The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization
IAEA

2162-28

Advanced Workshop on Anderson Localization, Nonlinearity and Turbulence: a Cross-Fertilization

23 August - 3 September, 2010

Kinematic Magnetic Dynamo in a Random Flow with Strong Average Shear

I.V. KOLOKOLOV

L.D. Landau Institute for Theoretical Physics
Moscow
Russia

Strada Costiera 11, 34151 Trieste, Italy - Tel.+39 040 2240 111; Fax +39 040 224 163 - sci_info@ictp.it

ИКИ, 9 июня 2010

Kinematic magnetic dynamo in a random flow with strong average shear

I. V. Kolokolov and V. V. Lebedev

J. Phys. A: **43**, 182001, 2010

We assume that the random flow is statistically homogeneous in space and time. The magnetic field grows from small fluctuations. If the velocity field is short-correlated in time then it is possible to derive closed equations for the magnetic induction correlation functions. In the isotropic case the pair correlation function has been analyzed by Kraichnan and Nagarajan (67) and Kazantsev (68). The complete statistical description of the magnetic field for a short-correlated smooth statistically isotropic flow has been done by Chertkov, Falkovich, Kolokolov, Vergassola (99). We are aiming to analyze a strongly anisotropic case, in the presence of strong shear. The flow appears to be effectively two-dimensional in this case.

colorred The hydrodynamic motion in the fluid is assumed to be random (turbulent) and the velocity statistics is assumed to be homogeneous in space and time. We examine the magnetic field growth from initial fluctuations distributed statistically homogeneously in space at the initial time $t = 0$. The correlation length of the initial fluctuations l is assumed to be smaller than the velocity correlation length η. If we consider hydrodynamic turbulence then a role of the velocity correlation length is played by the Kolmogorov scale. At scales less than η the velocity field $\boldsymbol{v}$ can be treated as smooth. The magnetic diffusive length r_d is assumed to be much smaller than l.

The magnetic growth (dynamo) can be characterized by moments of the magnetic induction (averaged over space):

$$\left\langle |\boldsymbol{B}(t)|^{2n} \right\rangle \propto \exp\left(\gamma_n t\right).$$

The exponential character of the growth is related to statistical homogeneity of the flow in space and time and to smoothness of the flow responsible for the growth.

The magnetic field evolution is governed by the equation

$$\partial_t \boldsymbol{B} = (\boldsymbol{B} \cdot \boldsymbol{\nabla})\boldsymbol{v} - (\boldsymbol{v} \cdot \boldsymbol{\nabla})\boldsymbol{B} + \kappa \nabla^2 \boldsymbol{B}.$$

Lyapunov exponent λ – average logarithmic rate of diverging close fluid particles, characterizes typical gradient of velocity. Diffusive magnetic length $r_d = \sqrt{\kappa/\lambda}$.

The initial magnetic field distribution in space can be thought as an ensemble of blobs of sizes $\sim l$. Then the blobs are distorted being stretched in one direction and compressed in another direction. At the diffusionless stage the magnetic field induction grows like a separation between close fluid particles. No reconnections occur at the stage.

Then the minimal size is stabilized at r_d, whereas the longitudinal size still increases. Due to reconnections the blobs overlap and as a result of summing a large number of random quantities an exponentially decaying in time factor appears. So the exponents γ_n at the diffusive stage are smaller than at the diffusionless stage.

The equation for the magnetic field can be formally solved in terms of Lagrangian trajectories propagating back in time

$$\boldsymbol{B}(\tau, \boldsymbol{r}) = \left\lfloor \hat{W}(\tau)\mathcal{B}[\boldsymbol{R}(0)] \right\rfloor,$$

$$\partial_t \boldsymbol{R} = \boldsymbol{v}(t, \boldsymbol{R}) + \boldsymbol{\xi}, \qquad \boldsymbol{R}(\tau) = \boldsymbol{r},$$

$$\lfloor \xi_i(t_1)\xi_j(t_2) \rfloor = 2\kappa\delta_{ij}\delta(t_1 - t_2),$$

where $\mathcal{B}(\boldsymbol{r})$ – initial magnetic field and $\boldsymbol{\xi}$ – Langevin forces.

The matrix $\hat{W}(t)$ is determined by the following equation

$$\partial_t \hat{W} = \hat{\Sigma} \hat{W}, \qquad \hat{W}(0) = 1,$$

where the last term represents the boundary condition. The matrix $\hat{\Sigma}(t)$ is the velocity gradients matrix, $\Sigma_{ji} = \partial_i v_j(t)$, taken at the spacial point $\boldsymbol{R}(t)$. We will call $\hat{W}$ an evolution matrix.

We use the decomposition

$$\hat{W} = \hat{O}_L \hat{D} \hat{O}_R,$$

where $\hat{O}_{R,L}$ are orthogonal matrices and $\hat{D}$ is a diagonal matrix

$$\hat{D} = \begin{pmatrix} e^{\rho_1} & 0 & 0 \\ 0 & e^{\rho_2} & 0 \\ 0 & 0 & e^{\rho_3} \end{pmatrix},$$

$\rho_1 + \rho_2 + \rho_3 = 0$ due to incompressibility.

In our case we can take $\rho_2 = 0$, then

$$\hat{D} = \begin{pmatrix} e^{\rho} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-\rho} \end{pmatrix}.$$

The quantity ρ typically linearly grows as time increases, one can estimate $\rho \sim \lambda t$. Therefore e^{ρ} is an exponentially growing factor at $t > \lambda^{-1}$, that explains the dynamo effect.

At the diffusionless stage $B^2 \propto e^{2\rho}$. It is correct if $\rho < \ln(l/r_d)$ since an initial separation between the Lagranginan trajectories is $\sim r_d e^{\rho}$. At the diffusion stage the main contribution to averaged B^2 are associated with events where the trajectories are separated less than l. That gives an additional small factor $e^{-\rho}$ that is $B^2 \propto e^{\rho}$ at the diffusion stage.

As an example, we consider the steady hyperbolic two-dimensional flow with $v_x = \lambda x$, $v_y = 0$, $v_z = -\lambda z$. Then the magnetic field blobs are stretched along the X-direction and compressed along the Z-direction. The evolution matrix is

$$\hat{W}(t) = \begin{pmatrix} e^{\lambda t} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & e^{-\lambda t} \end{pmatrix}.$$

We assume that initially, at $t = 0$, the magnetic field fluctuations are characterized by a pair correlation function $F_{ij}(\boldsymbol{r})$, then one obtains $\langle B_i(t, \boldsymbol{r}_1) B_j(t, \boldsymbol{r}_2)\rangle$

$$= \left\lfloor W_{ik} W_{jn} F_{kn}(e^{-\lambda t}x + U_x, y + U_y, ze^{\lambda t} + U_z) \right\rfloor,$$

where $\boldsymbol{r} = \boldsymbol{r}_1 - \boldsymbol{r}_2$ and

$$\lfloor U_x^2 \rfloor = 2\kappa/\lambda, \ \lfloor U_y^2 \rfloor = 4\kappa t, \ \lfloor U_z^2 \rfloor = 2e^{2\lambda t}\kappa/\lambda.$$

Then at the diffusion stage

$$\langle B_x^2 \rangle \propto e^{\lambda t}, \quad \langle B_y^2 \rangle \propto e^{-\lambda t}, \quad \langle B_z^2 \rangle \propto e^{-3\lambda t}.$$

We see the dynamo effect in accordance with our expectations, though the flow is two-dimensional. An error of Zel'dovich: It is impossible to neglect B_y though it is exponentially small, since it violates div $\boldsymbol{B} = 0$ because of $\partial/\partial x \propto e^{-\lambda t}$.

We examine the random flow with strong average shear, that is in the main approximation $v_x = \dot{\gamma} y$. The smallness of the velocity fluctuations means $\dot{\gamma} \gg \lambda$. In the situation the only relevant velocity gradient is $\partial_x v_y$. Keeping the term, we pass to the effectively two-dimensional flow. Of course, the magnetic field is assumed to have all three components.

The main assertions, concerning the situation

- The dynamo effect does exist and is determined by the exponents $\gamma_n \sim \lambda$.

- The principal magnetic field component is B_x, and $B_y \sim (\lambda/\dot{\gamma})B_x$.