The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

IAEA

2162-30

Advanced Workshop on Anderson Localization, Nonlinearity and Turbulence: a Cross-Fertilization

23 August - 3 September, 2010

Anderson Localization of Interacting 1D Bosons (Finite Temperature Transition for 1D Disordered Bosons)

Gora SHLYAPNIKOV

LPTMS, Orsay
France and University of Amsterdam
The Netherlands

Strada Costiera 11, 34151 Trieste, Italy - Tel.+39 040 2240 111; Fax +39 040 224 163 - sci_info@ictp.it

Finite temperature transition for 1D disordered bosons

Gora Shlyapnikov
LPTMS, Orsay, France
University of Amsterdam, The Netherlands

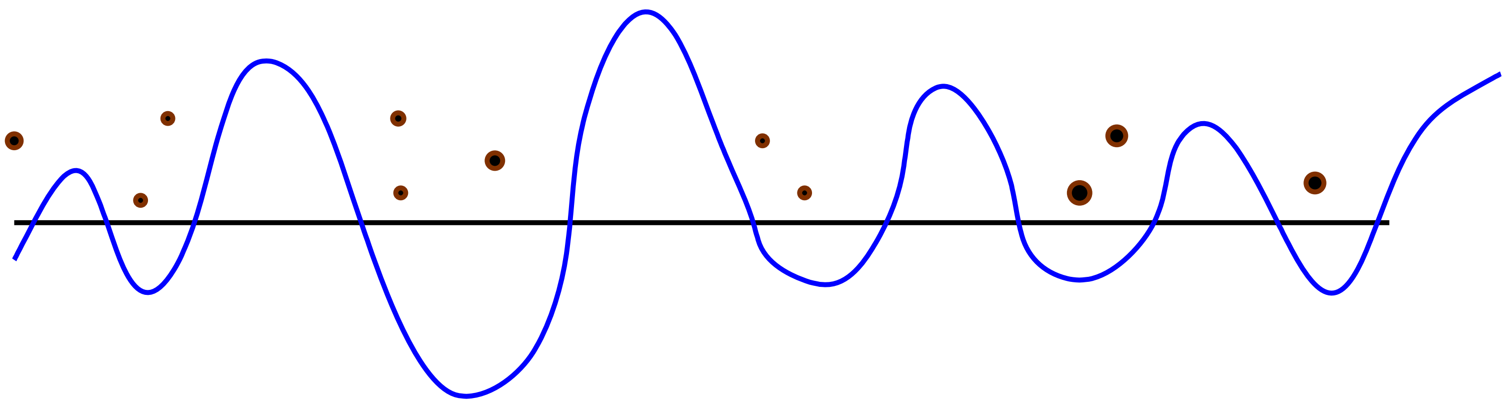
- Introduction.
- Many-body localization-delocalization transition
- Why a transition?
- Classical ansd degenerate bosons
- Phase diagram
- Gedanken expansion experiment
- Prospects

Collaborations I.L. Aleiner, B.L. Altshuler (Columbia University)

Trieste, August 31, 2010

Quantum gases in disorder

One-dimensional disordered bosons at finite temperature



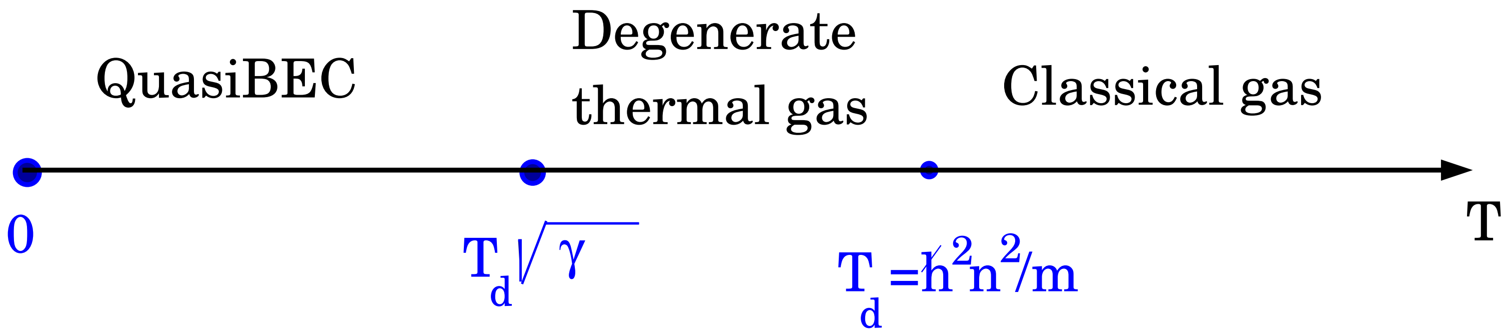
DOGMA → No finite temperature phase transitions in 1D
as all spatial correlations decay exponentially

There is a non-conventional phase transition between two distinct states

Fluid and Insulator

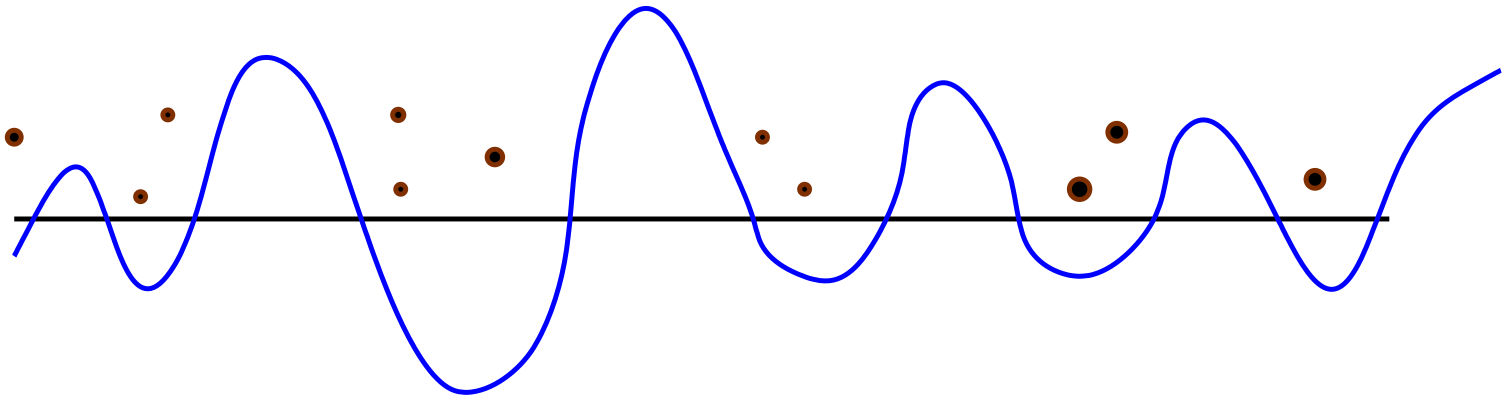
How to understand?

Interacting 1D Bose gas. No disorder $\Rightarrow$ Fluid phase



$$\gamma = \frac{mg}{\hbar^2 n} = \frac{ng}{T_d} \ll 1 \quad \rightarrow \quad \text{weakly interacting regime}$$

Disordered non-interacting 1D bosons



All single-particle states are localized at any energy $\rightarrow$ Anderson insulator

Old question

Interacting 1D bosons show one of the two types of behavior

”Old question”

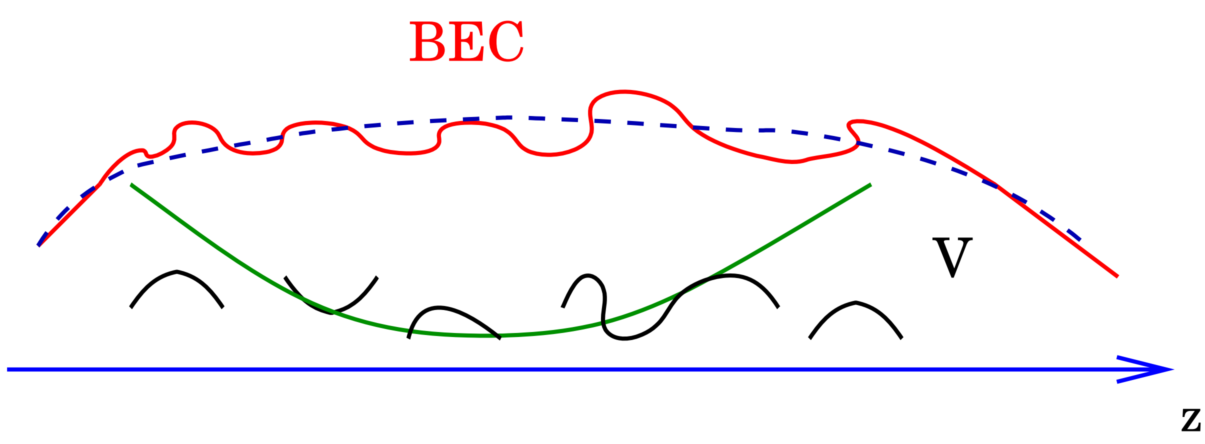
How does the interparticle interaction influences Anderson localization?

Crucial for charge transport in electronic systems

Appears in a new light for disordered ultracold bosons

Palaiseau, LENS experiments. More underway

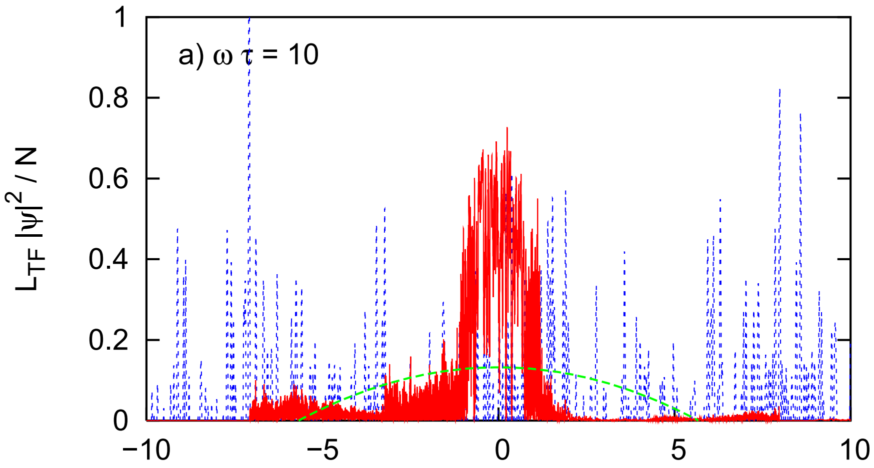
Experiments



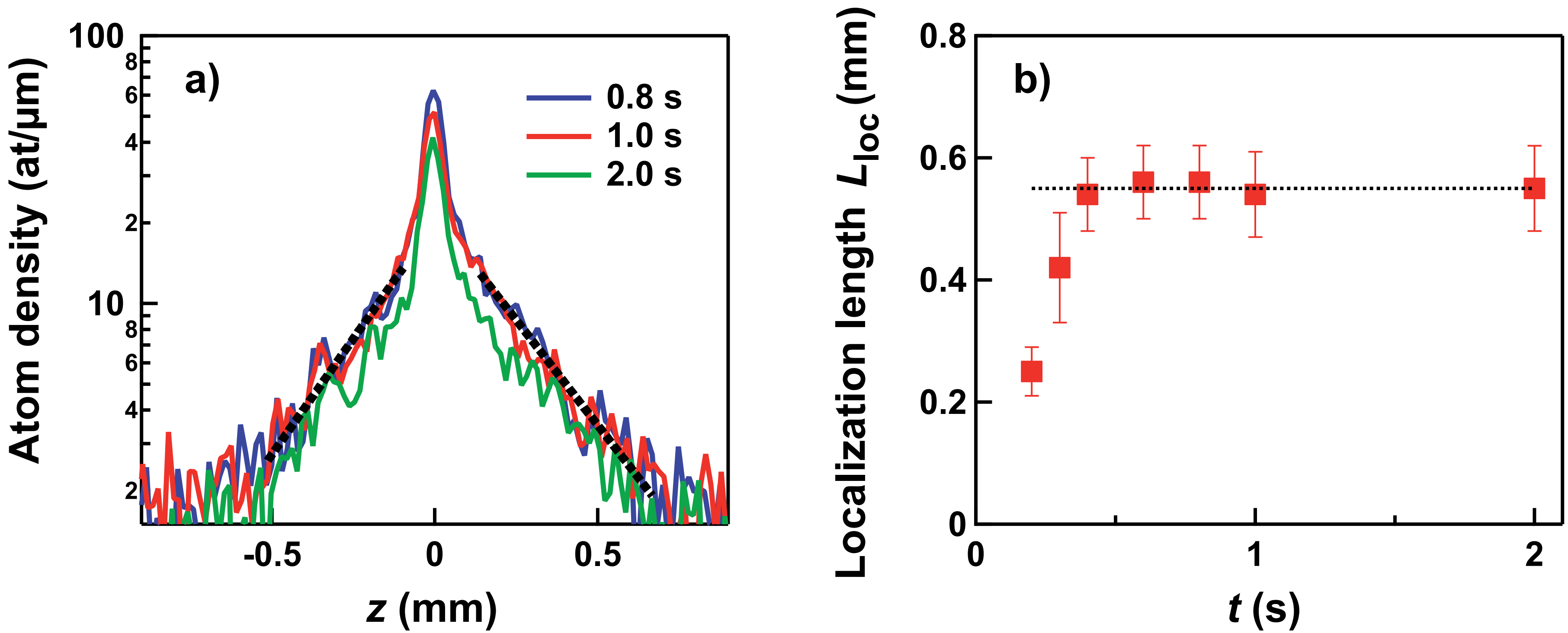
BEC in a harmonic + weak random potential $|V(z)| \ll ng \Rightarrow$ small density modulations of the static BEC. Switch off the harmonic trap, but keep the disorder $\Rightarrow$ What happens?

First experiments (Orsay, LENS, Rice)

The expansion stops and BEC gets stacked in between 2 large peaks



Orsay experiment

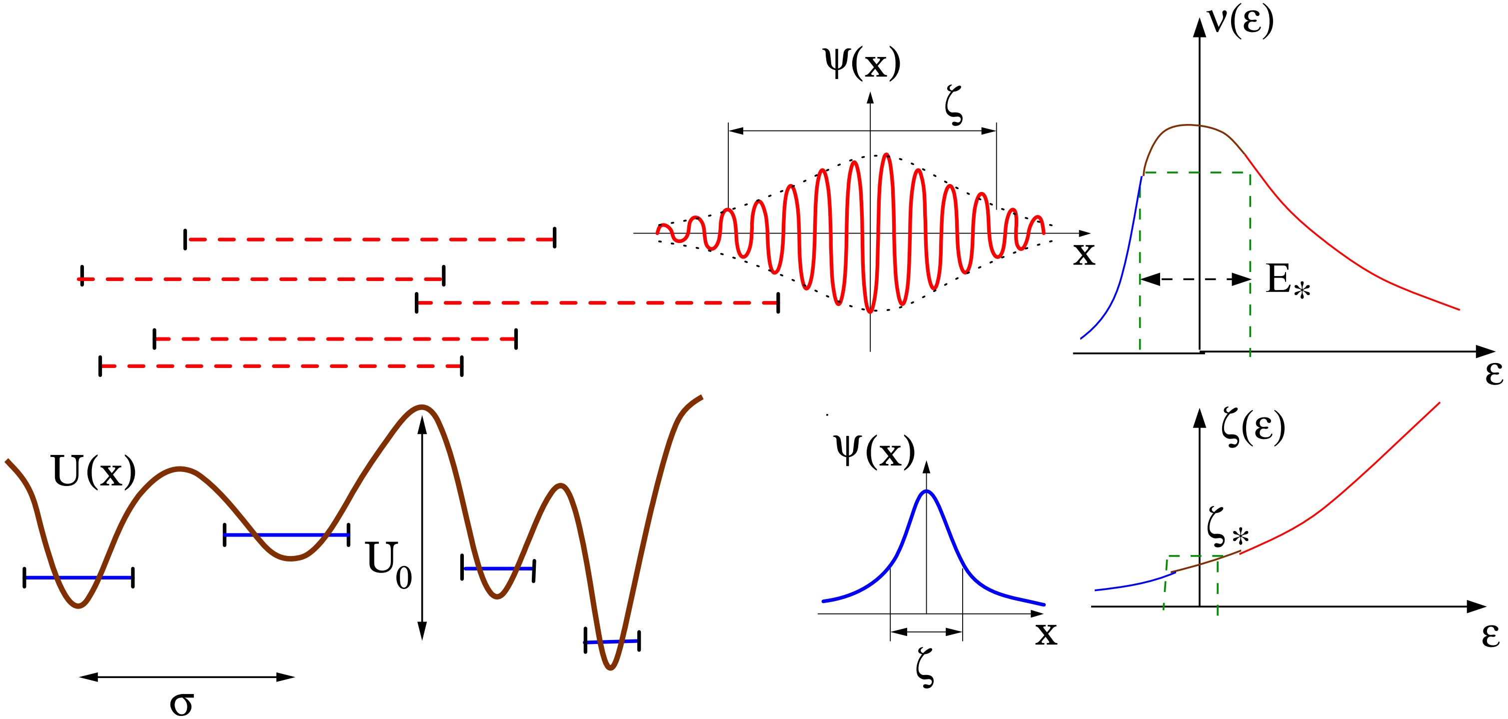


Single-particle localization

$$U_0 \ll \frac{\hbar^2}{m\sigma^2} \quad E_* \sim U_0(U_0\sigma^2 m/\hbar^2)^{1/3} \quad \zeta_* = \hbar/\sqrt{mE_*}$$

Minimize the energy $E \sim (\hbar^2/m\zeta^2 - U_0\sqrt{\sigma/\zeta})$

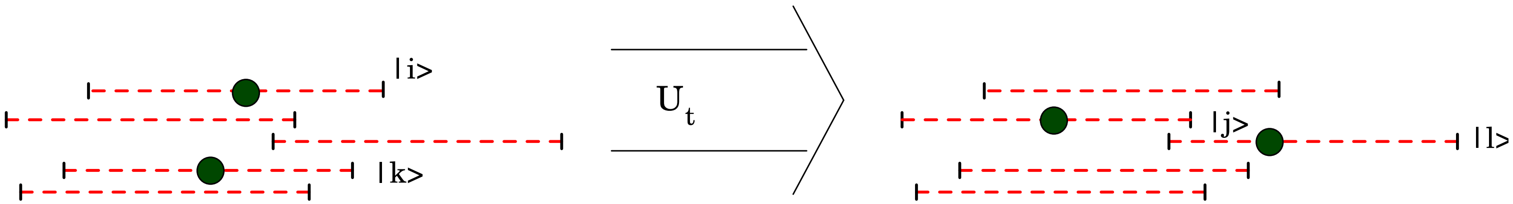
DOS $\quad \epsilon \gg E_* \to \nu(\epsilon) = \sqrt{m/2\pi^2\epsilon} \quad \epsilon \sim E_* \to \quad \nu_* \sim 1/E_*\zeta_*$



High-energy states ($\epsilon \gg E_*$), low-energy states ($\epsilon \sim E_*$), Lifshitz tail

Many-body localization-delocalization transition

Beat the discreteness!



Consider an occupied localized state $|i\rangle$ interacting with a particle in the state $|k\rangle$, which transfers them to the states $|j\rangle$ and $|l\rangle$ $\quad U_{ik,jl} \rightarrow U_t$

if $|i\rangle$ and $|k\rangle$ belong to the same localization volume and are neghbors in energy

Typical energy mismatch $\Delta_{ik,jl} = |\epsilon_i + \epsilon_k - \epsilon_j - \epsilon_l| \rightarrow \Delta_t$

$N_1 \rightarrow$ The number of processes $|i>, |k> \rightarrow |j>, |k>$ involving $|i>$

i.e. the number of channels for decay of a given excitation

$$\Delta_t(T_c) = U_t(T_c) N_1(T_c)$$

$T > T_c \rightarrow$ extended states $\qquad T < T_c \rightarrow$ localized states

Why a transition?

For $T < T_c$ the mixing of 1PE to all 3PE is weak. For $T > T_c$, a 1PE is strongly hybridized with at least one 3PE. This means the phase transition, not a crossover

Expand the 3PE connected with the original 1PE into a product of 1PE-s. At $T > T_c$ at least one constituent of 3PE is at resonance. Hence, each 3PE hybridizes with with at least one 5PE

Resonant path 1PE $\longrightarrow$ 5PE

It is reproduced for the mixing of $2n+3$-particle excitation to $2n+3$-particle excitations

$T > T_c \Rightarrow$ Resonant path 1PE$\longrightarrow$ $(2n+1)$-PE is formed with probability equal to one

Many-body states are extended $\Rightarrow$ linear combinations of 1PE's, 3PE,s etc. Fluid behavior

For $T < T_c$ the probability $p_{1\rightarrow 3}$ of hybridization of 1PE to 3PE is less than one.

Then $p_{1\rightarrow 2n+1} \simeq (p_{1\rightarrow 3})^n \longrightarrow 0$ for $n \longrightarrow \infty$

Insulating behavior Aleiner, Basko, Altshuler (2008)

Classical bosons

Weakly interacting regime

$$\gamma = \frac{mg}{\hbar^2 n} = \frac{ng}{T_d} \ll 1; \quad t = \frac{T}{ng}; \quad \kappa = \frac{E_*}{ng}$$

$T > T_d(t > \gamma^{-1}) \rightarrow$ High-energy states are occupied. Particle energy $\sim T$

$$U_t = g/\zeta(T),\ \Delta_t(T) \sim [\nu(T)\zeta(T)]^{-1},\ N_1(T) \sim \zeta(T)n$$

$$\nu(T) = 1/\sqrt{E_* T \zeta_*^2},\ \ \zeta(T) = \zeta_* T/E_*,$$

$$E_c \sim (ng)^{2/3} T^{1/3} \rightarrow \ \ \kappa_c \sim t^{1/3}$$

Degenerate bosons

$$T_d\sqrt{\gamma} < T < T_d(\gamma^{-1/2} < t < \gamma^{-1})$$

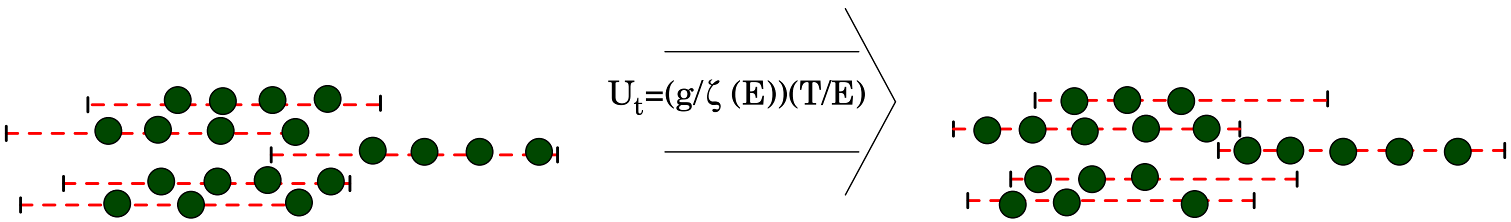
High-energy states. Particle energies $\sim T^2/T_d \ll T$. Energy scales T^2/T_d and T

Consider atoms with $\epsilon \sim E$ in the energy interval of width $\sim E$

$$\Delta_t(E) \sim [\nu(E)\zeta(E)]^{-1}, \quad N_1(E) \sim E\nu(E)\zeta(E), \quad U_t \sim (g/\zeta(E))(T/E)$$

$$E_c \sim (ng)^{1/3}T^{2/3}\gamma^{1/3} \quad \kappa_c \sim t^{2/3}\gamma^{1/3}$$

The result is independent of E



$$T \sim T_d\sqrt{\gamma}(t \sim \gamma^{-1/2}) \rightarrow \quad E_c \sim ng(\kappa_c \sim 1)$$

Low-temperature regime

$$T < T_D\sqrt{\gamma}(t < \gamma^{-1/2})$$

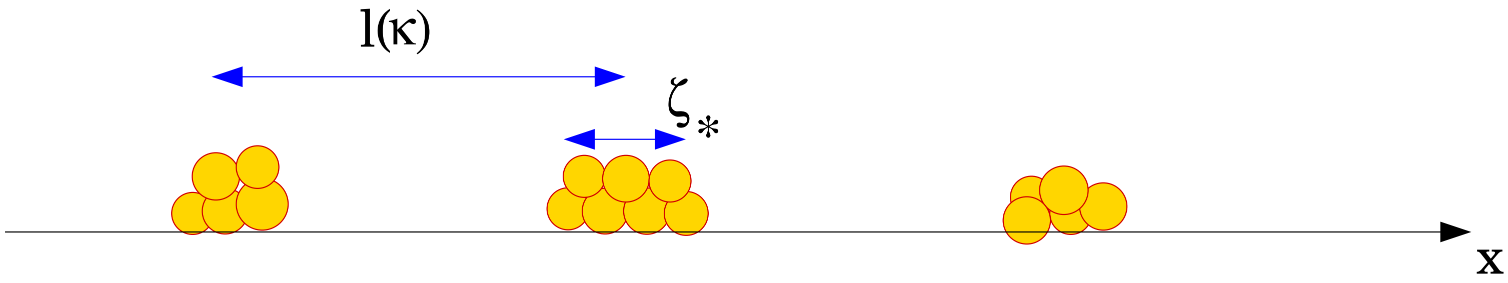
$T = 0 \Rightarrow$ For $\kappa \gg 1$ the boson density is fragmented. Lake $i \rightarrow N_i$ bosons w ith energy ϵ_i and $\zeta_i \approx \zeta_*$

Energy cost of bringing a boson to lake i is $E_i = \epsilon_i + gN_i/\zeta_* = \mu$. Bosons occupy states with $\epsilon_i < \mu$,

and $N_i \approx zeta_*(\mu - \epsilon_i)/g$. Low-energy states if $\mu < E_*$. DoS $\rightarrow \nu_* = 1/E_*\zeta_*$, and $n = \mu^2/2gE_*$

$\rightarrow \mu = E_*/\sqrt{\kappa} < E_*$ for $\kappa > 1$. Small fraction ($\sim \mu/E_*$) of low-energy states is occupied

Distance between neighboring lakes $l(\kappa) \sim \zeta_*\sqrt{\kappa}$. Lake size $\sim \zeta_*$, and $N_i \sim n\zeta_*\sqrt{\kappa} \sim 1/\sqrt{\gamma}$



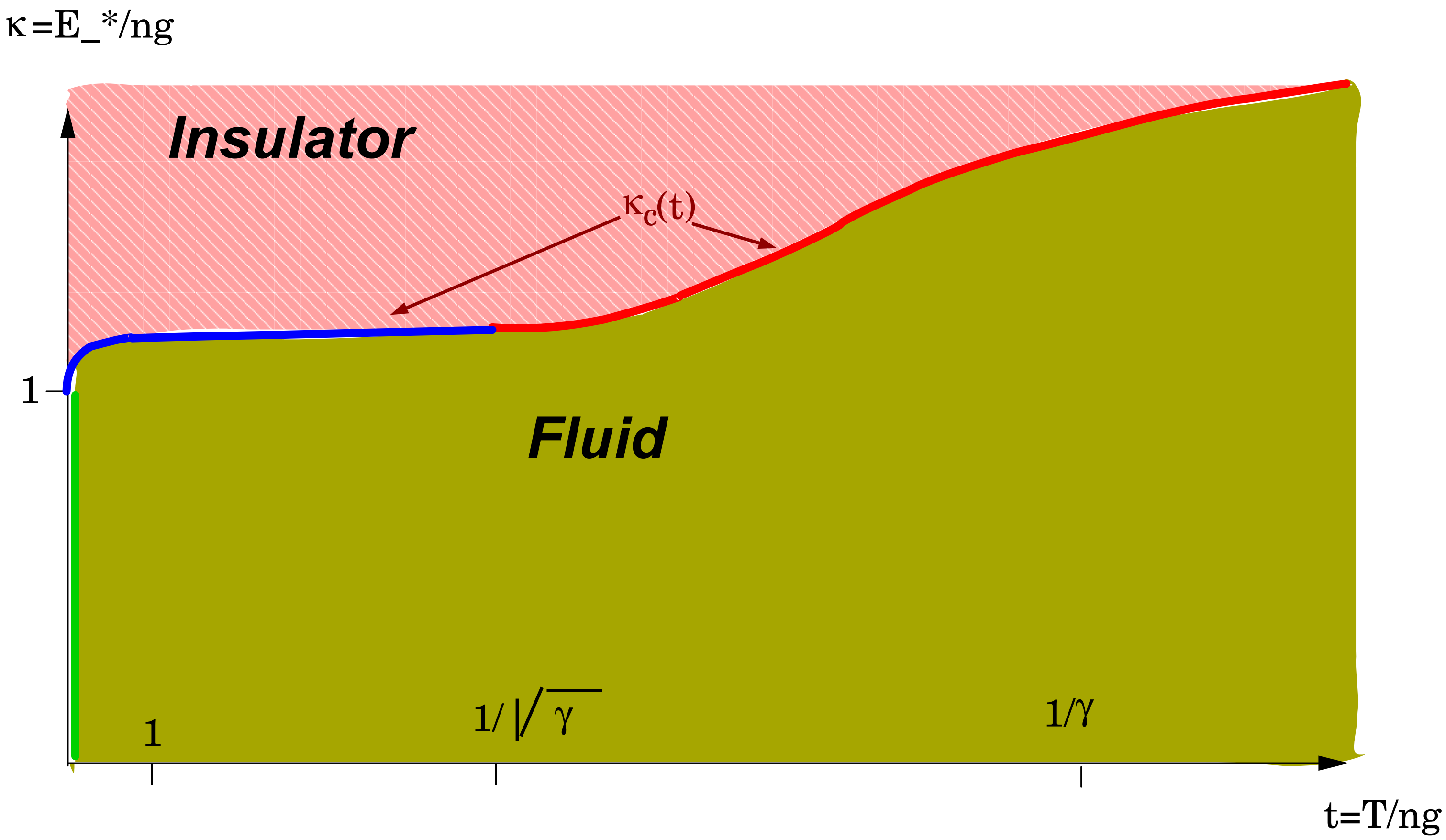
$l(\kappa) \gg \zeta_*$ for $\kappa \gg 1$ $\Rightarrow$ Insulator

$\kappa \rightarrow 1$ $\Rightarrow$ the interlake coupling drives the system to superfluid state

Critical value $\kappa_c \sim 1$ (KTT, Altman et al (2008), Falco et al (2009))

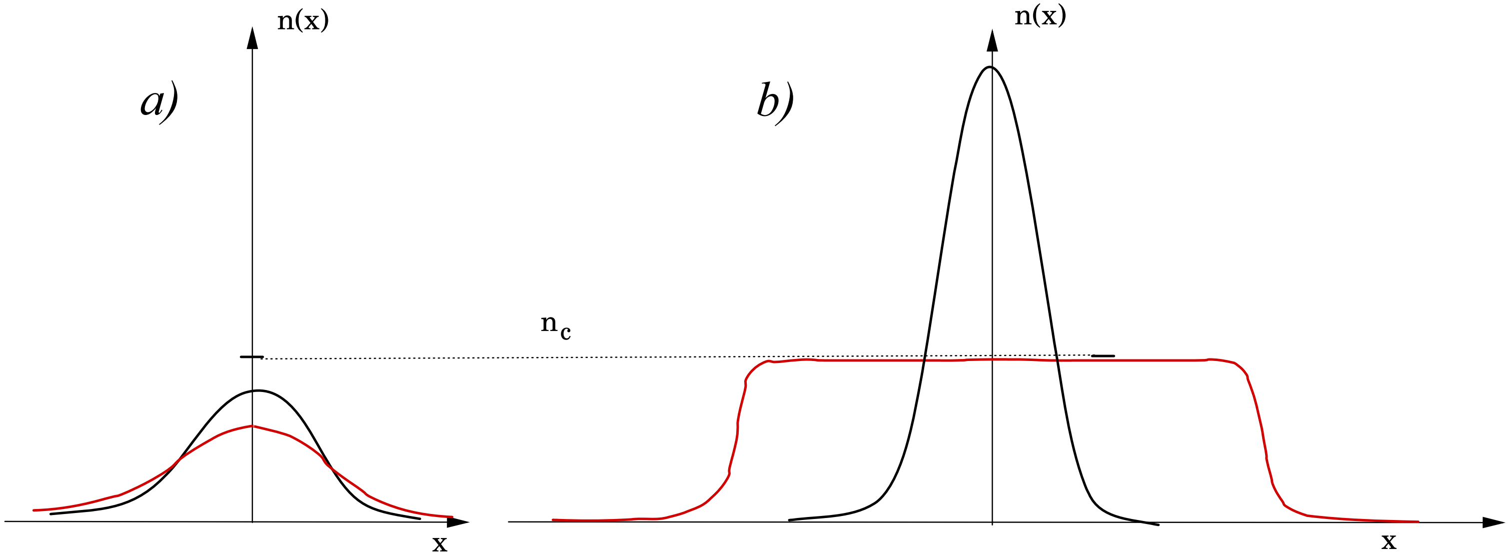
Also for $T \sim T_d\sqrt{\gamma}$ we have $\kappa_c \sim 1$. So, $\kappa_c \sim 1$ for all $T < T_d\sqrt{\gamma}(t < \gamma^{-1/2})$

Phase diagram



How to identify?

Expansion experiment



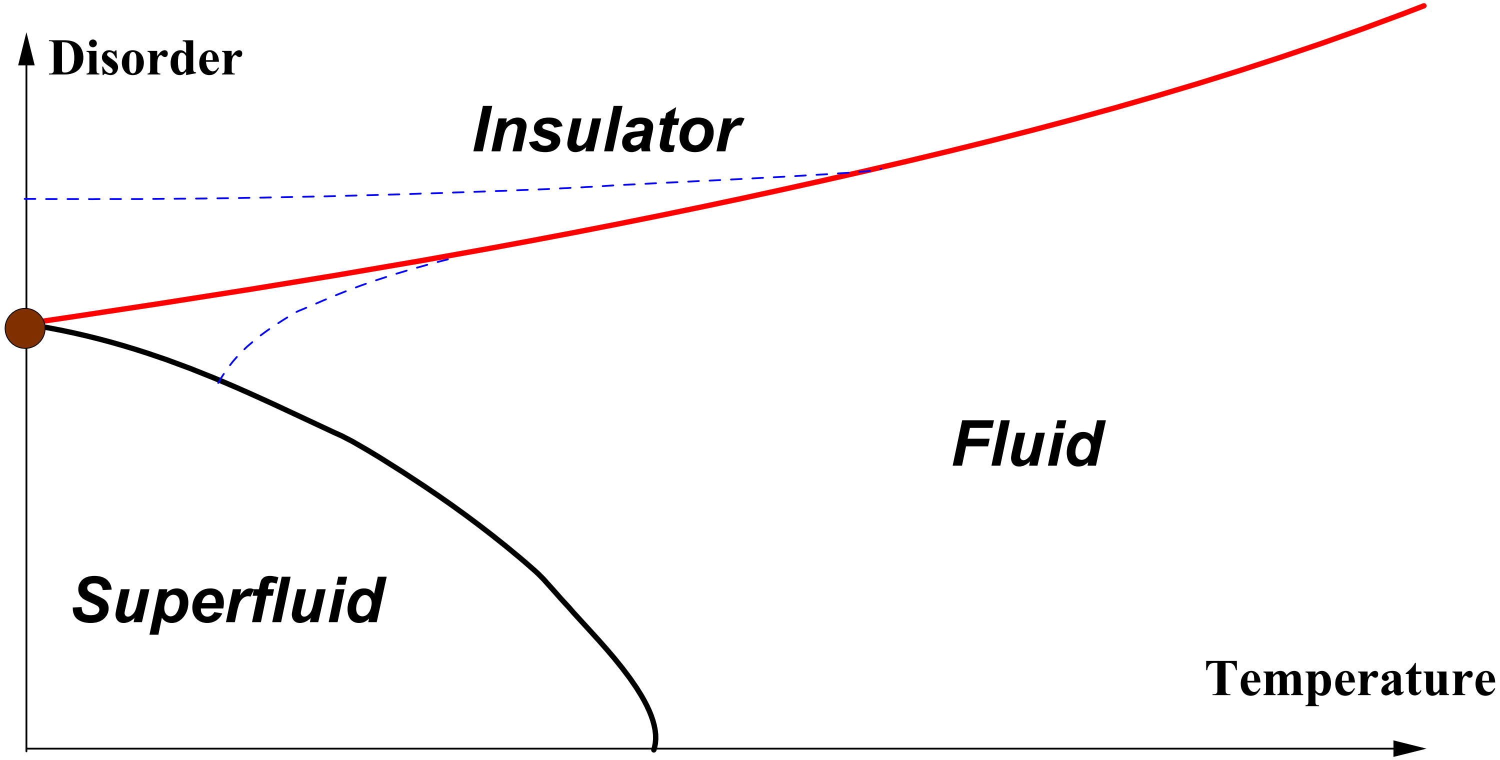
Slow diffusive expansion near the fluid-insulator transition $\tau \sim 1$ s

Prospects

- Strongly interacting bosons in 1D
- 1D interacting fermions
- Bosons and fermions in 2D and 3D
- Dynamics of expansion
- Critical regime

Example

Bosons in 2D and 3D



In 3D this is not a phase transition but a crossover