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Weakly Interacting Bose Gas in Disordered Environment

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Weakly interacting Bose gas in disordered environment*

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Outline

Introduction

Interacting Bose gas in weak random potential

Strong disorder

Bosons in traps

Bosons in lower dimensions

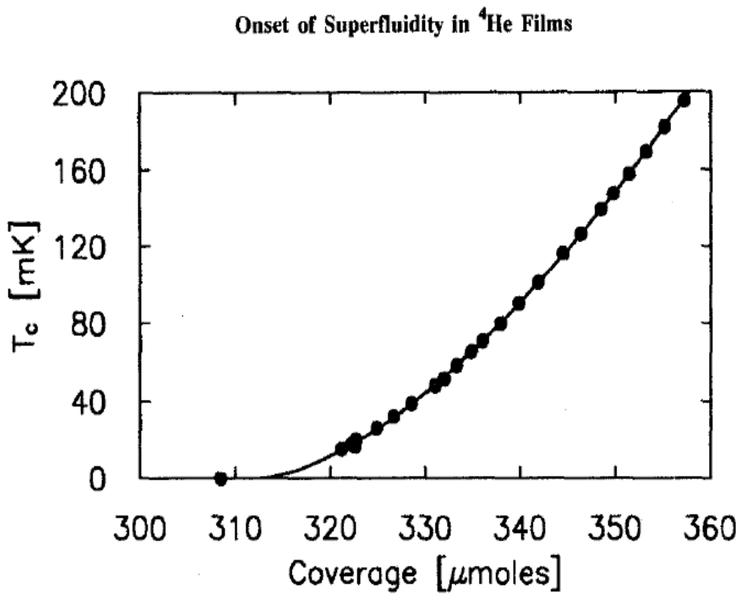
Introduction

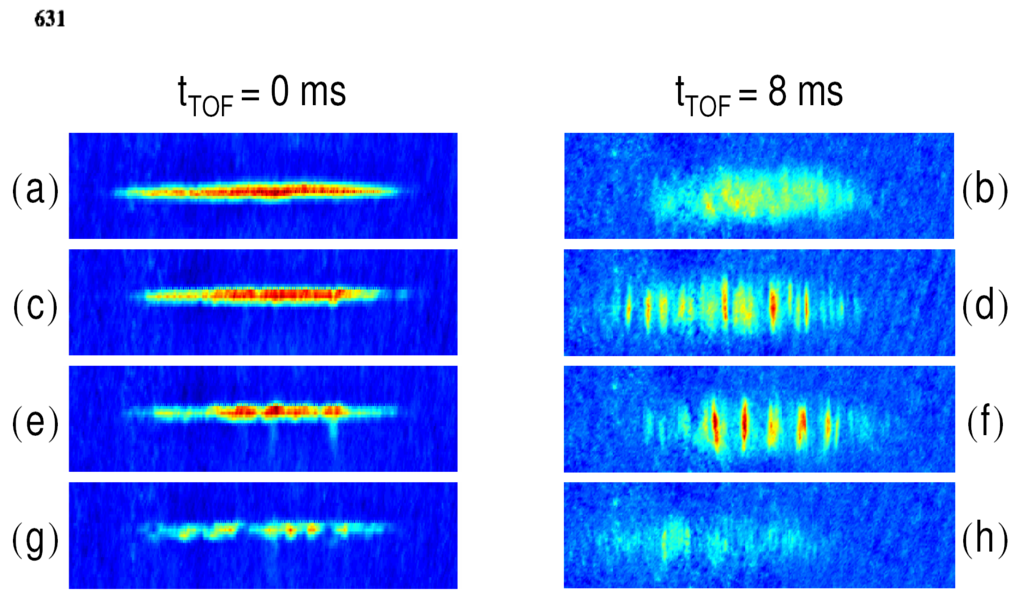
BEC : finite part of atoms in the state with minimal energy.

Examples: Superfluid ^{4}He, laser cooled atoms in a trap, SCs, excitons in semiconductors, BEC of spin waves

Disorder: Superfluid He in porous media (J.D. Reppy et al '92)
Cold atoms in speckle potential (R.G. Hulet et al. '08)
Disordered superconducting films (V.F. Gantmakher '09)

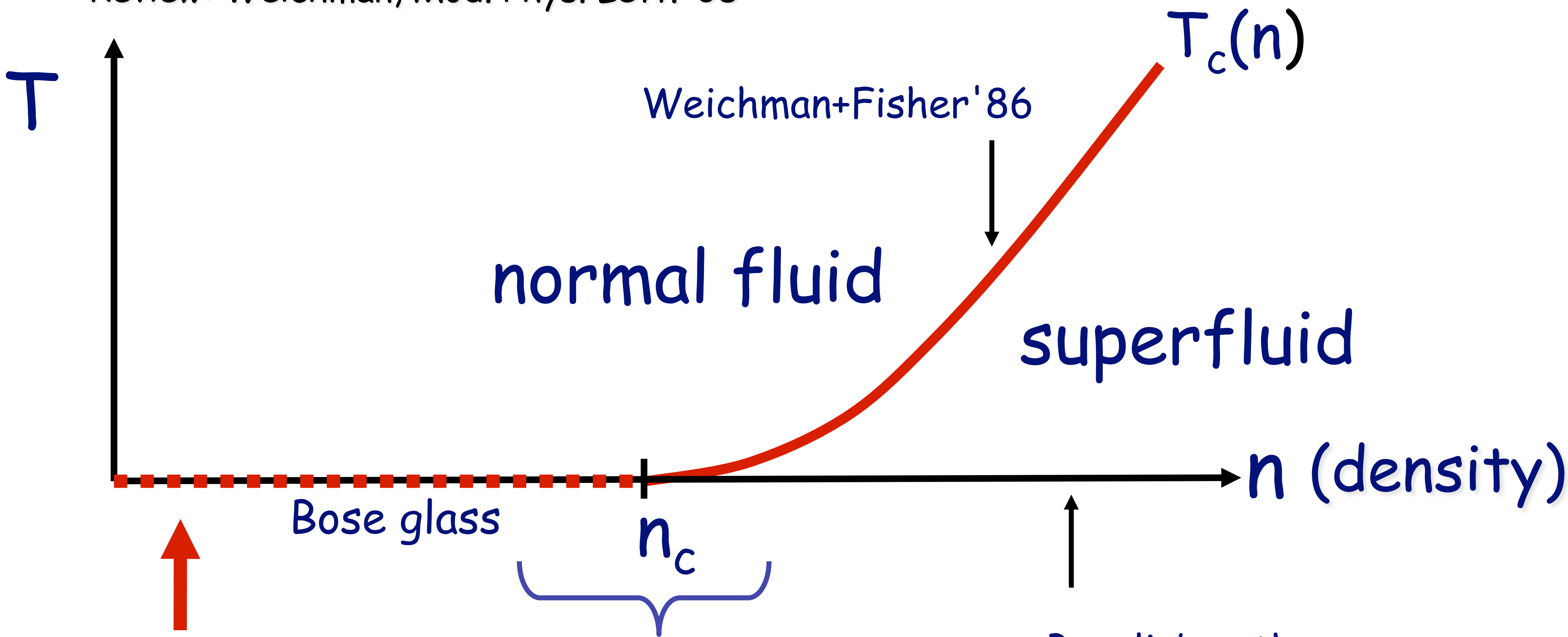
→ Breakdown of superfluidity at strong disorder





Bosons in disordered environment:

Review: Weichman, Mod. Phys. Lett. '08



this work

Shklovskii '08
Babichenko² '08
Shklovskii+Müller '08
Aleiner, Altshuler, Shlyapnikov '10

Halperin, Lee+ Ma'86

Giamarchi+Schulz '87 (d=1)

Fisher, Weichman, Grinstein+Fisher '89
$\rightarrow$ z=d

Bogoliubov theory
Huang+Meng '92

$$\rho_s \approx \rho_0[1 - c_1\sqrt{n_c/n}]$$

Gross-Pitaevskii equation

Classical particles

$$p = Tn$$

Fermions

$$p = E_F n$$

interaction → Fermi liquid

disorder → Localization

Ioffe-Regel $nL_c^3 \leq 1$

→ many body localization

Bosons

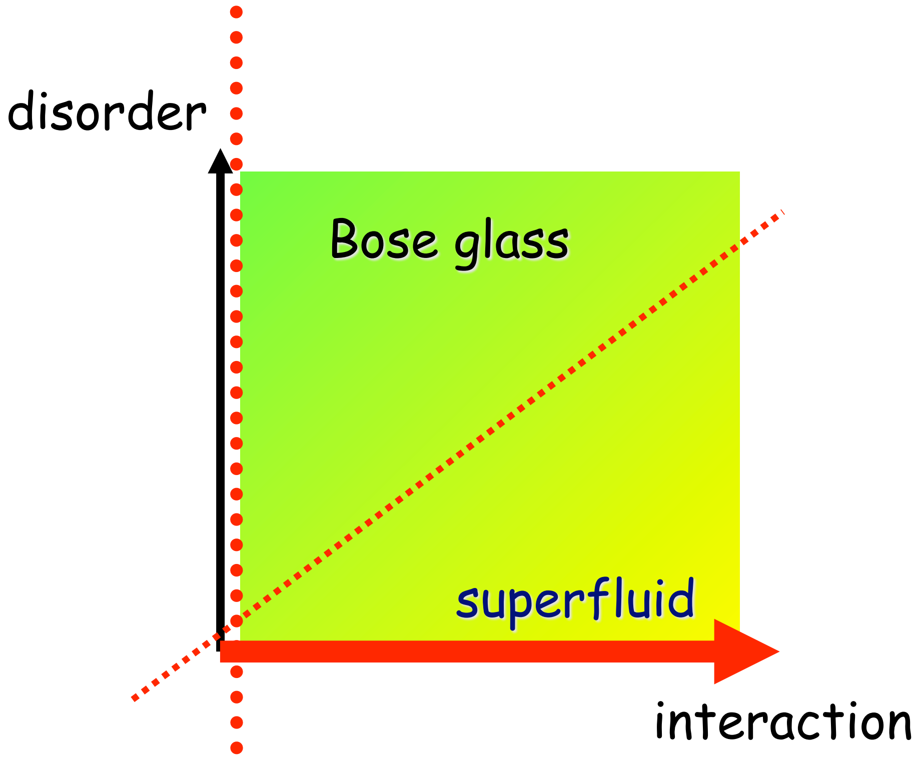
$$p = \frac{T}{\lambda_T^3} \qquad n_c = n - \frac{1}{\lambda_T^3}$$

interaction → Superfluid

disorder → Non-ergodic state

many body localization of bosons ?

No Disorder



BEC : Interacting bosons

$$\mathcal{H} = \frac{\hbar^2}{2m}\int d^3x \hat{\Psi}^\dagger[-\nabla^2 + 4\pi a\,\hat{\Psi}^\dagger\hat{\Psi}\,]\,\hat{\Psi}$$

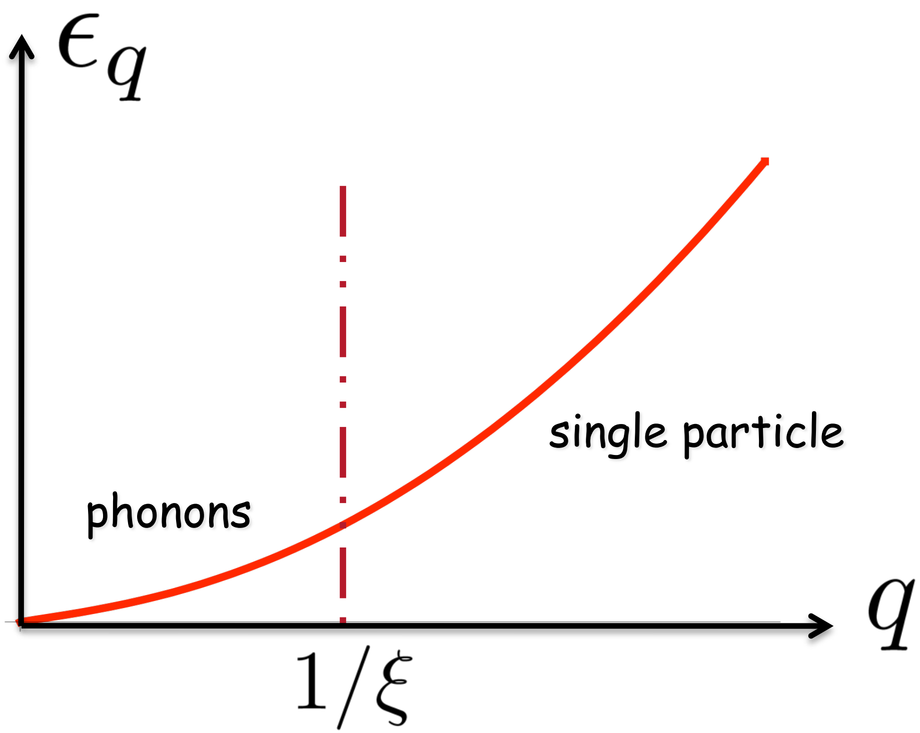
$$\hat{\Psi}^\dagger\hat{\Psi} = \hat{n}$$

Scattering length

$$\hat{\Psi}^{(\dagger)} = \Psi_0 + \delta\hat{\Psi}^{(\dagger)}$$

Bogoliubov transform $\Rightarrow$

$$\epsilon_q^2 = \frac{\hbar^2 q^2}{m}\left(\frac{\hbar^2}{m\xi^2} + \frac{\hbar^2 q^2}{4m}\right)$$

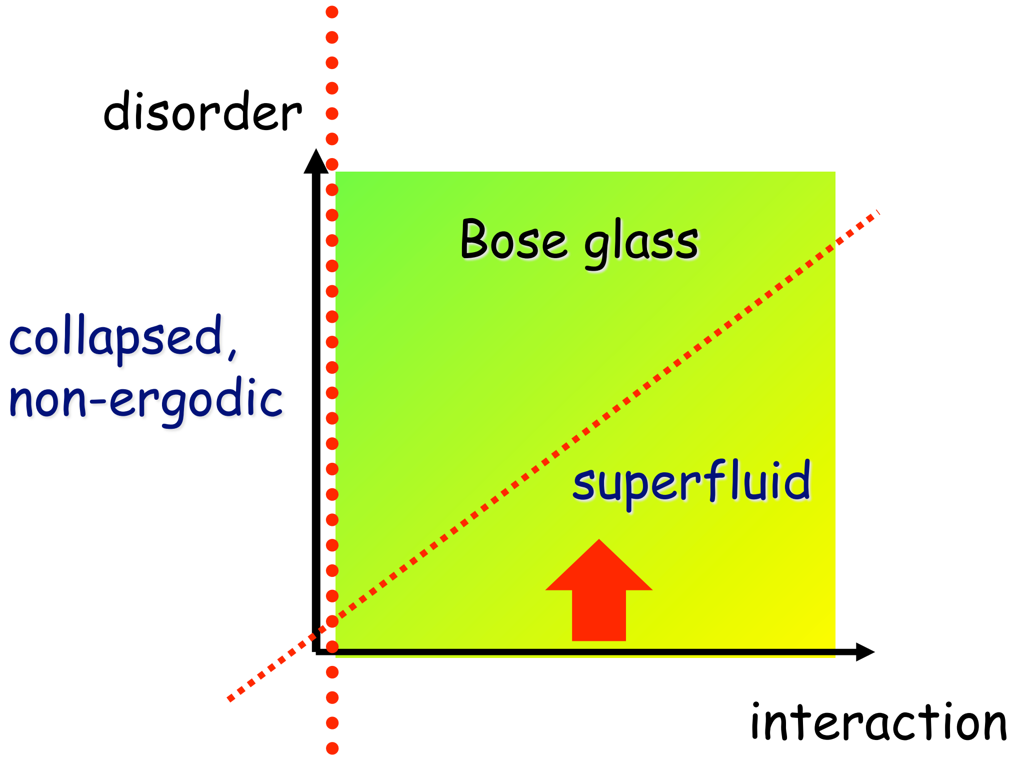


$$\mu = \frac{\hbar^2}{m\xi^2}\left(1 + \frac{c}{n^{1/3}\xi}\right)$$

Lee & Yang '57

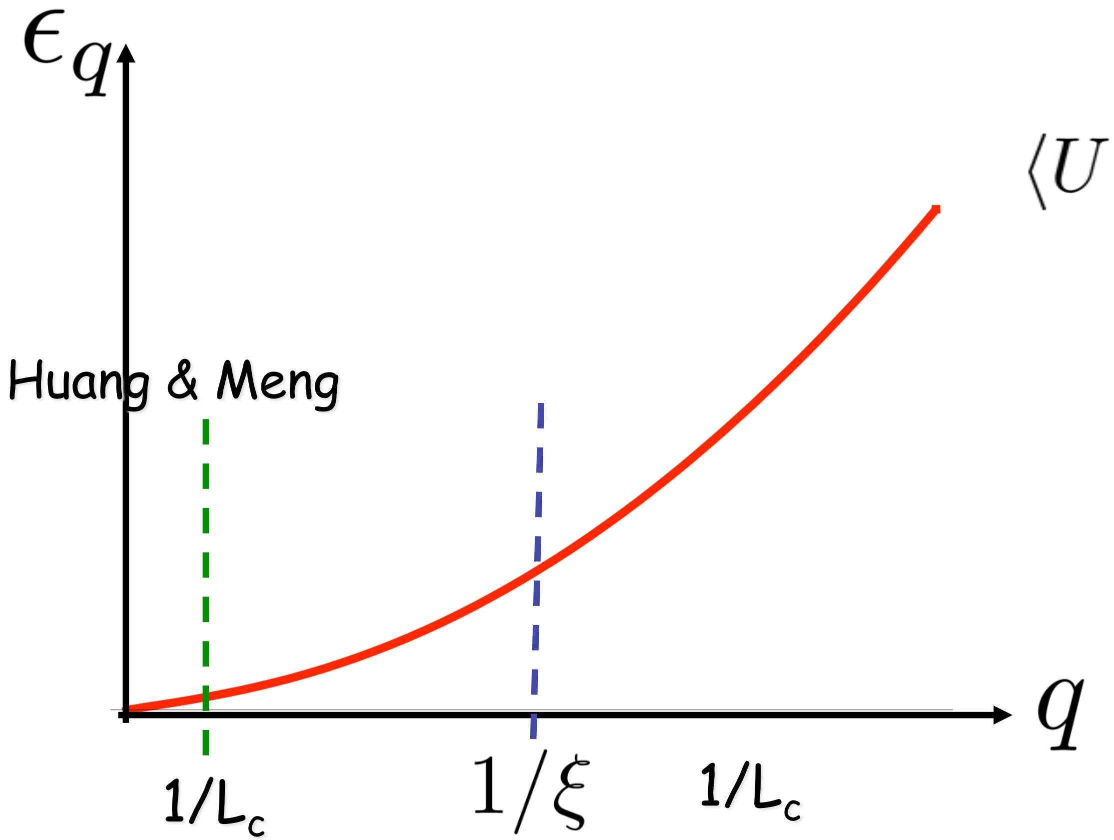
healing length $\xi = \frac{1}{\sqrt{4\pi a n}}$

Weak Disorder



(d) Weakly repulsive bosons in a weak random potential

$$\mathcal{H} = \int d^3x \Psi^\dagger \Big(-\frac{\hbar^2}{2m}\nabla^2 + U(\mathbf{x}) + \underbrace{\frac{2\pi\hbar^2 a}{m}}_{g} \Psi^\dagger \Psi \Big) \Psi$$



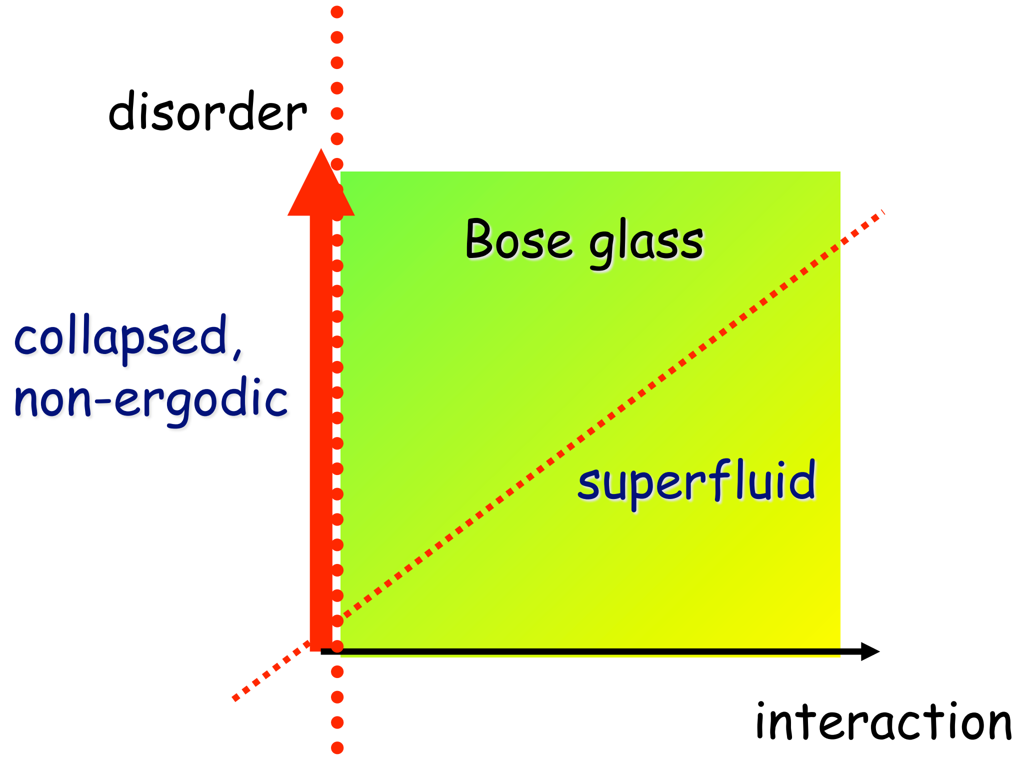
$$\langle U(\mathbf{x}) U(\mathbf{x}') \rangle = \kappa^2 \delta(\mathbf{x} - \mathbf{x}')$$

mean free path

$$L_c = \frac{\hbar^4}{m^2 \kappa^2}$$

$$\Delta\mu = \Delta\epsilon \approx \int_0^{\xi^{-1}} \frac{d^d q}{h^3} \frac{\kappa^2}{c_s q} \sim \frac{\hbar^2}{m\xi^2} \left(\frac{\xi}{L_c} \right)^{4-d}, \quad d > 1$$

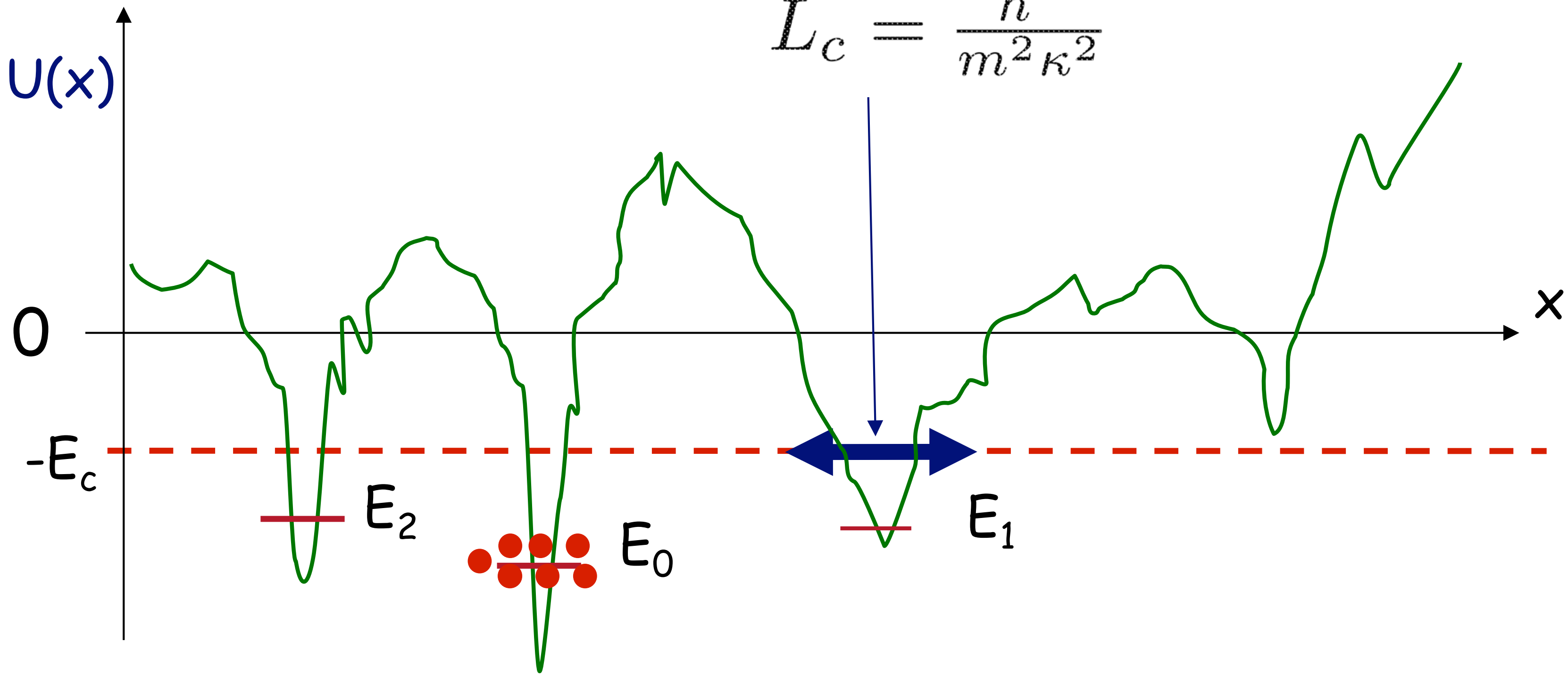
Strong Disorder, no Interaction



Ideal 3d Bose gas in random potential

$$\frac{\hbar^2}{2m}\nabla^2\psi + (E - U(\mathbf{x}))\psi = \mathbf{0} \quad \langle U(\mathbf{x})U(\mathbf{x}')\rangle = \kappa^2\delta(\mathbf{x}-\mathbf{x}')$$

$$L_c = \frac{\hbar^4}{m^2\kappa^2}$$

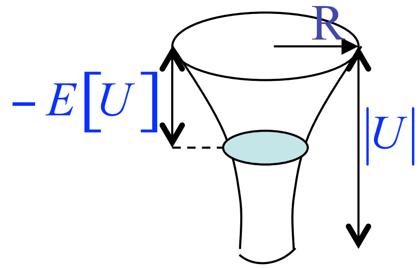


$$E_c = \frac{\hbar^2}{2mL_c{}^2}$$ energy scale of localized states

T=0: All particles in ground state $E_0 \approx -E_c \ln^{2/(4-d)}(L_0/L_c)$

Single particle density of states DOS $E \to -\infty$

$$\nu(E) = \int \mathcal{D}\mathcal{U}(\mathbf{r}) \mathrm{Tr}\delta(E - \hat{H}) e^{-\int d^d\mathbf{r} U^2(\mathbf{r})/2\kappa^2}$$



Consider potential fluctuation of depth U and width R

probability $W[U] \sim \exp[-U^2 R^d / 2\kappa^2]$

$\to$ localized state of energy $E \sim \hbar^2/(2mR^2) - U$

Contribution of DOS at energy E -> max $W[E - \hbar^2/(2mR^2)]$

Maximize W with respect to R $\Rightarrow$ $R = L_c(E_c/|E|)^{1/2}$

$$E = -E_c \frac{L_c^2}{R^2}$$ $\to$ $$\nu \sim \exp\{-(|E|/E_c)^{(4-d)/2}\}$$

a=0: Density of states, search for the optimal fluctuation of random potential

$$\nu(E) = \int DU\ Tr\,\delta(E-\hat{H})e^{-\int d^3rU^2/2\kappa^2}$$

$$\sim \exp\left[-\int d^3rU^2/2\kappa^2 + \lambda(E - min_\Psi\langle\Psi|H|\Psi\rangle)\right]$$

$$\rightarrow U(\mathbf{r}) = -\lambda\kappa^2|\Psi|^2$$

$$\rightarrow \hat{H}\Psi = E\Psi \quad \rightarrow \Psi$$

non-linear Schroedinger equation

I.M. Lifshitz '66,
Zittartz and Langer '66,
Halperin and Lax,'66
Cardy '78

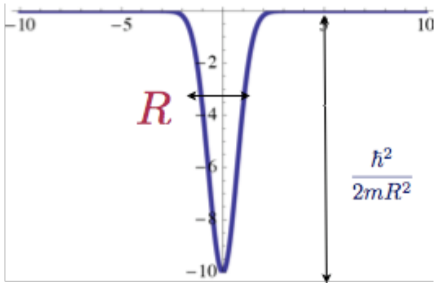
simplification $\Psi(\mathbf{r}) \sim e^{-r^2/2R^2}$

$$\langle\Psi|\hat{H}|\Psi\rangle(R,\lambda) = E \quad \rightarrow \lambda(E,R)$$

$$\int d^3r\frac{U^2}{2\kappa^2} = \Phi(R,\lambda(E,R))$$

$$\rightarrow min_R \rightarrow E = E(R)$$

$\frac{\hbar^2}{mR^2}$, $U \sim E_b e^{-r^2/R^2}$, $\nu \sim e^{-L_c/R}$



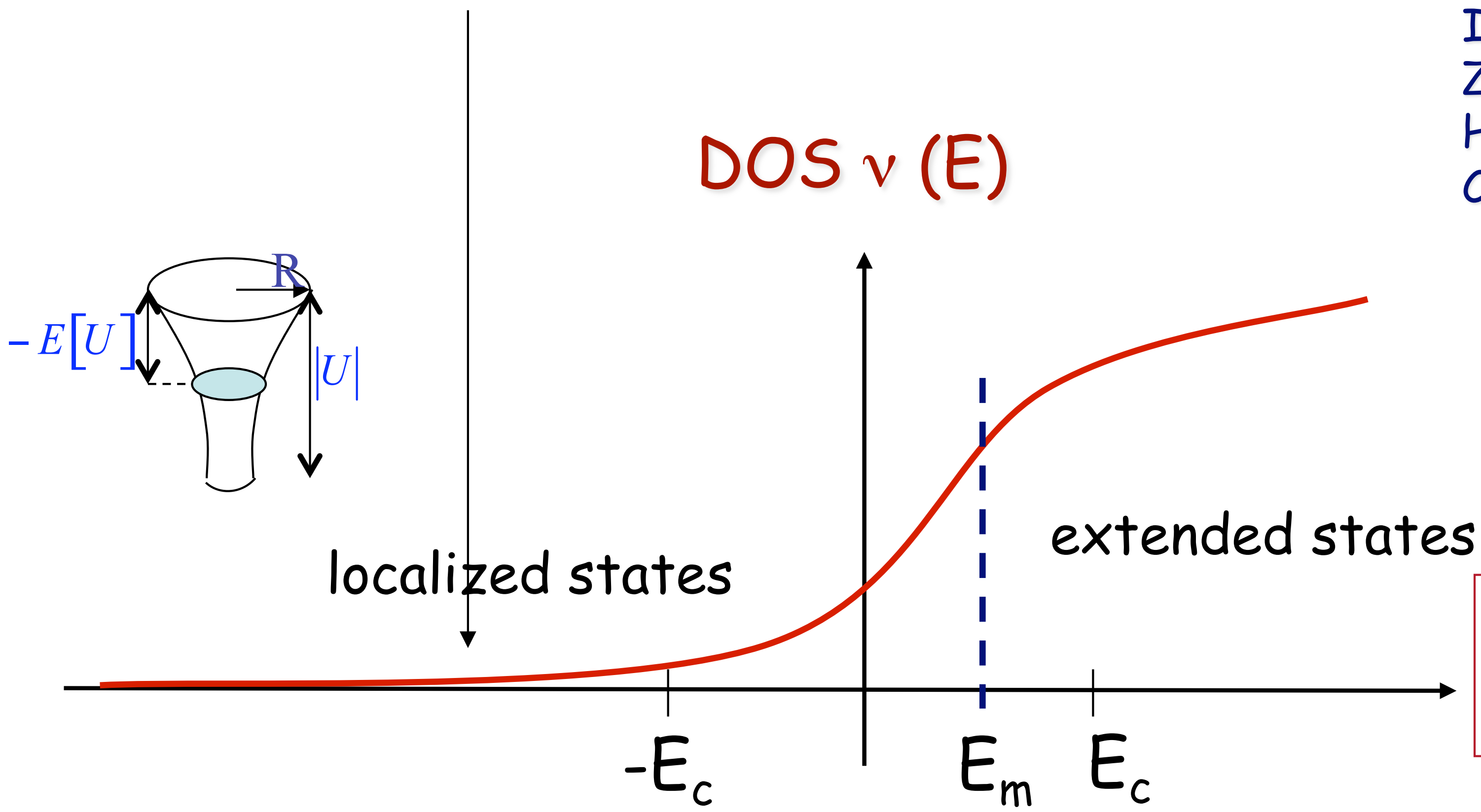
$$\rightarrow \nu(E) \sim e^{-\Phi(R)}$$

Ideal Bose gas in random potential

DOS for $E \ll -E_c$ dominated by wells of width $R \sim \hbar/\sqrt{m|E|} \ll L_c$

$$\nu(E) = \frac{1}{V}\langle \delta(E - E[U(\mathbf{x})])\rangle \sim |E|^{3/2} e^{-\sqrt{|E|/E_c}}$$

I.M. Lifshitz '66,
Zittartz and Langer '66,
Halperin and Lax,'66
Cardy '78



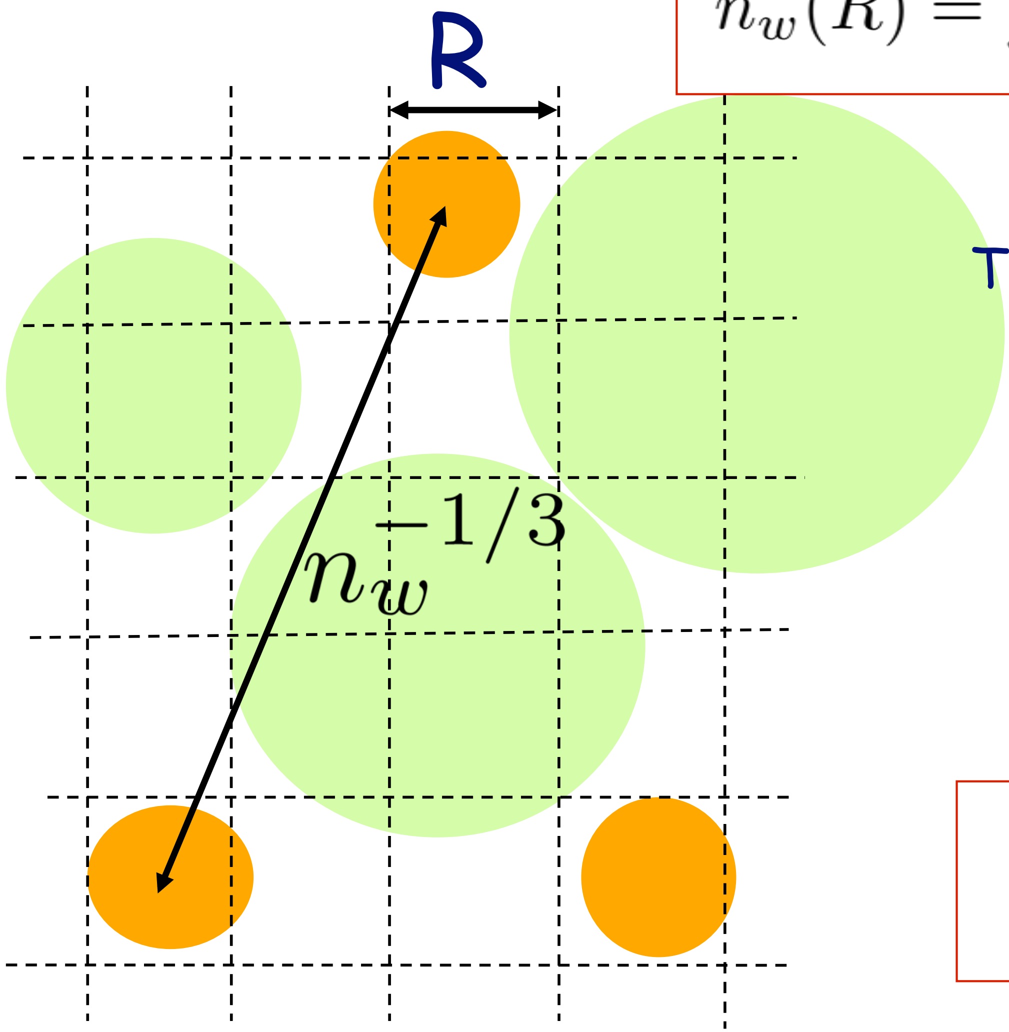
Width of well

$E \sim -\hbar^2/mR^2$

Ideal Bose gas in random potential

Spatial density $n_W(R)$ of wells with radius < R<< L_c (E <- $h^2/(2mR^2)$<<E_c)

$$n_w(R) = \int_{-\infty}^{-\frac{\hbar^2}{2mR^2}} dE\ \nu(E) \sim \frac{L_c}{R^4} e^{-L_c/R}$$



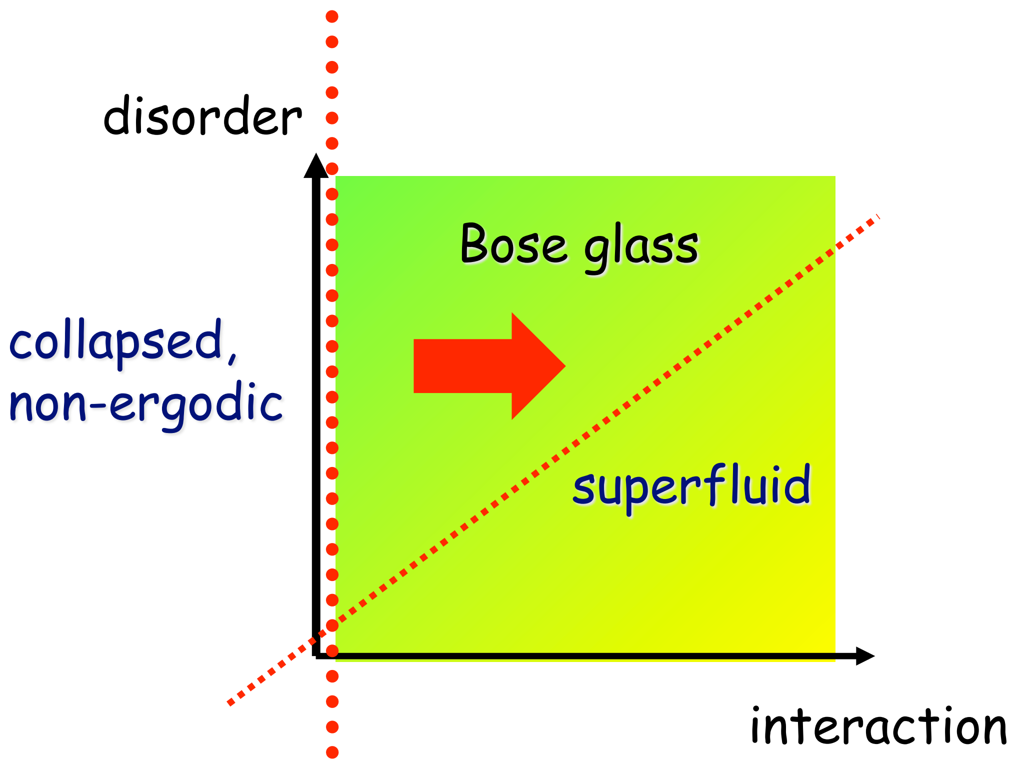
Tunneling amplitude t(R) between wells with radius < R :

$$t(R) = \exp\left(-\frac{1}{\hbar}\int |p|dl\right)$$

$$\frac{1}{\hbar}\int |p|dl \approx n_w^{-1/3}/R \sim e^{L_c/3R}$$

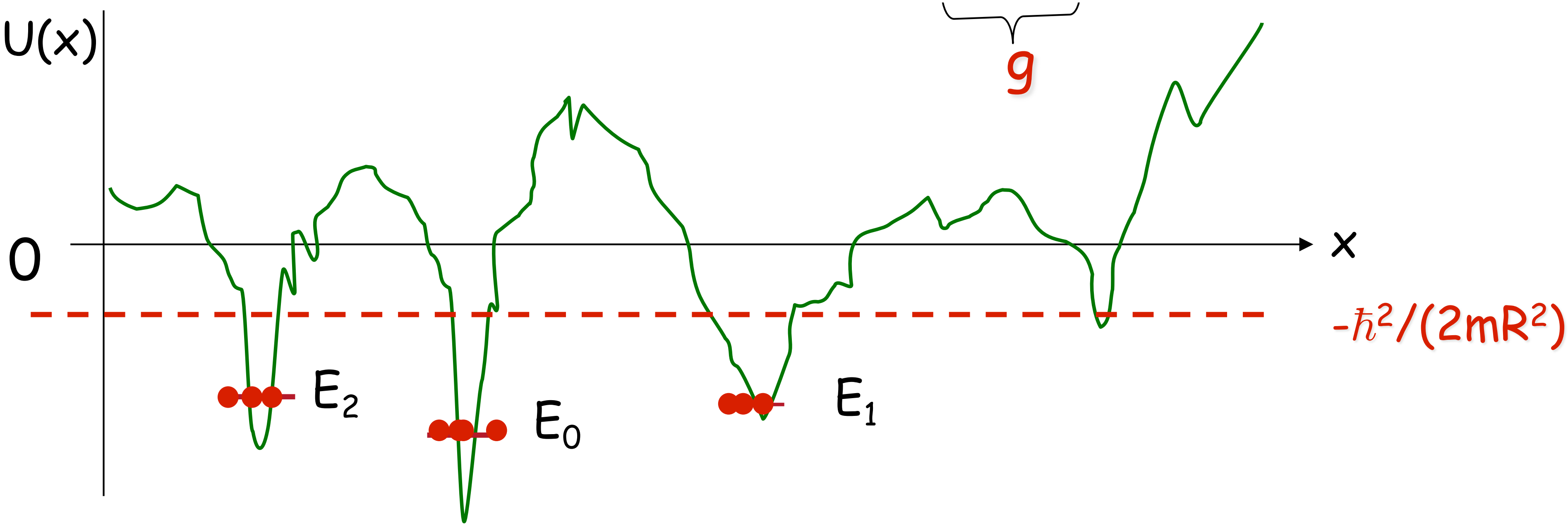
$$t(R) \sim e^{-(\frac{R}{L_c}e^{L_c/R})^{1/3}}$$

Strong Disorder + Interaction



Weakly repulsive bosons in a random potential

$$\mathcal{H} = \int d^3x \Psi^\dagger \Big(-\frac{\hbar^2}{2m}\nabla^2 + U(\mathbf{x}) + \underbrace{\frac{2\pi\hbar^2 a}{m}}_{g} \Psi^\dagger\Psi \Big)\Psi$$



Assume that all potential wells with radii up to R are filled:

⇒ number of particles per well of size R : $N_w(R) = n/n_w(R) \gg 1$

⇒ repulsion energy per particle: $E_g(R) \approx g\, N_w/R^3 \sim g\, n\, e^{Lc/R}$

⇒ total energy per particle: $\mu(R) = -\hbar^2/(2mR^2) + E_g(R)$

Weakly repulsive bosons in a random potential

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⇒ total energy per particle: $\mu(R) = -\hbar^2/(2mR^2) + E_g(R)$

Mininization over R: ⇒ $R(n)=L_c/\ln(n_c/n)$,

$n \ll n_c \approx 1/(3L_c^2 a)$

(non-interacting Fermions: Ioffe-Regel $a \to L_c$)

$$\mu(n) = -\frac{\hbar^2}{2mR^2(n)} = -\frac{1}{2}E_c(\ln\frac{n_c}{n})^2$$

$$\frac{n_c}{n} \approx \frac{\xi^2}{L_c^2} \approx \frac{E_c}{gn}$$

Babichenko & Babichenko 09

Variable hopping conductivity:

Absence of interaction: probability that two localized states have the same energy is zero.

Switch on interaction: energy levels split by amount gn_p. If $n \ll n_c$ wave function is still localized .

$\rightarrow$ T=0 conductivity (response to external force) in Bose-glass is still zero.

Tunneling probability between wells of distance L is $\sim$ exp{-2L/R}

$\rightarrow$ hopping probability P(T) $\sim$ exp{-2L/R-ΔE/T}

$\Delta E\, \nu(E)\, L^3 \approx 1$, use relation R(n) and maximize P(T) with respect to hopping distance L $\Rightarrow$

$$\sigma(T) \sim e^{-C[E_c n_c/(Tn)]^{1/4}}$$

Preliminary conclusions

⇒ At $n \ll n_c$ Bose gas decays into fragments,
particle density in fragments each of density $n_c \sim 1/(aL_c^2)$

⇒ tunneling exponentially suppressed: $t(n) \sim e^{-c(n_c/n)^{1/3}}$

⇒ particle number in fragments $\sim L_c/a \gg 1$ well defined

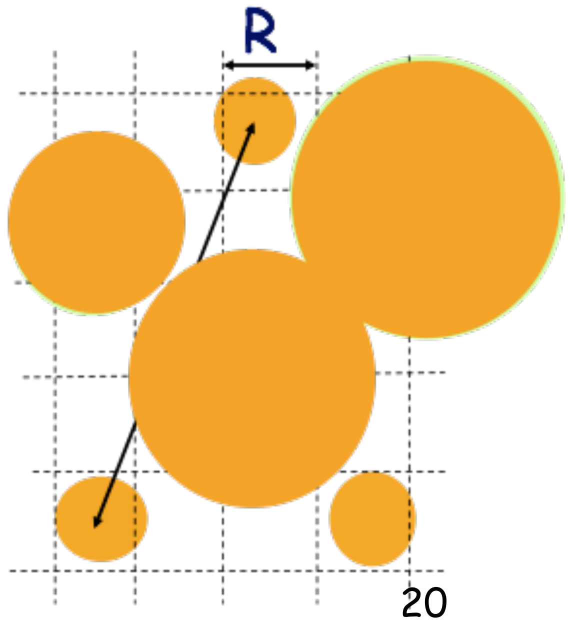
⇒ phase uncertain, no phase coherence ⇒ no superfluidity

⇒ finite compressibility $\frac{\partial n}{\partial \mu} = \frac{n}{E_c} \ln(n_c/n)$ „Bose glass"

⇒ charged bosons VRH

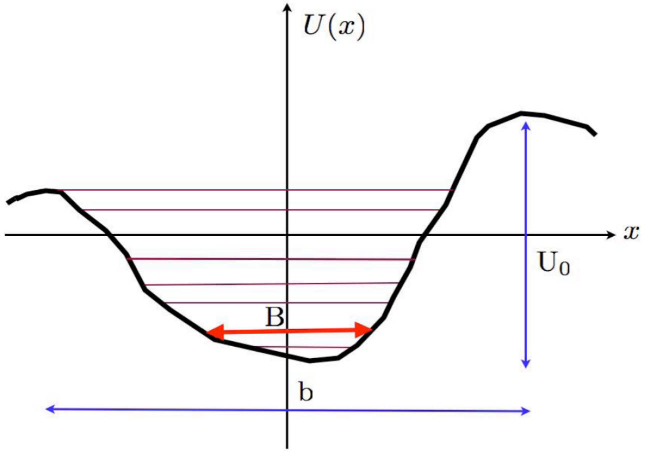
$$R(T) \sim e^{(T_0/T)^{1/4}}, \quad T_0 = E_c n_c/n$$

For $n \approx n_c$ i.e. fragments merge → transition to superfluid



Correlated disorder

$$\langle U(\mathbf{x})U(\mathbf{x}')\rangle = \frac{U_0^2}{b^3} e^{-|\mathbf{x}-\mathbf{x}'|/b}$$



$\Rightarrow$ 2 length scales b , $B=(\hbar^2/(mU_0))^{1/2}$

b << B $\Rightarrow$ uncorrelated disorder

$$\nu(E) \sim |E|^3 \exp(-E^2/2U_0^2)$$

Keldysh & Proshko '63
Kane '63
Shklovskii and Efros '70
John & Stephen '84

b >> B $\Rightarrow$ new results

$$\frac{L_c}{b} = \frac{B^4}{b^4}$$

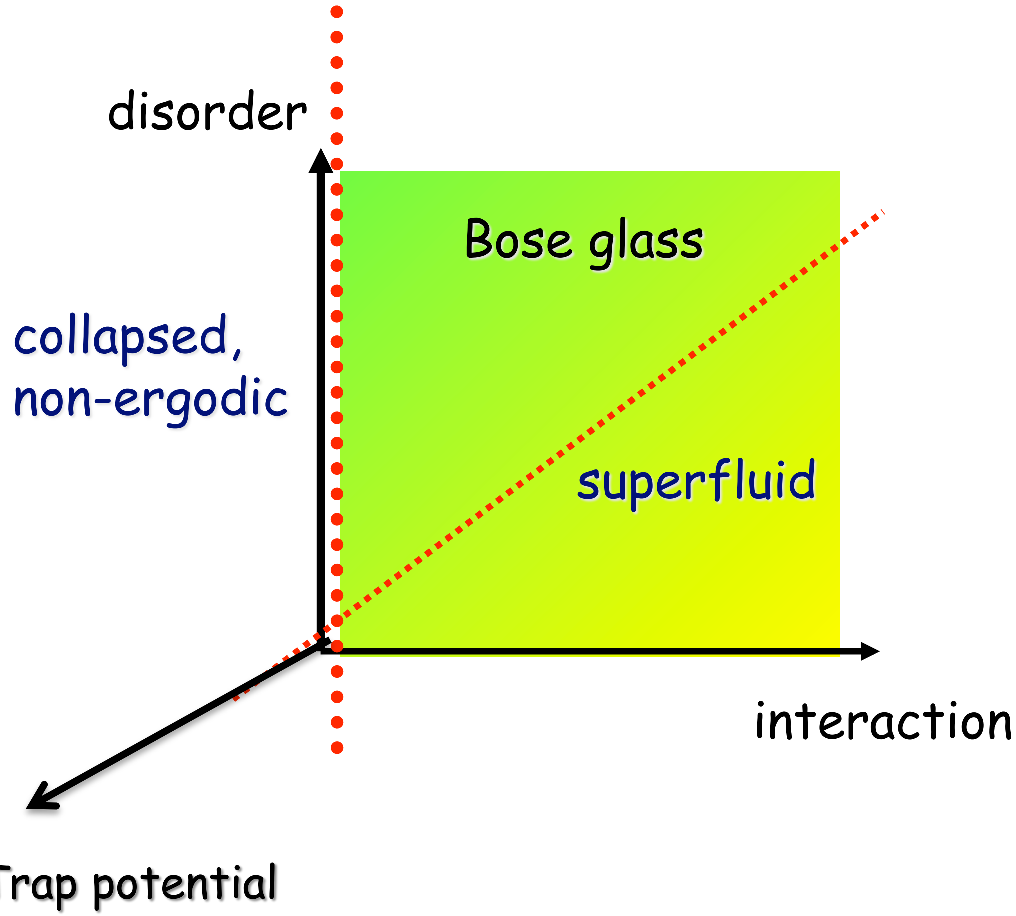
$$\mu(b,n) \approx -U_0\sqrt{2\ln(\frac{n_c}{n})}$$

$n \ll n_c \sim 1/(B^2 a)$

$n_w(E)=b^{-3}\exp\{E^2/2U_0^2\}$

Percolation at E=-0.9U

Strong Disorder + Interaction + Trap



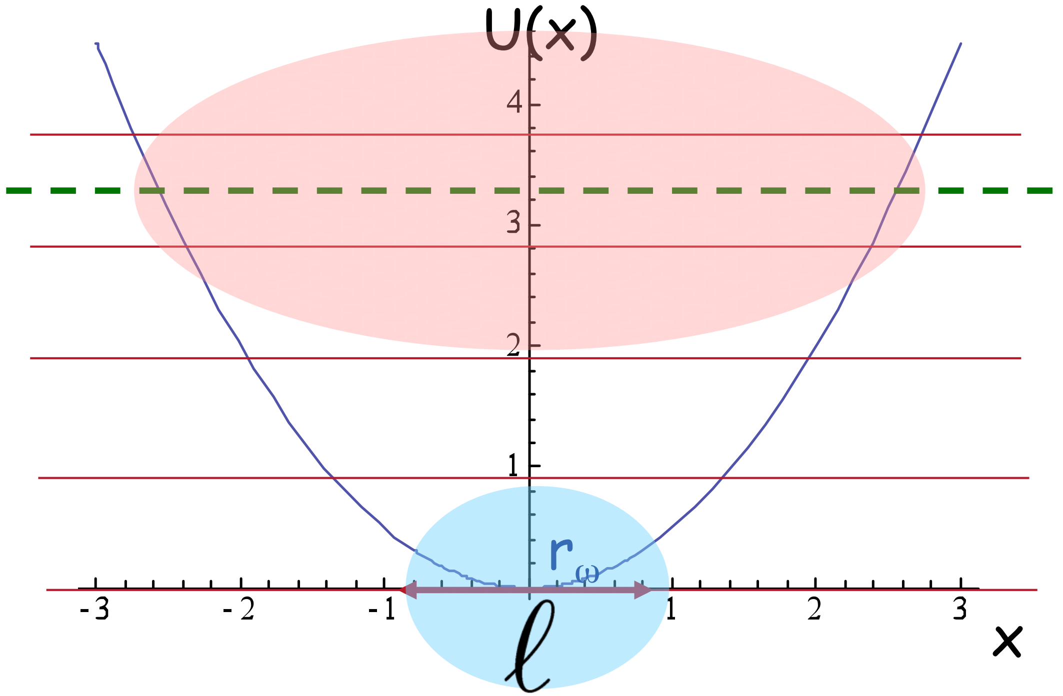
Ideal quantum gas in a harmonic trap

- oscillator length $\ell = (\hbar/m\omega)^{1/2}$ ($\approx$ 1000nm), $\hbar\omega \approx$ nK

- Bosons: T=0: all particles in ground state

- T_c : $\lambda_T{}^3 n \sim \lambda_T{}^3 N/R^3 \approx 1$ → $T_{c0} \sim \hbar\omega\, N^{1/3}$

$\lambda_T = (h^2/\,Tm)^{1/2}$



$N \approx 10^3 \ldots 10^8$

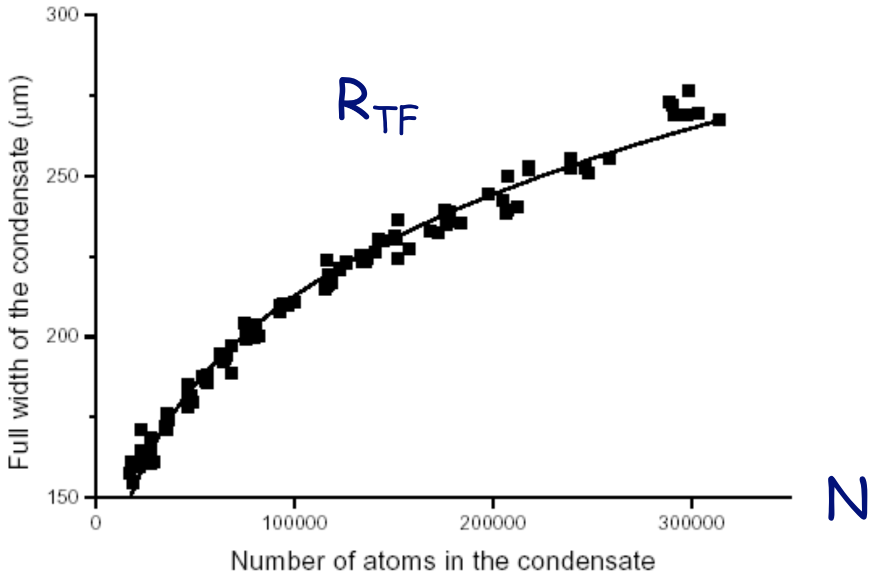
Fermions: $\varepsilon_F \sim T_{c0}$

$R_F \approx N^{1/6}\ell$

Weakly interacting bosons in a harmonic trap

- $\mu(R) = \frac{\hbar^2}{2m}\left[\frac{1}{R^2} + \frac{R^2}{\ell^4} + 3a\frac{N}{R^3}\right]$ $\Rightarrow$ min!

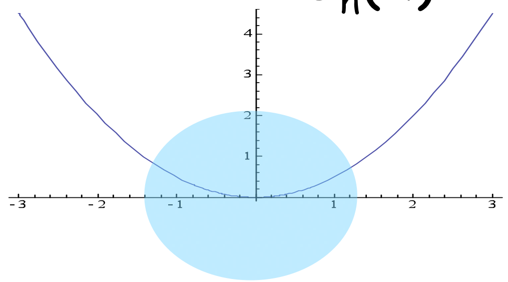
- T=0: $R_{TF} \approx \ell\,[1 + c_1\, aN/\ell\,]^{1/5}$ "Thomas-Fermi"



$aN \gg \ell$: $R \gg \xi \rightarrow$ SF

Bosons in traps (uncorrelated disorder)

oscillator length $\ell = (\hbar/m\omega)^{1/2}$ ($\approx$ 1000nm), $\hbar\omega \approx$ nK



$$\mu = -E_c \ln^2(n_c/n(r)) + \frac{\hbar^2 r^2}{2m\ell^4}$$

$$n(r) = n_c \exp\left(- \sqrt{\frac{L_c^2 r^2}{\ell^4} - \frac{\mu}{E_c}} \right)$$

$$N = \int d^d r n(r)$$

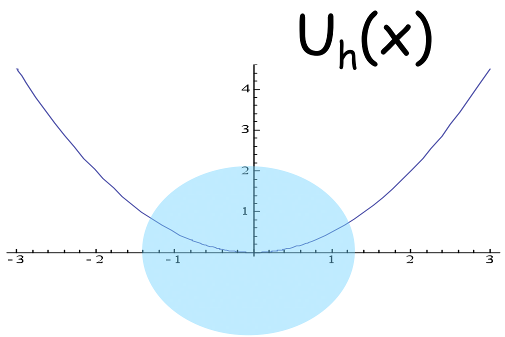
$$n(r) = n_c \left(\frac{n}{n_c}\right)^{\sqrt{1+r^2/r_F^2}},$$

$$r_F = \frac{\ell^2}{L_c} \ln \Gamma$$

$$\frac{n_c}{n} = \Gamma = \frac{\ell^6}{3NaL_c}$$

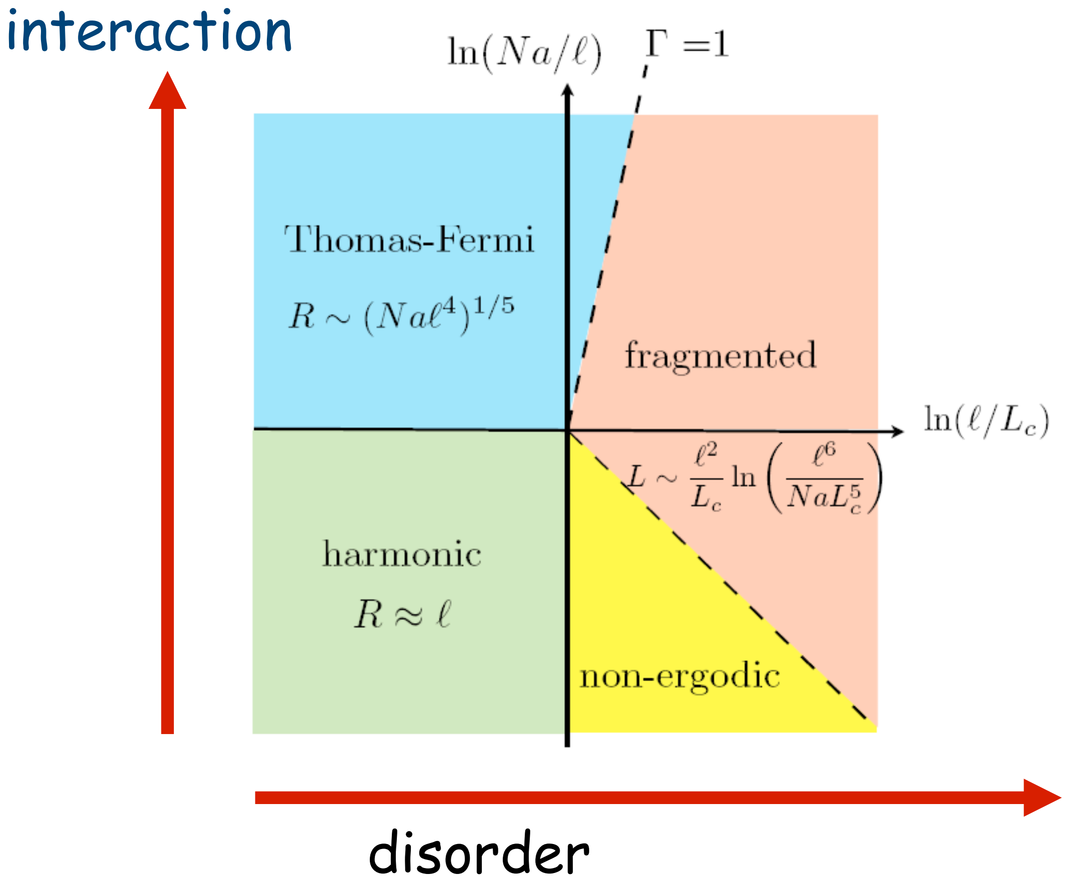
Bosons in traps (uncorrelated disorder)

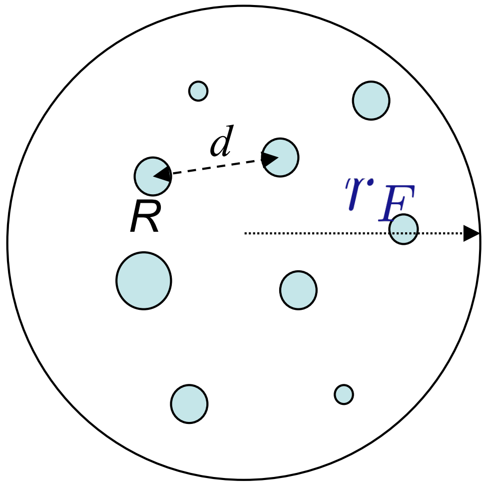
oscillator length $\ell = (\hbar/m\omega)^{1/2}$ ($\approx$ 1000nm), $\hbar\omega \approx$ nK



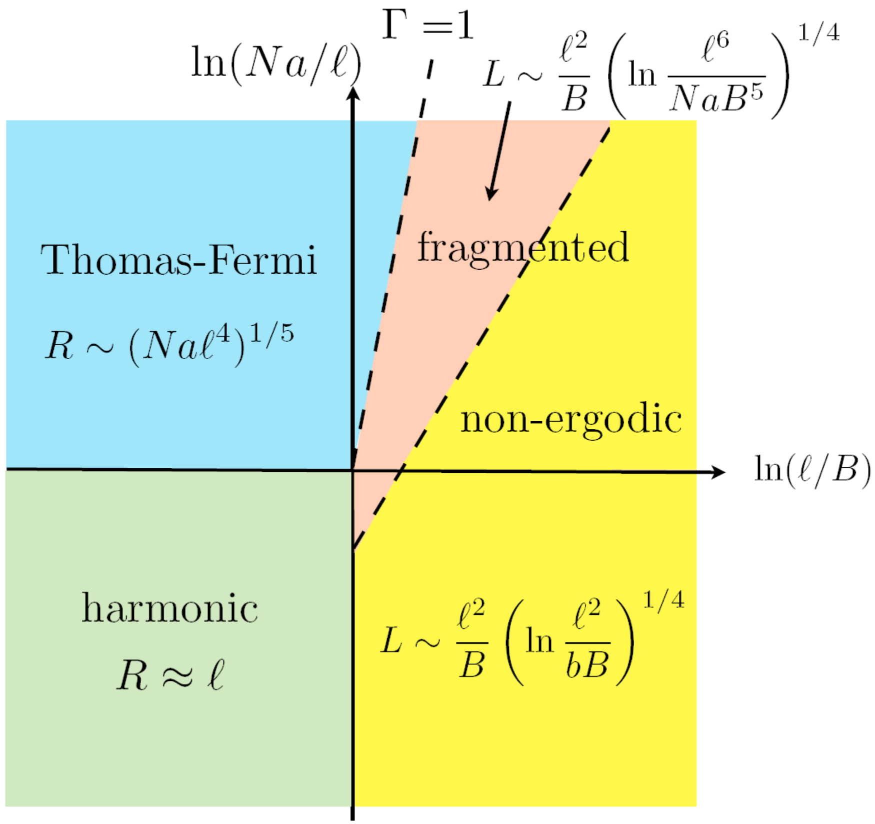
$$\Gamma = \frac{\ell^6}{3NaL_c}$$

fragmented state





Bosons in traps (correlated disorder, d=3)



$$\Gamma = \frac{\ell^6}{NaB^5}$$

Generalization to d<3 dimensions

What is different?

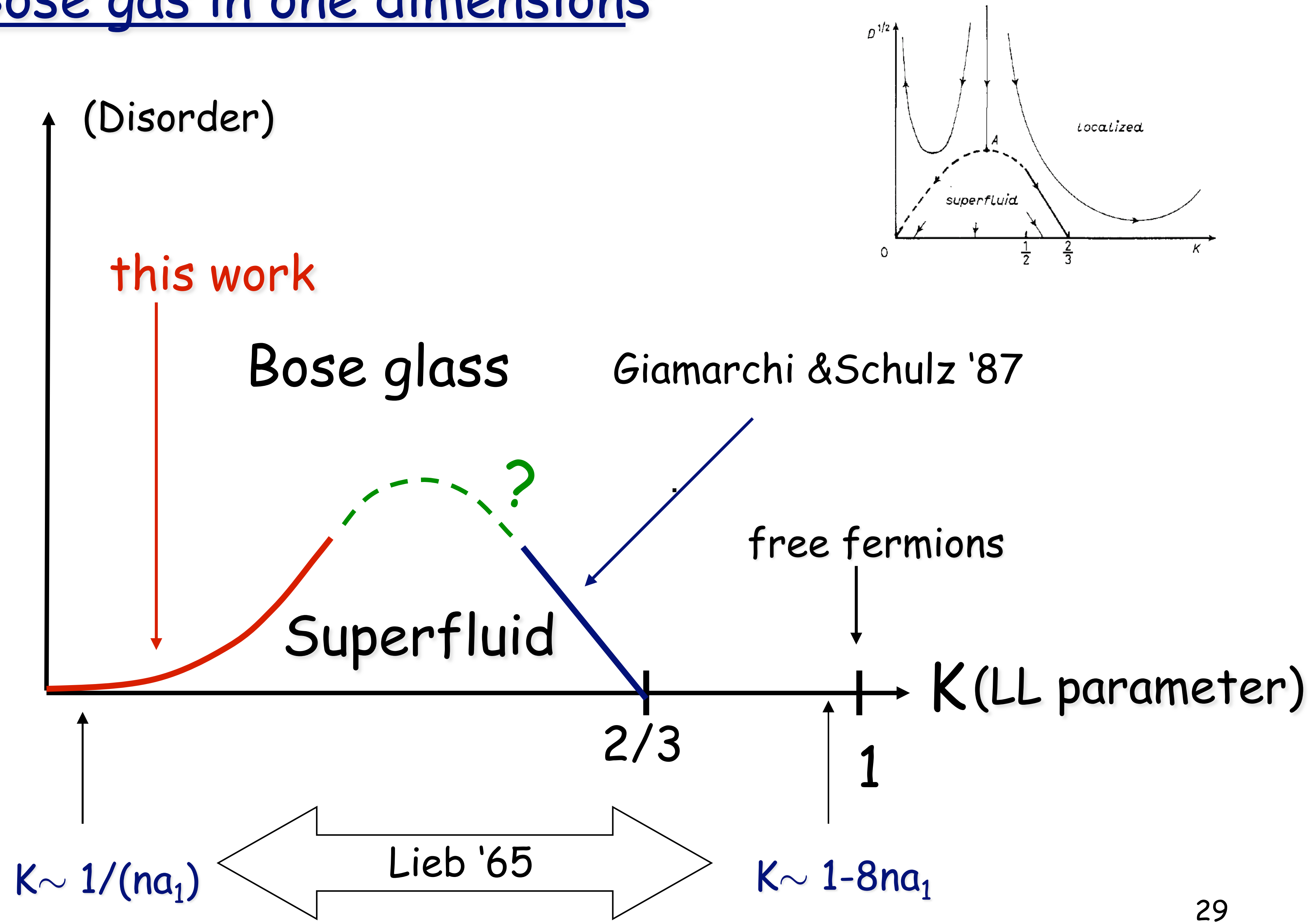
DOS,

$a \rightarrow a_d^{d-2} = a\, r_\perp^{d-3}$

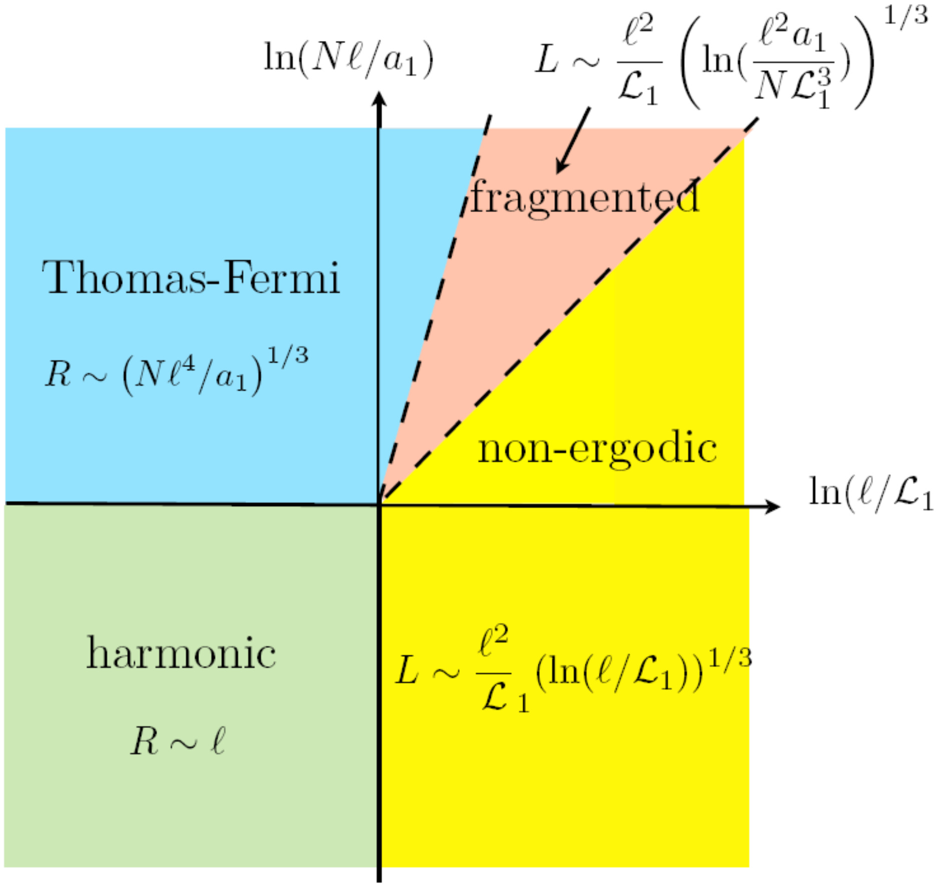
ξ, L_c, E_c

$n/n_c \sim n/L_c^2 a_d^{d-2}$

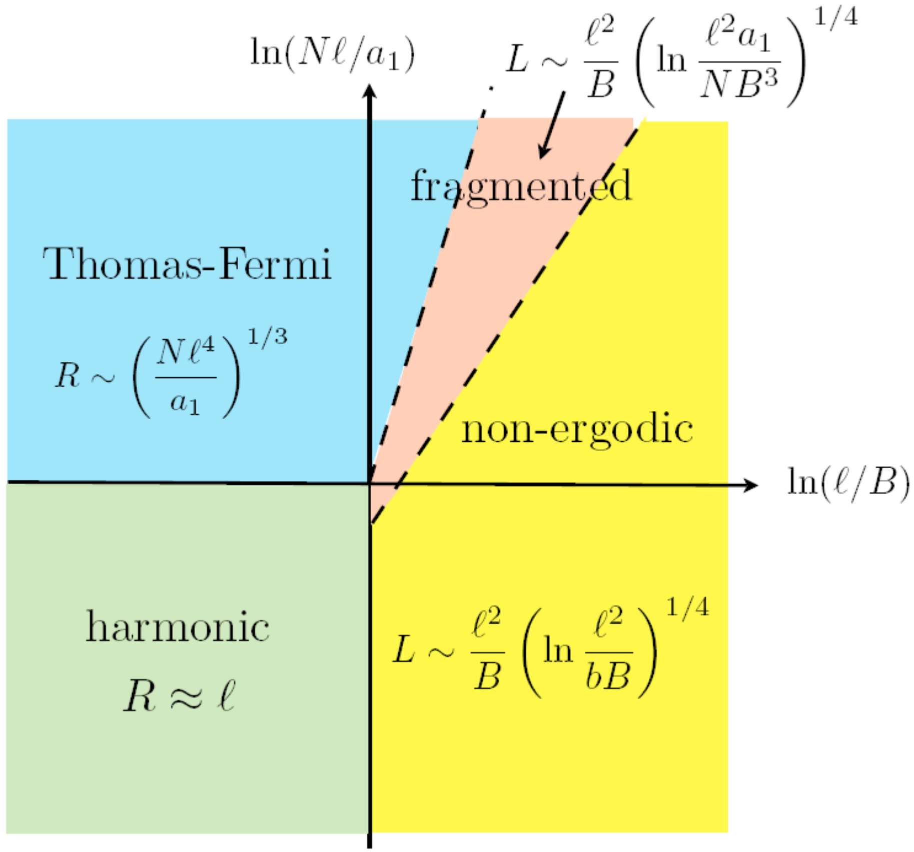
Bose gas in one dimensions



Bose gas in 1 dimensions: parabolic trap



Uncorrelated disorder



Correlated disorder

Prediction which could be tested

1. Cloud size as function of these parameters in fragmented state?

2. Cross-over from non-ergodic to ergodic state at critical n $N_c = L_c/3a$, $N_c = b^3/(3aB^2)$, number of particles in fragments?

3. Time of flight spectroscopy, $\Delta p \sim \hbar/R$, fragmented state should be visible.

4. Ground state reachable? According to our estimates ($L_c \approx 1\ \mu m$) relaxation time $\approx$ 0.06s. Easier in lower dimension.

Changeable parameter: N, ω, a, U_0, b

Conclusions

- Semi-quantitative analysis of the phase states of a weakly interacting strongly diluted Bose gas in a random Gaussian potential.
- The system is charcterized by the mean free path L_c and the scattering length a (or a, U_0 and B for correlated disorder)
- At particle density $n \ll n_c \approx 1/(aL_c^2)$ the Bose particles occupy deep potential wells and exponentially weakly tunnel to other wells. The number of particles in each well is defined, but phases are uncertain.
- At average particle density $n \approx n_c$ the transition to the superfluid proceeds.
- In a trap the oscillator length l appears as a new length scale. Four different regimes are found, depending on the mutual strength of L_c, aN and l , respectively.
- All results can be extended to lower dimensions and to correlated disorder.