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International Centre for Theoretical Physics



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Turbulence: a Cross-Fertilization**

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**Four-wave Mixing Induced Turbulent Spectral Broadening in CW Fiber Lasers**

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# **Four-wave mixing induced turbulent spectral broadening in CW fiber lasers**

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Novosibirsk, Russia**

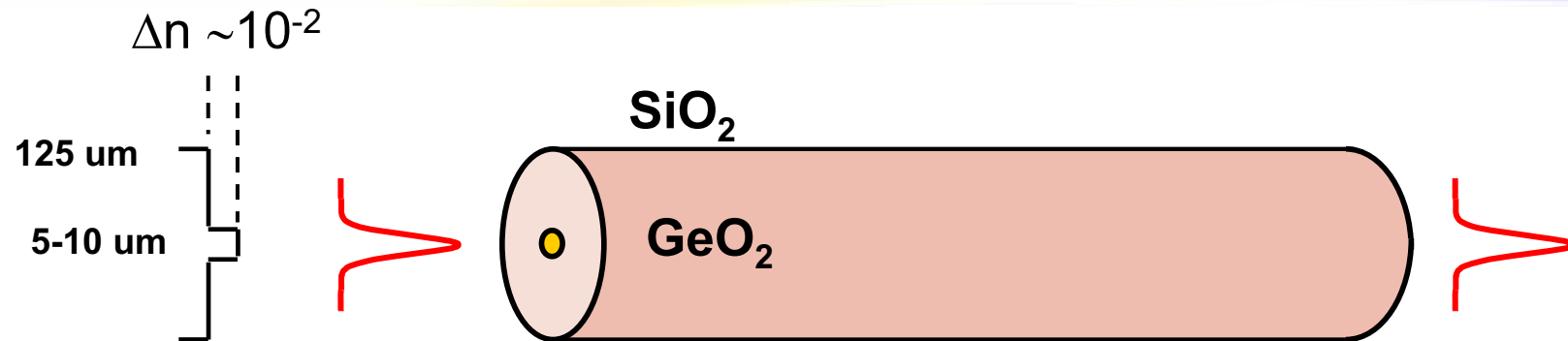
**Together with: S. Babin, E. Podivilov, S. Turitsyn, S. Smirnov**

# Outline

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- + Optical fiber and CW fiber lasers
- + Weak wave turbulence approach in fiber optics
- + CW lasing due to disorder in a fiber
- + 1D light localization in a fiber

# Optical fiber

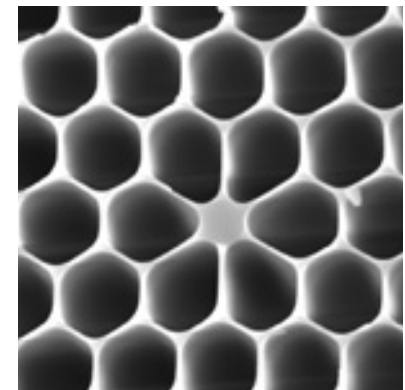


- Small linear losses of only 0.2 dB/km at 1550 nm (C. Cao, Nobel prize `09)
- Dispersion (managed including sign, slope, ZDW point position and number)
- Kerr nonlinearity  $1\text{-}100 \text{ km}^{-1}\text{W}^{-1}$ 
  - Self-phase modulation (SPM)
  - Cross-phase modulation (XPM)
  - Modulation instability (MI)
- Stimulated Brillouin scattering (SBS)
- Stimulated Raman scattering (SRS)

Fiber length scales from 1 m to thousand of kms.

Light intensity in fiber core up to  $I \sim 10^6\text{-}10^8 \text{ W/cm}^2$

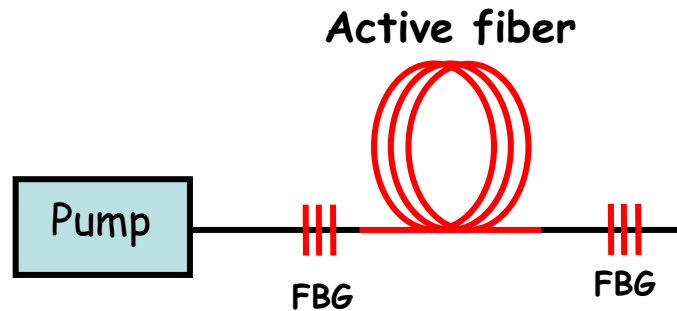
PCF



# CW fiber laser and its spectrum

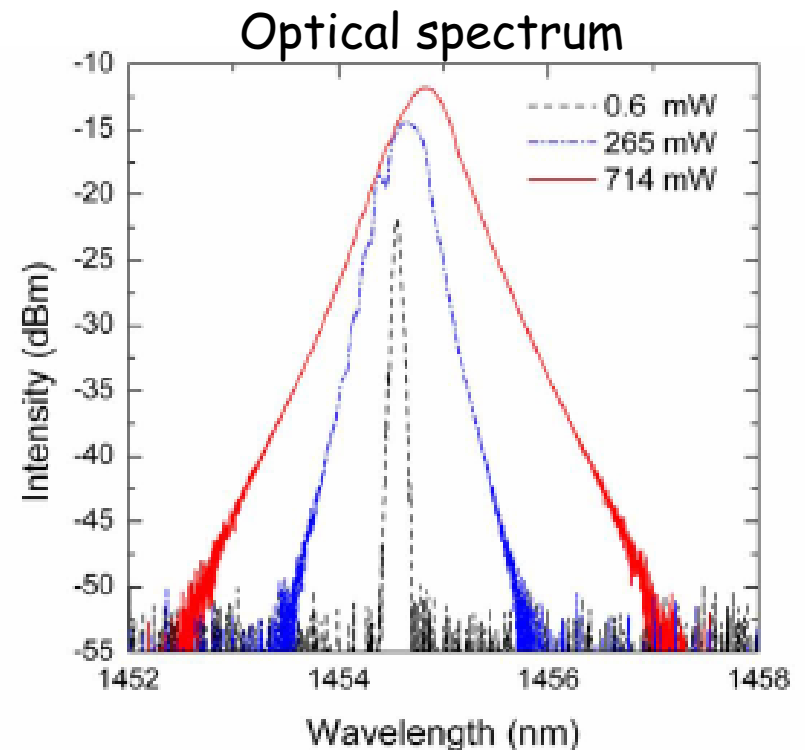
Fiber laser =

pumped active doped (Yb, Er, ...) fiber or SRS, SBS in passive fiber  
+ Mirrors (fiber Bragg gratings FBGs)



Typical performances of CW fiber lasers:

- Output power 1 - 100W
- Spectral width 0.1-1 nm
- Efficiency (wall plug) - 30%



Problem:  
Spectrum is broadened

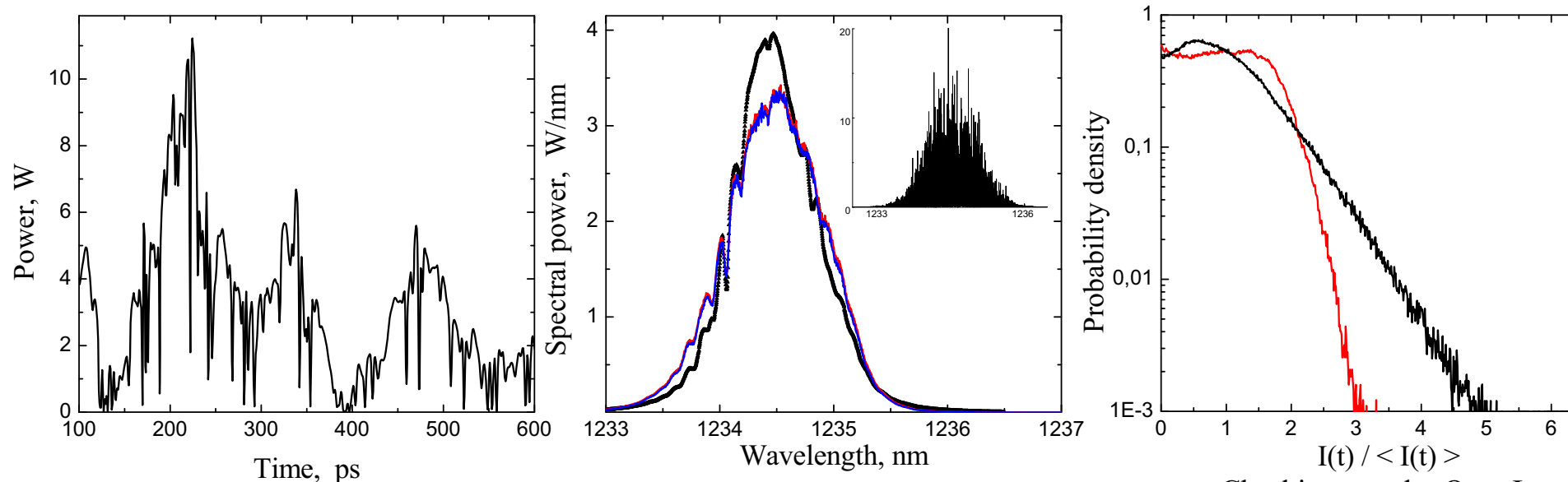
# NLSE based modeling

$$\frac{\partial A_p^\pm}{\partial z} + \frac{i}{2} \beta_{2p} \frac{\partial^2 A_p^\pm}{\partial t^2} + \frac{\alpha_p}{2} A_p^\pm = i\gamma_p \left( |A_p^\pm|^2 + 2|A_s^\pm|^2 \right) A_p^\pm - \frac{g_p}{2} \left( |A_s^\pm|^2 + \langle |A_s^\mp|^2 \rangle \right) A_p^\pm$$

$$\frac{\partial A_s^\pm}{\partial z} + \left( \frac{1}{v_s} - \frac{1}{v_p} \right) \frac{\partial A_s^\pm}{\partial t} + \frac{i}{2} \beta_{2s} \frac{\partial^2 A_s^\pm}{\partial t^2} + \frac{\alpha_s}{2} A_s^\pm = i\gamma_s \left( |A_s^\pm|^2 + 2|A_p^\pm|^2 \right) A_s^\pm + \frac{g_s}{2} \left( |A_p^\pm|^2 + \langle |A_p^\mp|^2 \rangle \right) A_s^\pm$$

+ Boundary conditions at FBGs

NLSE can reveal fast time evolution  $I(t)$ , statistical properties of radiation (both  $\mathcal{P}(I(t))$  and  $\mathcal{P}(I(\omega))$ ), as well as provide generation spectrum

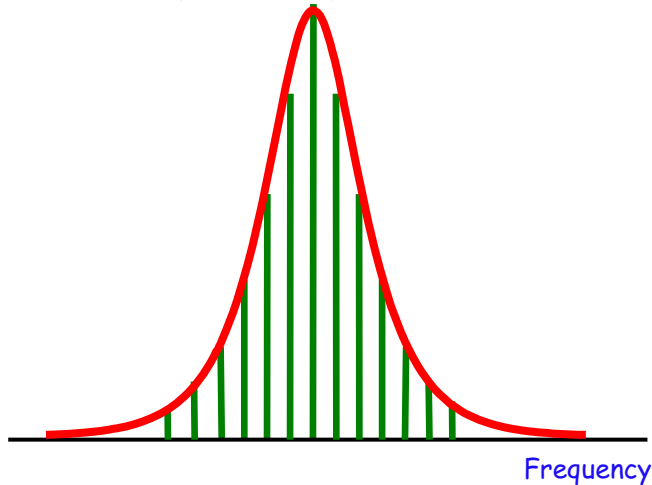


Disadvantage: no analytical insight

Churkin et al, Opt Lett (2010) in press

# Mode structure

Optical spectrum



Generation is strongly multimode

In the cavity of length  $L = 10 \text{ m} - 300 \text{ km}$   
modes are separated by  $10 \text{ MHz} - 250 \text{ Hz}$

Typical spectrum width  $0.1 \text{ nm} - 1 \text{ nm}$   
 $\Rightarrow 10^3 - 10^8$  longitudinal modes.

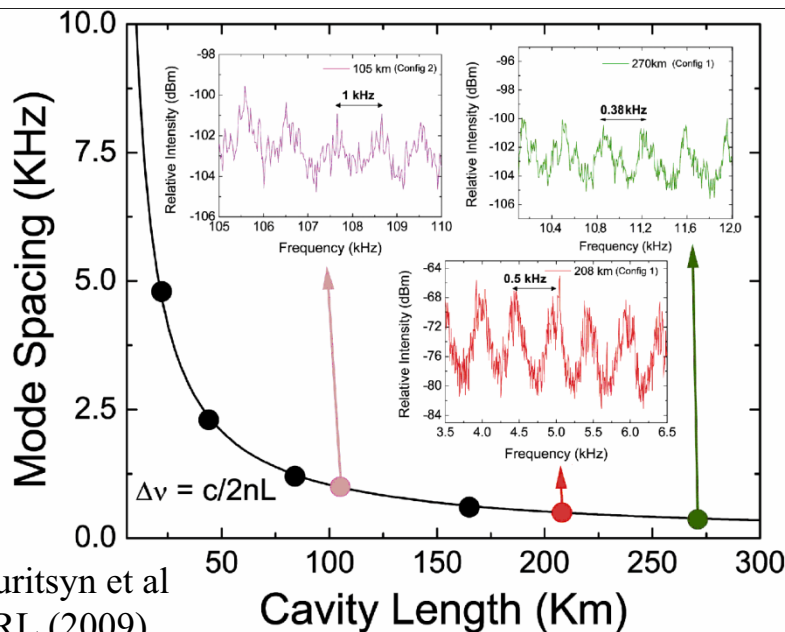
RF peaks are broadened due to  
nonlinear dephasing.

RF peak width depends linearly on  
power.

Modeless spectrum at high power

The amplitudes and phases of modes  
change their values stochastically in  
numerous FWM processes  $\Rightarrow$

statistical description



# Turbulence-induced spectral broadening

## analytical approach: wave kinetic equation

Starting point - Generalized NLSE for modes amplitudes

Technique of averaging and splitting of correlation functions under number of assumptions  
 => 1 D wave kinetic equation for a spectrum power density

Nonlinear attenuation

Generation spectrum

Nonlinear gain - origin of the spectral broadening

$$[\delta(\Omega) + \alpha 2L + \delta_{NL}] I(\Omega) =$$

$$= 2g_R \bar{P} I(\Omega) + \frac{\delta_{NL}}{I^2} \int I(\Omega_1) I(\Omega_2) I(\Omega_1 + \Omega_2 - \Omega) d\Omega_1 d\Omega_2$$

$$\delta_{NL} = \sqrt{\frac{2}{3}} \frac{\gamma L}{\sqrt{1 + (4\beta L / 3\delta_2)^2}}$$

SPM, XPM are averaged to zero

Podivilov et al JOSA B (2007)

⚡ Nonlinear homogeneous attenuation: longitudinal mode of frequency  $\Omega$  scatters to the modes of other frequencies.

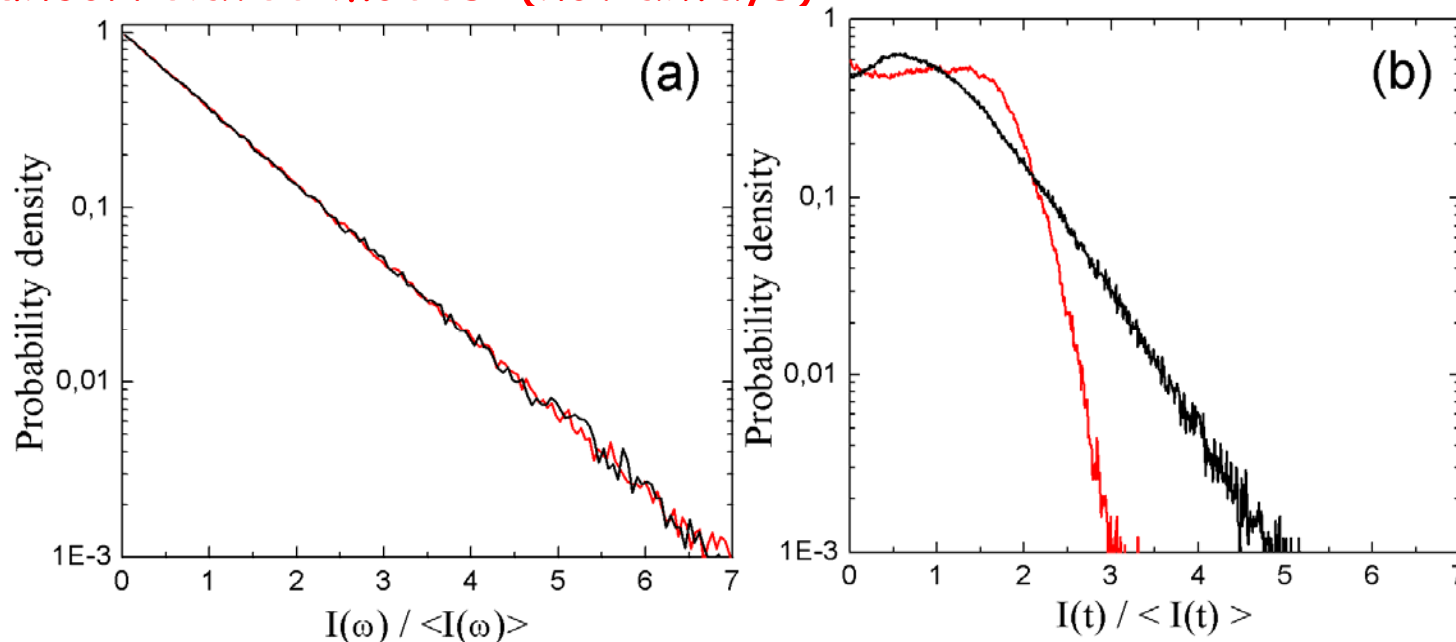
⚡ Nonlinear inhomogeneous gain: longitudinal modes of frequencies  $\Omega_1$  and  $\Omega_2$  scatters to the mode of frequency  $\Omega$ .



# Assumptions

+ Gaussian statistics for  $A_n(t)$  (i.e. exp for intensities)

+/- uncorrelated modes (not always)



+ FWM with pump wave is neglected, OK high above threshold

+ dephasing time  $<$  than round-trip time  $T_{rt}$ , OK for  $I > 1$  W

+ total intra-cavity power,  $I(z) = \text{const}$ , OK

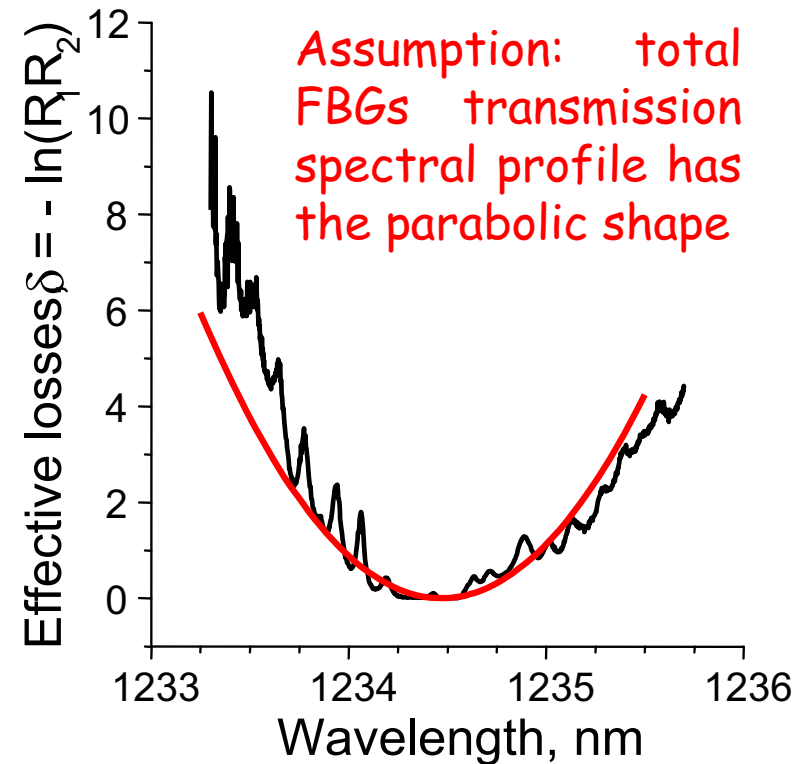
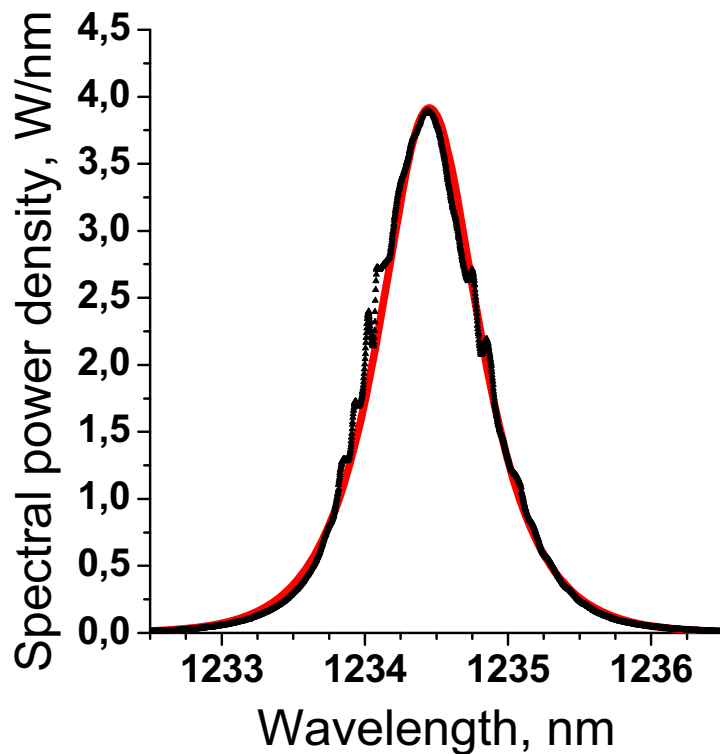
+ nonlinearity is smaller than dispersion,  $\gamma IL \ll 4\beta L \overline{\Omega^2}$  OK

# Generation spectrum

## analytical theory and experiment

$$I(\Omega) = \frac{2I}{\pi \Gamma \cosh(2\Omega/\Gamma)}$$

$$\Gamma = \frac{2}{\pi} \sqrt{\frac{2\delta_{NL}}{\delta_2}}$$



# Initial spectral broadening

Mechanism near the threshold - nondegenerate FWM involving generated wave and pump wave

Generated wave spectrum Zero dispersion  $g = 2g_R \bar{P}L - \alpha 2L$

$$(g - T(\Omega))I(\Omega) = \varepsilon_1 (\gamma 2L)^2 F_0 I(\Omega) - \varepsilon_1 (\gamma 2L)^2 \int_{-\infty}^{\infty} d\Omega' F(\Omega') I(\Omega - \Omega')$$

↑ Total FBGs transmission profile
↑ pump wave correlation time is small (dimensionless)
↑ pump wave rf-spectrum

- ✚ First term (nonlinear attenuation): Stokes wave ( $\Omega$ ) scatters on pump wave to the Stokes wave (some frequency).
- ✚ Second term: Stokes wave (some frequency) scatters on pump wave to the Stokes wave ( $\Omega$ ).

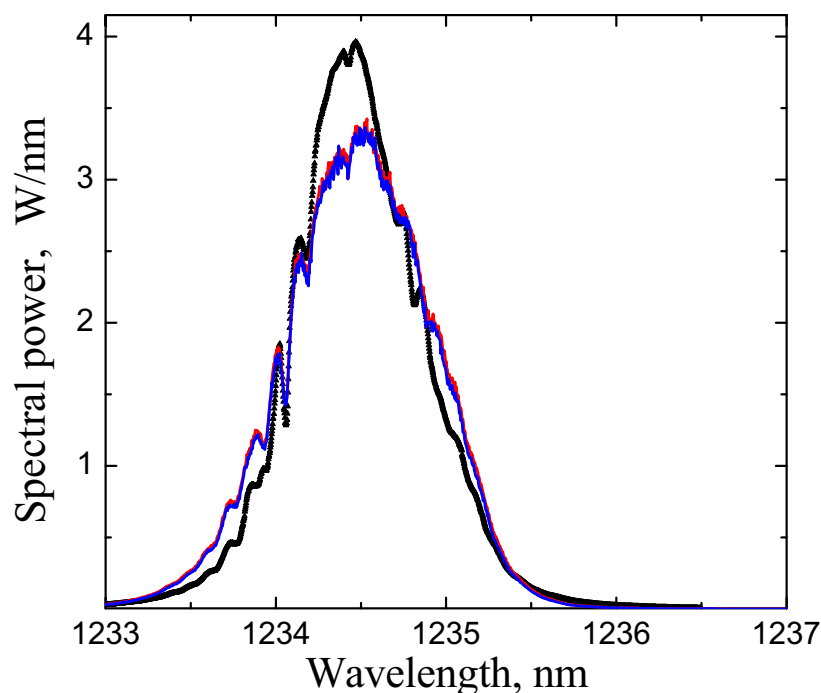
# Initial spectral broadening nondegenerate FWM with pump wave??

## Wave kinetic equation:

intensity fluctuations from the pump wave are transferred to the generated wave;

generated spectrum width near the threshold is proportional to the pump wave spectrum width.

Experimental confirmations of noise transfer is exist

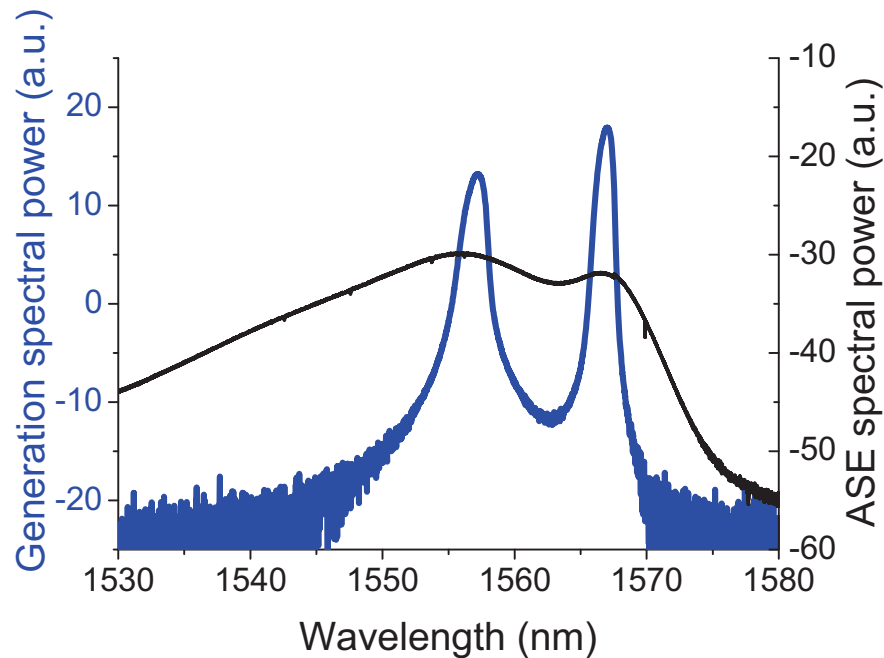
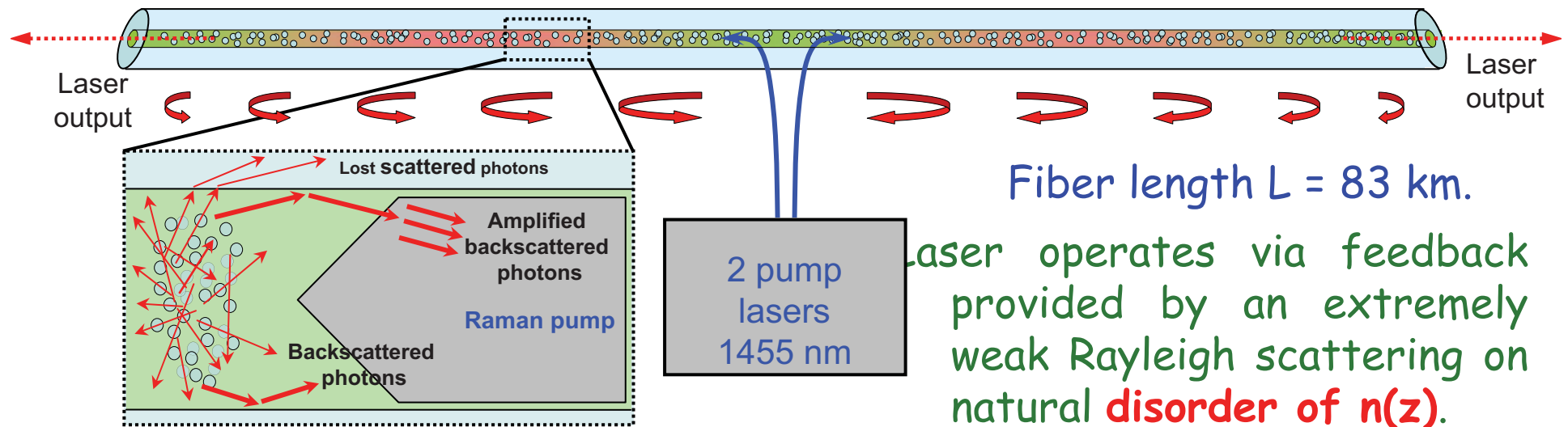


## Direct NLSE numerics :

single frequency pump wave (no amplitude or phase fluctuations) gives same spectrum as multimode pump wave...

Additional mechanisms should exist

# Random distributed feedback fiber laser (RDFB)

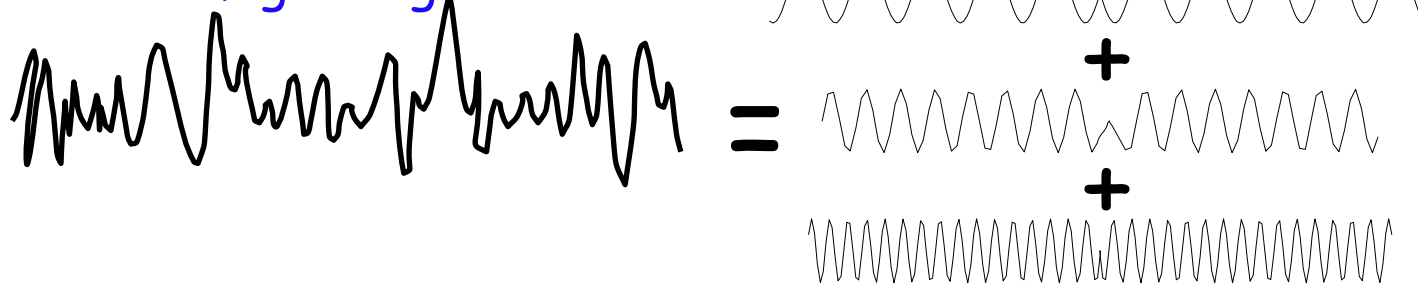


- CW operation
- Narrow modeless spectrum
- 35 dB ASE suppression
- $TEM_{00}$  profile

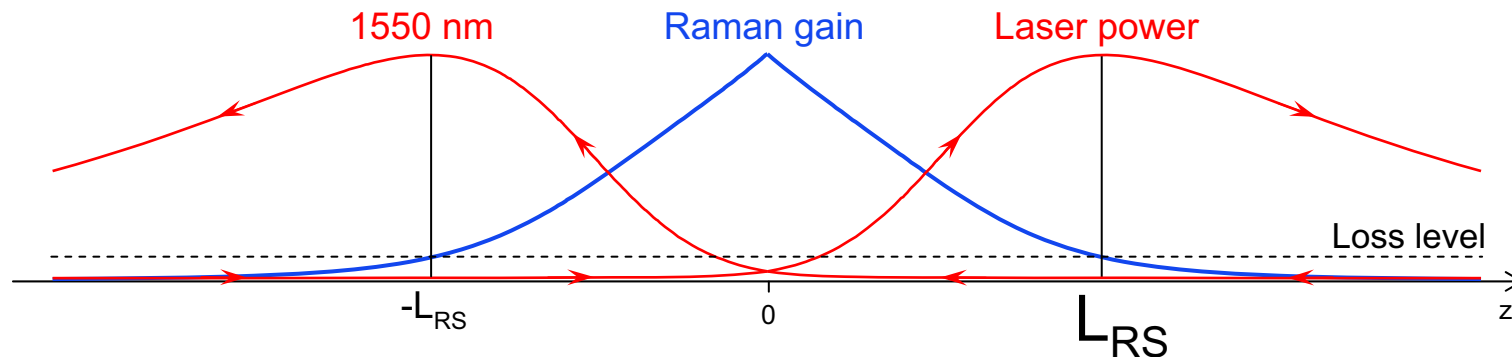
Spectrum formation and broadening in the absence of defined longitudinal modes.

# RDFB mode structure

"Frozen" random grating

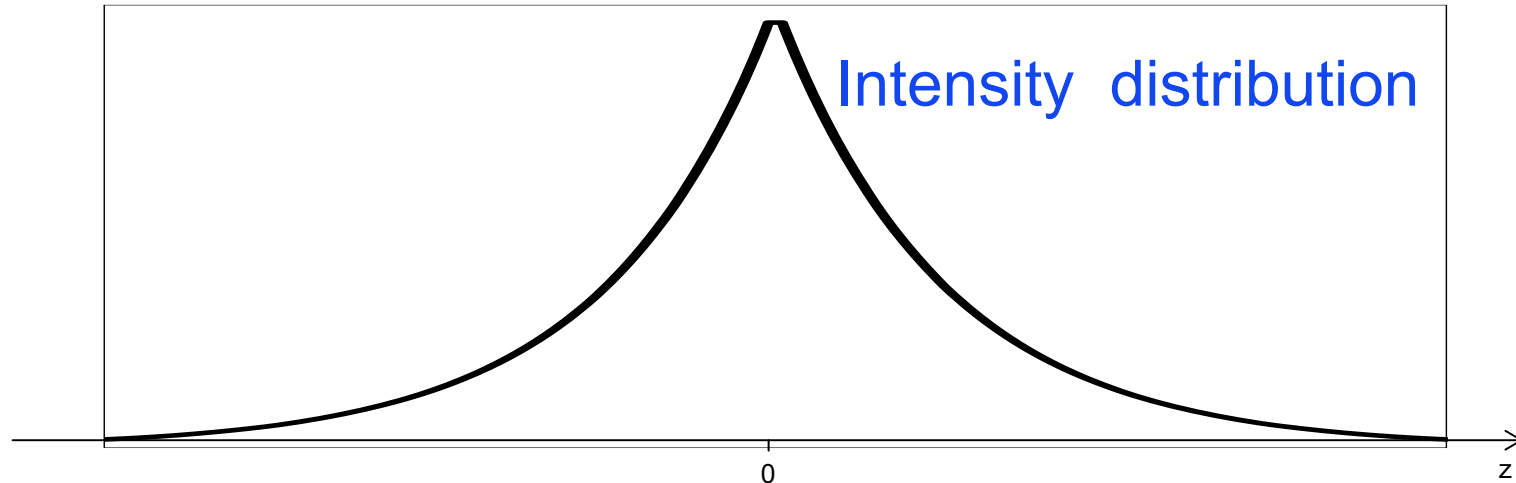


Multitude of weak randomly distributed modes with arbitrary phase and amplitude



Longitudinal distribution of the generated power is defined by the gain/loss profile, but not by the random scattering.

# 1D light localization



Random Rayleigh scattering in a 1D fiber waveguide should lead to the localization of light.

The scattering is an extremely weak. Thus the localization length should be extremely big,  $L_{loc} \sim 1/\varepsilon$ , where  $\varepsilon$  is the scattering strength,  $\varepsilon \sim 5 * 10^{-5} \text{ km}^{-1}$  in standard single mode fibers.

Linear losses  $\alpha$  are  $\sim 1000$  times higher than backscattering  $\varepsilon$ . Losses have to be compensated by gain with accuracy  $\varepsilon/\alpha \sim 0.001$ .

Nonlinearity can be managed by adjusting the intensity of coupled light.

# Summary

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- ✚ Weak wave turbulence approach is fruitful in fiber optics: conventional CW fiber laser generation spectrum is described.
- ✚ Natural extremely weak disorder in a fiber can provide stable random generation.
- ✚ If losses are compensated with high accuracy, 1D light localization is possible in a fiber. Nonlinearity can be controlled.