The Abdus Salam
International Centre for Theoretical Physics

2162-5

Advanced Workshop on Anderson Localization, Nonlinearity and Turbulence: a Cross-Fertilization

23 August - 3 September, 2010

INTRODUCTORY
Anderson Localization - Introduction

Boris ALTSHULER
Columbia University, Dept. of Physics, New York
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Anderson localization - introduction

Boris Altshuler

Physics Department, Columbia University

Advanced Workshop on Anderson Localization, Nonlinearity and Turbulence: a Cross-Fertilization

August 23 – September 03, 2010

ICTP

The Abdus Salam
International Centre for Theoretical Physics

UNESCO
United Nations
Educational, Scientific and
Cultural Organization

IAEA
International Atomic Energy Agency

One-particle Localization

>50 years of Anderson Localization

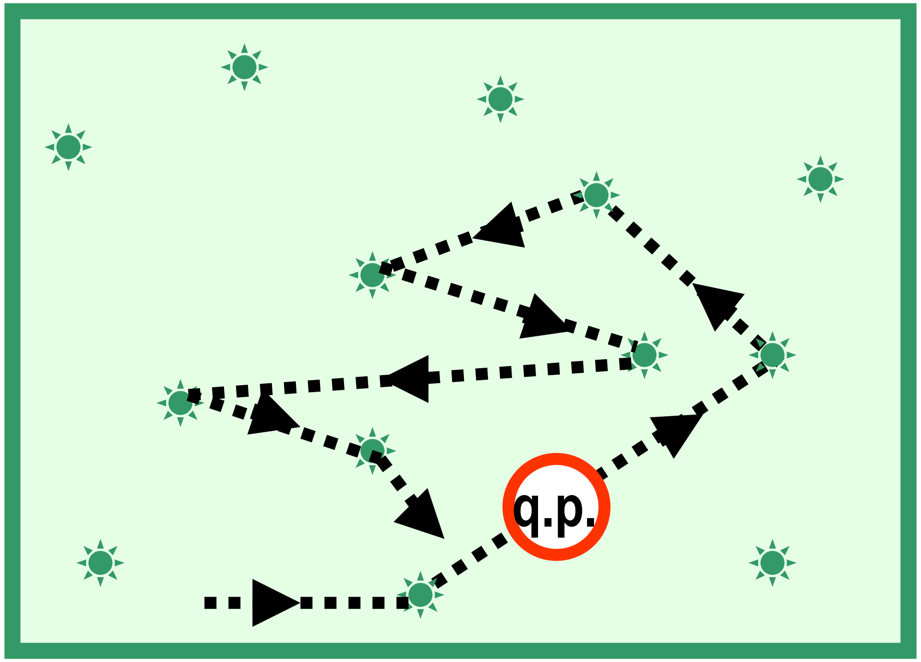
PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

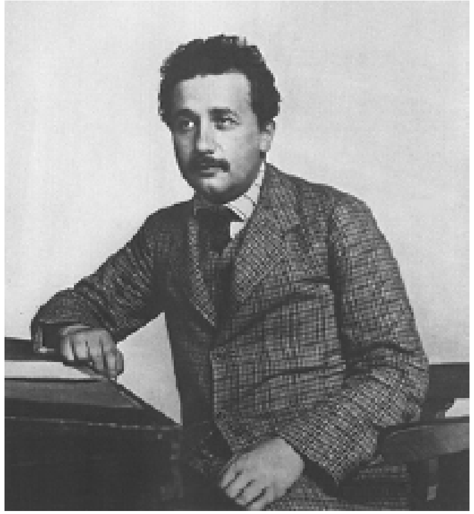
Absence of Diffusion in Certain Random Lattices

P. W. ANDERSON
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received October 10, 1957)

This paper presents a simple model for such processes as spin diffusion or conduction in the "impurity band." These processes involve transport in a lattice which is in some sense random, and in them diffusion is expected to take place via quantum jumps between localized sites. In this simple model the essential randomness is introduced by requiring the energy to vary randomly from site to site. It is shown that at low enough densities no diffusion at all can take place, and the criteria for transport to occur are given.

- One quantum particle
- Random potential (e.g., impurities) Elastic scattering





Einstein (1905):

$$\langle r^2 \rangle = Dt$$

diffusion constant

Random walk

⇓

always diffusion

as long as the system has no memory

Anderson(1958):

For quantum particles

⇓

not always!

It might be that

$$\langle r^2 \rangle \xrightarrow[t \to \infty]{} const$$

$$D = 0$$

Quantum interference ⇒ memory

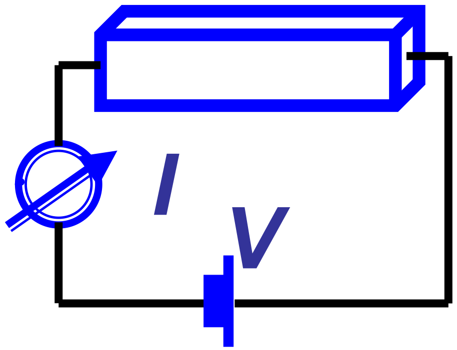
Einstein Relation (1905)

$$\sigma = e^2 D \nu \qquad \nu \equiv \frac{dn}{d\mu}$$

Conductivity

Diffusion Constant

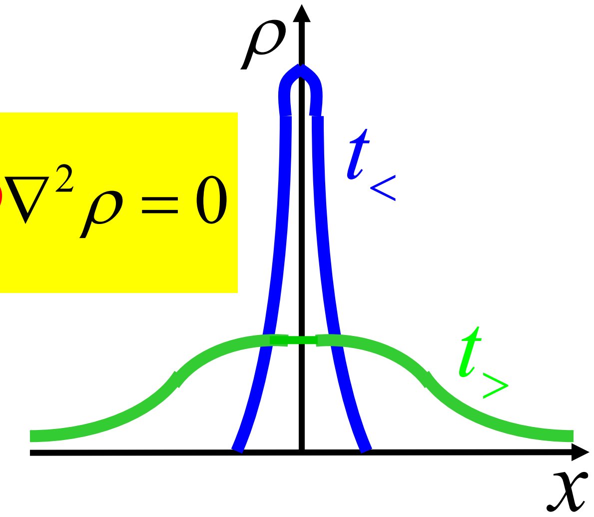
Density of states



$$G = \left(\frac{I}{V}\right)_{V=0} \; ;$$

$$\sigma = G\frac{L}{A}$$

$$\frac{\partial \rho}{\partial t} - D\nabla^2 \rho = 0$$



Einstein Relation (1905)

$$\sigma = e^2 D \nu \qquad \nu \equiv \frac{dn}{d\mu}$$

Conductivity

Diffusion Constant

Density of states

No diffusion – no conductivity

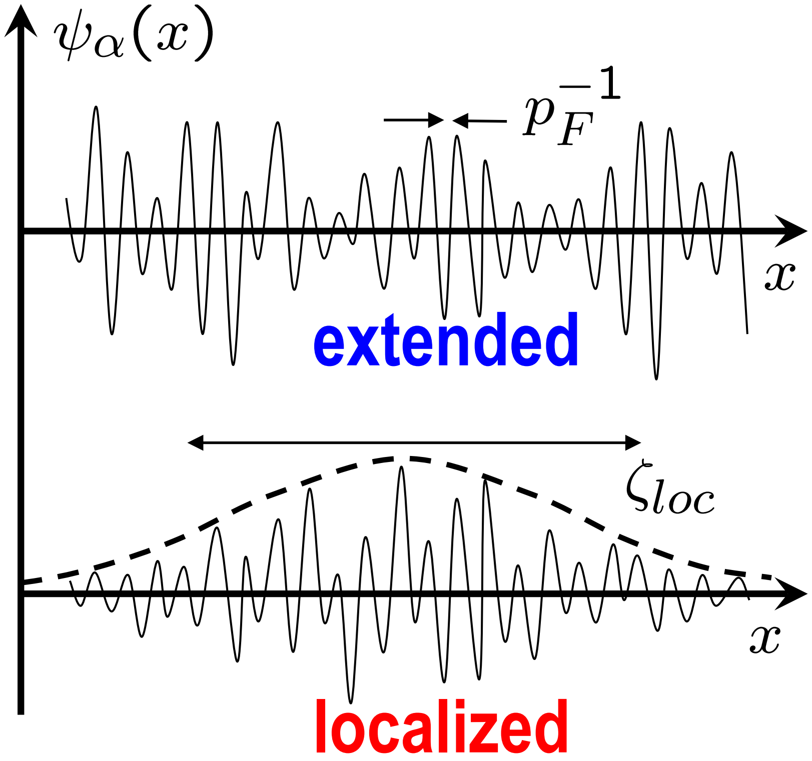
Localized states – insulator
Extended states – metal

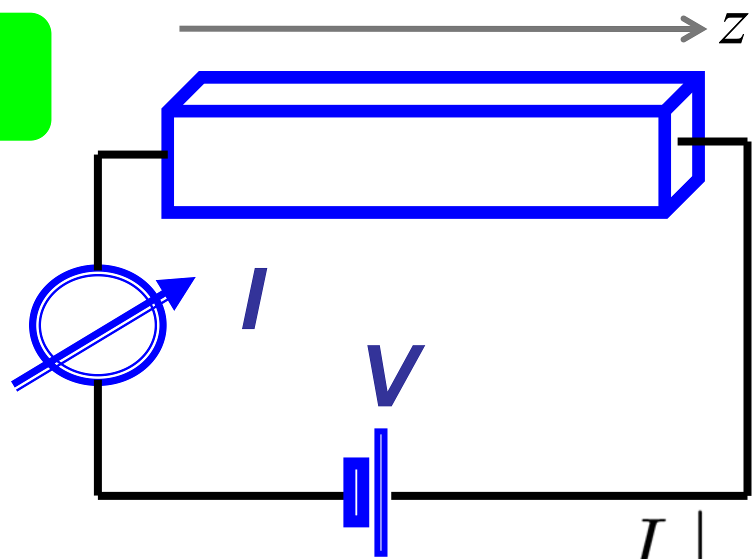
Metal – insulator transition

Localization of single-electron wave-functions:

$$\left[-\frac{\nabla^2}{2m}+U(\boldsymbol{r})-\epsilon_F\right]\psi_\alpha(\boldsymbol{r})=\xi_\alpha\psi_\alpha(\boldsymbol{r})$$

Disorder





Conductance $G=\left.\frac{I}{V}\right|_{V\to 0}$

$$=\begin{cases}\sigma\dfrac{L_xL_y}{L_z} & \text{extended}\\ \propto\exp\left(\dfrac{-L_z}{\varsigma_{loc}}\right) & \text{localized}\end{cases}$$

Experiment

Spin Diffusion

Feher, G., Phys. Rev. 114, 1219 (1959); Feher, G. & Gere, E. A., Phys. Rev. 114, 1245 (1959).

Light

Wiersma, D.S., Bartolini, P., Lagendijk, A. & Righini R. "Localization of light in a disordered medium", *Nature* 390, 671-673 (1997).

Scheffold, F., Lenke, R., Tweer, R. & Maret, G. "Localization or classical diffusion of light", *Nature* 398,206-270 (1999).

Schwartz, T., Bartal, G., Fishman, S. & Segev, M. "Transport and Anderson localization in disordered two dimensional photonic lattices". *Nature* 446, 52-55 (2007).

C.M. Aegerter, M.Störzer, S.Fiebig, W. Bührer, and G. Maret : JOSA A, 24, #10, A23, (2007)

Microwave

Dalichaouch, R., Armstrong, J.P., Schultz, S.,Platzman, P.M. & McCall, S.L. "Microwave localization by 2-dimensional random scattering". *Nature* 354, 53, (1991).

Chabanov, A.A., Stoytchev, M. & Genack, A.Z. Statistical signatures of photon localization. *Nature* 404, 850, (2000).

Pradhan, P., Sridar, S, "Correlations due to localization in quantum eigenfunctions od disordered microwave cavities", PRL 85, (2000)

Sound

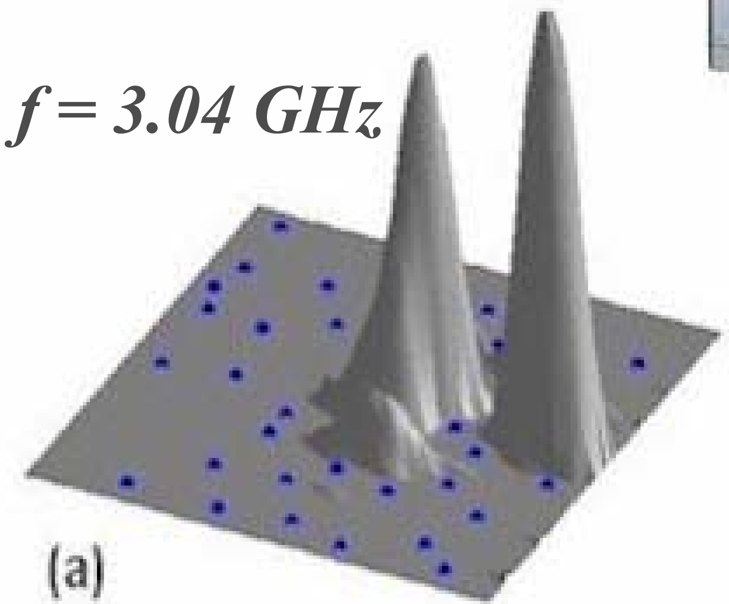
Weaver, R.L. Anderson localization of ultrasound. *Wave Motion* 12, 129-142 (1990).

VOLUME 85, NUMBER 11 PHYSICAL REVIEW LETTERS 11 SEPTEMBER 2000

Correlations due to Localization in Quantum Eigenfunctions of Disordered Microwave Cavities

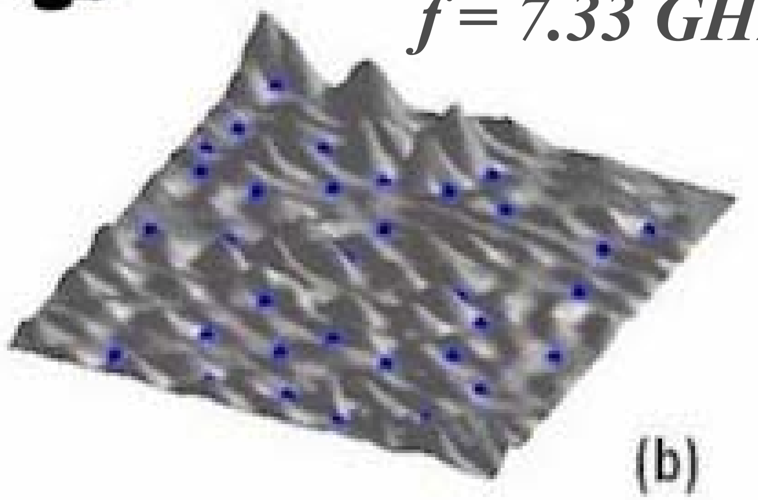
Prabhakar Pradhan and S. Sridhar
Department of Physics, Northeastern University, Boston, Massachusetts 02115
(Received 28 February 2000)

***f* = 3.04 GHz**



Localized State
Anderson Insulator

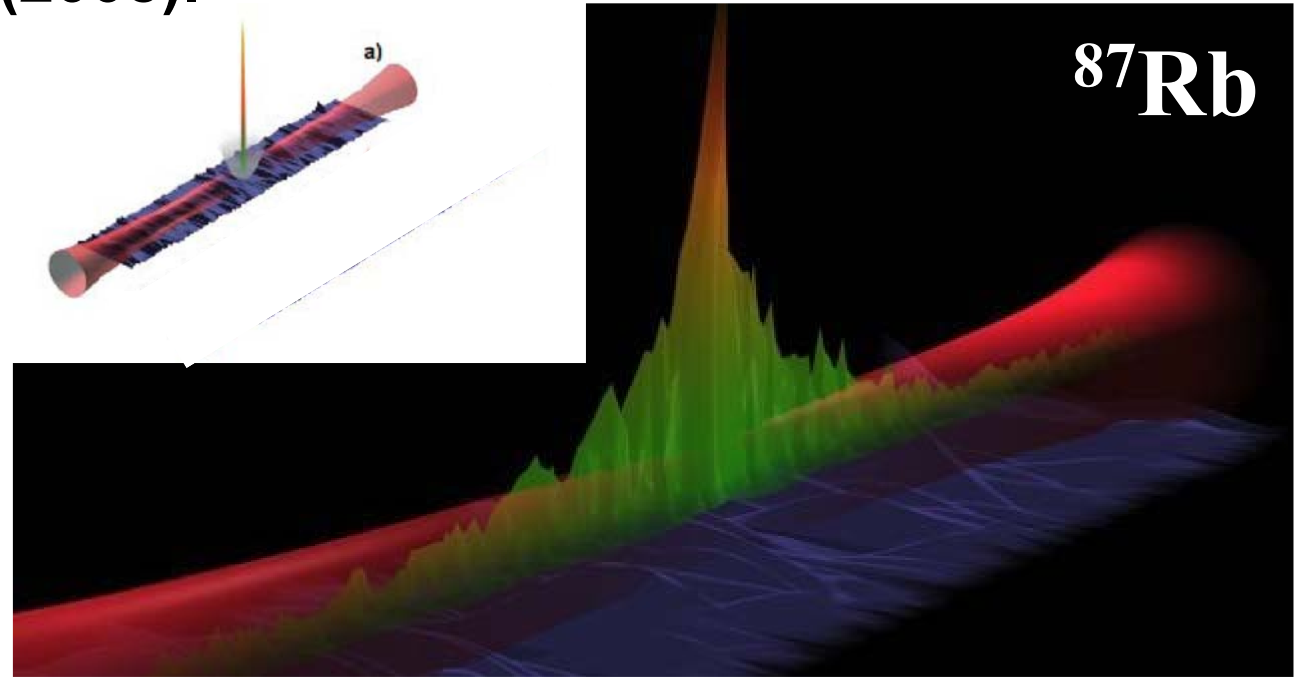
***f* = 7.33 GHz**



Extended State
Anderson Metal

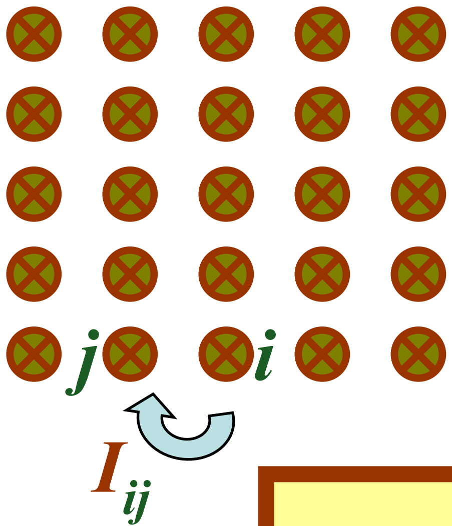
Localization of cold atoms

Billy et al. "Direct observation of Anderson localization of matter waves in a controlled disorder". Nature 453, 891- 894 (2008).



Roati et al. "Anderson localization of a non-interacting Bose-Einstein condensate". Nature 453, 895-898 (2008).

Anderson Model



- *Lattice - tight binding model*
- *Onsite energies* ε_i *- random*
- *Hopping matrix elements* I_{ij}

$-W < \varepsilon_i < W$

uniformly distributed

$$I_{ij} = \begin{cases} I & i \text{ and } j \text{ are nearest neighbors} \\ 0 & \text{otherwise} \end{cases}$$

Anderson Transition

$I < I_c$

Insulator

All eigenstates are localized

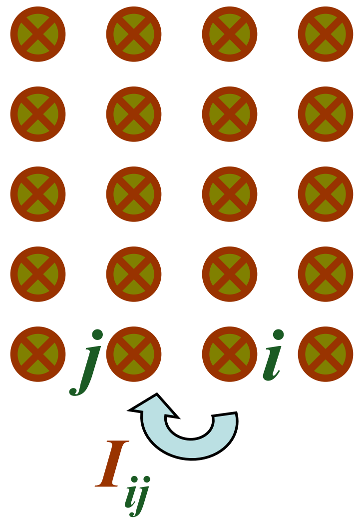
Localization length ξ

$I > I_c$

Metal

There appear states extended all over the whole system

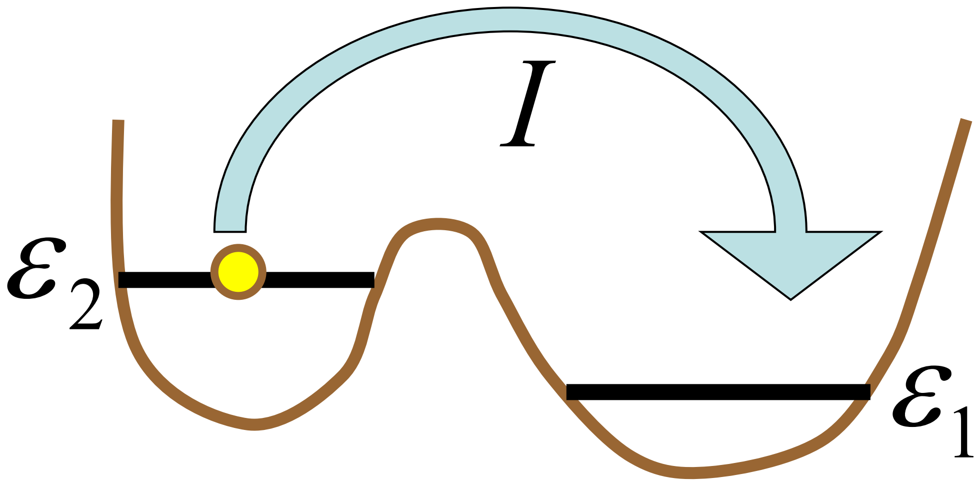
Q: Why arbitrary weak hopping I is not sufficient for the existence of the diffusion ?



Einstein (1905): Marcovian (no memory) process → diffusion

Quantum mechanics is not marcovian
There is memory in quantum propagation !

Why ?



Hamiltonian

$$\hat{H} = \begin{pmatrix} \varepsilon_1 & I \\ I & \varepsilon_2 \end{pmatrix} \xrightarrow{\text{diagonalize}} \hat{H} = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}$$

$$E_2 - E_1 = \sqrt{(\varepsilon_2 - \varepsilon_1)^2 + I^2}$$

$$\hat{H} = \begin{pmatrix} \varepsilon_1 & I \\ I & \varepsilon_2 \end{pmatrix}$$

$$E_2 - E_1 = \sqrt{(\varepsilon_2 - \varepsilon_1)^2 + I^2} \approx \begin{matrix} \varepsilon_2 - \varepsilon_1 & \varepsilon_2 - \varepsilon_1 >> I \\ I & \varepsilon_2 - \varepsilon_1 << I \end{matrix}$$

What about the eigenfunctions ?

$$\phi_1, \varepsilon_1; \phi_2, \varepsilon_2 \quad \Leftarrow \quad \psi_1, E_1; \psi_2, E_2$$

$$\varepsilon_2 - \varepsilon_1 >> I$$

$$\psi_{1,2} = \varphi_{1,2} + O\left(\frac{I}{\varepsilon_2 - \varepsilon_1}\right)\varphi_{2,1}$$

Off-resonance

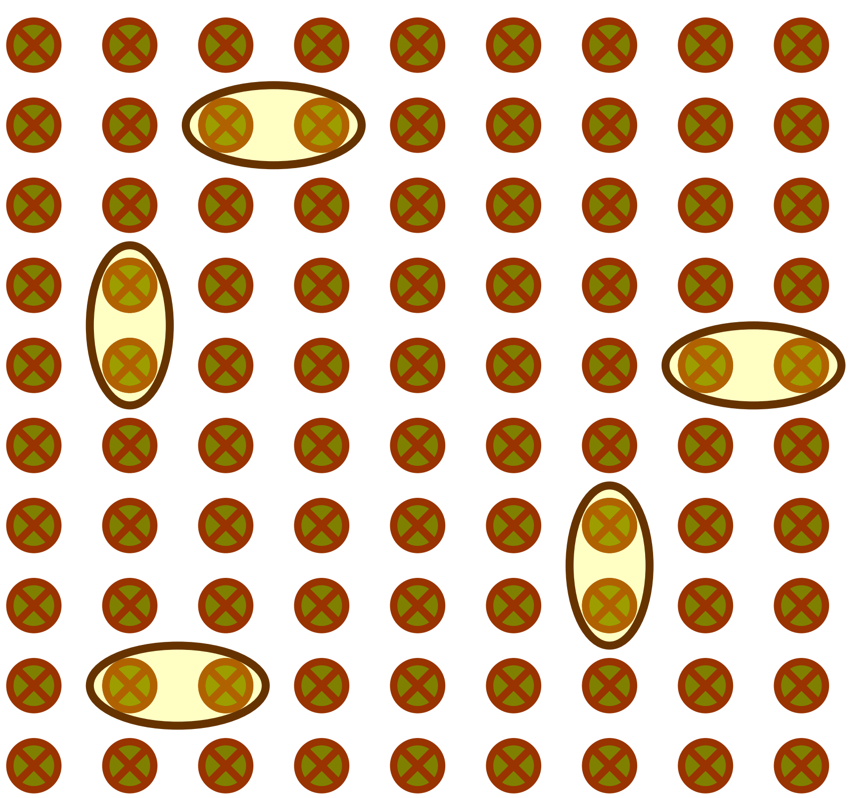
Eigenfunctions are close to the original on-site wave functions

$$\varepsilon_2 - \varepsilon_1 << I$$

$$\psi_{1,2} \approx \varphi_{1,2} \pm \varphi_{2,1}$$

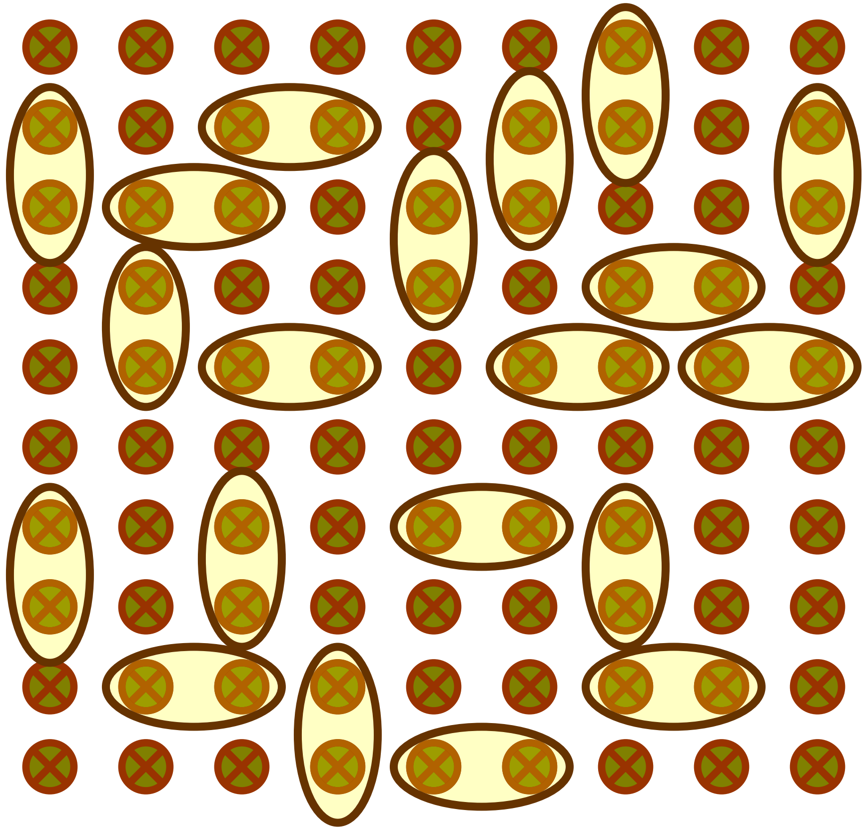
Resonance

In both eigenstates the probability is equally shared between the sites



Anderson insulator

Few isolated resonances



Anderson metal

There are many resonances
and they overlap

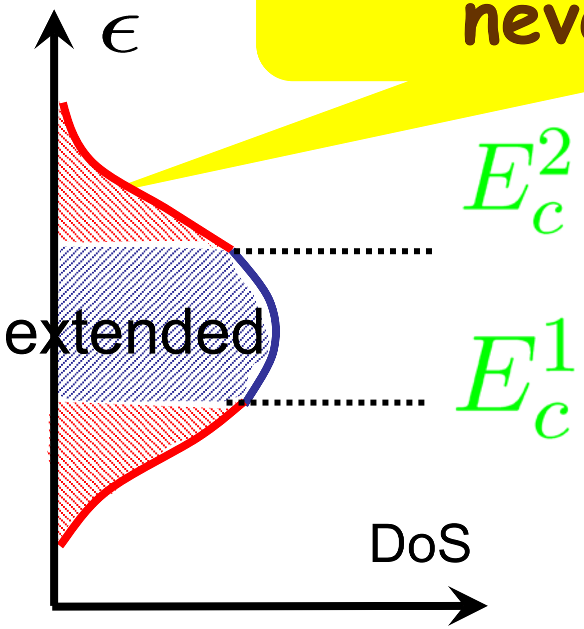
Transition: Typically each site is in resonance with some other one

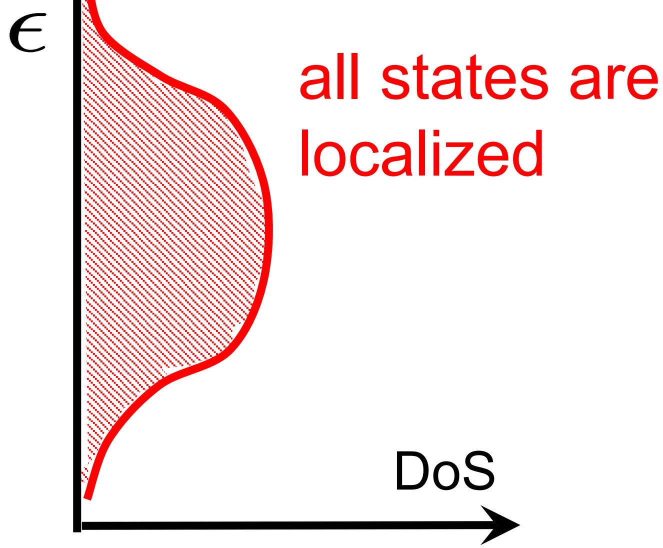
Anderson Transition

$I > I_c$

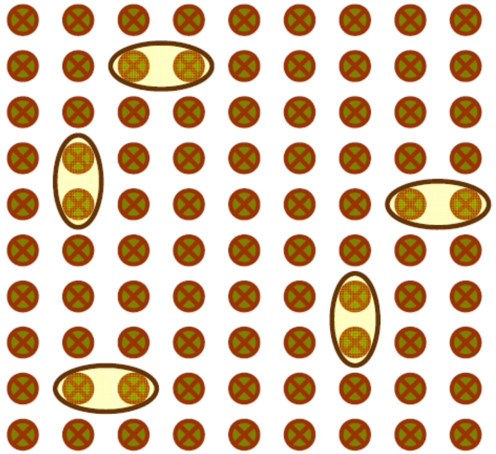
$I < I_c$

localized and extended never coexist!

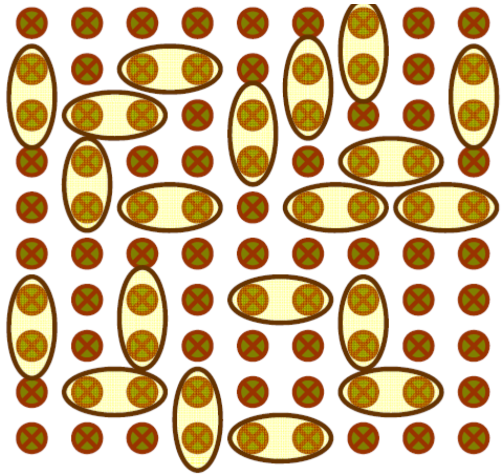




E_c - mobility edges (one particle)



Anderson insulator
Few isolated resonances



Anderson metal
There are many resonances and they overlap

Transition: Typically each site is in resonance with some other one

Condition for Localization:

$$I < \frac{\text{energy mismatch}}{\text{\# of n.neighbors}}$$

energy mismatch $= \left|\varepsilon_i - \varepsilon_j\right|_{typ} = W$

\# of nearest neighbors $= 2d$

A bit more precise:

$$\frac{I_c}{W} \simeq \left(\frac{1}{2d}\right)\left(\frac{1}{\ln d}\right)$$

Logarithm is due to the resonances, which are not nearest neighbors

Condition for Localization:

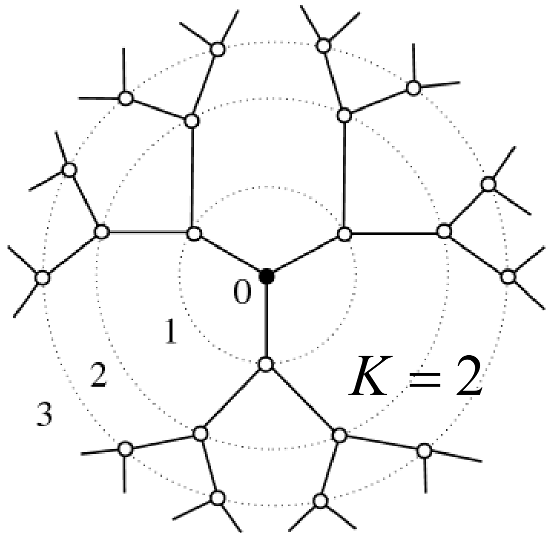
$$\frac{I_c}{W} \simeq \left(\frac{1}{2d}\right)\left(\frac{1}{\ln d}\right)$$

Q: Is it correct?

A1: Is exact on the Cayley tree

$$I_c = \frac{W}{K \ln K},$$

K is the branching number



J. Phys. C : Solid State Phys., Vol. 6, 1973. Printed in Great Britain. © 1973

Anderson Model on a Cayley tree

A selfconsistent theory of localization

R Abou-Chacra†, P W Anderson‡§ and D J Thouless†

† Department of Mathematical Physics, University of Birmingham, Birmingham, B15 2TT

‡ Cavendish Laboratory, Cambridge, England and Bell Laboratories, Murray Hill, New Jersey, 07974, USA

Received 12 January 1973

Abstract. A new basis has been found for the theory of localization of electrons in disordered systems. The method is based on a selfconsistent solution of the equation for the self energy in second order perturbation theory, whose solution may be purely real almost everywhere (localized states) or complex everywhere (nonlocalized states). The equations used are exact for a Bethe lattice. The selfconsistency condition gives a nonlinear integral equation in two variables for the probability distribution of the real and imaginary parts of the self energy. A simple approximation for the stability limit of localized states gives Anderson's 'upper limit approximation'. Exact solution of the stability problem in a special case gives results very close to Anderson's best estimate. A general and simple formula for the stability limit is derived; this formula should be valid for smooth distribution of site energies away from the band edge. Results of Monte Carlo calculations of the selfconsistency problem are described which confirm and go beyond the analytical results. The relation of this theory to the old Anderson theory is examined, and it is concluded that the present theory is similar but better.

Condition for Localization:

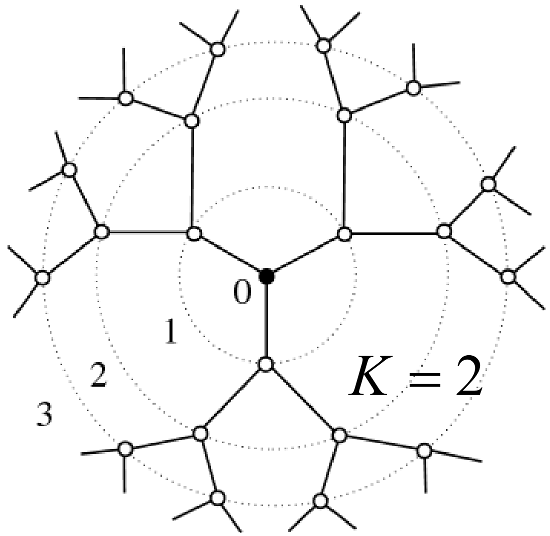
$$\frac{I_c}{W} \simeq \left(\frac{1}{2d}\right)\left(\frac{1}{\ln d}\right)$$

Q: Is it correct?

A1: Is exact on the Cayley tree

$$I_c = \frac{W}{K \ln K},$$

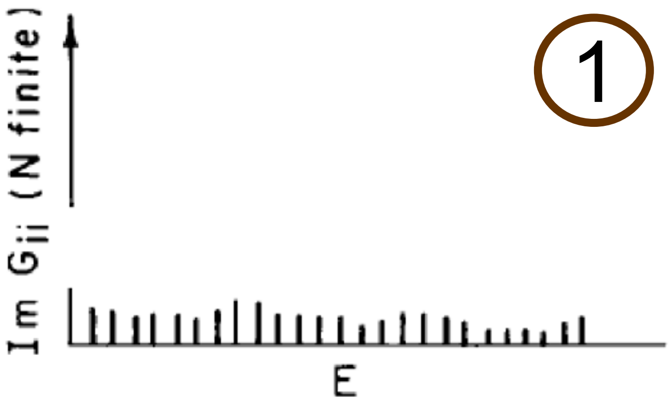
K is the branching number

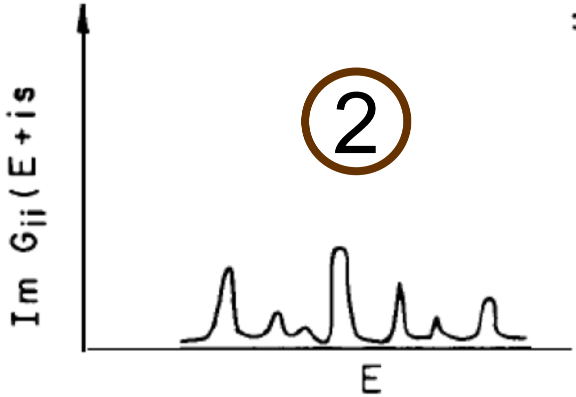


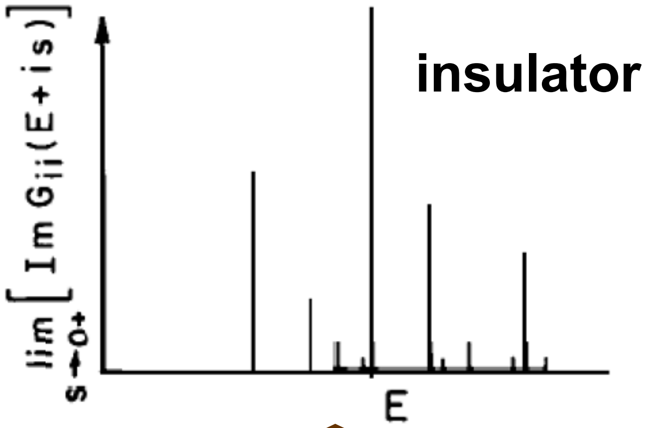
A1': Is a good approximation at high dimensions.
Is qualitatively correct for $d \geq 3$

Anderson's recipe:

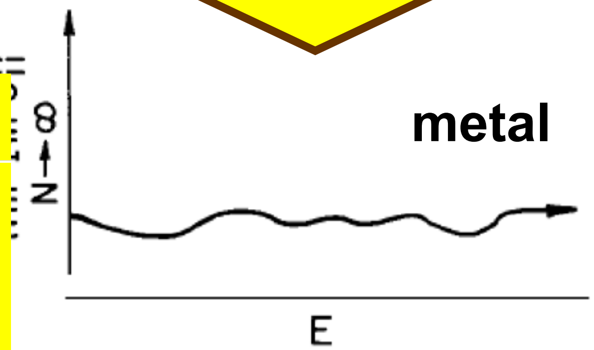
1. take descrete spectrum E_μ of H_0
2. Add an infinitesimal Im part $i\eta$ to E_μ
3. Evaluate $Im\Sigma_\mu$





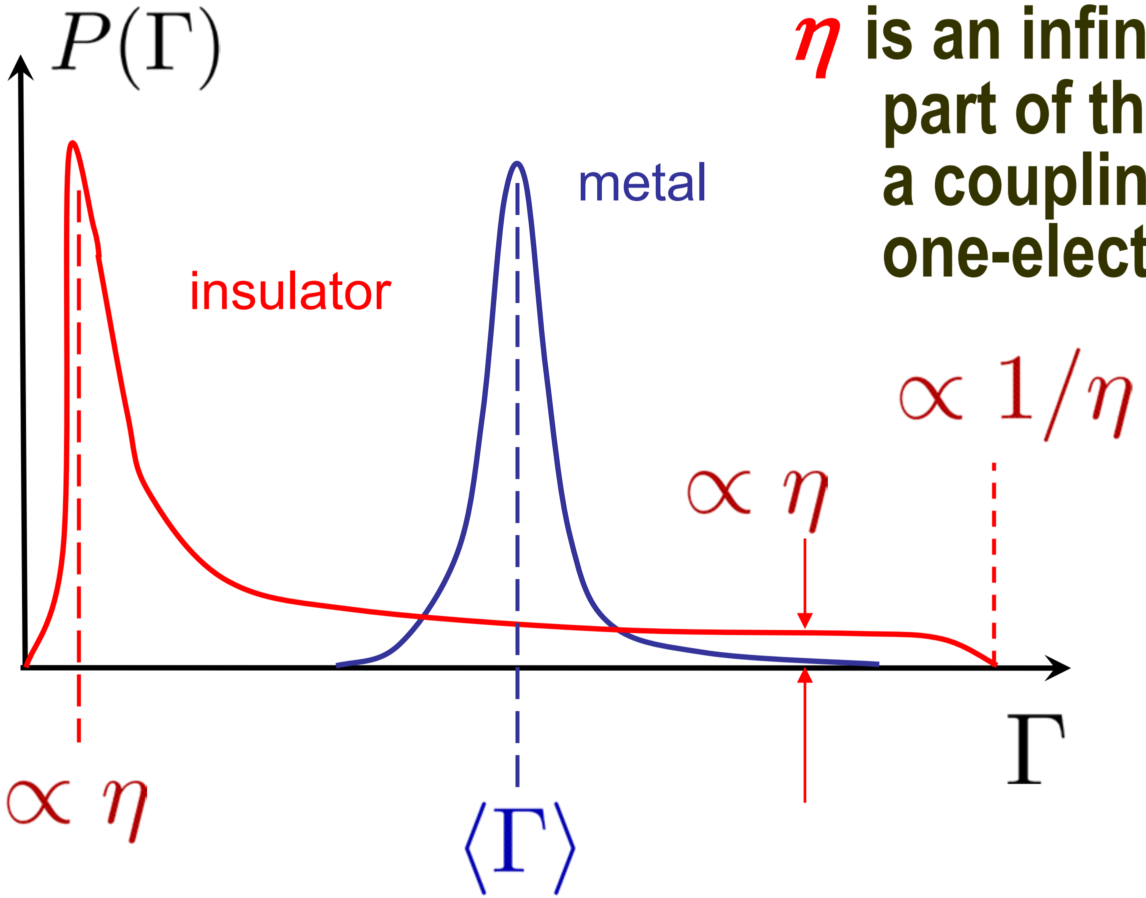


4 **limits** 1) $N \to \infty$ 2) $\eta \to 0$



4. take limit $\eta \to 0$ but only **after** $N \to \infty$
5. "What we really need to know is the *probability distribution* of $Im\Sigma$, *not* its average…" **!**

Probability Distribution of $\Gamma = Im\ \Sigma$



η is an infinitesimal width (*Im* part of the self-energy due to a coupling with a bath) of one-electron eigenstates

Look for:

$$\lim_{\eta \to +0} \lim_{V \to \infty} P(\Gamma > 0) = \begin{cases} > 0; & metal \\ 0; & insulator \end{cases}$$

Condition for Localization:

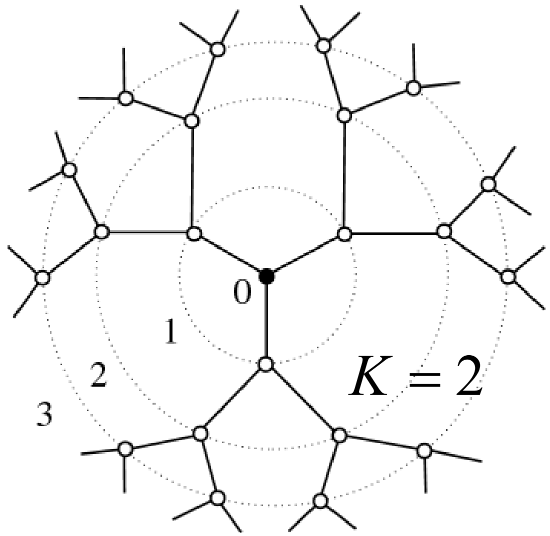
$$\frac{I_c}{W} \simeq \left(\frac{1}{2d}\right)\left(\frac{1}{\ln d}\right)$$

Q: Is it correct?

A1: Is exact on the Cayley tree

$$I_c = \frac{W}{K \ln K},$$

K is the branching number



A1': Is a good approximation at high dimensions. Is qualitatively correct for $d \geq 3$

A2: For low dimensions – **NO**. $I_c = \infty$ for $d = 1, 2$
All states are localized. Reason – loop trajectories

1D Localization

Exactly solved:
all states are localized

Gertsenshtein & Vasil'ev, 1959

Conjectured:

Mott & Twose, 1961

-
-
-

Condition for Localization:

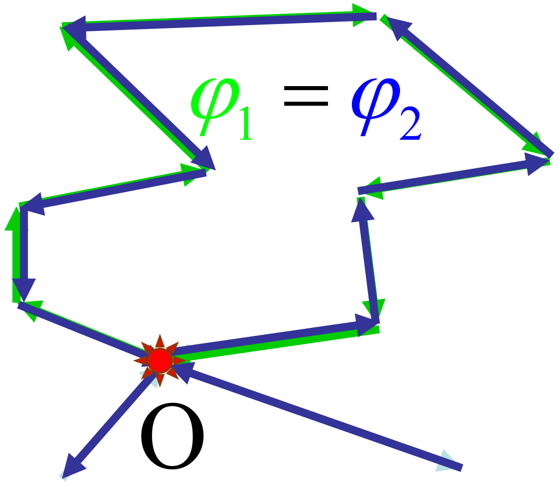
$$\frac{I_c}{W} \simeq \left(\frac{1}{2d}\right)\left(\frac{1}{\ln d}\right)$$

Q: Is it correct?

A2: For low dimensions – NO. $I_c = \infty$ for $d = 1, 2$
All states are localized. Reason – loop trajectories

$$\varphi = \oint \vec{p}\, d\vec{r}$$

Phase accumulated when traveling along the loop

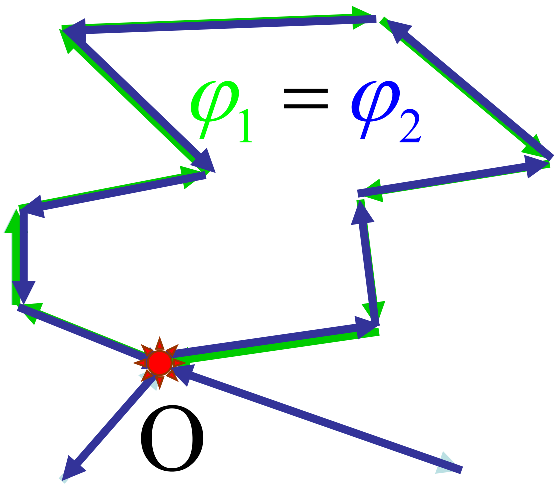


The particle can go around the loop in two directions

Memory!

Einstein: there is no diffusion at too short scales – there is memory, i.e., the process is not marcovian.

Due to the localization effects diffusion description fails at large scales.
Quantum interference ⟶ Memory



Large scales are important ⟶ diffusion constant depends on the system size

Einstein relation for the conductivity σ

$$\sigma = e^2 D\nu \qquad \nu \equiv \frac{dn}{d\mu}$$

Conductance

$$G = \sigma L^{d-2}$$

for a cubic sample of the size L

$$G = \frac{e^2}{h}\underbrace{\left(\nu L^d\right)\frac{Dh}{L^2}}_{g(L)}$$

$$g(L) = \frac{hD/L^2}{1/\nu L^d} = \frac{\text{Thouless energy}}{\text{mean level spacing}}$$

Dimensionless Thouless conductance

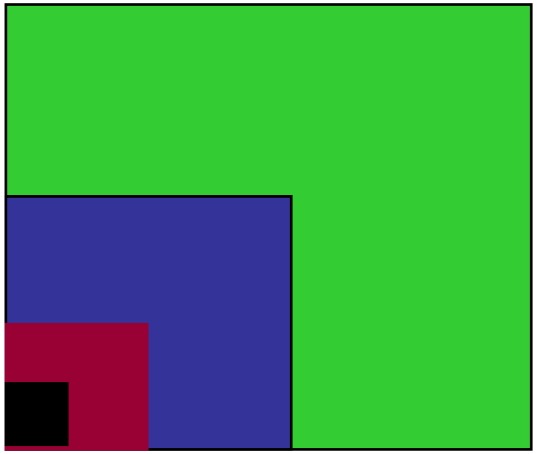
Scaling theory of Localization

(Abrahams, Anderson, Licciardello and Ramakrishnan 1979)

$$g = E_T / \delta_1$$

Dimensionless Thouless conductance

$$g = Gh/e^2$$



$$L = 2L = 4L = 8L \ldots.$$

without quantum corrections

$$E_T \propto L^{-2} \qquad \delta_1 \propto L^{-d}$$

$$\frac{E_T}{\delta_1} \rightarrow \frac{E_T}{\delta_1} \rightarrow \frac{E_T}{\delta_1} \rightarrow \frac{E_T}{\delta_1}$$

$$g \rightarrow g \rightarrow g \rightarrow g$$

$$\frac{d(\log g)}{d(\log L)} = \beta(g)$$

$$\frac{d(\log g)}{d(\log L)} = \beta(g)$$

$\beta -$ function is

Universal, i.e., material independent

But

It depends on the global symmetries, e.g., it is different with and without T-invariance (in orthogonal and unitary ensembles)

Limits:

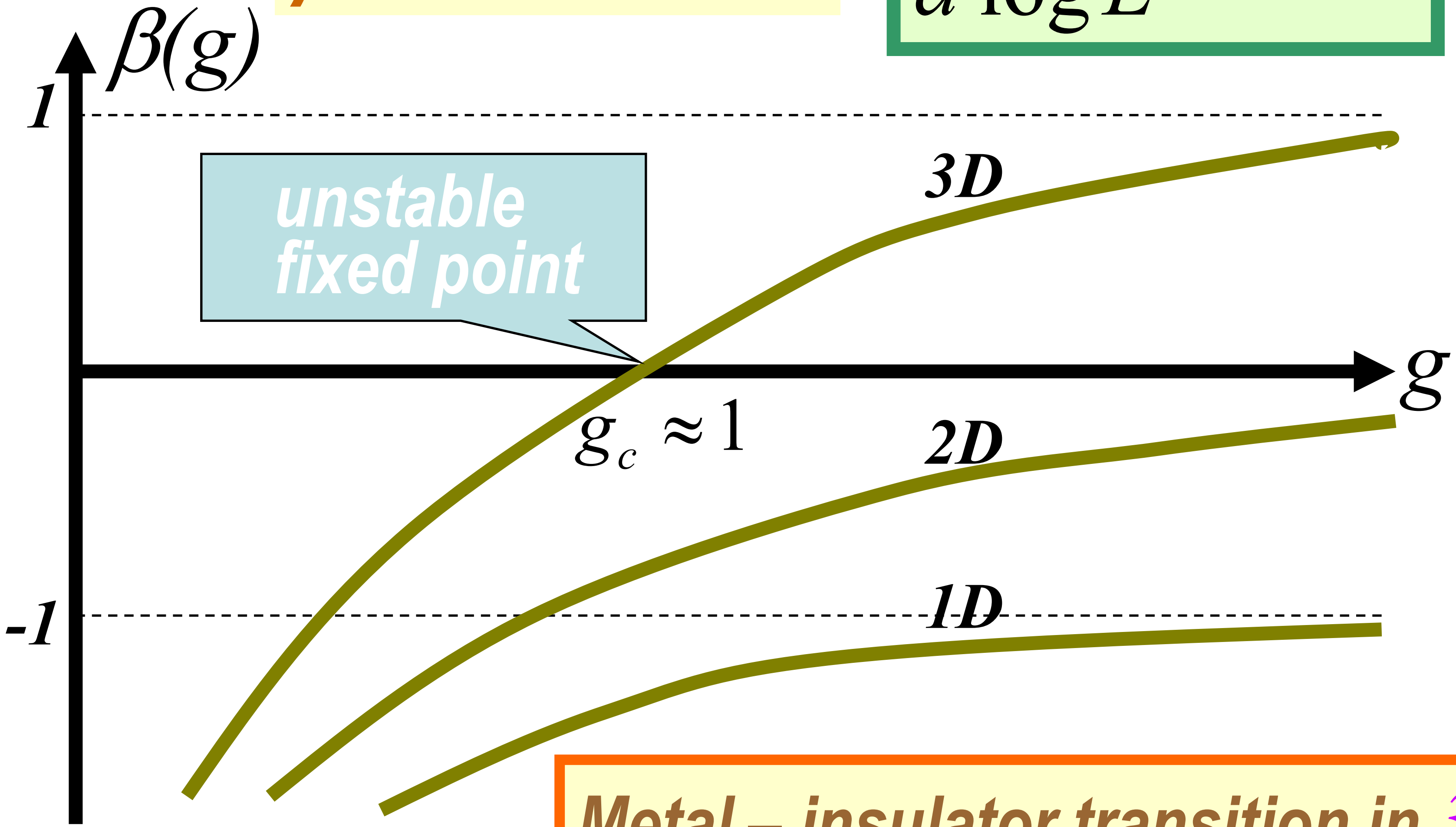
$$g >> 1 \quad g \propto L^{d-2} \quad \beta(g) = (d-2) + O\left(\frac{1}{g}\right)$$

$$g << 1 \quad g \propto e^{-L/\xi} \quad \beta(g) \approx \log g < 0$$

$$\begin{array}{ll} > 0 & d > 2 \\ ?? & d = 2 \\ < 0 & d < 2 \end{array}$$

β - function

$$\frac{d\log g}{d\log L} = \beta(g)$$



Metal – insulator transition in 3D

All states are localized for d=1,2

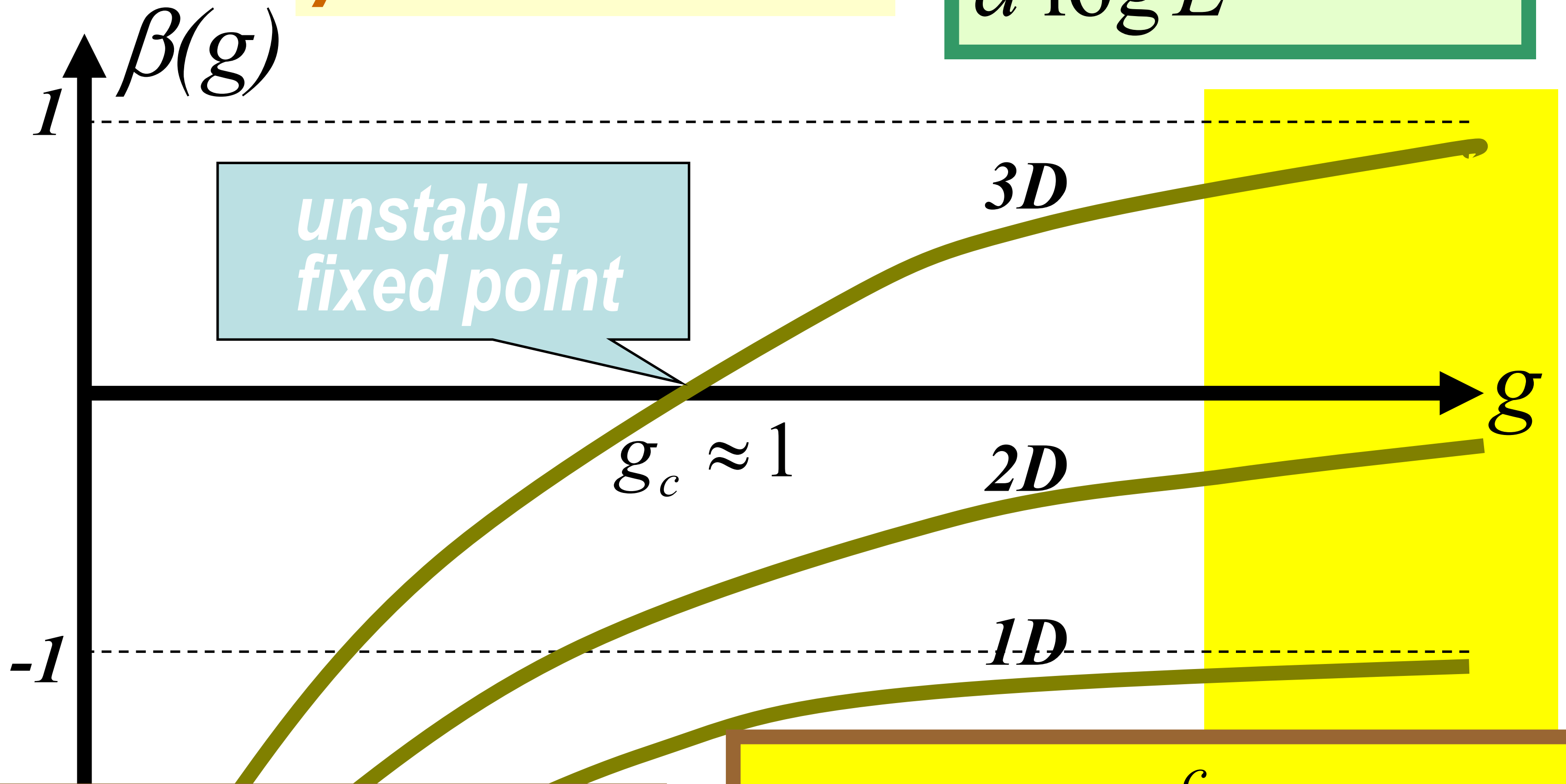
RG approach

Effective Field Theory of Localization –

Nonlinear σ - model

β - function

$$\frac{d\log g}{d\log L} = \beta(g)$$



$$\beta(g) = d - 2 + \frac{c_d}{g}$$

$c_d = ?\quad \pm ?$

$$g(L) = \sigma_{cl} L^{d-2} - \begin{cases} \dfrac{c_d}{d-2} & d \neq 2 \\ c_2 \log\left(\dfrac{L}{l}\right) & d = 2 \end{cases}$$

$$P(t) = \lambda\!\!\!^{-}{}^{d-1} \int_{\tau}^{t} \frac{v_F dt'}{(Dt')^{d/2}}$$

$$\frac{\delta g}{g} \approx P(t_{\max})$$

$$\frac{\delta g}{g} \approx -\frac{\lambda\!\!\!^{-} v_F}{D} \log \frac{L^2}{D\tau} = -\frac{2\lambda\!\!\!^{-} v_F}{D} \log \frac{L}{l}$$

$$\lambda\!\!\!^{-} v_F = \frac{1}{\pi \nu}$$

$$g = \nu D \hbar$$

$$\delta g = -\frac{2}{\pi} \log \frac{L}{l}$$

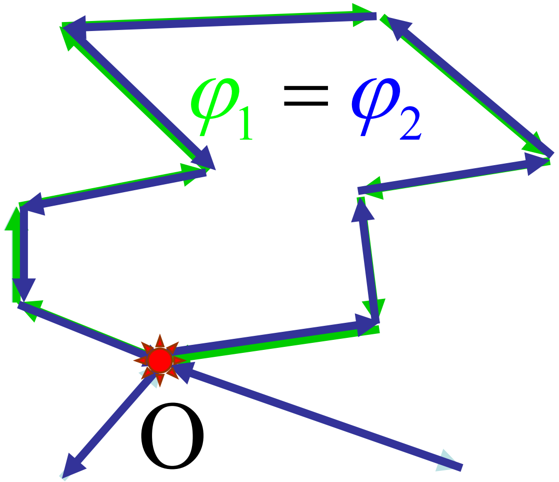
$$\beta(g) = -\frac{2}{\pi g}$$

Universal !!!

For d=1,2 all states are localized.

$$\varphi = \oint \vec{p}\,d\vec{r}$$

Phase accumulated when traveling along the loop



The particle can go around the loop in two directions

Memory!

Weak Localization:

The localization length ς can be large

Inelastic processes lead to dephasing, which is characterized by the dephasing length L_φ

If $\varsigma >> L_\varphi$, then only small corrections to a conventional metallic behavior

Т. 18 Журнал экспериментальной и теоретической физики. Вып.
1948

ОБ ИЗМЕНЕНИИ ЭЛЕКТРИЧЕСКОГО СОПРОТИВЛЕНИЯ ТЕЛЛУРА В МАГНИТНОМ ПОЛЕ ПРИ НИЗКИХ ТЕМПЕРАТУРАХ

Р. А. Ченцов

R.A. Chentsov ***"On the variation of electrical resistivity of tellurium in magnetic field at low temperatures"***, *Zh. Exp. Theor. Fiz. v.18, 375-385, (1948).*

Таблица 2

Уменьшение сопротивления теллура в магнитном поле

Образец	Температура (°K)	Максимальное уменьшение сопротивления
Te-1	2,13	$0{,}7 \cdot 10^{-3}$
Te-2	2,15	$1{,}0 \cdot 10^{-3}$
Te-4	1,96	$1{,}1 \cdot 10^{-3}$
Te-5	1,96	$0{,}5 \cdot 10^{-3}$

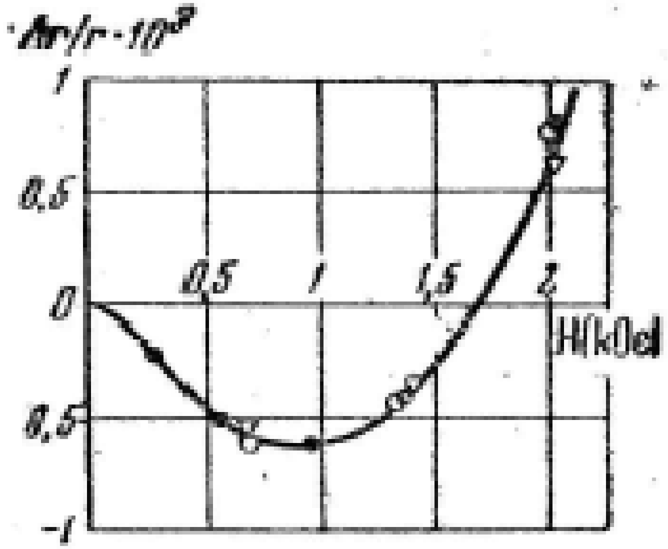
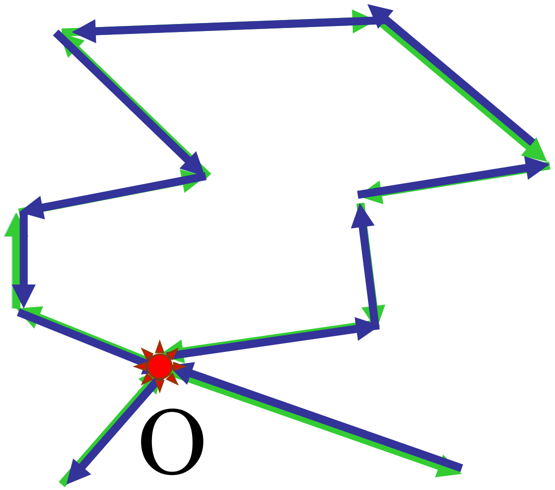
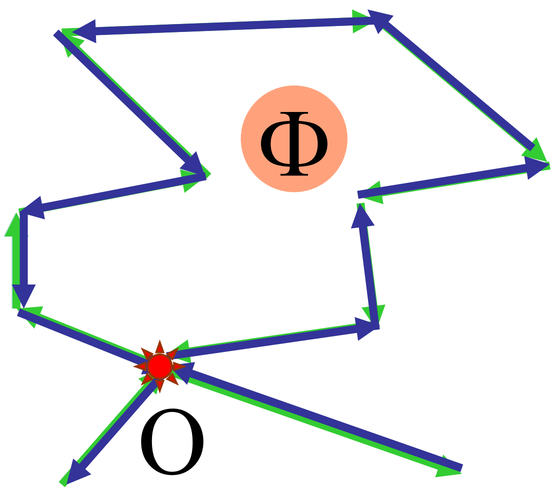


Рис. 2

Magnetoresistance





No magnetic field

$$\varphi_1 = \varphi_2$$

With magnetic field H

$$\varphi_1 - \varphi_2 = 2 * 2\pi\, \Phi/\Phi_0$$

Length Scales

Magnetic length $L_H = (hc/eH)^{1/2}$

Dephasing length $L_\varphi = (D\ \tau_\varphi)^{1/2}$

$$\delta g(H) = f_d\left(\frac{L_H}{L_\varphi}\right)$$

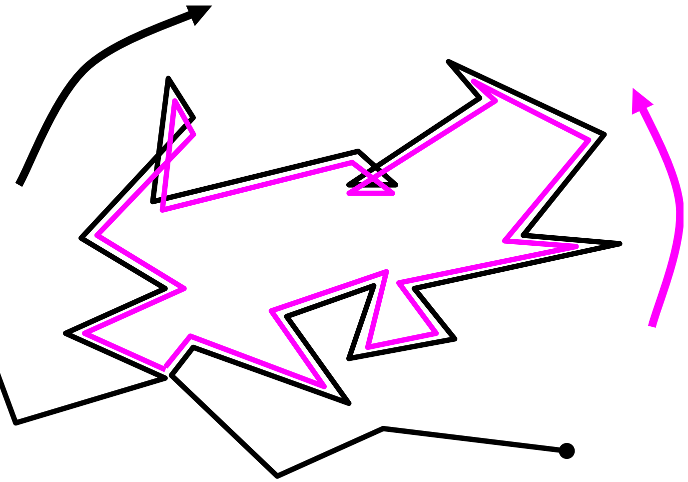
Universal functions

Magnetoresistance measurements allow to study inelastic collisions of electrons with phonons and other electrons

Weak Localization

Negative Magnetoresistance

Chentsov (1949)



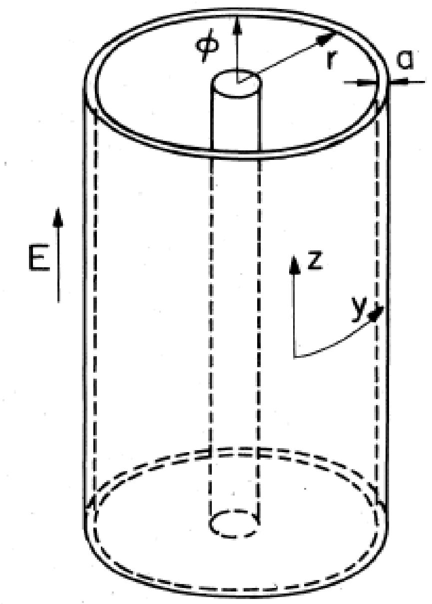
Aharonov-Bohm effect

Theory

B.A., Aronov & Spivak (1981)

Experiment

Sharvin & Sharvin (1981)



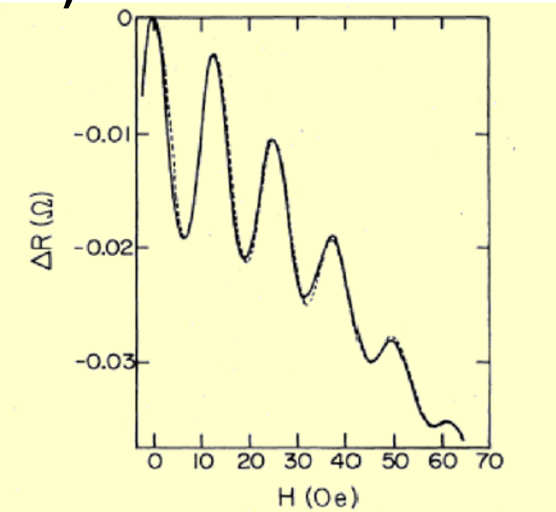
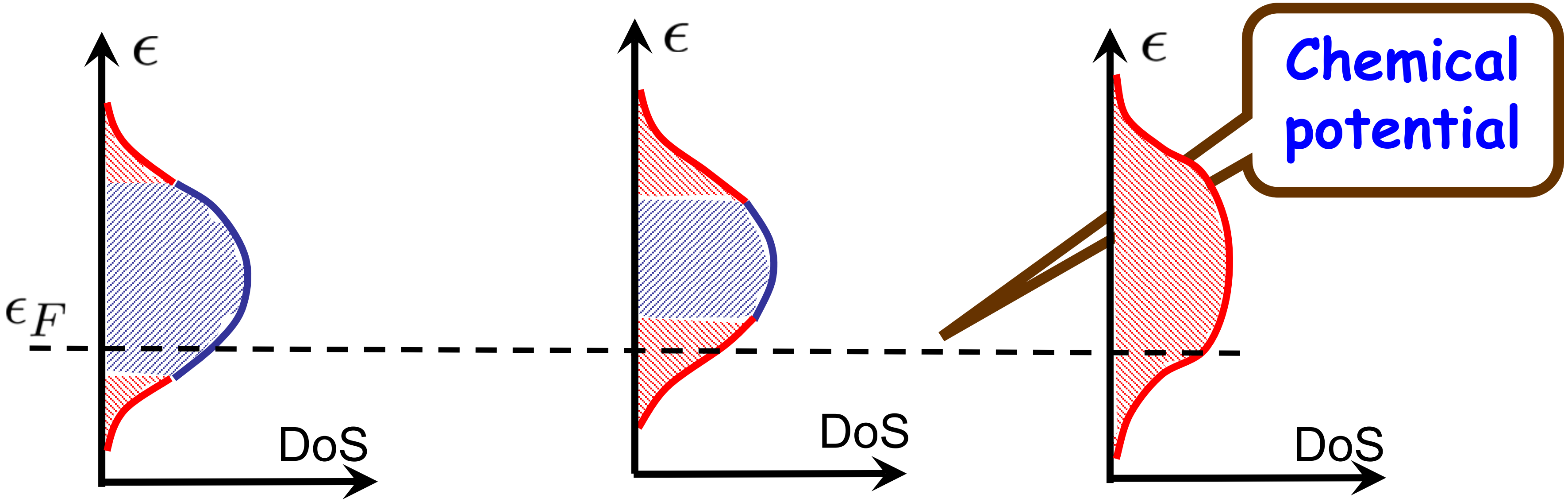


FIG. 8. Longitudinal magnetoresistance $\Delta R(H)$ at $T=1.1$ K for a cylindrical lithium film evaporated onto a 1-cm-long quartz filament. $R_{4.2}=2$ kΩ, $R_{300}/R_{4.2}=2.8$. Solid line: averaged from four experimental curves. Dashed line: calculated for $L_\varphi=2.2$ μm, $\tau_\varphi/\tau_{so}=0$, filament diameter $d=1.31$ μm, film thickness 127 nm. Filament diameter measured with scanning electron microscope yields $d=1.30\pm0.03$ μm (Altshuler *et al.*, 1982; Sharvin, 1984).

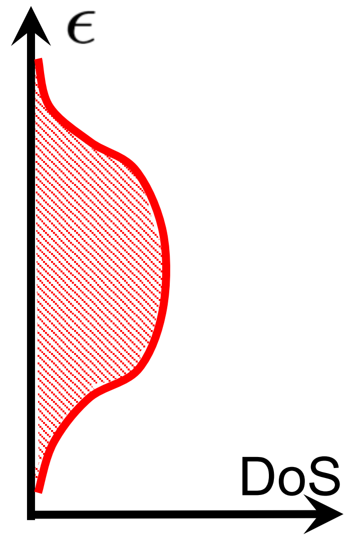
Temperature dependence of the conductivity
one-electron picture



$$\sigma(T \to 0) > 0 \qquad \sigma(T) \propto e^{-\frac{E_c - \varepsilon_F}{T}} \qquad \sigma(T) = 0 \ \ \forall T$$

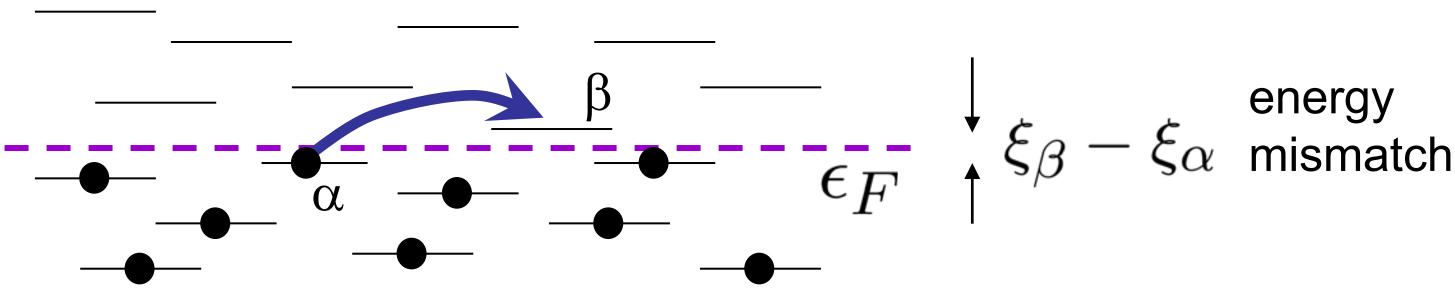
Temperature dependence of the conductivity
one-electron picture

Assume that all the states are localized; e.g. $d = 1,2$



$$\sigma(T) = 0 \quad \forall T$$

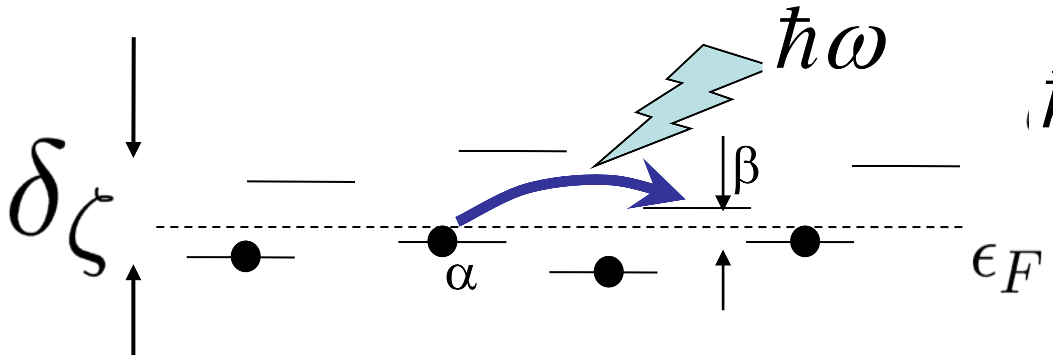
Inelastic processes
transitions between localized states



$$T = 0 \quad \Rightarrow \quad \sigma = 0$$

(**any mechanism**)

Phonon-assisted hopping



$$\hbar\omega = \varepsilon_\alpha - \varepsilon_\beta$$

$$\sigma(T=0) = 0$$

Variable Range Hopping

N.F. Mott (1968)

$$\sigma(T) \propto T^{\gamma} \exp\left[-\left(\frac{\delta_\zeta}{T}\right)^{\frac{1}{d+1}}\right]$$

Mechanism-dependent prefactor

Optimized phase volume

Any bath with a continuous spectrum of delocalized excitations down to $\omega = 0$ will give the same exponential

Spectral statistics and Localization

RANDOM MATRIX THEORY

Spectral statistics

$N \times N$ *ensemble of Hermitian matrices with random matrix element* $N \to \infty$

E_α - spectrum (set of eigenvalues)

$\delta_1 \equiv \langle E_{\alpha+1} - E_\alpha \rangle$ - mean level spacing, determines the density of states

$\langle ...... \rangle$ - ensemble averaging

$s \equiv \frac{E_{\alpha+1} - E_\alpha}{\delta_1}$ - spacing between nearest neighbors

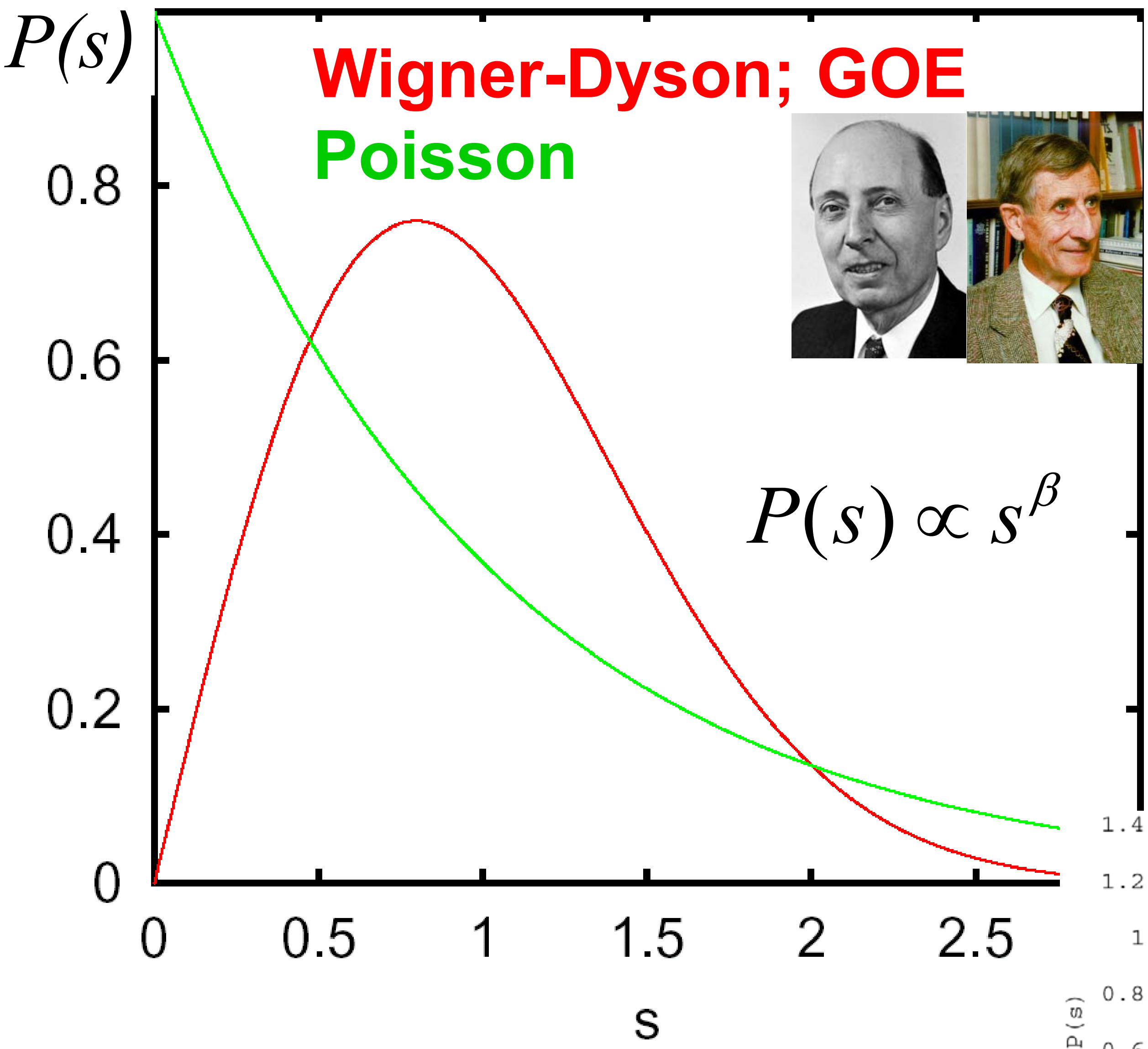
$P(s)$ - distribution function of nearest neighbors spacing between

Spectral Rigidity

Level repulsion

$$P(s=0)=0$$

$$P(s \ll 1) \propto s^\beta \qquad \beta=1,2,4$$

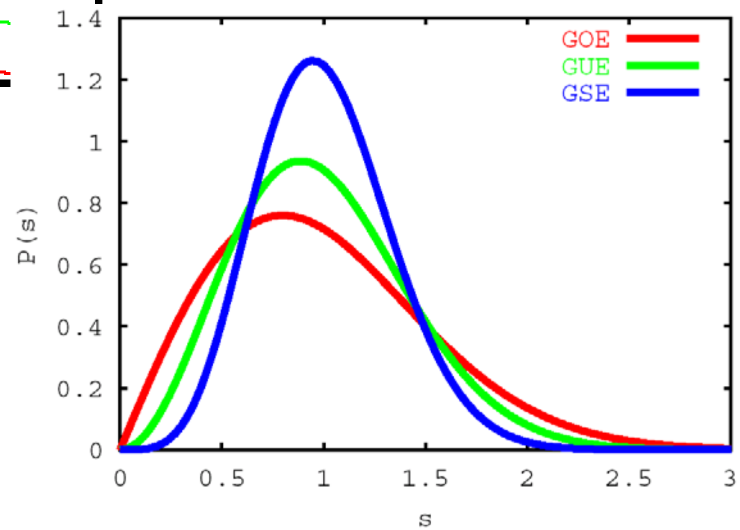


Gaussian
Orthogonal
Ensemble

Orthogonal
$\beta=1$

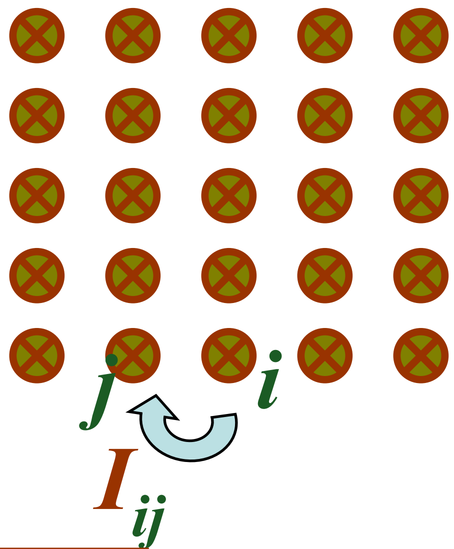
Unitary
$\beta=2$

Simplectic
$\beta=4$



Poisson – completely uncorrelated levels

Anderson Model



- *Lattice - tight binding model*
- *Onsite energies* ε_i *- random*
- *Hopping matrix elements* I_{ij}

$$-W < \varepsilon_i < W$$

uniformly distributed

Is there much in common between Random Matrices and Hamiltonians with random potential ?

Q: What are the spectral statistics of a finite size Anderson model ?

Anderson Transition

Strong disorder

$I < I_c$

Insulator

All eigenstates are localized

Localization length ξ

The eigenstates, which are localized at different places will not repel each other

↓

Poisson spectral statistics

Weak disorder

$I > I_c$

Metal

There appear states extended all over the whole system

Any two extended eigenstates repel each other

↓

Wigner – Dyson spectral statistics

Anderson Localization and Spectral Statistics

Localized states
Insulator

Poisson spectral statistics

Extended states
Metal

Wigner-Dyson spectral statistics

Extended states: Level repulsion, anticrossings, Wigner-Dyson spectral statistics

Localized states: Poisson spectral statistics

Invariant (basis independent) definition

In general:
Localization in the space of quantum numbers.
KAM tori ⟺ localized states.

Glossary

Classical	Quantum
Integrable $H_0 = H_0\left(\vec{I}\right)$	**Integrable** $\hat{H}_0 = \sum_\mu E_\mu \lvert\mu\rangle\langle\mu\rvert, \quad \lvert\mu\rangle = \lvert\vec{I}\rangle$
KAM	**Localized**
Ergodic - distributed all over the energy shell **Chaotic**	**Extended ?**

Many-Body Localization

BA, Gefen, Kamenev & Levitov, 1997
Basko, Aleiner & BA, 2005. . .

Example: Random Ising model in the perpendicular field

Will not discuss today in detail

$$\hat{H} = \sum_{i=1}^{N} B_i \hat{\sigma}_i^z + \sum_{i \neq j} J_{ij} \hat{\sigma}_i^z \hat{\sigma}_j^z + I \sum_{i=1}^{N} \hat{\sigma}_i^x \equiv \hat{H}_0 + I \sum_{i=1}^{N} \hat{\sigma}_i^x$$

Random Ising model in a parallel field

Perpendicular field

$\vec{\sigma}_i$ - Pauli matrices, $\sigma_i^z = \pm \frac{1}{2}$

$i = 1, 2, \ldots, N; \quad N >> 1$

Without perpendicular field all σ_i^z commute with the Hamiltonian, i.e. they are integrals of motion

$$\hat{H}=\sum_{i=1}^{N}B_i\hat{\sigma}_i^z+\sum_{i\neq j}J_{ij}\hat{\sigma}_i^z\hat{\sigma}_j^z+I\sum_{i=1}^{N}\hat{\sigma}_i^x\equiv\hat{H}_0+I\sum_{i=1}^{N}\hat{\sigma}_i^x$$

Random Ising model in a parallel field

Perpendicular field

$\vec{\sigma}_i$ - Pauli matrices

$i=1,2,\ldots,N;\quad N>>1$

Without perpendicular field all σ_i^z commute with the Hamiltonian, i.e. they are integrals of motion

Anderson Model on *N*-dimensional cube

$\left\{\sigma_i^z\right\}$ determines a site

$H_0\left(\left\{\sigma_i\right\}\right)$

onsite energy

$\hat{\sigma}^x=\hat{\sigma}^++\hat{\sigma}^-$

hoping between nearest neighbors

$$\hat{H} = \sum_{i=1}^{N} B_i \hat{\sigma}_i^z + \sum_{i \neq j} J_{ij} \hat{\sigma}_i^z \hat{\sigma}_j^z + I \sum_{i=1}^{N} \hat{\sigma}_i^x \equiv \hat{H}_0 + I \sum_{i=1}^{N} \hat{\sigma}_i^x$$

Anderson Model on *N*-dimensional cube

Usually:

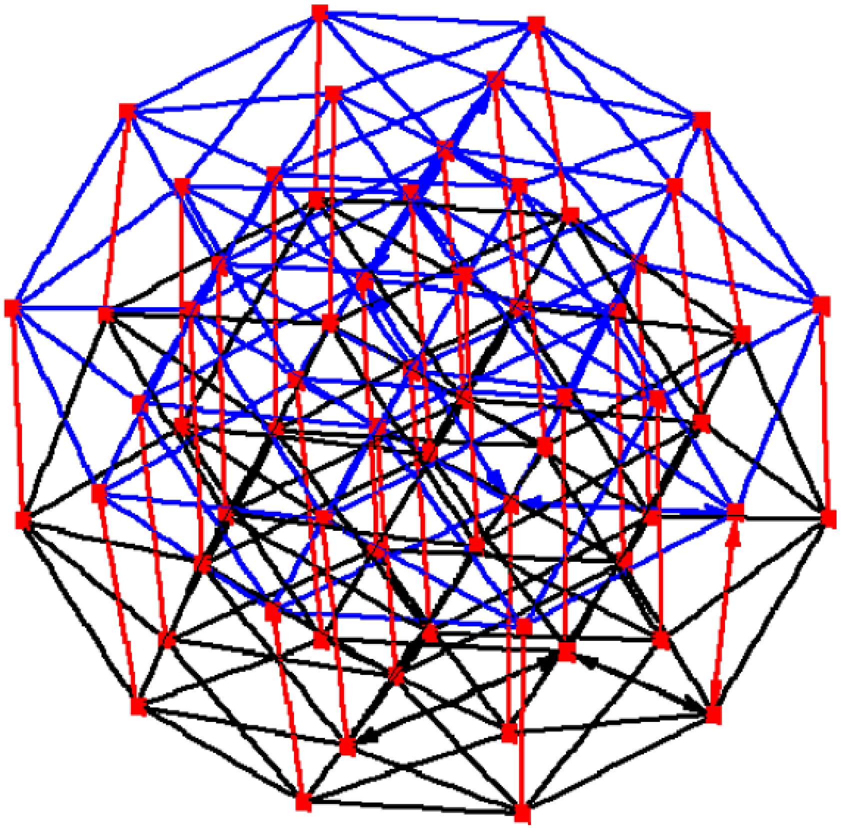
# of dimensions $d \to const$

system linear size $L \to \infty$

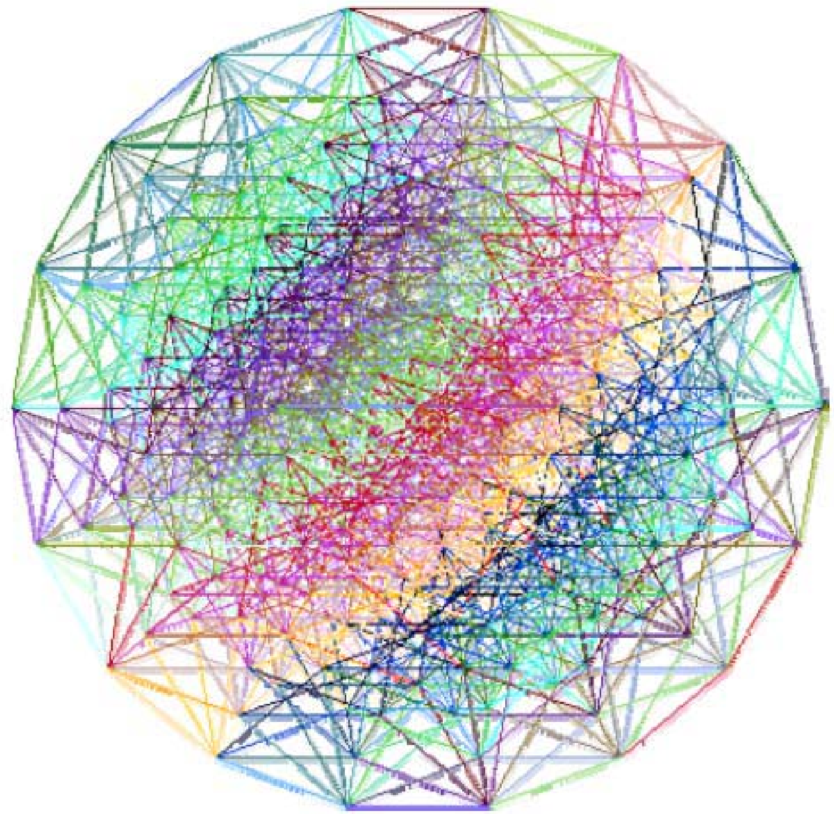
Here:

# of dimensions $d = N \to \infty$

system linear size $L = 1$



6-dimensional cube



9-dimensional cube

$$\hat{H} = \sum_{i=1}^{N} B_i \hat{\sigma}_i^z + \sum_{i \neq j} J_{ij} \hat{\sigma}_i^z \hat{\sigma}_j^z + I \sum_{i=1}^{N} \hat{\sigma}_i^x \equiv \hat{H}_0 + I \sum_{i=1}^{N} \hat{\sigma}_i^x$$

Anderson Model on *N*-dimensional cube

Localization:
- No relaxation
- No equipartition
- No temperature
- No thermodynamics

Glass ??