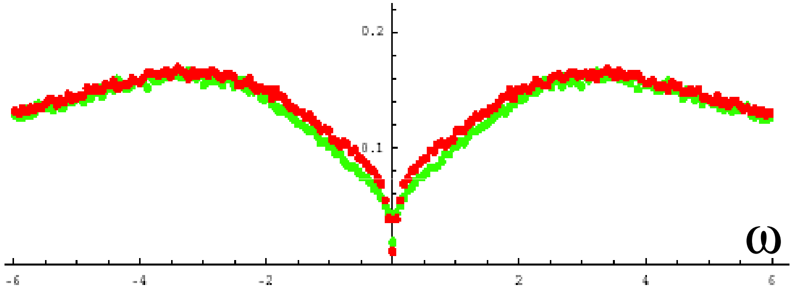
# Can also look at finite freq. conductivity vs. size

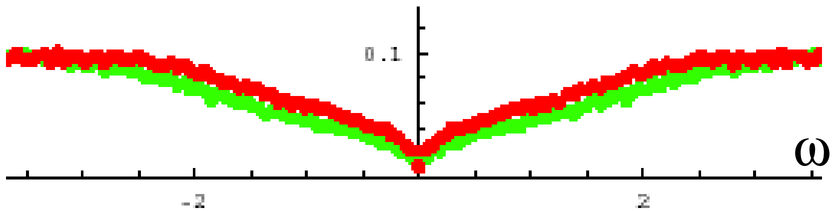
Weak disorder →




← stronger disorder

yet stronger disorder →




Preliminary comments:

- non-Mott AC behavior
- stronger finite size effects than either clean interacting model or Anderson insulator
- decent statistics/self-averaging of finite frequency response – events are not rare
- is it nonlinear hydrodynamics similar to clean interacting model? PRB 73, 035113 (2006)

# Upshot of quantum numerics

- Good news – crossover sharpens with system size, indicative of a phase transition
- Bad news – no straightforward fit to simple one parameter scaling, leaving room for other, non-critical interpretations...e.g. breakdown of the insulator.
- Quite generally, some theoretical expectation is needed to interpret numerics, e.g. one parameter scaling, expected non-Mott AC law
- How can one study and ”prove” existence of many-body insulator numerically without studying criticality?

See also, Pal/Huse **arXiv:1003.2613**

# The other $10^6 question:
## Is quantum mechanics REQURED for many-body localization?

- Lore: macroscopic, hydrodynamic phenomena can be consistently understood by coarse-graining quantum dynamics into effective classical models (usually with diffusion and noise terms)
- Many-body exceptions: superconductors and superfluids
- Anderson/Thouless/Basko etal: as one coarsegrains, the effective D(L) is reduced to zero and conventional classical description fails
- Is many-body localization transition an intrinsically quantum phase transition (at infinite T?!) or can one find and use effective classical models to study this phenomenon more efficiently?

# Any classical many-body localization?

- Finite temperature transport in all quantum models can usually be discussed in terms of classical diffusive models (possibly even strongly non-linear, e.g. PRB 73, 035113 (2006) )….
- Are all generic classical models diffusive?

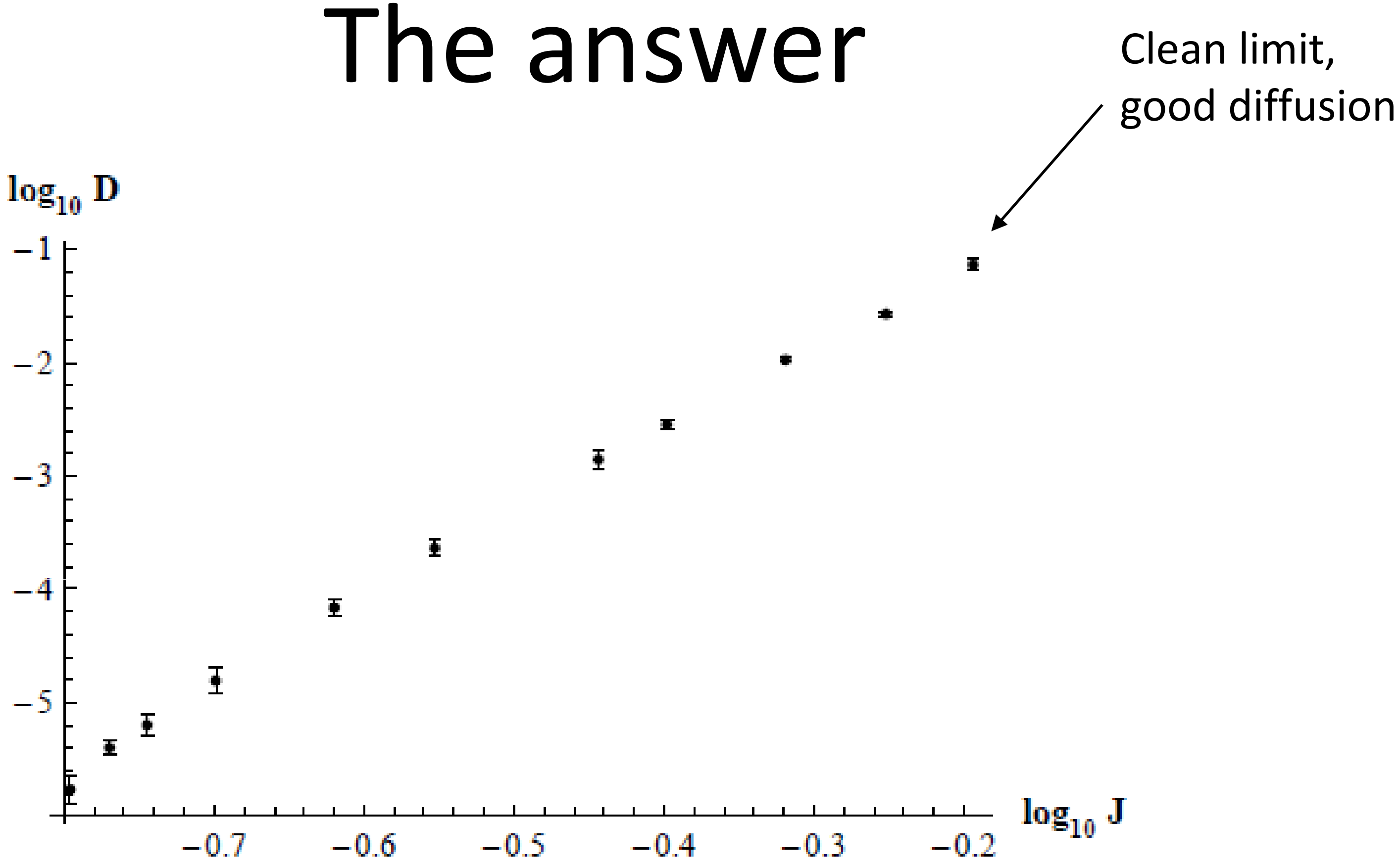
$$H = J\sum_j \vec{S}_j \cdot \vec{S}_{j+1} + \sum_j \vec{S}_j \cdot \vec{h}_j \equiv \sum_j e_j$$
$$\{S_i^a, S_j^b\} = \delta_{ij}\epsilon_{abc}S_j^c$$

**Two extreme regimes:**

- **J=0 – each spin precesses in it's own random field $h_j$ (orientation and magnitude) – no transport, but the model is "integrable" and not generic**
- **$h_j$=0 – spin and energy diffusion (e.g. numerics D. Landau et.al.)**

  - **How are these regimes "connected"?**
  - **I.e. what is (energy) conductivity/diffusion constant as a function of J/|h|, esp. as J → 0?**

# The answer




- Surprise:

an ENORMOUS suppression of diffusion already for moderate disorder

- Conjectures -- no transition, results are not dimension specific.

# How to measure diffusion near equilibrium: current or density?

- Definition via slow density relaxation:

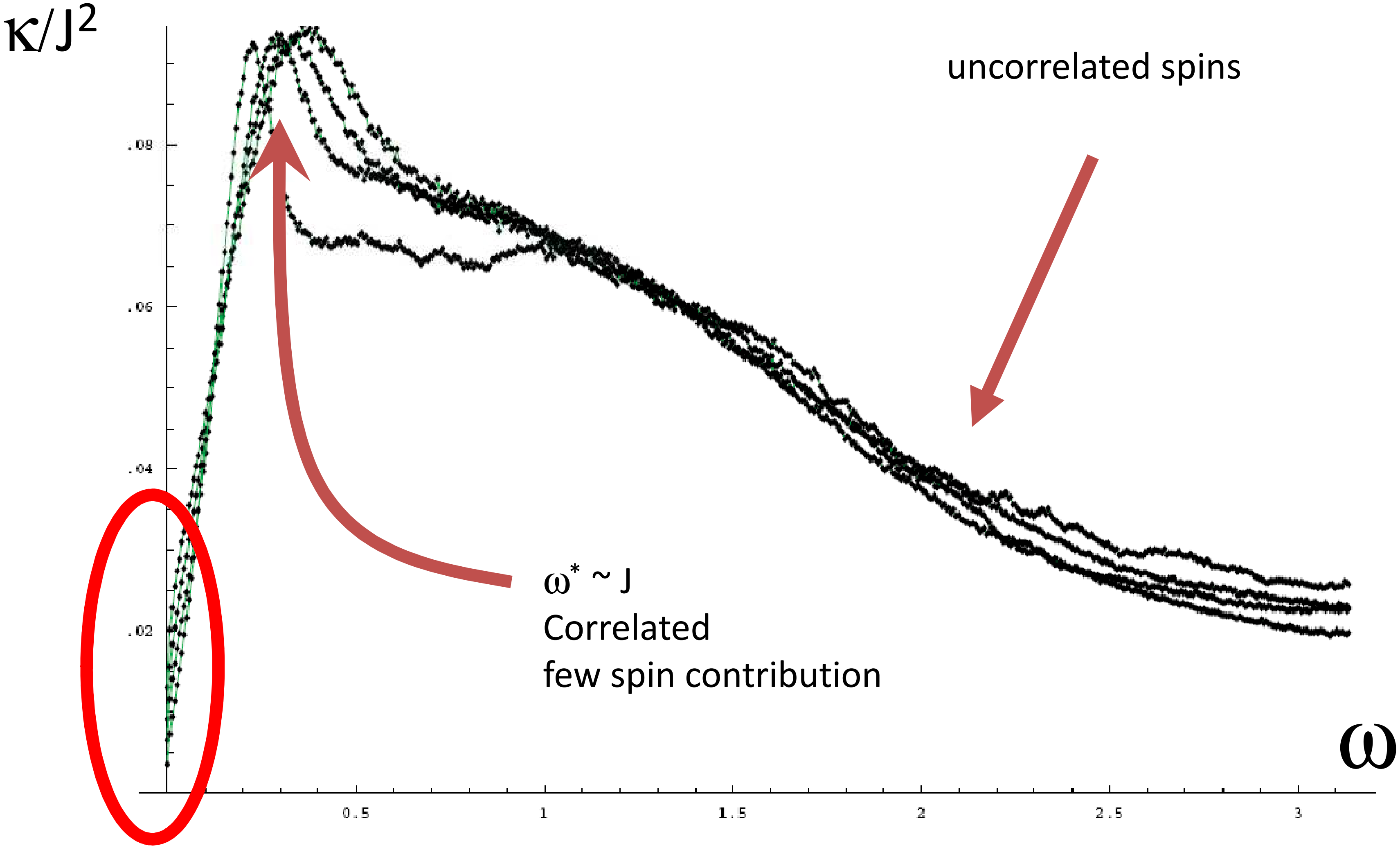
$$G(q,t) \equiv \langle e(q,t)e(q,0)\rangle / \langle e(q,0)e(q,0)\rangle \equiv e^{-Dq^2t}$$

- Often easier obtained via conductivity (Kubo+Einstein), by integrating “fast” current relaxation

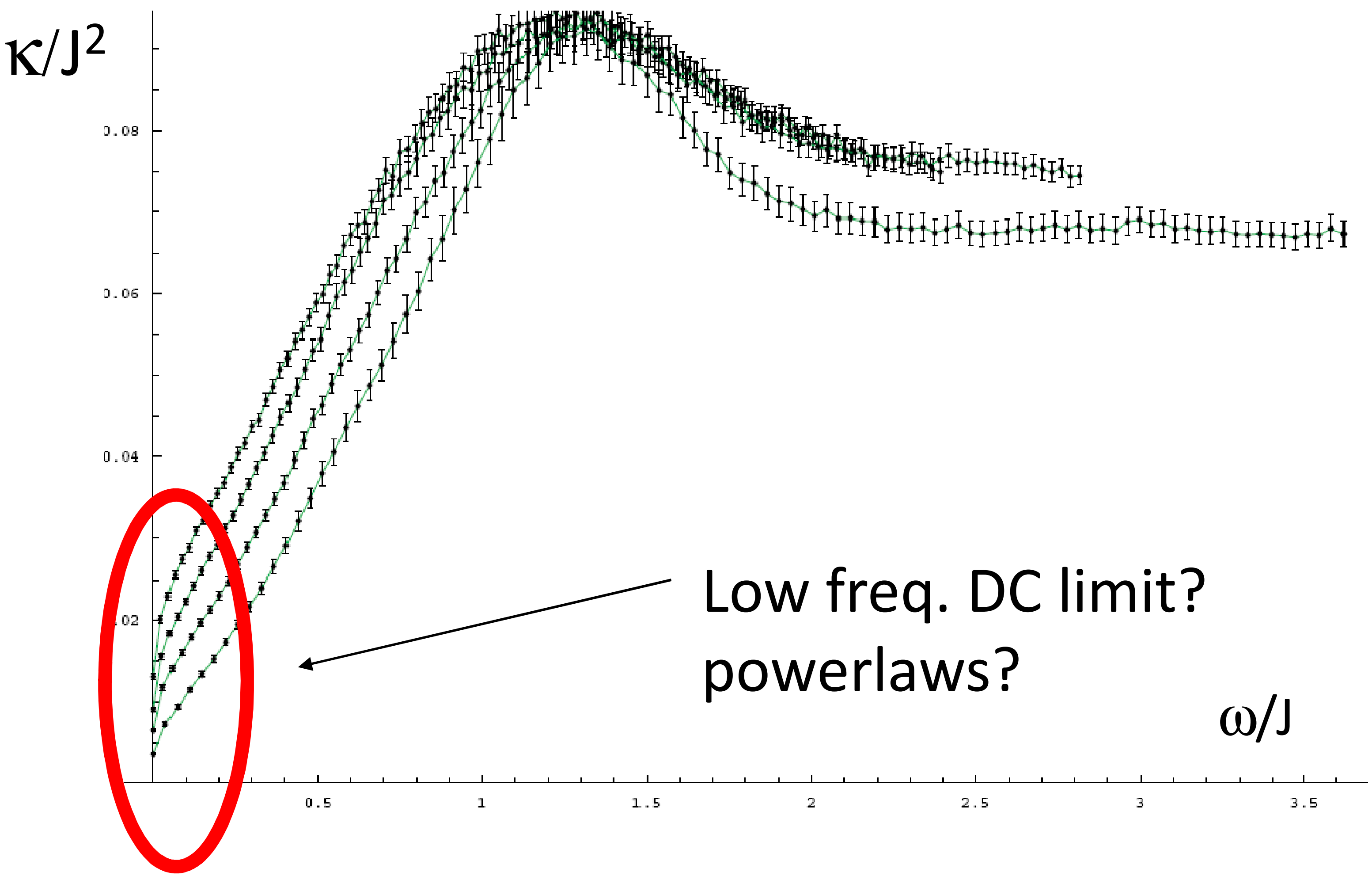
$$\kappa(\omega) = \int dr dr' dt dt' e^{i\omega(t-t')} \langle j(r,t) j(r',t')\rangle / (L t_{max})$$

- Localization -- the integral vanishes!
- The challenge – accurately compute the long-time tail of <jj>
- Will use an “exact simulation” with discrete time steps (local map?) – highly efficient for going to long times and only accumulating roundoff errors

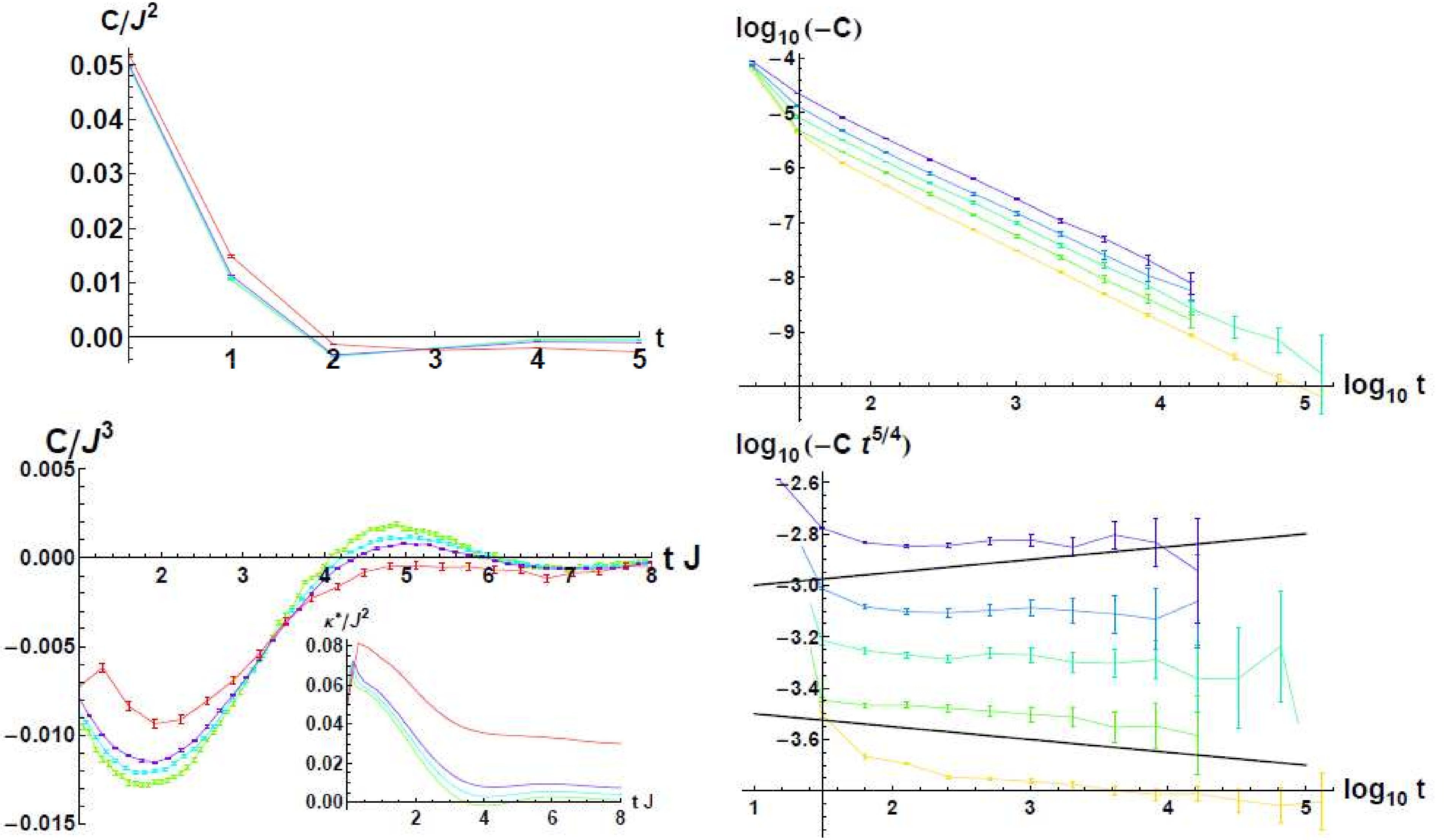
# Optical conductivity vs. J



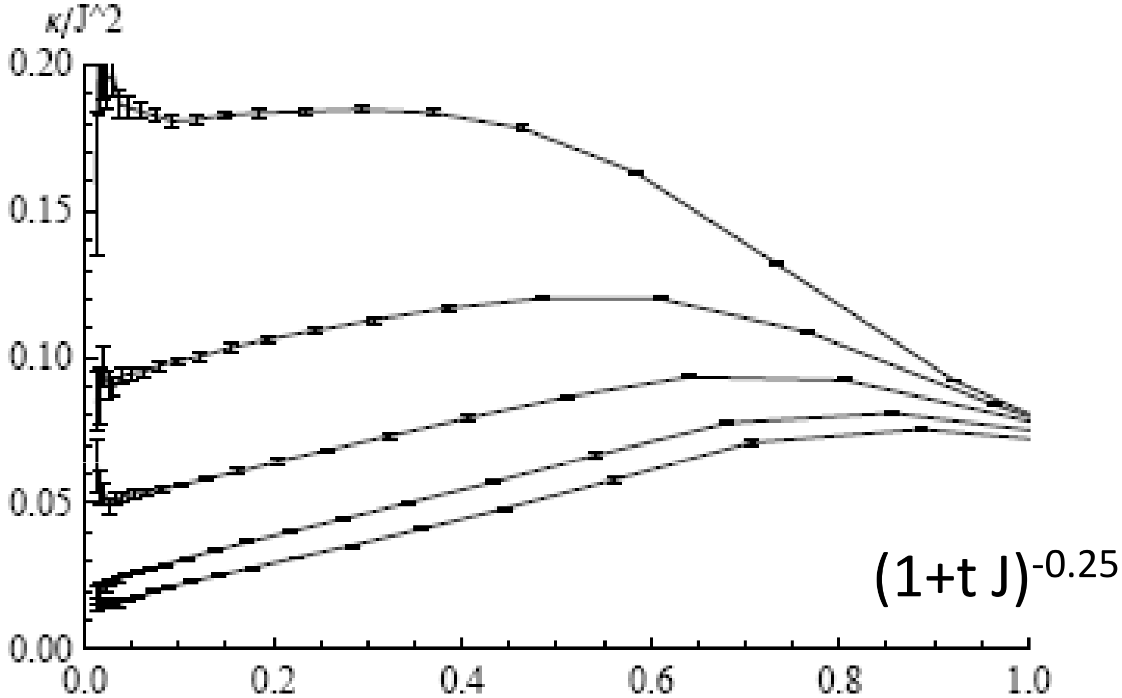
# Optical conductivity vs. J



# Localization and long time tails: $C(t) \equiv \langle j(q=0,t)j(q=0,0)\rangle$



# Extrapolation past long time tails: diffusion



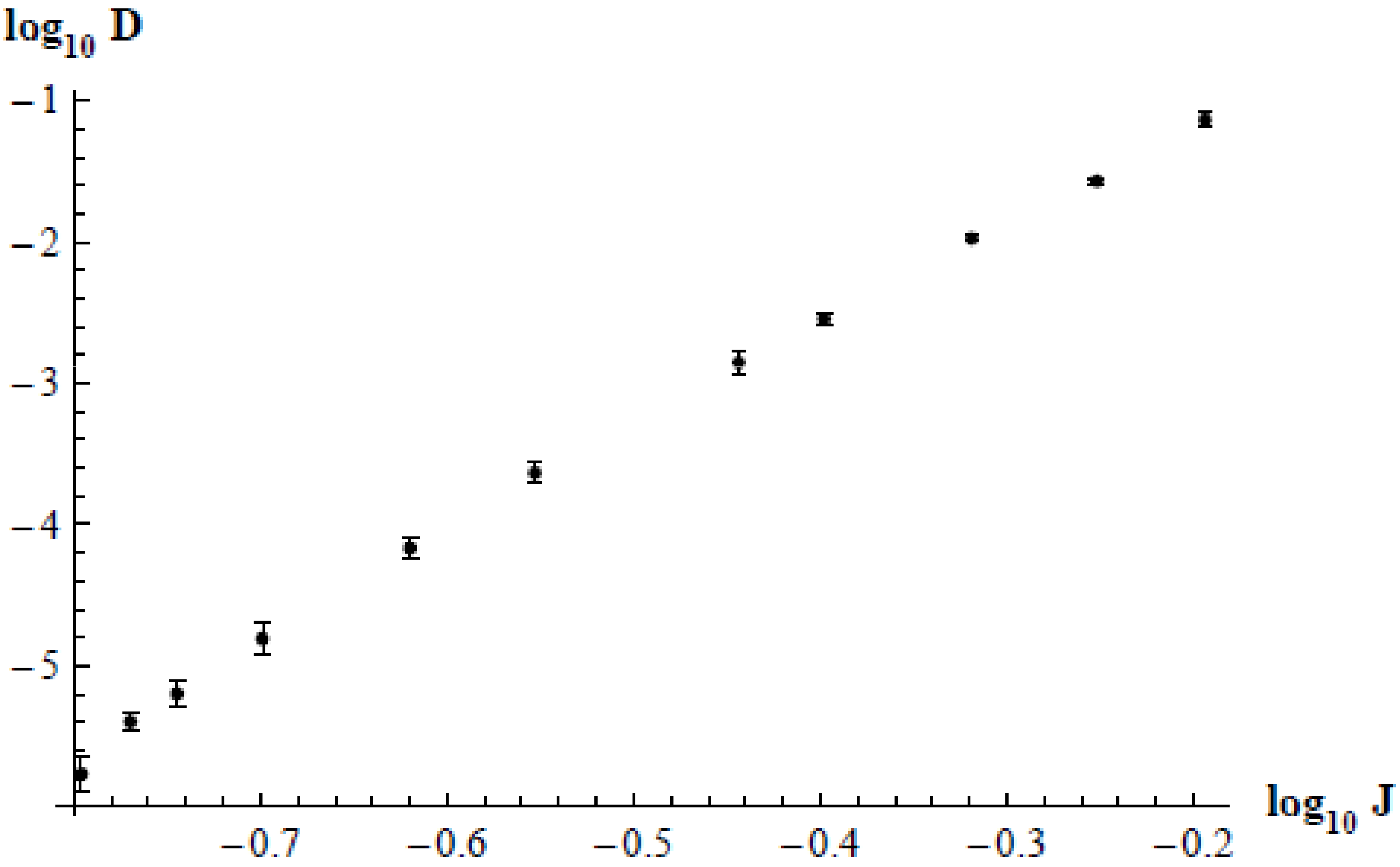

Appear robust w.r.t. to changes in disorder strength or type

A very strong violation of mode-coupling prediction (Ernst etal 1973–90's) which has been confirmed for a wide range of disordered stochastic systems

$$\kappa(\omega) = k(0) + |\omega|^{1/2} + \ldots$$

What is different here? – strong nonlinearity, apparently

# Are these small numbers real or computer artifacts?

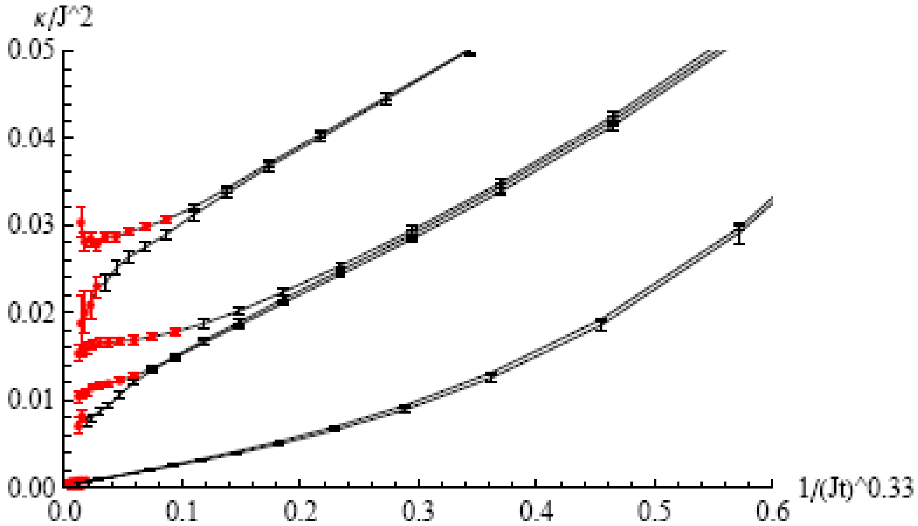


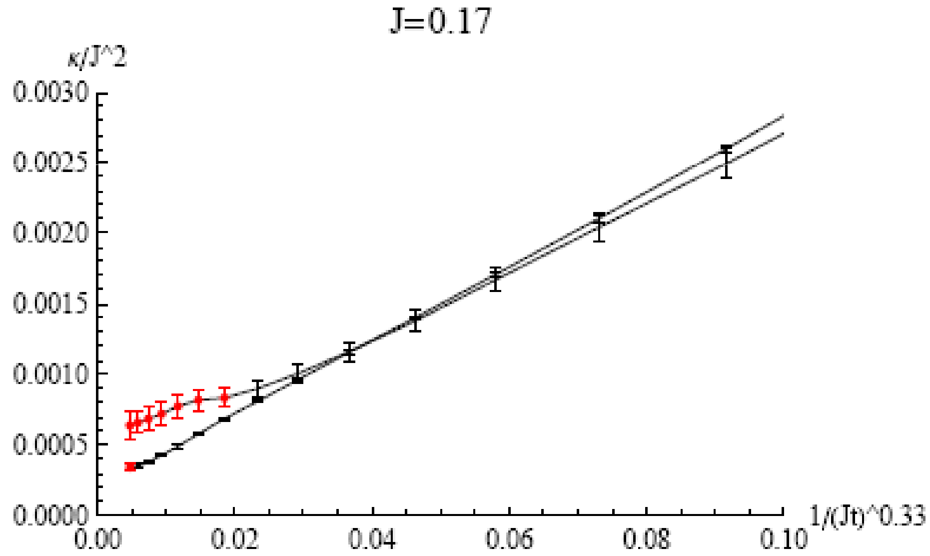

Stability w.r.t. roundoff errors can be studied in detail – these numbers are real.

# Is this really diffusion? finite size/time effects?

$$\omega < \omega_L = CD/L^2$$

J=0.4,0.32,0.17




Red data – finite size effects in small rings (10~20 spins)

Interestingly, C=10 throughout, despite large changes in D and inverse RC times

# Microscopics of the insulating regime

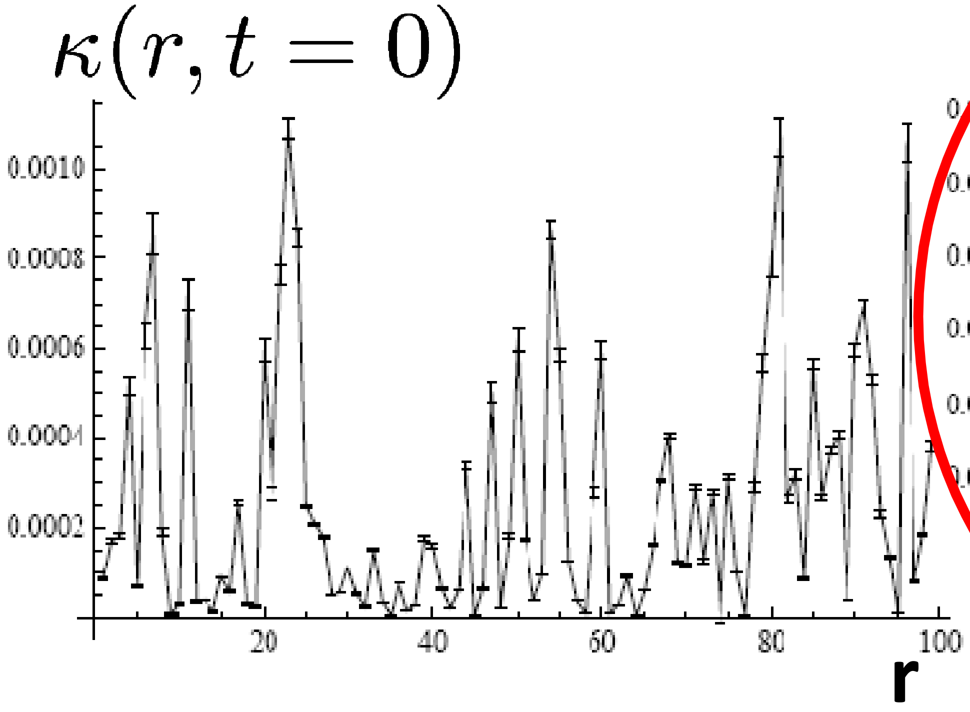
Define a sample specific semilocal Kubo conductivity

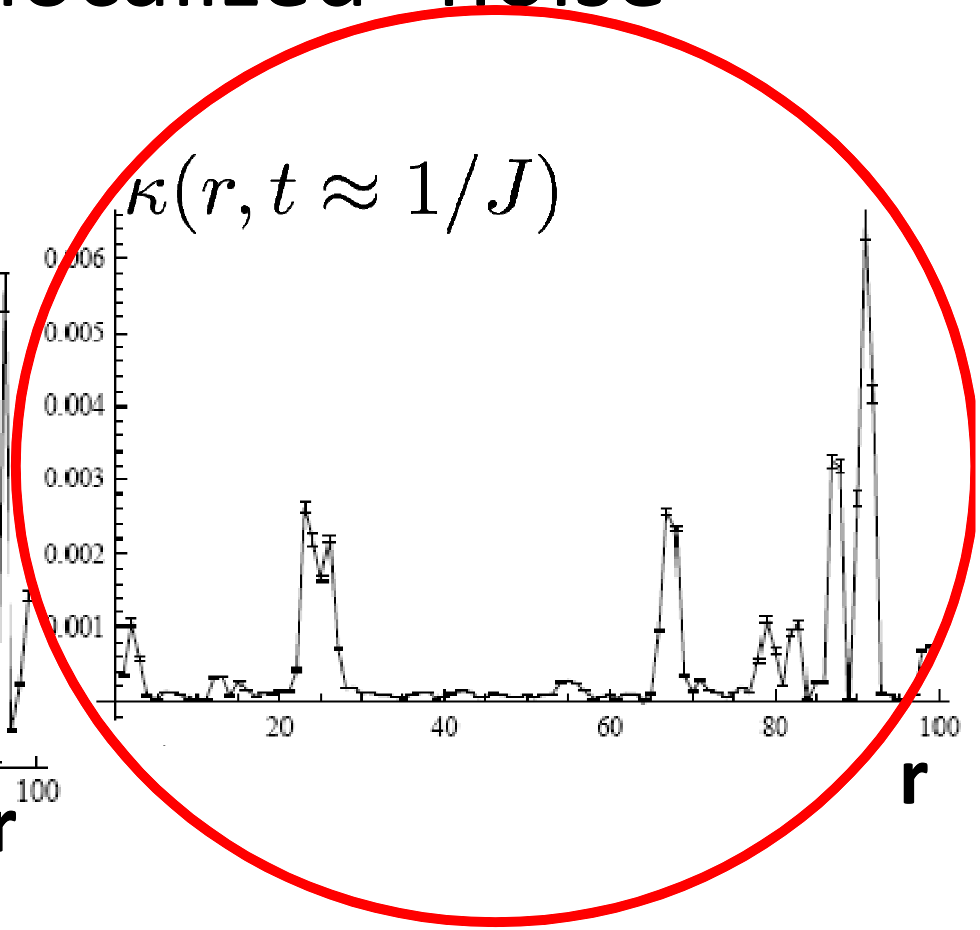
$$\kappa(r, t_{max}) = \int dr' dt dt' \langle j(r,t) j(r',t') \rangle / (L t_{max})$$

Useful facts:

- positive definite for all t
- short times – computable analytically, “follows” disorder (i.i.d. etc)
- Intermediate times??????
- “infinite” time, DC – uniform (follows from energy conservation), the magnitude specifies sample specific DC conductivity

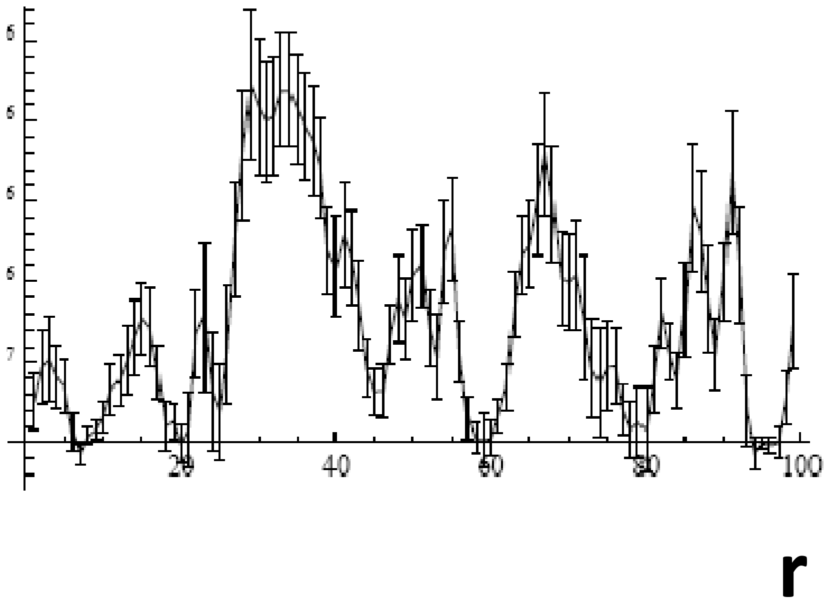
# Semilocal Kubo: localized “noise”



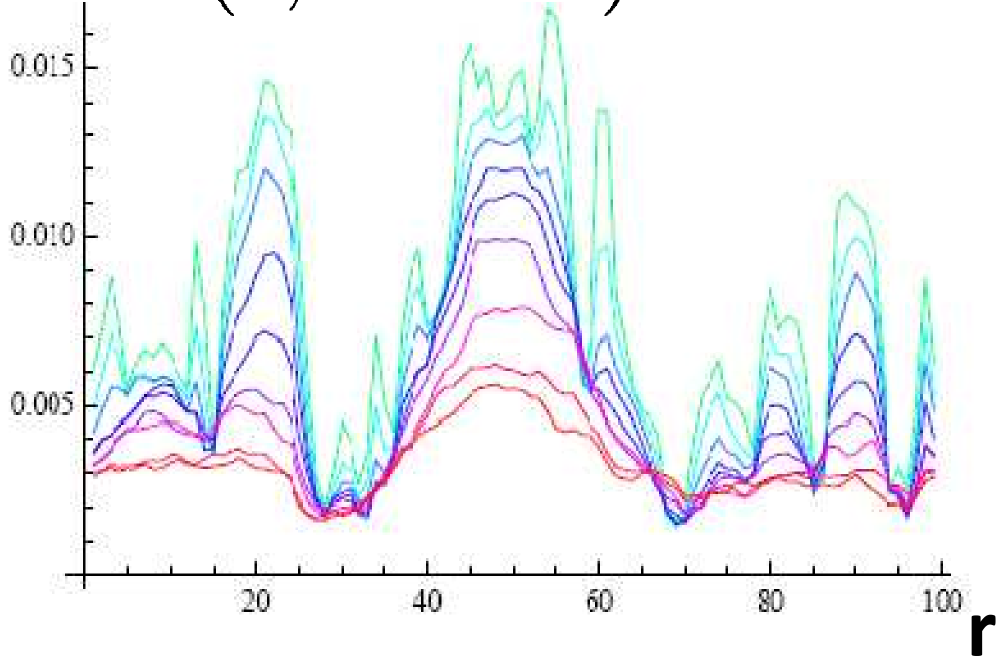



# Longer time tails and approach to diffusion

$\kappa(r, t \gg 1/J)$




$\kappa(r, t \to \infty)$



# New (?) classical phenomena

- Very strong suppression of diffusion already at infinite temperature
- this phenomenon is clearly unrelated to anything thermodynamic – the free energy is trivial
- Strong non-analytic corrections to diffusion at finite frequency – at odds with existing (and otherwise successful) theory; also accompanied with coarsening like spatial current-current correlations
- A sort of localized chaos originates from isolated resonances of 2~3 spins at short times but ultimately leaks out into detuned bulk of sample. Mechanism? Effective model/description?

# Physical properties of non-Anderson insulators

- Anderson insulators are relatively well understood, e.g. Mott AC law, infinite lifetimes for local excitations. Are interacting insulators (beyond Efros-Shklovskii) the same?
- Analogy: Fermi gas is distinguishable from Fermi liquid (via FL parameters, zero sound, transport)
- Some thoughts/examples:

  (i) current bi-stability near Tc (Basko etal);

  (ii) non-Mott AC behavior $\sigma(\omega) \gg \omega^2$

  (iii) dephasing by Hartree

# Examples of non-Mott AC laws

- If $\langle x^2 \rangle \sim t^{1-x}$ then $\sigma(\omega) \sim \omega^x$
- Many linear/single particle examples with 0<x<1:

  Quantum Hall effect (Meden/Sinova/Girvin): delocalized states at a single energy (a set of measure zero), at finite T

  $$\sigma(\omega) \sim \omega^{\frac{1}{2\nu}}$$

  also long range hopping problems; random classical transmission lines with broad disorder distributions
- Sub-diffusive spreading in non-linear problems – talks over past 5 days (including experiments?)
- Numerical evidence from earlier in this talk

# Dephasing by Hartree

- Consider a very simple many-body Hamiltonian

$$H = \sum_j U_j \sigma_j^z + \sum_{ij} K_{ij} \sigma_i^z \sigma_j^z$$

  with interactions rapidly decaying in space (e.g. exponentially). Single site correlations computed with <u>generic</u> initial configuration show rapid dephasing – a large number of larmor frequencies originating from non-dynamic eigenstate-specific shifts via interactions (see also Linden et.al. PRE 79, 061103 (2009))

# Broad conclusions and outlook

- Numerical studies of dynamics with strong disorder and interactions are not only feasible but they tend to produce interesting and often surprising results, quantitatively AND qualitatively, especially, if effort is taken to shrink intermediate scales, e.g. by going to high temperature
- Presented examples: disordered spinless fermions and classical spins
- Some thoughts towards experimental search for non-Anderson (interacting) insulators: anomalous low frequency current dynamics and state dependent dephasing of local excitations