The Abdus Salam
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Advanced Workshop on
"Anderson Localization, Nonlinearity and Turbulence: A Cross-Fertilization"

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D.M. BASKO
Lab. de Physique et Modelisation des Milleux Condenses
Univ. Joseph Fourier and CNRS
38042 Grenoble, France

TITLE:
``Weak chaos in the disordered nonlinear Schrödinger chain: Destruction of Anderson localization by Arnold diffusion"

ABSTRACT:
The subject of this study is the long-time dynamics of a strongly disordered chain of weakly coupled nonlinear classical oscillators, with the focus on the regime where the average energy per oscillator is finite (so that the thermodynamic limit is well-defined), but small (so that the nonlinear frequency shifts are small compared to the disorder). The main result is an explicit expression for the macroscopic transport coefficients [1].
The key to the solution is the analysis of the spatial structure of chaos. It is often assumed that upon thermalization chaos has no spatial structure, and all sites of the chain are more or less equally chaotic; here it is argued not to be the case. For strong disorder and weak nonlinearity, chaos is concentrated on a small number of rare chaotic spots. A chaotic spot is a collection of resonantly coupled oscillators, in which one can separate a collective slow degree of freedom performing chaotic motion. This chaotic motion acts as a stochastic pump, i. e. drives the exchange of energy between other oscillators, non-resonantly coupled to the spot, which corresponds to the Arnold diffusion. This represents the main mechanism for relaxation and thermalization of the oscillators, as well as for the transport of conserved quantities. An important role is played by the fact that chaotic spots can migrate along the chain, as the collective degree of freedom may get in and out of the chaotic region of its phase space. This migration bears analogy with the variable-range hopping of electrons in strongly disordered solids.
The main technical challenge in this work is the analysis of high orders of the perturbation theory, which is necessary both to separate the collective degree of freedom performing the chaotic motion, and to couple other oscillators to this degree of freedom. The perturbation theory diverges because of resonances. Here, the probability for resonances to occur is estimated in each order of perturbation theory. The subsequent treatment of each resonance and description of the associated chaotic motion is based on earlier works [2].
[1] D. M. Basko, arXiv:1005.5033. [2] B. V. Chirikov, Phys. Rep. 52, 263 (1979).