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Advanced Workshop on
"Anderson Localization, Nonlinearity and Turbulence: A Cross-Fertilization"

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TITLE:
`` q-Breathers, FPU problem, and anomalous conductivity"

ABSTRACT:

From the present perspective, the FPU problem consist of three major ingredients. First, for suitable parameter ranges of the energy, system size, and nonlinearity the low-frequency excitations are localized in q-space of the normal modes. Second, recurrence of energy to an initially excited low-frequency mode is observed. Third, different thresholds upon tuning the parameters are seen: the weak stochasticity threshold, which separates regular and chaotic dynamics, yet possibly preserving the localization character in q-space, and the strong stochasticity threshold, separating localized from delocalized dynamics in q-space.
In this talk I review the recent progress in the theory of q-breathers, exact time-periodic low-frequency solutions in the nonlinear FPU system, that allows to explain qualitatively and semiquantitatively the above aspects of the FPU problem. These solutions are exponentially localized in the q-space of the normal modes and preserve stability for small enough nonlinearity. They continue from their trivial counterparts for zero nonlinearity at finite energy. The FPU trajectories computed 50 years ago can be regarded as perturbations of the exact q-breather orbits. The stability threshold of QB solutions coincides with the weak chaos threshold. The delocalization threshold estimate of q-breathers shows identical scaling properties as the estimate of equipartition from second-order nonlinear resonance overlap.
These results spurred diverse research in the field. Among the latests developments are the analysis of the FPU trajectories on extremely large times and the final route to equipartition, the discovery of q-breathers in 2D and 3D acoustic lattices, semi-quantum and quantum extended discrete systems as generic objects. Dynamical localization in the q-space was also shown to persist in transient processes and thermal equilibrium. Quasi-periodical orbits were demonstrated to continue as q-tori, also exponentially localized in the q-space.
Finally, I present the theory of q-breathers in disordered nonlinear chains in some detail and discuss the insights its gives into the mechanisms behind anomalous heat conductivity of nonlinear disordered arrays.