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Advanced Workshop on Anderson Localization, Nonlinearity and Turbulence: a Cross-Fertilization

23 August - 3 September, 2010

Phase Diagram of Electron System in Vicinity of Superconductor-Insulator Transition

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Phase diagram of electron system in vicinity of superconductor-insulator transition

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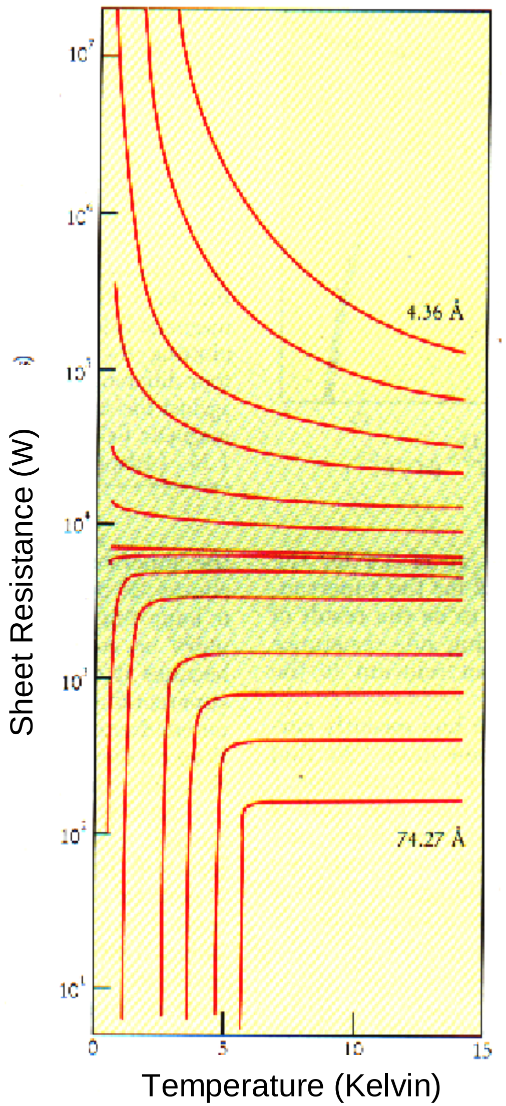
Anderson Localization, Trieste, August 31, 2010

Outline

- Experimental facts
- Theoretical works
- Model
- Thin film in parallel magnetic field
- Thin film in perpendicular magnetic field
- Thick film
- Resistance

2D SC/I quantum phase transition



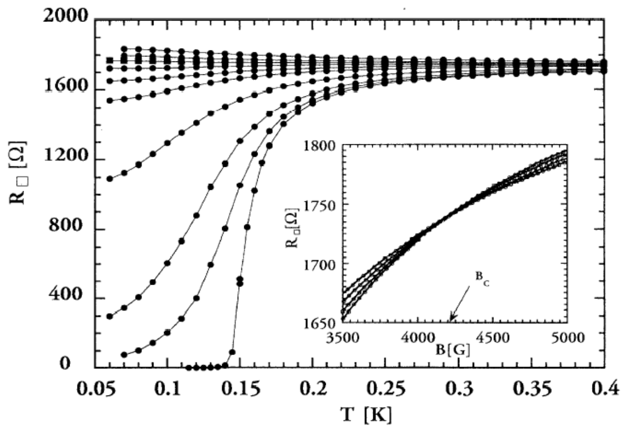


FIG. 1. Zero bias resistance of sample 2 plotted versus temperature at $B = 0, 0.5, 1.0, 2.0, 3.0, 4.0, 4.4, 4.5, 5.5, 6$ kG. In the inset, $R_\square(B, T, E = 0)$ for the same sample measured versus field, at $T = 80, 90, 100, 110$ mK.

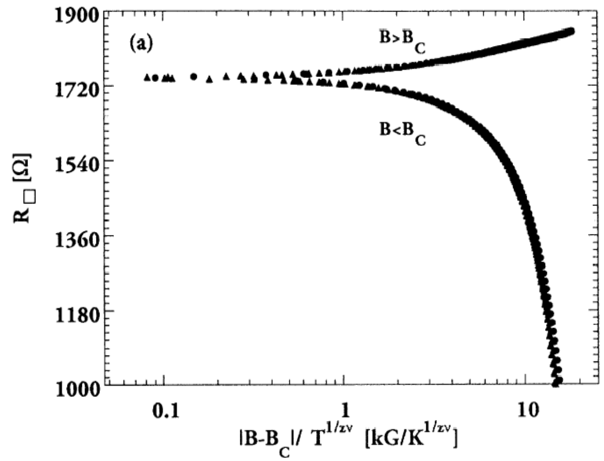


FIG. 3. Top: Scaling of $R_\square(B, T, E = 0)$ for sample 2 measured at $T = 80, 90, 100, 110$ mK ($B_c = 4.19$ kG, $\nu z = 1.36$).

Yazdani and Kapitulnik, **Phys. Rev. Lett. 74, 3037–3040 (1995)**

Two-Dimensional a-MoGe Thin Films

Goldman and Markovic, Physics Today 1998, amorphous very thin Bi films (near 10 Å)

TiN films

PRL **99**, 257003 (2007) PHYSICAL REVIEW LETTERS week ending 21 DECEMBER 2007

Localized Superconductivity in the Quantum-Critical Region of the Disorder-Driven Superconductor-Insulator Transition in TiN Thin Films

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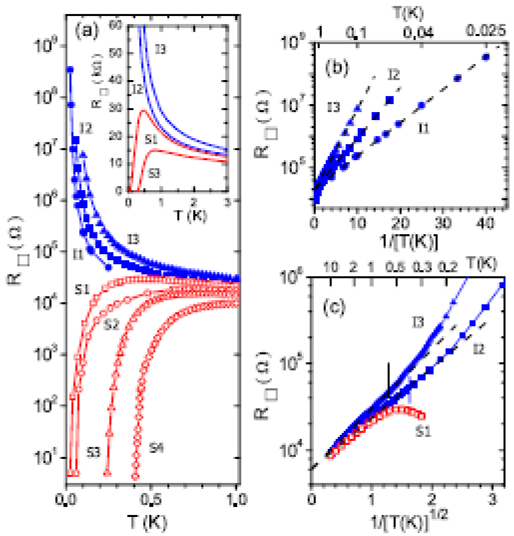

FIG. 1 (color online). Temperature dependences of $R_\square$ taken at zero magnetic field for the samples near the localization threshold. (a) $\log R_\square$ versus T. Inset: some part of the $R_\square$ data in a linear scale. (b) $\log R_\square$ versus $1/T$ for samples I1, I2, and I3. Dashed lines represent Eq. (1) and fit perfectly at low temperatures. All curves saturate at the same $R_\square \approx 20\ \mathrm{k\Omega}$ at high temperatures. (c) $R_\square$ versus $1/T^{1/2}$; dashed lines are given by $R_\square = R_1 \exp(T_1/T)^{1/2}$ which (with $R_1 \sim 6\ \mathrm{k\Omega}$) well fit the data at high temperatures. Vertical strokes mark T_0, determined by the fit to the Arrhenius formula of Eq. (1).

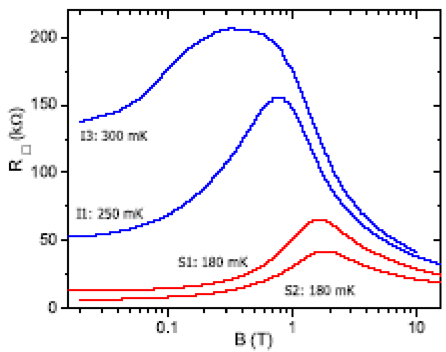

FIG. 2 (color online). Magnetoresistance isotherms for superconducting (S1, S2) and insulating samples (I1, I3) at similar temperatures. All curves converge above 2 T.

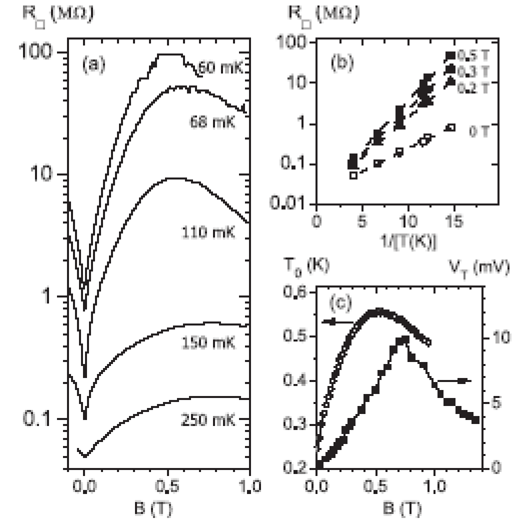

FIG. 3. (a) Sheet resistance of sample I1 as a function of the magnetic field at some temperatures listed. (b) R versus $1/T$ at $B = 0$ (open circles), 0.2 (triangles), 0.3 (filled circles), and 0.5 T (squares). The dashed lines are given by Eq. (1). (c) T_0 (left axis), calculated from fits to Eq. (1), and the threshold voltage V_T (right axis) as a function of B.

Giant negative magnetoresistance (GNM)

VOLUME 85, NUMBER 1 PHYSICAL REVIEW LETTERS 3 JULY 2000

Tenfold Magnetoconductance in a Nonmagnetic Metal Film

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tronomy, Louisiana State University, Baton Rouge, Louisiana 70806
(Received 9 November 1999)

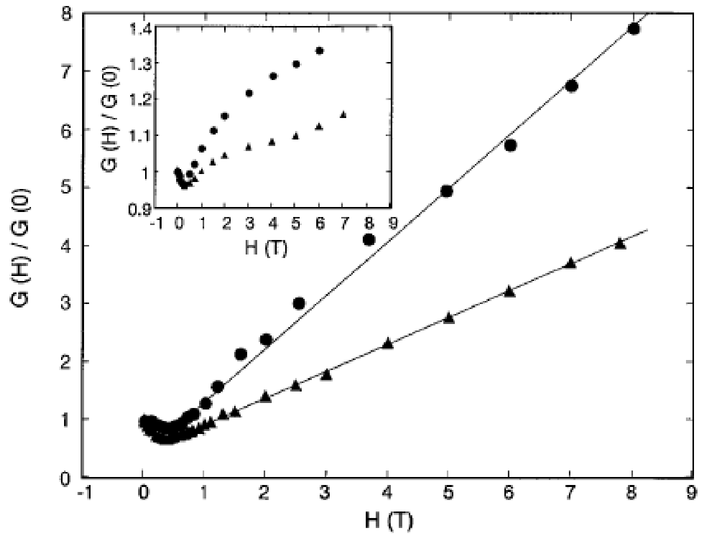
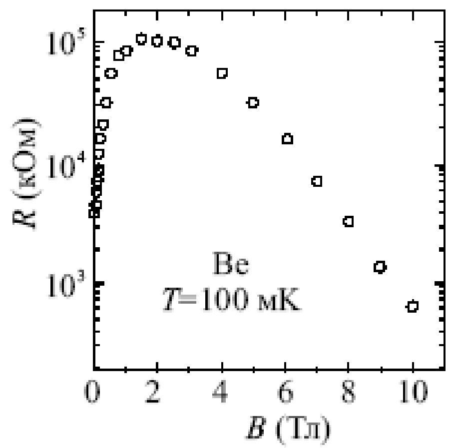


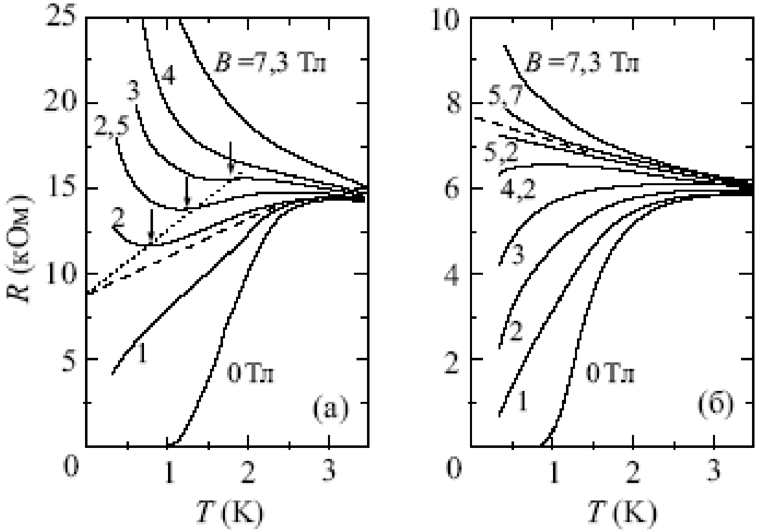
FIG. 1. Relative magnetoconductance of a 3 MΩ Be film at 50 mK. Circles: field perpendicular to film surface. Triangles: field parallel to film surface. The solid lines are linear fits to the data above 1 T with slopes of 1/(1.1 T) and 1/(2.2 T) for the perpendicular and parallel data, respectively. Inset: relative magnetoconductance of a 16 kΩ Be film.



Wenhao Wu, AIP Conference Proceeding (LT24) **850**, 995 (2006)

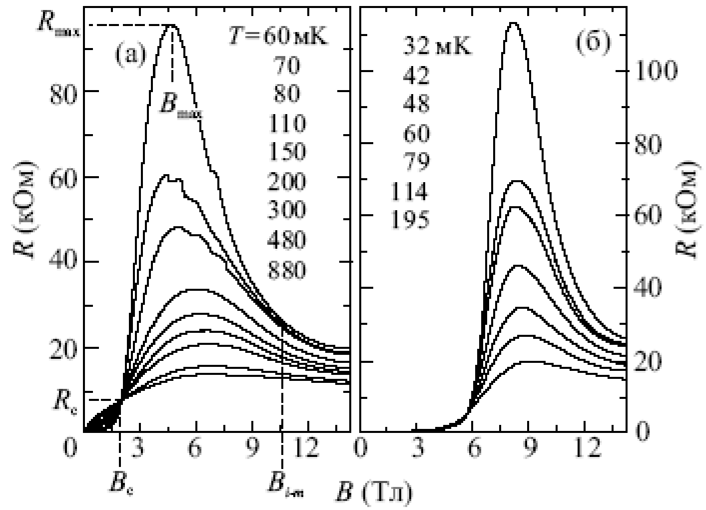
Be thin films

Amorphous films In_2O_{3-x}



Temperature dependence of resistance
in two samples at different magnetic fields.
Extracted from the works:

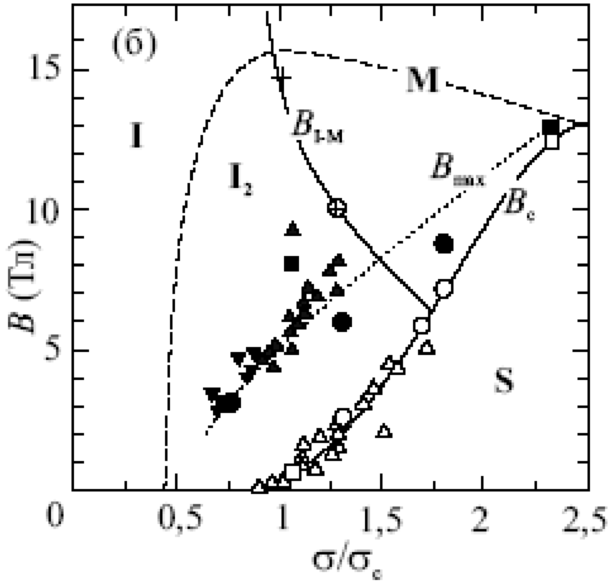
V.F. Gantmakher, Int. J. Modern Phys. **12**, 3151 (1998)
V.F. Gantmakher, M.V. Golubkov, V.T. Dolgopolov, A.A. Shashkin, and G.E. Tsydynzhapov, Pis'ma v ZhETF **71**, 231 (2000) [JETP Lett. **71**, 160 (2000)]



Magnetic field dependence of resistance
in one sample at different starting
deviations from the SIT on the insulating
side.

V.F. Gantmakher, M.V. Golubkov, V.T. Dolgopolov, A.A. Shashkin, and G.E. Tsydynzhapov, Pis'ma v ZhETF **71**, 693 (2000) [JETP Lett. **71**, 473 (2000)]

Temperature behavior of Conductivity: activation at small fields *B*<10T, Mott VRH behavior at larger *B:* $\sigma \sim \exp\left[-(T_0/T)^{1/4}\right]$



Phase diagram of a superconductor near the SIT transition.
The dashed line separates the region of existence of a glass state.
The dotted curve corresponds to a maximum of resistance.

V.F. Gantmakher and V. Dolgopolov, UFN (Russian Physics, Uspekhi, Jan. 2010.

Theoretical works

A. M. Finkelstein, JETP Letters **45**, 46 (1987).

2d , no magnetic field. Coulomb interaction + disorder suppresses superconductivity.

No reasons for GNM.

K. B. Efetov, JETP **78**, 1015 (1981).

Cooper pairs are bound in granules and can tunnel between them. Depending on relative strength of interaction and hopping amplitude the S or I phase is realized.

No real grains in films, the interaction and hopping are not independent (FIKQ)

M.P.A. Fisher, Phys. Rev. Lett. 65, 923 (1990).

2d. Duality between vortices and CP. In superconductors CP are free, vortices are bound. In insulators CP are bound, vortices are free. Universal resistance at SIT transition.

Experiments do not confirm the duality and universal resistance.

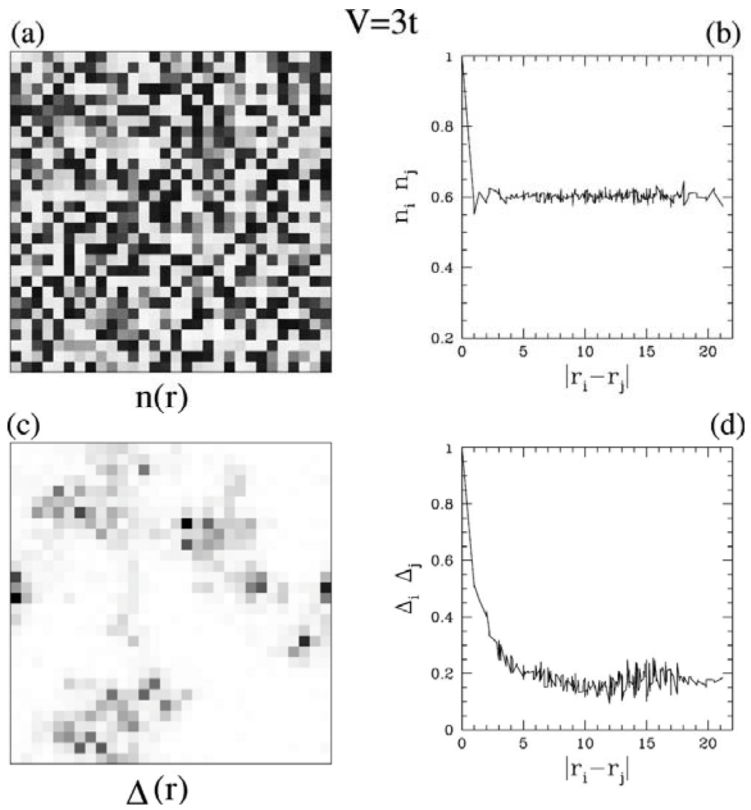
PHYSICAL REVIEW B, VOLUME 65, 014501

Inhomogeneous pairing in highly disordered *s-wave superconductors*

Amit Ghosal, Mohit Randeria, and Nandini Trivedi
Department of Theoretical Physics, Tata Institute of Fundamental Research, Homi Bhabha Road, Colaba, Mumbai 400005, India

~Received 13 March 2001; published 29 November 2001!

Numerical solution of Bogoliubov-de Gennes equations on a 2d lattice with random distribution of single particle levels. Islands of Cooper pairs. Comparatively homogeneous electron density. Localization.



M.V. Feigel'man, L.B. Ioffe, V.E. Kravtsov, E. Cuevas, Fractal superconductivity near localization threshold, **arXiv:1002.0859;** Annals of Physics **325**, 1368 (2010).

Cooper pairs in insulating phase. Enhanced gap. Important role of the fractal states near localization threshold.

Purpose of this work:

Explanation of the anomalous magnetic behavior

Construction of complete phase diagram

We show that

There exist 3 different non-superconducting phases:
Bosonic insulator, Fermionic insulator and metal

Transitions between these phases are due either to Zeeman depairing or to squeezing of Cooper pairs by potential wells of disordered potential.

arXiv:1001.5431v1 [cond-mat.supr-con] 29 Jan 2010

Model

- Cooper pairs survive in insulator phase. CP have a fixedbinding energy Δ
- Near SIT $k_F l \sim 1, \quad l \sim 1nm \quad n_e \sim 10^{21} cm^{-3}$
- Fluctuations of electron density on the distance ξ are small (Ghosal et al)
- Number of CP is not conserved, but their average density is well defined

$$n = \left\langle \left| \psi \right|^2 \right\rangle = \frac{\Delta}{E_F} n_e$$

- CP density *n* is 3 to 4 orders of magnitude smaller than the electron density. A weak long-scale random potential can localize them and form SC droplets
- Coulomb forces on a distance > $n_e^{-1/d}$ are weak due to screening.
- Random potential acting on CP is uncorrelated Gaussian

$$\left\langle U(\mathbf{x}) U(\mathbf{x}') \right\rangle = \kappa^2 \delta(\mathbf{x} - \mathbf{x}')$$

- CP can be destroyed either by paramagnetic effect (Zeeman energy exceeds binding energy) or due to squeezing (the size of the droplet becomes of the order of the CP size).

Hamiltonian

$$H_k = \frac{1}{2m_k}\left(\mathbf{p} - \frac{e_k}{c}\mathbf{A}\right)^2 + U_k(\mathbf{x}) - g\mu_B \mathbf{s}_k \mathbf{B} \qquad k = b, f$$

$$\mathbf{A} = \frac{1}{2}\mathbf{B}\times\mathbf{r} \qquad e_b = 2e_f;\ m_b = 2m_f \qquad s_b = 0;\ s_f = 1/2$$

$$U_b \approx 2U_f$$ Random potentials are due to stray electric fields

Parallel field, very thin film $h \ll \ell = \sqrt{c\hbar / eB}$

→ Diamagnetic effect is negligible

Depairing field $B_c = \dfrac{\Delta}{g\mu_B}$

Random field

$$\langle U(\mathbf{x})U(\mathbf{x}')\rangle = \kappa^2 \delta(\mathbf{x}-\mathbf{x}')$$

Characteristic disorder length: $\mathcal{L}_k = \left(\frac{\hbar^4}{m_k{}^2 \kappa_k{}^2}\right)^{\frac{1}{4-d}}$

$$m_b = 2m_f \qquad \kappa_b \approx 2\kappa_f \qquad \mathcal{L}_f = 4\mathcal{L}_b \text{ in 2d} \qquad \mathcal{L}_f = 16\mathcal{L}_b \text{ in 3d}$$

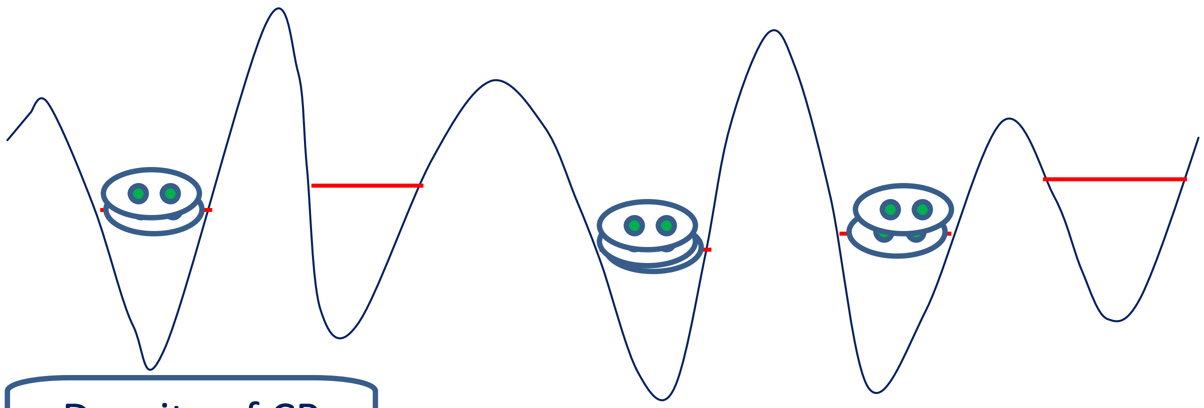
Electrons feel weaker random potential than CP

Characteristic disorder energy: $\mathcal{E}_k = \frac{\hbar^2}{2m\mathcal{L}_k^2}$

Assumption: $\mathcal{L}_b$? ξ

Idea for GNM: CP pairs fill localized states of the random potential forming Bose-insulator. High magnetic field causes depairing. The appearing fermions are weakly localized.

Random field and localized CP $B = 0$



Bosonic insulator

Density of CP

$$\mu = -\frac{\hbar^2}{4mR^2} + b\frac{n}{n_w(R)\pi R^2}$$

GL interaction
L interaction===

$$n_w(E) = \mathcal{L}^{-d} \exp\left[-\left(\frac{E}{\mathcal{E}}\right)^{(4-d)/2}\right]$$

Density of potential wells supporting energy not exceeding E_b

"Lifshitz tails"

$$\mu = E = -\mathcal{E}\left[\ln\frac{n_c}{n}\right]^{\frac{2}{4-d}} \qquad n_c \sim \frac{1}{\mathcal{L}^2 a^{d-2}}$$

I.M. Lifshitz, JETP, 1966
J. Zittartz and J. Langer, Phys. Rev. 1966
B.I. Halperin and M. Lax, Phys. Rev. 1967

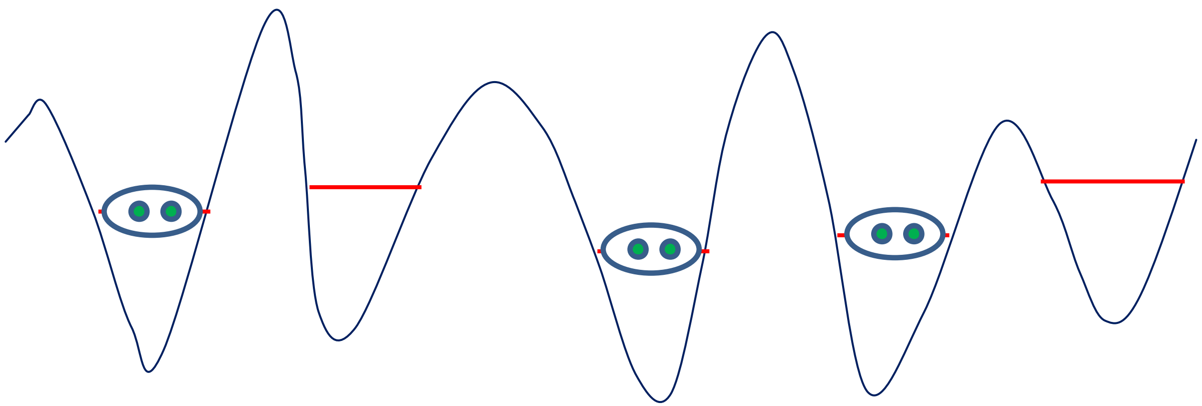
FNP, Phys. Rev. B, Sept. 2009, Talk by T. Nattermann

Random field and CP $B \neq 0$

Parallel field, very thin film $h \ll \ell = \sqrt{c\hbar / eB}$ $d = 2$

No diamagnetic effect $E_b = -\mathcal{E}_b \left[\ln \frac{n_{cb}}{n} \right]$ $n_{cb} = \frac{\pi \mathcal{E}_b}{b}$

Transition from Bosonic to Fermionic insulator (BFT)



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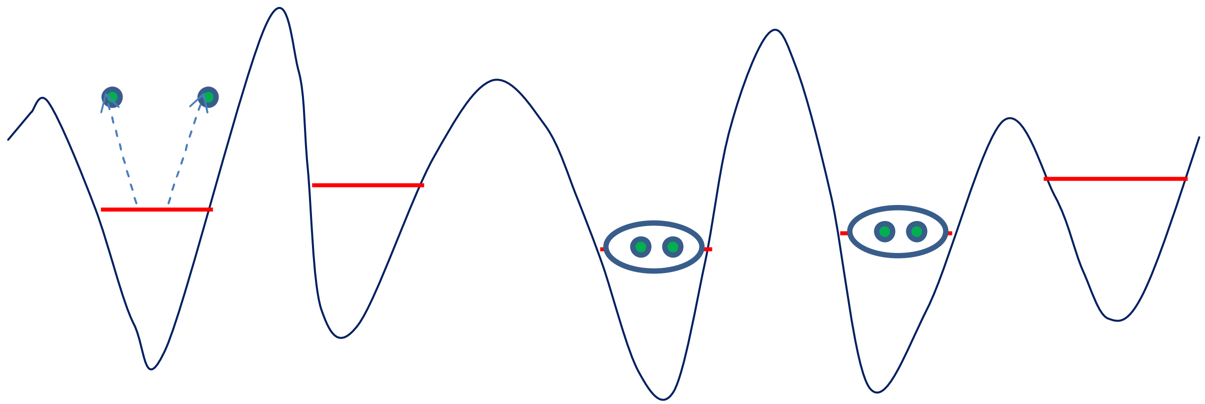
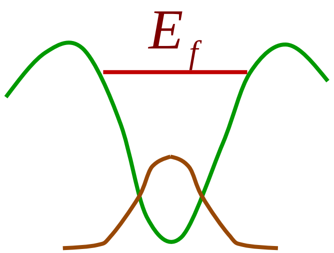
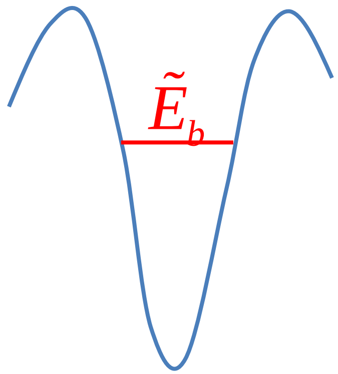


Energy balance: $E_b - 2\Delta = 2\left(E_f - \frac{g\mu_B B}{2} \right)$ $E_f = ?$





$$U_b = 2U_f$$

Basic requirement: $\tilde{E}_b\left(E_f\right) = E_b$

If $\tilde{E}_b < E_b$, it must be occupied

If $\tilde{E}_b > E_b$, E_f is not minimal.

Variational approach:

$$\psi_f = \sqrt{\frac{\alpha_f}{\pi}} \exp\left(-\frac{\alpha_f r^2}{2}\right) \qquad U_f = -\lambda \psi_f^2$$

Minimization of $\frac{1}{2}\int U_f^2 d^2x$ at fixed value of $E_f = \int\left[\frac{\hbar^2}{2m_f}\left(\nabla\psi_f\right)^2 + U\psi_f^2\right]d^2x$

$$\alpha_f = -2mE_f / \hbar^2 \qquad \lambda = 2\pi\hbar^2 / m_f$$

Gaussian variational function for CP ⟶ $E_f = \frac{2}{9}E_b$

The BFT line: $$E_b - 2\Delta = 2\left(E_f - \frac{g\mu_B B}{2}\right)$$

$$E_f = \frac{2}{9}E_b \qquad E_b = -\mathcal{E}_b\left[\ln\frac{n_{cb}}{n}\right]$$

$$B_{BFT} = B_c + \frac{5}{9}\frac{\mathcal{E}_b}{g\mu_B}\ln\frac{n_{cb}}{n}$$

How the density of fermions grows above the BFT?

Equilibrium condition: $$\frac{d\varepsilon}{dn_f} = 0$$ $\varepsilon(n, n_f)$ - energy per unit area

$$d\varepsilon = (E_b - 2\Delta)dn_b + \left(E_f - \frac{g\mu_b B}{2}\right)dn_f$$

Conservation of number of electrons: $$dn_b = -\frac{dn_f}{2}$$

$$\frac{d\varepsilon}{dn_f} = 0 \Rightarrow E_b - 2\Delta = 2\left(E_f - \frac{g\mu_B B}{2}\right)$$ Now it is an equation determining n_f

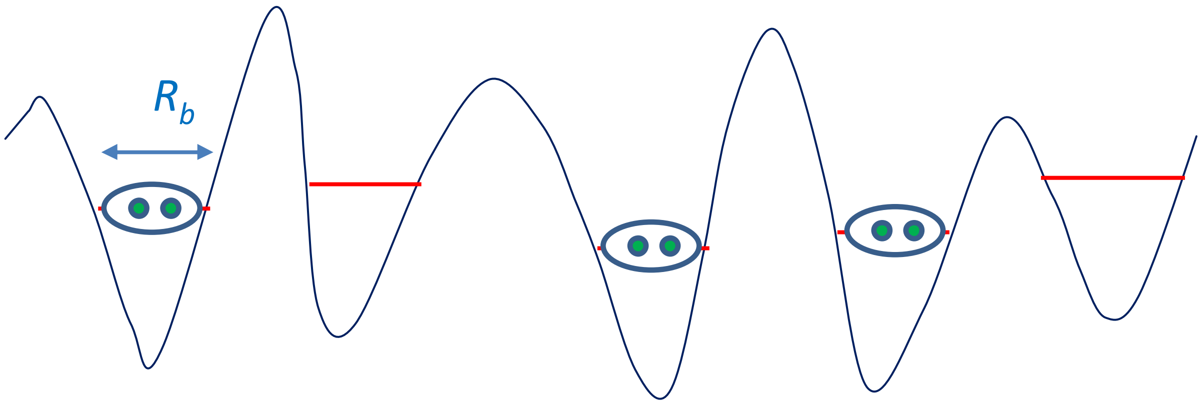
Metal-Insulator Transition (MIT)

$$n_f = n_{cf} = \mathcal{L}_f^{-2} \leq \frac{n_{cb}}{16} \qquad n_b = n - \frac{n_{cf}}{2} \qquad E_f \approx 0$$

$$B_{MIT} = B_c + \frac{\mathcal{E}_b}{g\mu_B} \ln \frac{n_{cb}}{n - n_{cf}/2}$$

Squeezing transition

$$R_b = \frac{\hbar}{\sqrt{-2m_b E_b}} = \xi$$



$$n_{sq} = n_{cb} \exp\left(-\frac{\mathcal{L}^2}{\xi^2}\right)$$

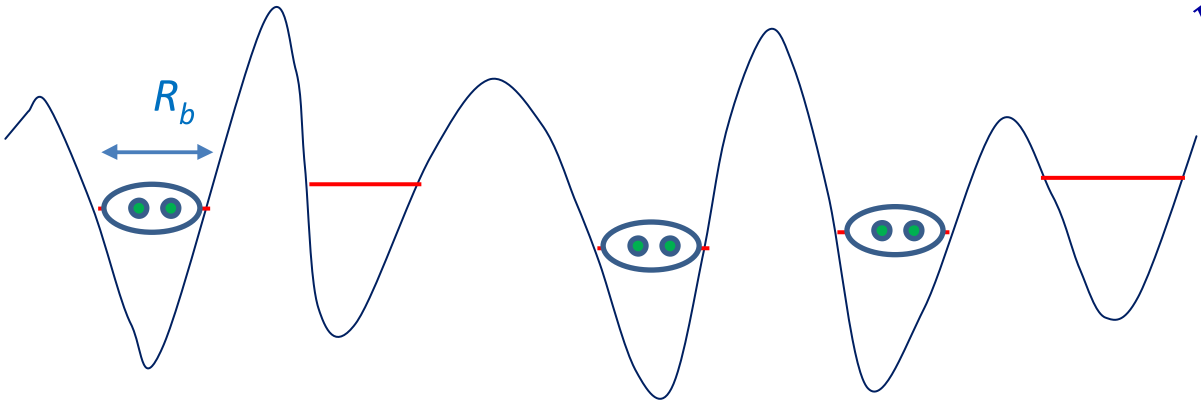
Metal-Insulator Transition (MIT)

$$n_f = n_{cf} = \mathcal{L}_f^{-2} \le \frac{n_{cb}}{16} \qquad n_b = n - \frac{n_{cf}}{2} \qquad E_f \approx 0$$

$$B_{MIT} = B_c + \frac{\mathcal{E}_b}{g\mu_B} \ln \frac{n_{cb}}{n - n_{cf}/2}$$

Squeezing transition

$$R_b = \frac{\hbar}{\sqrt{-2m_b E_b}} = \xi = \sqrt{\xi_0 l}$$

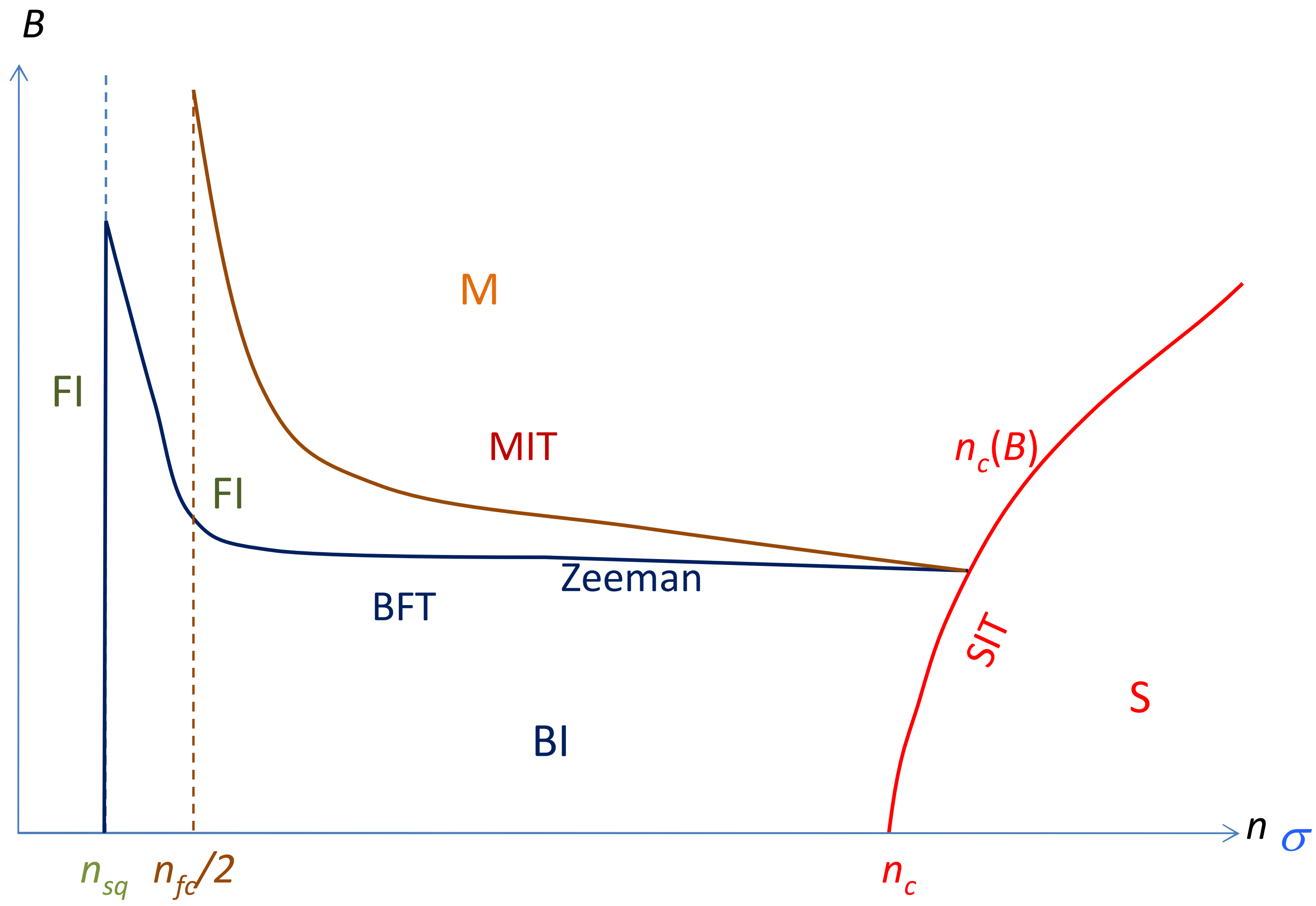


$$n_{sq} = n_{cb} \exp\left(-\frac{\mathcal{L}^2}{\xi^2}\right)$$

$$\mathcal{E}_b = \frac{\hbar^2}{2m\mathcal{L}_b^2};\ \mathcal{L}_b \sim l \ \rightarrow \ \frac{n_{cb}}{n} = \frac{\pi \mathcal{E}_b}{bn} = \frac{l_c^2}{l^2} = \frac{\sigma_c^2}{\sigma^2}$$

Phase diagram in 2d, parallel field.

$n_{sq} < n_{fc} / 2$



Perpendicular field, very thin film $h \ll \ell = \sqrt{c\hbar / eB}$ $d = 2$

Variational approach:

Minimization of $\frac{1}{2}\int U_k^2 d^2x$ at fixed value of

$$E_k = \int \left[\frac{\hbar^2}{2m_k} (\nabla \psi_k)^2 + U\psi_k^2 + \frac{\hbar^2 r^2 \psi_k^2}{8m_k \ell_k^4} \right] d^2x; \quad k = b, f; \quad \ell_k = \sqrt{\frac{\hbar c}{e_k B}}$$

$$\psi_k = \sqrt{\frac{\alpha_k}{\pi}} \exp\left(-\frac{\alpha_k r^2}{2} \right) \qquad U_k = -\lambda \psi_k^2$$

Result of minimization: $E_k = -\frac{\hbar^2 \alpha_k}{2m_k} \left(1 - \frac{3}{4\alpha_k^2 \ell_k^4} \right)$

Density of potential wells supporting energy level not exceeding E_k:

$$n_w(E_k) = \alpha_k e^{-\mathcal{L}_k^2 a_k (1-(4a_k l_k^2)^{-2})^2}$$

Interpolation between Lifshitz and Ioffe-Larkin, 1981.

Perpendicular field, very thin film

Result of minimization: $E_k = -\frac{\hbar^2 \alpha_k}{2m_k}\left(1 - \frac{3}{4\alpha_k^2 \ell_k^4}\right)$

Density of potential wells supporting energy level not exceeding E_{k}:

$$n_w(E_k) = \alpha_k e^{-\mathcal{L}_k^2 a_k (1-(4a_k l_k^2)^{-2})^2}$$

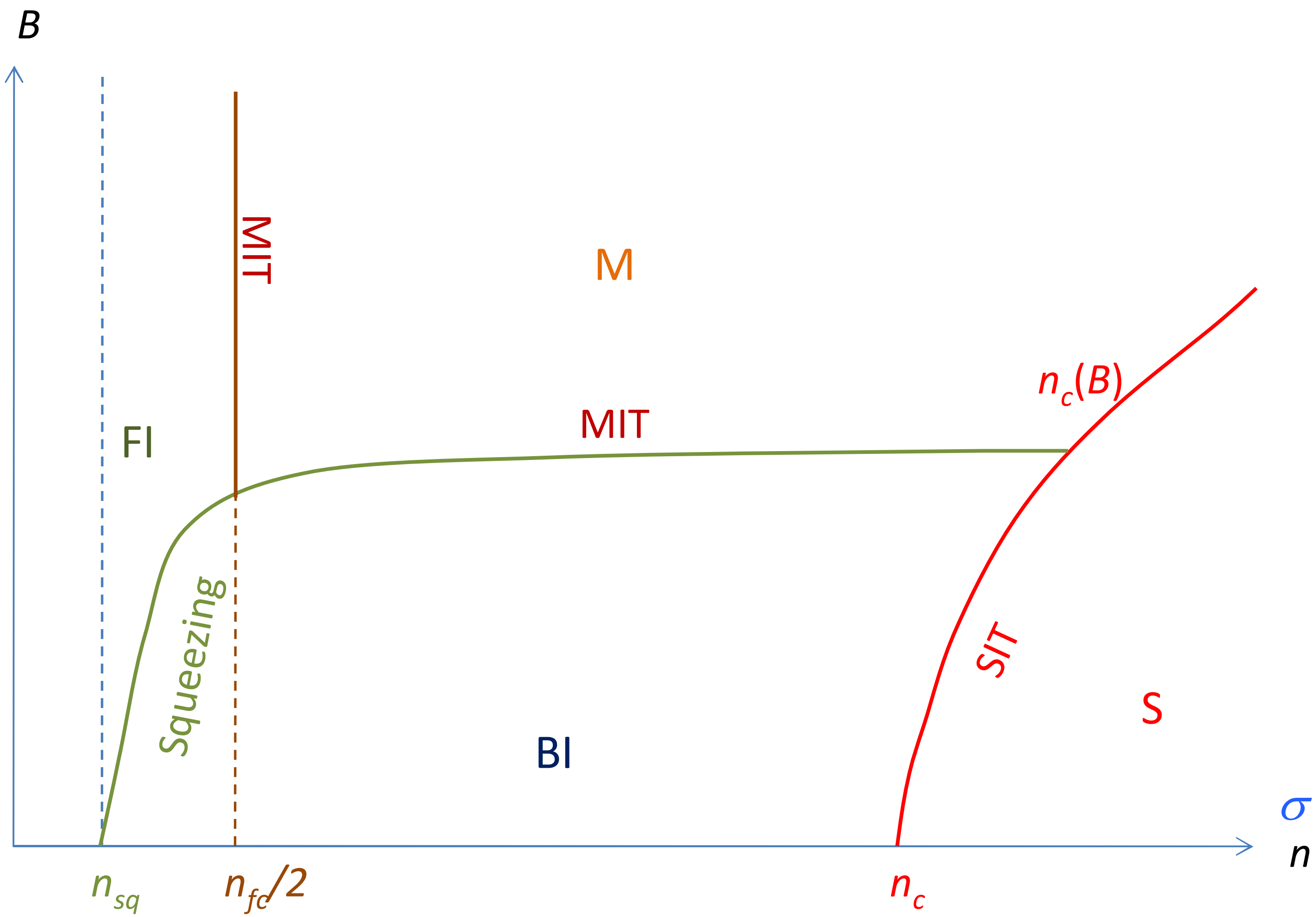
In the BI phase $E_b = -\mathcal{E}_b \ln \frac{n_{cb}}{n}$

In FI (mixed) phase $E_b = -\mathcal{E}_b \ln \frac{n_{cb}}{n_b}; \quad n_b = n - \frac{n_f}{2}$

Squeezing line: $\alpha_b = \xi^{-2} = (\xi_0 l)^{-1}$ $\quad B_{BFT}^{\perp}(n) = \frac{B_c}{k_F l}\left[1 - \frac{\xi}{\mathcal{L}}\left(\ln \frac{1}{n\xi^2}\right)^{1/2}\right]^{1/2}; \quad B_c = \frac{\Phi_0}{2\pi\xi^2}$

Paramagnetic (Zeeman) transition line: $E_b - 2\Delta = 2\left(E_f - \frac{g\mu_B B}{2}\right)$

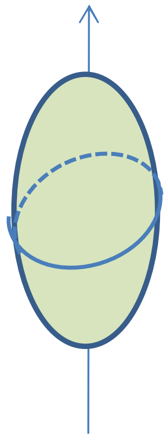
The paramagnetic line is located at higher fields if $k_F l = 1$ and at lower fields if $k_F l$ <<1 reducing the anisotropy. Explanations of discrepancy between E. Belejec *et al*. (PRL **88**, 206802 (2002) and Y.M. Xiong *et al*. (PRB **79**, 020510 (2009).



2d, perpendicular field. $n_{sq} < n_{fc} / 2$

Thick film (3-d case)

The optimal potential well and the wave function are anisotropic: prolonged
In the direction of magnetic field



$$\psi_k(\mathbf{x}) = \frac{\alpha_k^{1/2}\beta_k^{1/4}}{\pi^{3/4}} \exp\left(-\frac{\alpha_k \rho^2 + \beta_k z^2}{2}\right); \quad \rho^2 = x^2 + y^2$$

$$U_k(\mathbf{x}) = -\lambda_k \psi_k^2(\mathbf{x})$$

Minimization of $\frac{1}{2}\int U_k^2 d^2 x$ at fixed value of

$$E_k = \int \left[\frac{\hbar^2}{2m_k} (\nabla \psi_k)^2 + U\psi_k^2 + \frac{\hbar^2 \rho^2 \psi_k^2}{8m_k \ell_k^4} \right] d^3 x; \quad k = b, f; \quad \ell_k = \sqrt{\frac{\hbar c}{e_k B}}$$

Minimization results: $\beta = \alpha - \frac{1}{\alpha \ell^4}$ $E = \frac{\hbar^2}{2m}\left(-\frac{\alpha}{2} + \frac{5}{8\ell^4 \alpha}\right)$

$$n_w(E) = \alpha^{3/2}\left(1 - \frac{1}{\alpha^2 \ell^4}\right)^{1/2} \exp\left[-\pi^{3/2} \mathcal{L} \alpha^{1/2}\left(1 - \frac{1}{\alpha^2 \ell^4}\right)^{3/2}\right]$$

Interpolation between Lifshitz and Ioffe-Larkin 3d results

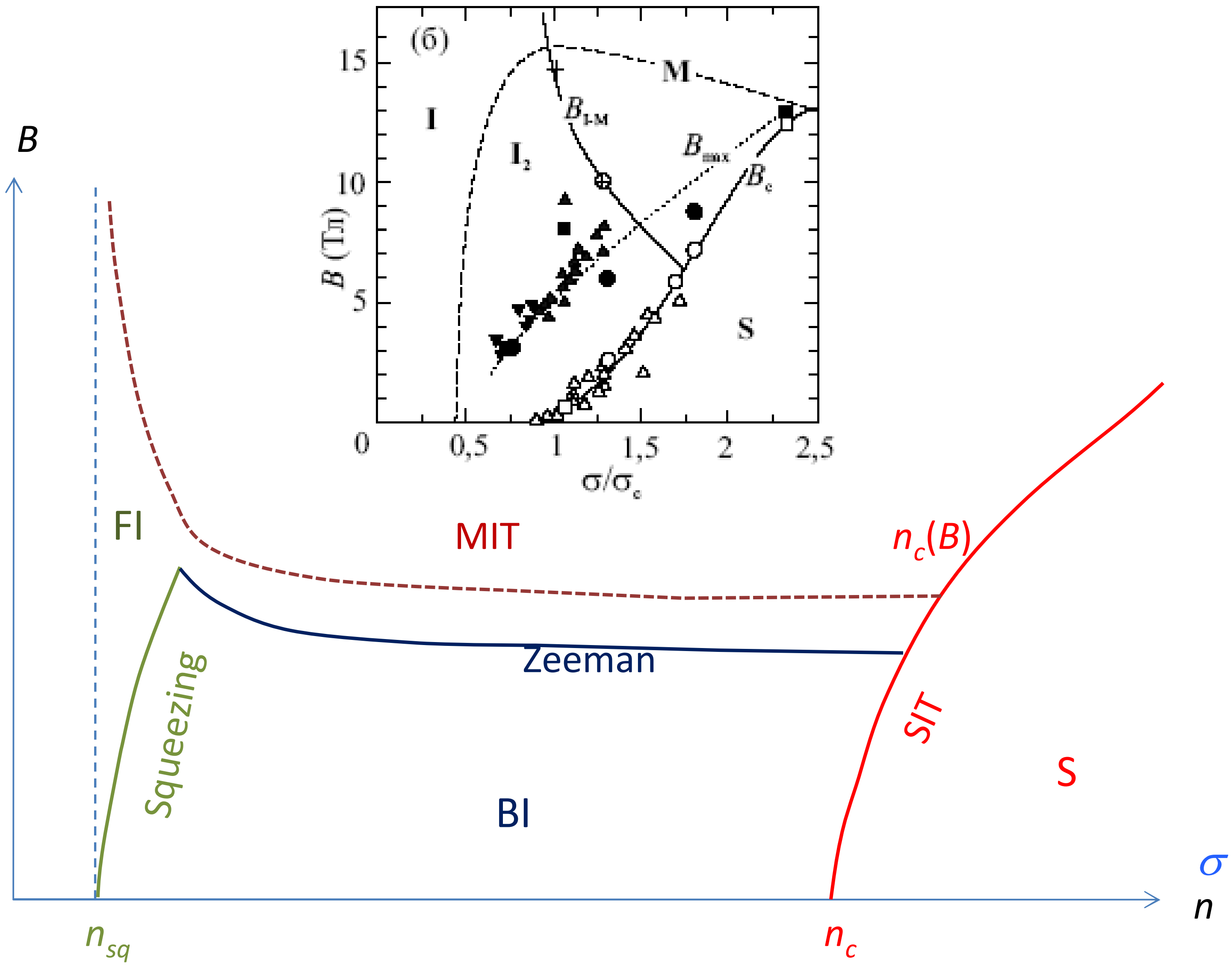
Squeezing transition: Size of CP equals to the longitudinal size of the well

$$B_{sq} = B_c \frac{\sqrt{1-x}}{x}; \qquad x = \frac{\xi}{\mathcal{L}} \ln \frac{n_{cb}}{n} \qquad \xi = \alpha_b^{-1/2}\left[1 - 1/\left(\alpha_b^2 \ell_b^4\right)\right]^{-1/2}$$

Paramagnetic BFT transition: $E_b - \Delta = 2\left(E_f - \frac{g\mu_B}{2}\right)$ $n_f = 0$

MIT: $E_b - \Delta = 2\left(E_f - \frac{g\mu_B}{2}\right); \quad n_f = n_{cf} = \mathcal{L}_f^{-3}$ $B_{MIT} = \frac{B_c}{(gm/m_0) - 2}\left(1 + 0.25x^2\right)$

MIT does not exist if $gm/m_0 \le 2$



3d. The MIT transition does not proceed if $gm/m_0<2$

SIT line

Strong overlapping of wave functions ⟹ Exponent in the density is about 1

3d: $\mathcal{L}\alpha^{1/2}\left(1-\frac{1}{\alpha^2\ell^4}\right)^{3/2}\sim 1$ $\quad B_{SIT}=\frac{\Phi_0 n^{2/3}}{2\pi}\sqrt{1-\left(\frac{n_{cb}}{n}\right)^{2/9}}$

$$n_c=\left(3\mathcal{L}^2 a\right)^{-1} \qquad \text{(3d)}$$

$$\mu=-\mathcal{E}\ln^2\frac{n_c}{n}$$

Resistance

At low magnetic field the Cooper pairs are localized. The excitations are fermions They have a gap in spectrum. After BFT carriers are electrons with zero gap.

Activation behavior of resistance in InO at weak field: $\rho = \rho_0 \exp\left(\frac{\tilde{\Delta}}{T}\right)$

Variable Range Hopping of Fermions at $B>B_{\mathrm{BFT}}$

$$\rho = \rho_0 \exp\left[-\left(T_0/T\right)^{1/(d+1)}\right] \quad T_0 ; \frac{\alpha^{d/2}}{\nu} : \frac{\mathcal{E}}{n\mathcal{L}^d} : \mathcal{L}^{-d-2}$$

Small for fermions

Phase diagram is isotropic, but resistance is anisotropic

Conclusions

- Phase diagram depends on dimensionality. In thin films it depends on the magnetic field direction.
- In all considered situations there are 4 interplaying phases: Bose Insulator, Fermi Insulator, Metal and Superconductor
- Transitions between them are due either to paramagnetic depairing or to squeezing of Cooper pairs by the random potential well in magnetic field
- Negative magnetoresistance appears due to proliferation of fermions which are weakly confined by the random field.
- In thick film or bulk the phase diagram does not depend on direction of magnetic field, however the resistivity must be anisotropic.

Bi amorphous films

PRL **103,** 157001 (2009) PHYSICAL REVIEW LETTERS week ending 9 OCTOBER 2009

Observation of Giant Positive Magnetoresistance in a Cooper Pair Insulator

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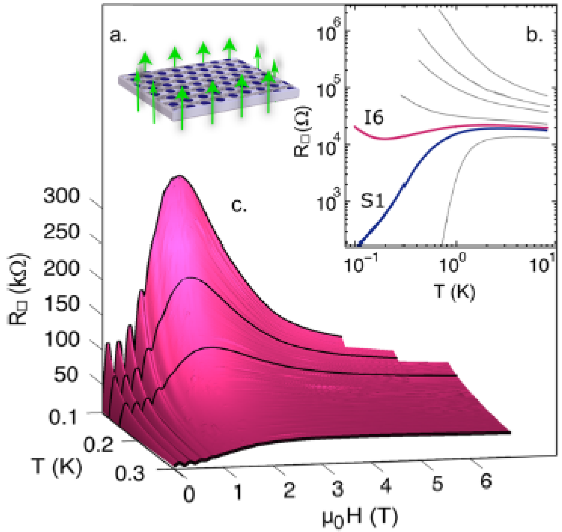

FIG. 1 (color online). (a) SEM image of the nanohoneycomb substrate. The hole center to center spacing and radii are 100 ± 5 and 27 ± 3 nm, respectively. Arrows denote $\vec{H}$. (b) Sheet resistance as a function of temperature, $R_\square(T)$, of NHC films produced through a series of Bi evaporations. The film I6 is the last insulating film and S1 is the first superconducting film in the series. (c) Surface plot of $R_\square(T, H)$ for film I6, which has a normal state sheet resistance of 19.6 kΩ and 1.1 nm Bi thickness. The solid lines are isotherms.

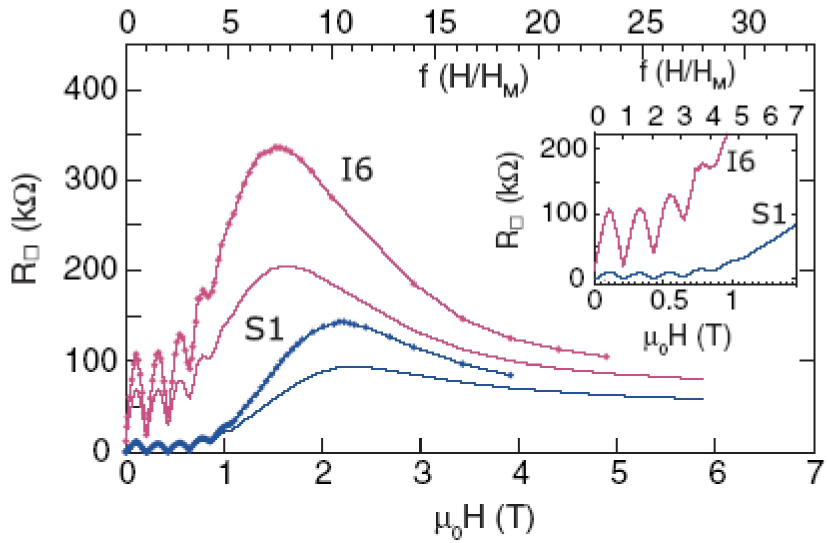

FIG. 2 (color online). Sheet resistance as a function of *H* at 100 and 120 mK for films I6 and S1. The lines are spline fits to the data points (shown as symbols on the 100 mK traces). Inset: Magnified view of the low *H* data.

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LETTERS

Nature of the superconductor–insulator transition in disordered superconductors

Yonatan Dubi[1], Yigal Meir[1,2] & Yshai Avishai[1,2]

PHYSICAL REVIEW B **78**, 024502 (2008)

Island formation in disordered superconducting thin films at finite magnetic fields

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(Received 28 December 2007; published 1 July 2008)

The same as Ghosal et al., two magnetic fields. Disappearance of superconducting islands in large magnetic field.
No Fermi excitations, chemical potential is not determined.
No explanation of GNM. No phase diagram.

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LETTERS

Superinsulator and quantum synchronization

Valerii M. Vinokur[1], Tatyana I. Baturina[1,2,3], Mikhail V. Fistul[4], Aleksey Yu. Mironov[2,3], Mikhail R. Baklanov[5] & Christoph Strunk[3]

Infinite barrier for charge transfer

2d, a regular set of Josephson junctions.

VOLUME 75, NUMBER 25 PHYSICAL REVIEW LETTERS 18 DECEMBER 1995

Logarithmic Divergence of both In-Plane and Out-of-Plane Normal-State Resistivities of Superconducting $La_{2-x}Sr_xCuO_4$ in the Zero-Temperature Limit

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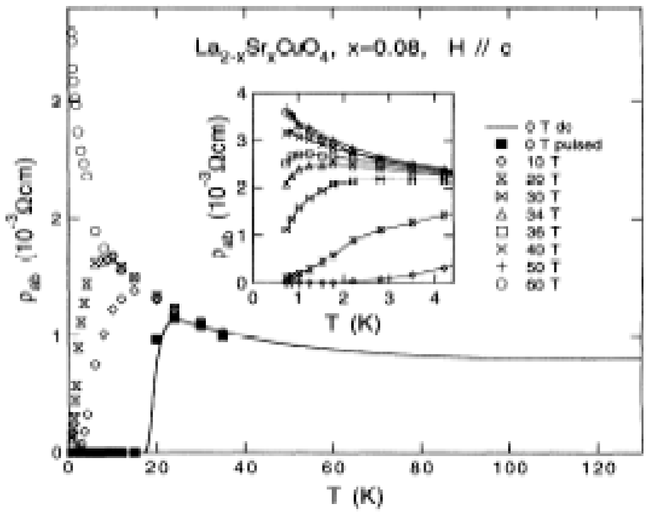

FIG. 2. Temperature dependence of ρ_{ab} in 0, 10, 20, and 60 T, obtained from the pulsed magnetic field data. The solid line shows the zero-field resistive transition. The inset contains the low-temperature data.

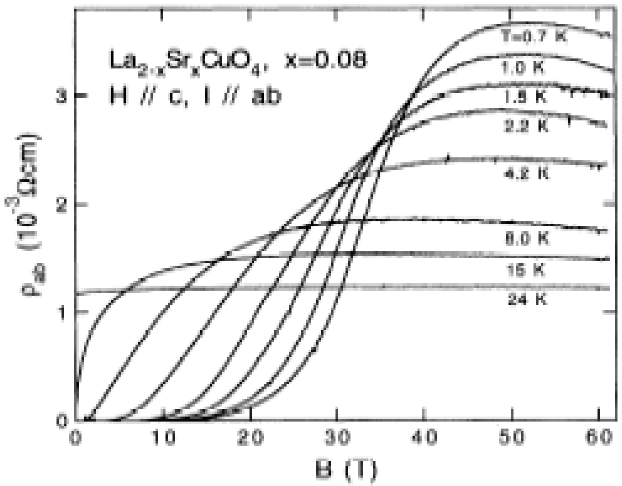

FIG. 1. In-plane resistivity ρ_{ab} versus magnetic field for the $x = 0.08$ $La_{2-x}Sr_xCuO_4$ single crystal at various temperatures.

• CP number is not conserved, but their average number is well defined.

Ginzburg-Landau Hamiltonian

$$H_{GL} = a|\psi|^2 + \frac{b}{2}|\psi|^4 + c|\nabla\psi|^2$$

Assumptions:

1) Smooth fluctuations $|\nabla\psi|/|\psi| = \xi^{-1}$

2) Islands of CP $|\psi|^2 = \begin{cases} -a/b & \text{if } a<0 \\ 0 & \text{if } a>0 \end{cases}$

3) Gaussian random potential $a(\mathbf{r}) = \bar{a} + u(\mathbf{r});\quad \langle u^2(\mathbf{r})\rangle = u_0^2$

Consequence: $\langle|\psi|^2\rangle = -\frac{\bar{a}}{b} f\left(\frac{\bar{a}}{u_0}\right);\qquad f(z) = 1 - \mathrm{Ei}(z) + \frac{\exp(-z^2/2)}{\sqrt{2\pi z}}$

$$f(z) \approx \begin{cases} 1 & z\to\infty \\ \left(\sqrt{2\pi z}\right)^{-1} & z\to 0 \end{cases}$$