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Advanced Workshop on Anderson Localization, Nonlinearity and Turbulence: a Cross-Fertilization

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Experimental Observation of the Anderson Transition and its Critical State

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Experimental Observation of the Anderson Transition and its Critical State


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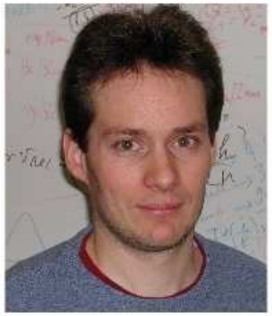
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Outline

Quantum transport/localization in disordered or chaotic systems

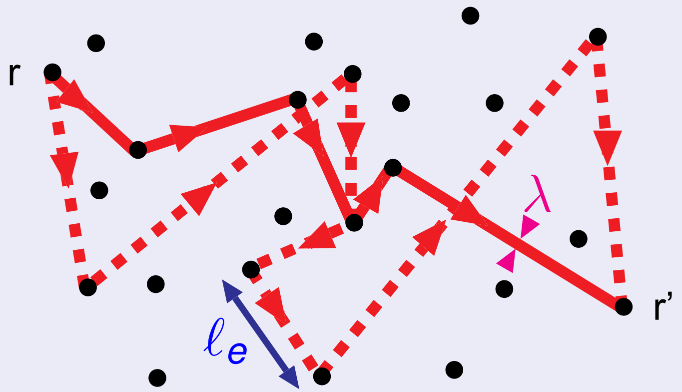
The Kicked Rotor with cold atoms: a very practical tool for studying Anderson localization

Experimental observation of the Anderson transition with atomic matter waves

Critical State of the Anderson transition: Between a Metal and an Insulator

Interplay between disorder and interference effects

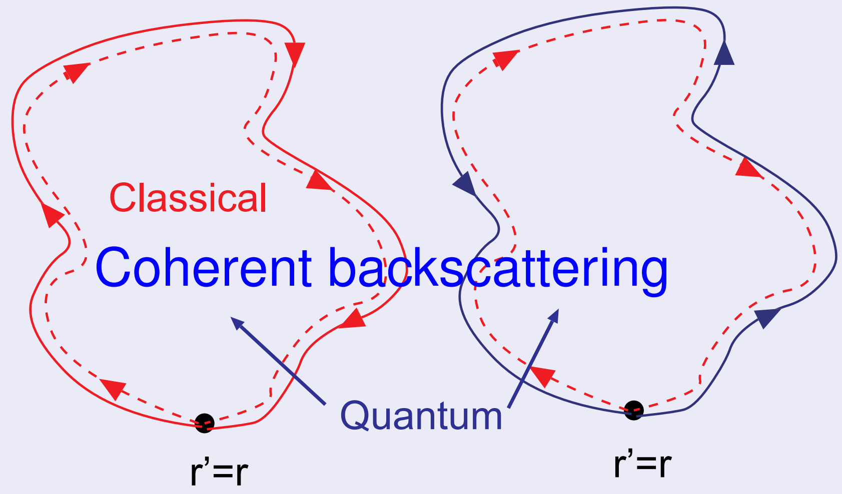
Characteristic length scales



Mesoscopic regime: $L_\phi \gg L \gg \ell_e$

$\ell_e/\lambda \gg 1 \equiv$ "weak disorder"

An interference effect



The Anderson *Metal*-*Insulator* Transition in 3D

Scale Invariance

Localization length $\ell \sim |K_c - K|^{-\nu}$

$D \sim |K - K_c|^s$ diffusion constant

$s = \nu$ **Universal**

Insulator $\equiv$ Localized

Metal $\equiv$ Diffusive transport

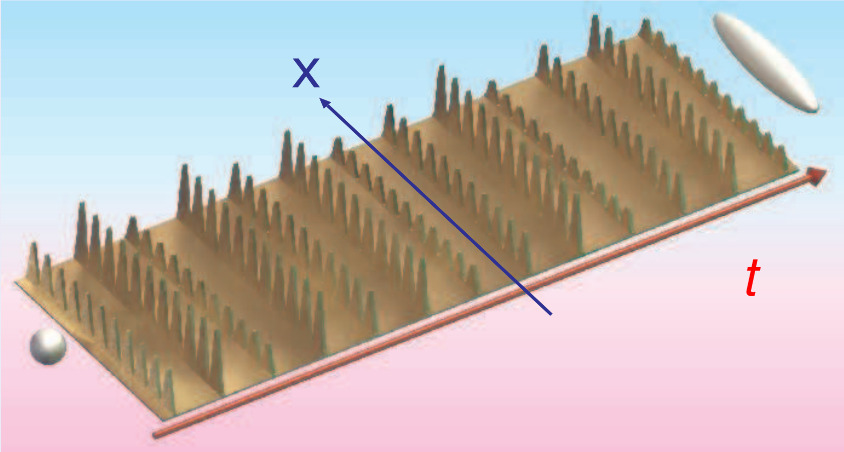
K_c

K control parameter (e.g. $= \ell_e/\lambda$)

The Kicked Rotor with cold atoms: a very practical tool for studying Anderson localization

The Kicked Rotor

The kicked rotor

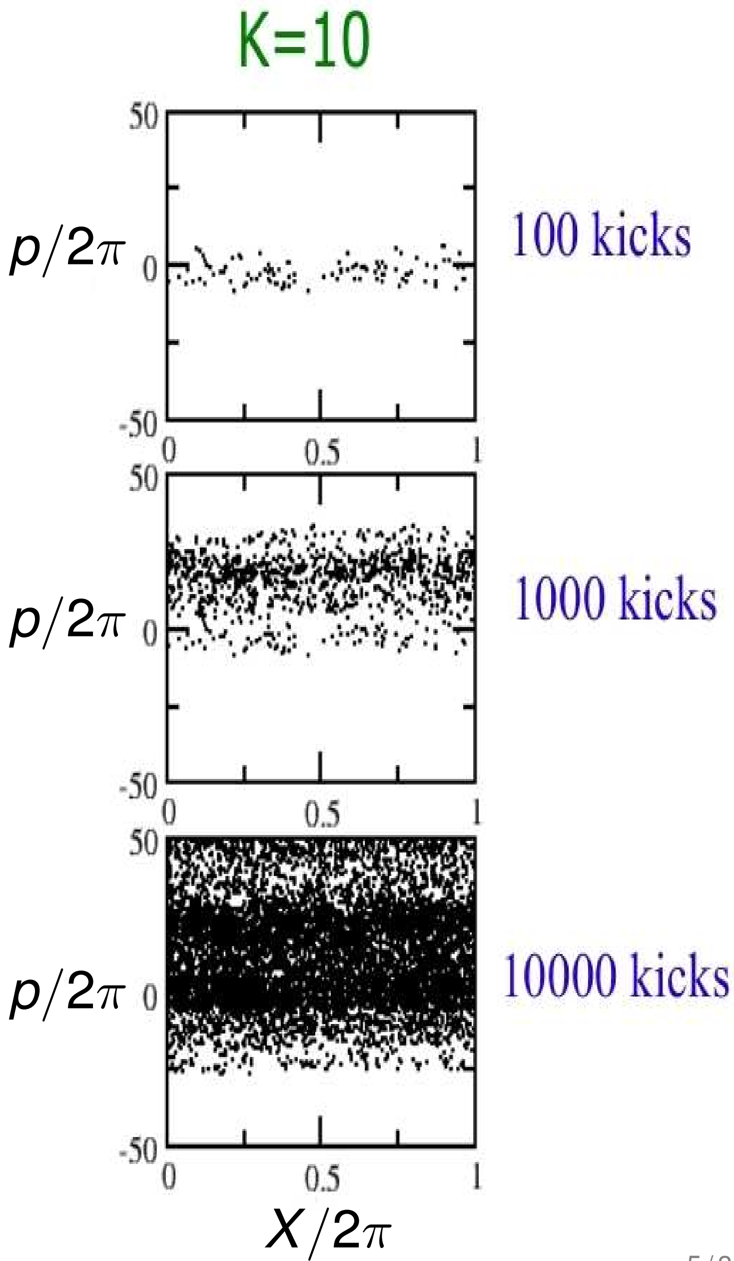


$$H = \frac{p^2}{2} + K \cos x \sum_n \delta(t-n)$$

Classical CHAOTIC DIFFUSION in momentum space, $K \gg 1$

$$p_{n+1} = p_n + K \sin \theta_n$$
$$\theta_{n+1} = \theta_n + p_{n+1}$$

- Looks like a random walk (although perfectly deterministic)
- On average, $\langle p^2 \rangle \sim Dt$



Quantum dynamics vs. Classical dynamics

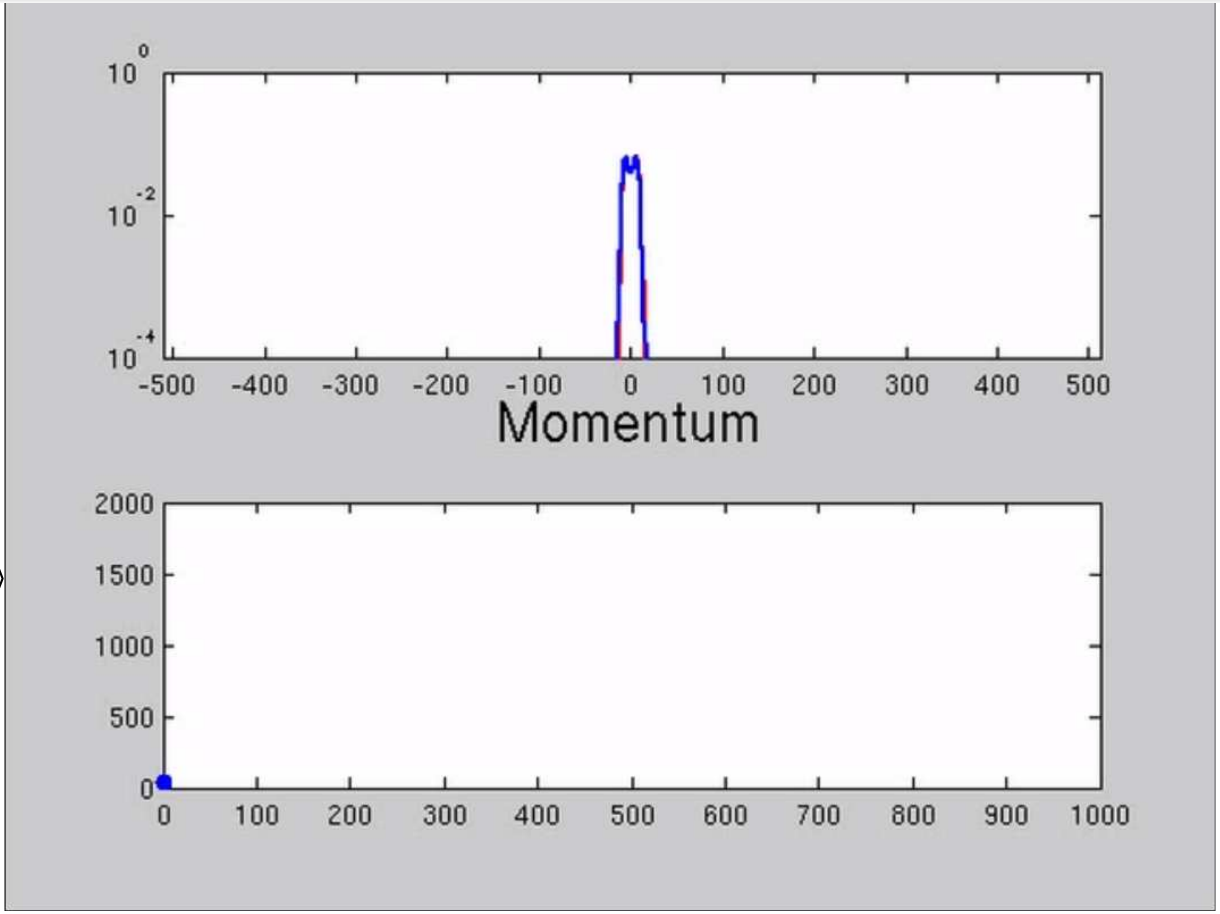
Interplay between chaos and interference effects

- Initially peaked state $\Rightarrow$ Chaotic diffusive expansion?
- $t > t_\ell$, dynamical localization [G. Casati et al. (1979)]
- $\equiv$ Anderson localization in 1D disordered systems [Fishman et al. (1982)]

$|\psi(p,t)|^2$

(log scale)

$\langle p^2 \rangle$



t

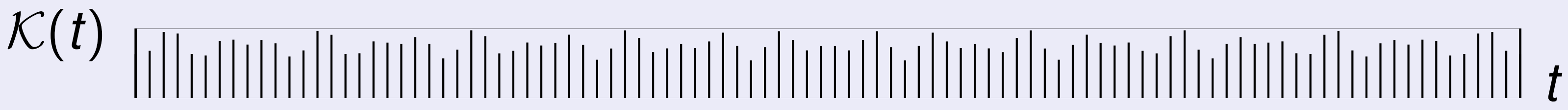
Quasi-periodicity and effective dimensionality

The quasiperiodic Kicked Rotor [Shepelyansky (1987)]

$$H_{\mathrm{qp}} = \frac{\hat{p}^2}{2} + \mathcal{K}(t)\cos\hat{\theta}\sum_n \delta(t-n)$$

- quasi-periodic modulation with two new frequencies:

$$\mathcal{K}(t) = K\left[1 + \varepsilon\cos(\omega_2 t + \varphi_2)\cos(\omega_3 t + \varphi_3)\right]$$

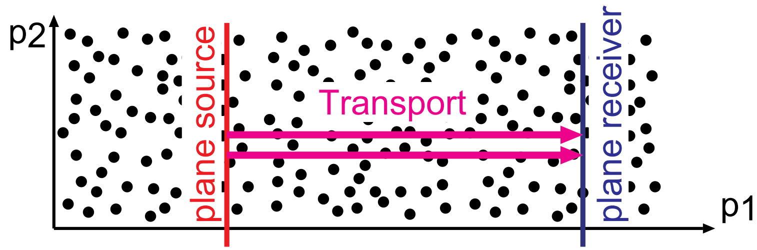


- dynamics **strictly identical** to that of a 3D Kicked "Rotor"

$$H_3 = \frac{{p_1}^2}{2} + \omega_2 p_2 + \omega_3 p_3 + K\cos\theta_1\left[1 + \varepsilon\cos\theta_2\cos\theta_3\right]\sum_n \delta(t-n)$$

with an initial condition taken as a plane source

$$\psi_3(\theta_1,\theta_2,\theta_3;t=0) = \psi_{\mathrm{qp}}(\theta_1,t=0)\delta(\theta_2-\varphi_2)\delta(\theta_3-\varphi_3)$$

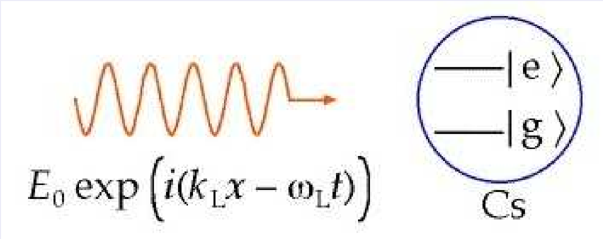


Experimental observation of the Anderson transition with atomic matter waves

Experimental realization with cold atoms [Moore et al. (1995)]

Quantum chaos group of PHLAM laboratory, Lille: JC Garreau, P Szriftgiser, J Chabé, H Lignier

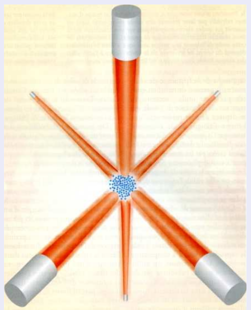
Atom-light interactions



$$\Delta_L = \omega_L - \omega_0$$

- Spontaneous emission
  dissipative, rate$\sim \Gamma\Omega^2/\Delta_L^2$
- Stimulated emission
  dipole potential,
  amplitude$\sim \Omega^2/\Delta_L$

I. Cooling and trapping



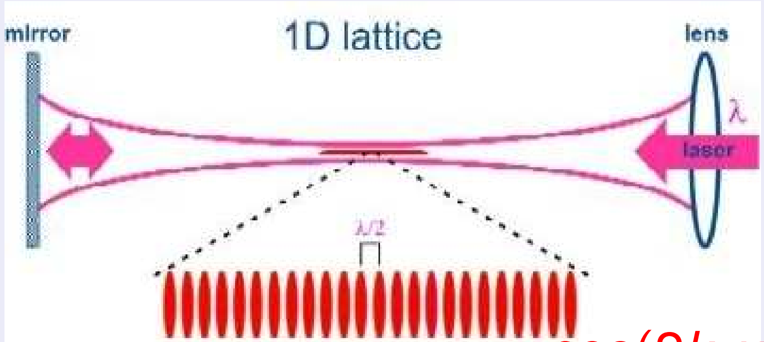
- MOT

$\Rightarrow$ narrow initial distribution

$\Rightarrow$ negligible interactions

II. Pulse sequence

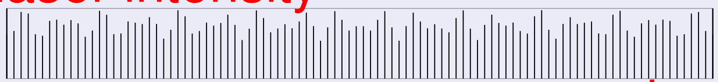
- Standing wave, $\Delta_L \gg \Gamma$



$\propto \cos(2k_L x)$

- Temporal forcing

laser intensity



time

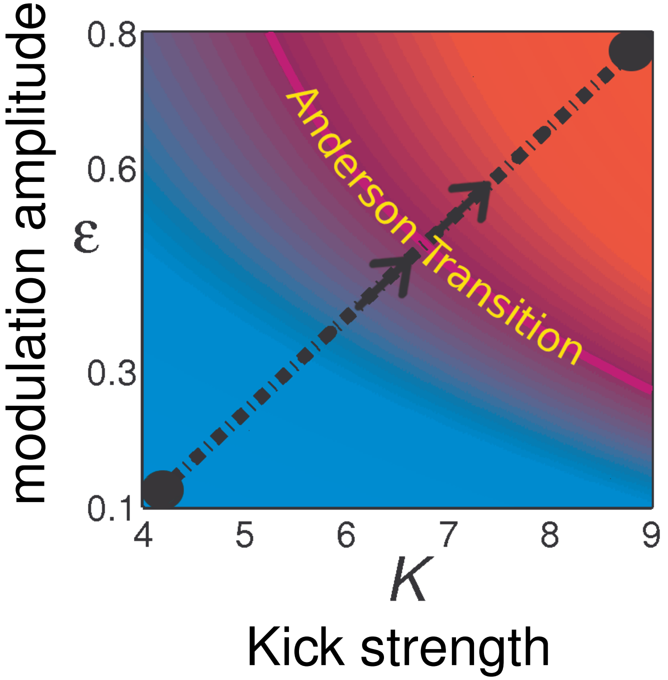
- Limitations

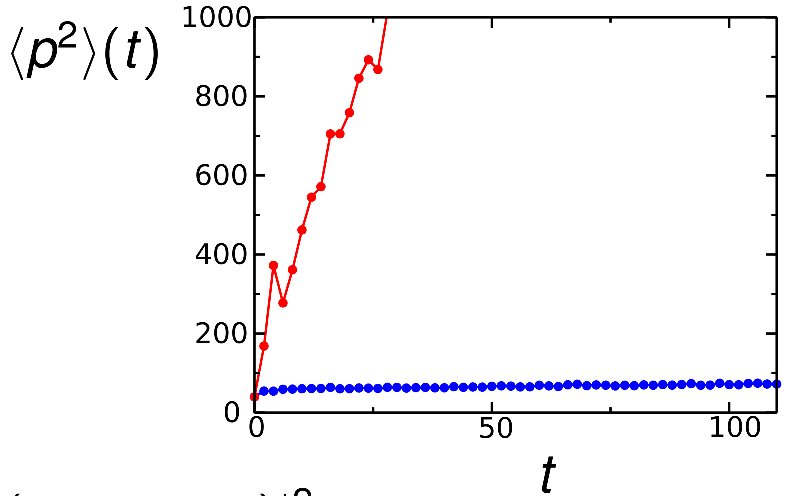
gravity, decoherence $\Rightarrow t \leq 160$

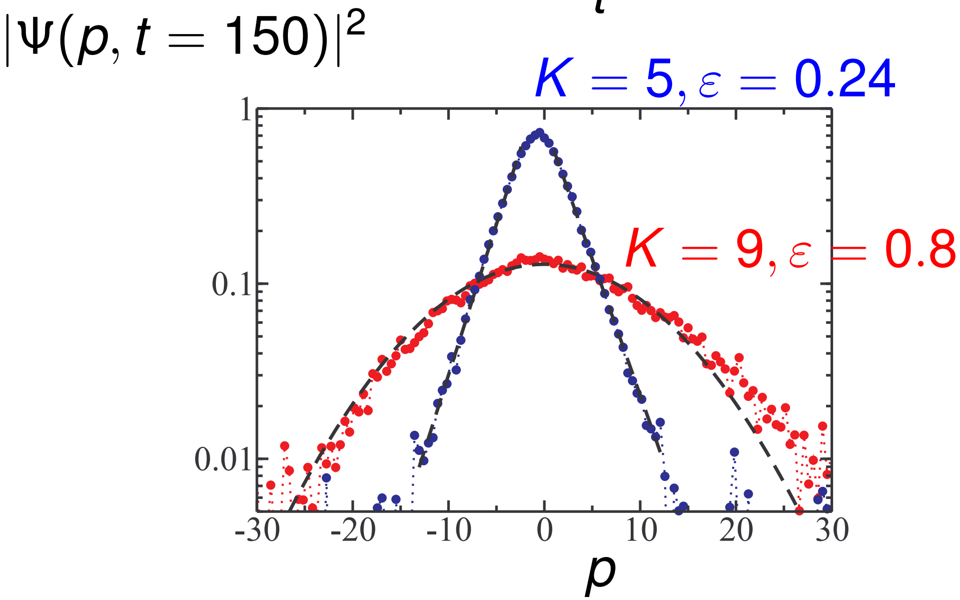
III. Watching the wave-function

Raman velocimetry $\Rightarrow |\Psi(p)|^2$

Experimental observation of localized/diffusive dynamics



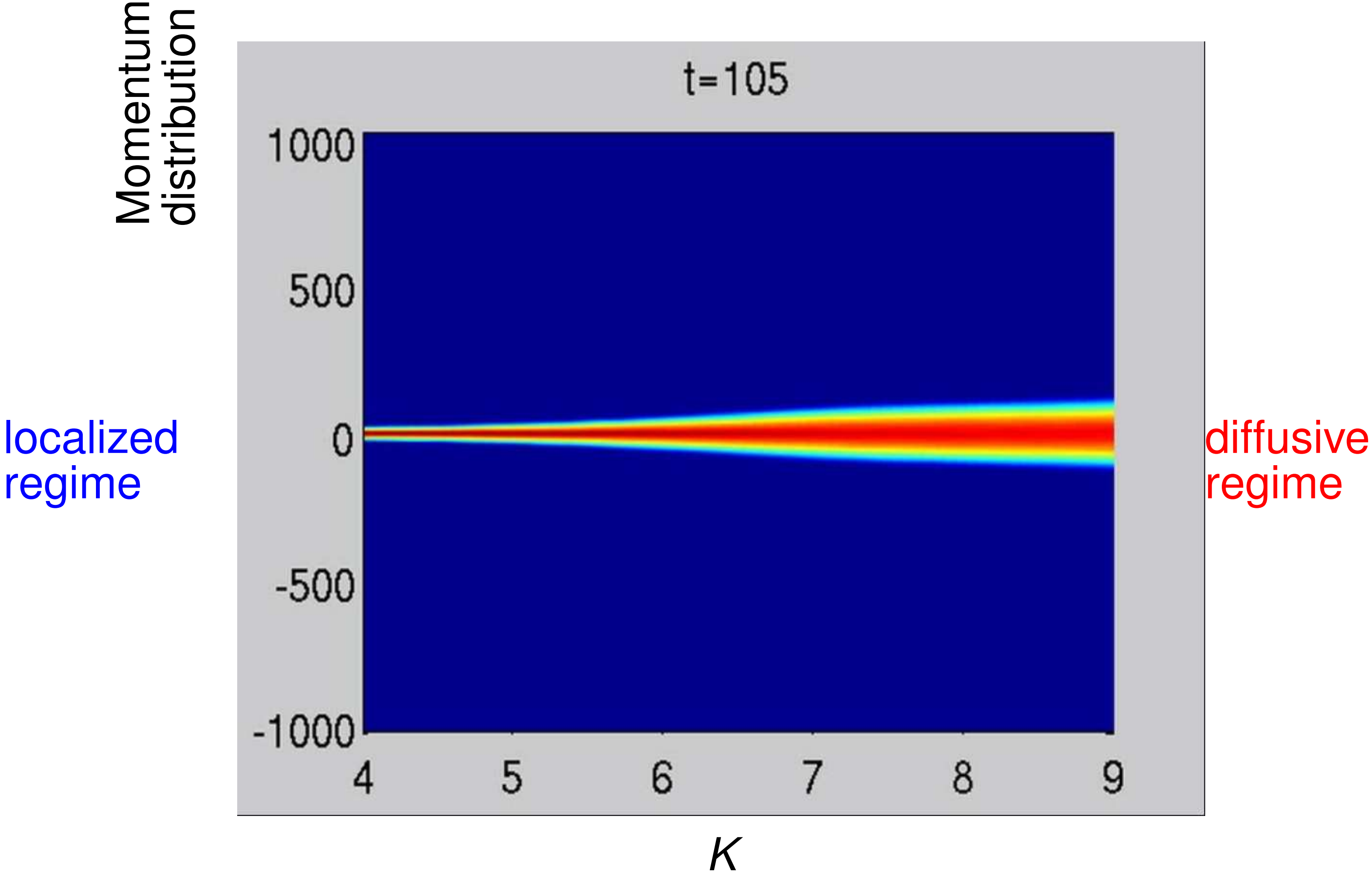




Finite-time limitations on a continuous transition

How to determine K_c at 150 kicks? seems easier at long times!

- for $t \ll t_\ell$ not yet localized ($\approx$ "not yet diffusive" distribution) but t_ℓ diverges at the transition



How to unambiguously identify the transition?

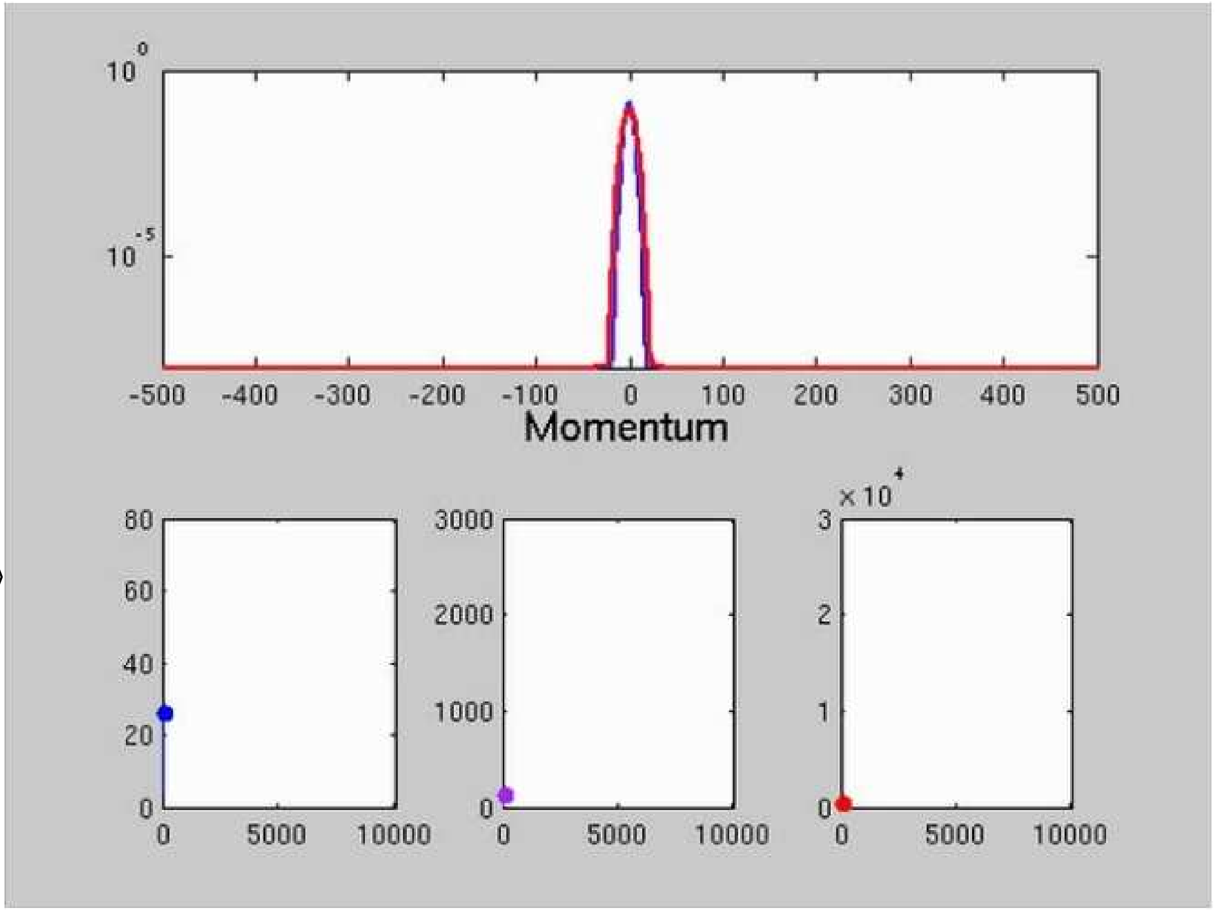
What characterizes the criticality?

- No caracteristic time $\Rightarrow$ algebraic dependence of $\langle p^2 \rangle \sim t^\gamma$

$|\Psi(p)|^2$

(log scale)

$\langle p^2 \rangle$



Time t (number of kicks)

K=9
diffusive
regime
$\langle p^2 \rangle \sim Dt$

$K = K_c$
critical
regime

K=4
localized
regime
$\langle p^2 \rangle \sim \ell^2$

Scaling law in time domain

Finite time effects

- Sharp transition only observable when $t \to \infty$
- At finite time, smooth crossover
- $\equiv$ finite-size effects? [Pichard et al. (1981); MacKinnon et al. (1981)]
- Renormalization flow in time domain?

One parameter scaling hypothesis

$$\langle p^2 \rangle \sim t^{k_1} F\left[(K - K_c)\, t^{k_2}\right]$$

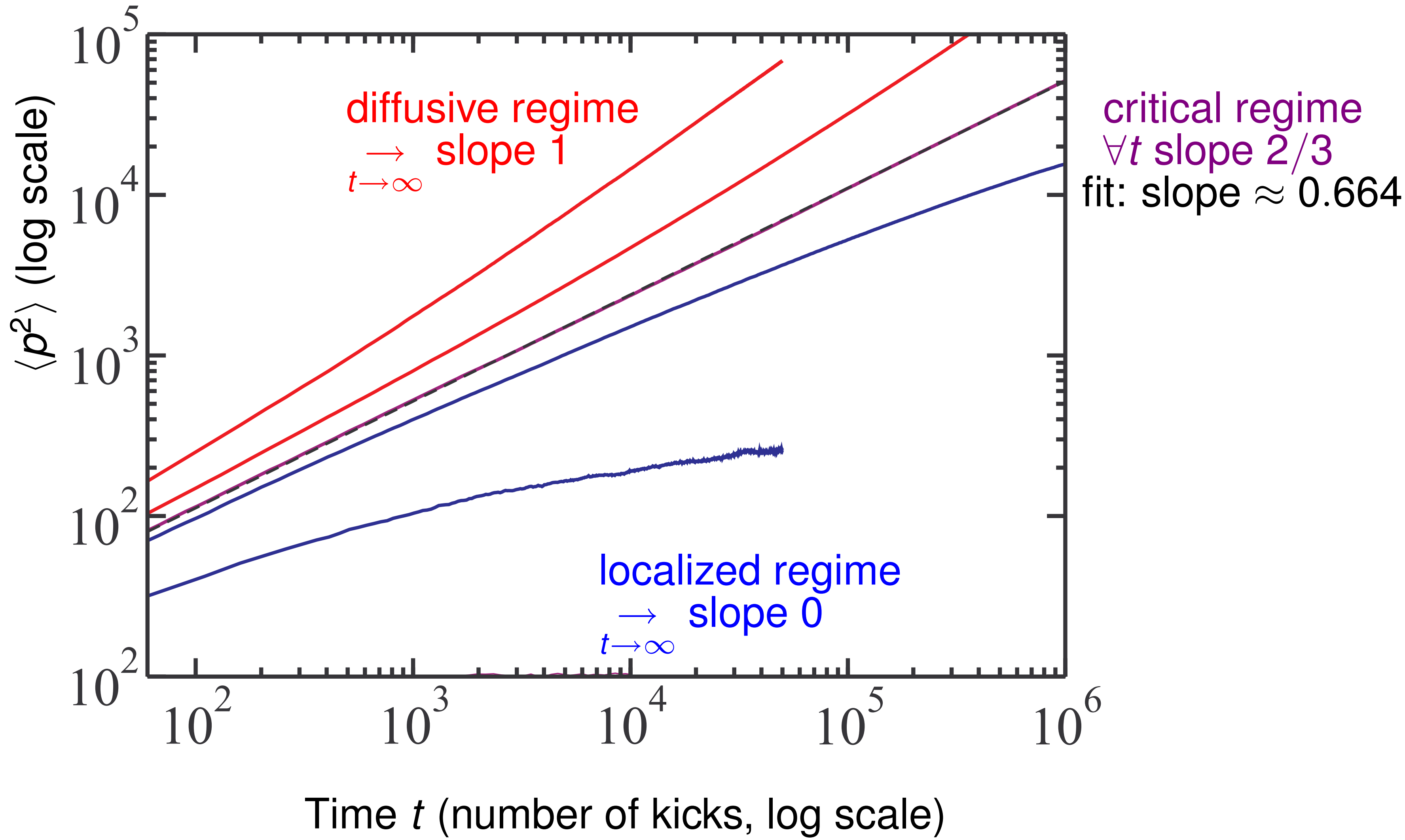
Asymptotic behaviours ($t \to \infty$):

- Localized, $K \lesssim K_c$: $\langle p^2 \rangle \sim \ell^2 \sim |K_c - K|^{-2\nu}$
- Diffusive, $K \gtrsim K_c$: $\langle p^2 \rangle \sim Dt \sim |K - K_c|^{\nu} t$

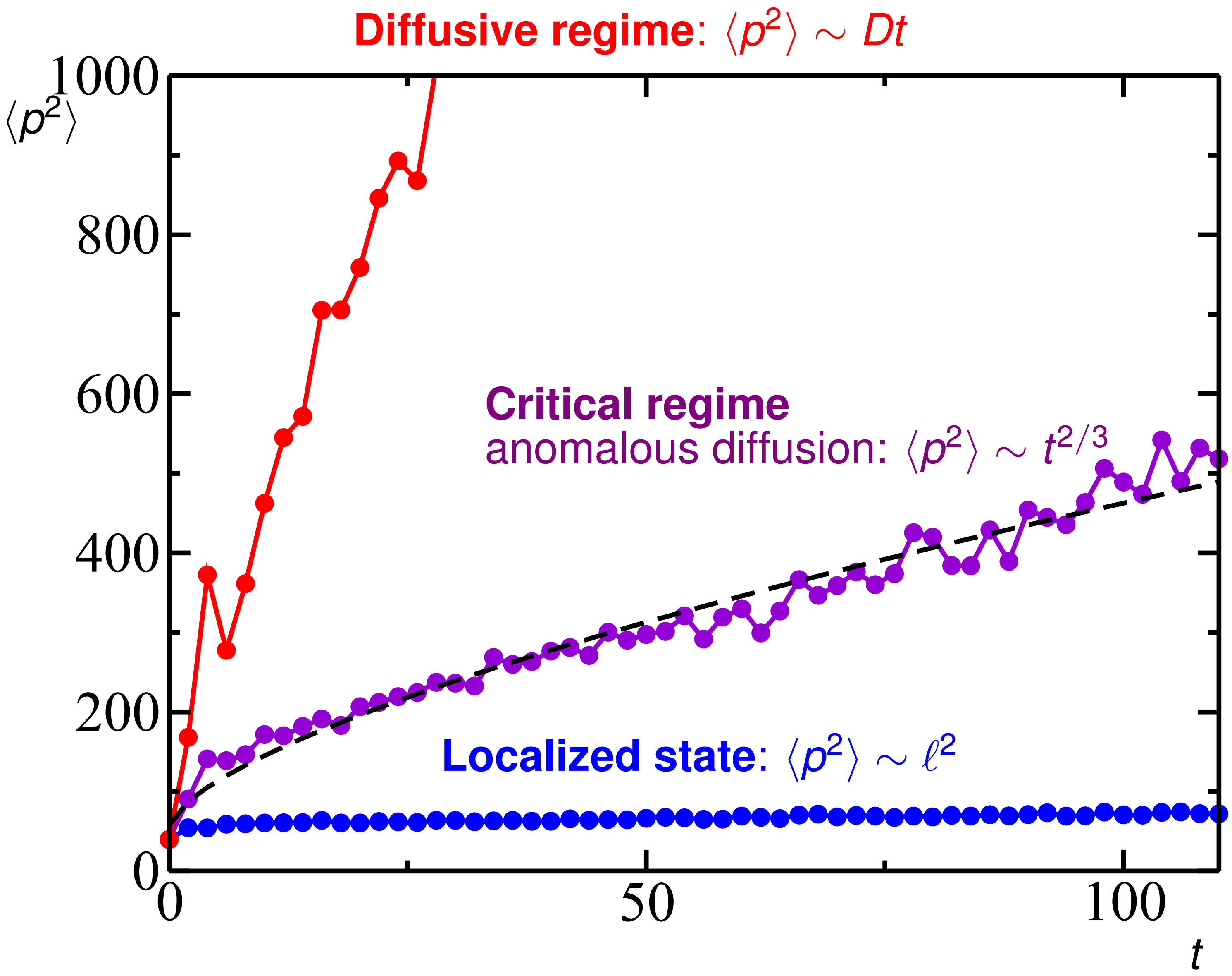
$$\langle p^2 \rangle \sim t^{2/3} F\left[(K - K_c)\, t^{1/3\nu}\right]$$

Critical anomalous diffusion

- $\langle p^2 \rangle \sim t^{2/3} F\left[(K - K_c)\, t^{1/3\nu}\right] \Rightarrow \langle p^2 \rangle \sim t^{2/3}$ at $K = K_c$



Critical anomalous diffusion: experimental observation



How to verify $\langle p^2 \rangle \sim t^{2/3} F\left[(K - K_c)\, t^{1/3\nu}\right]$?

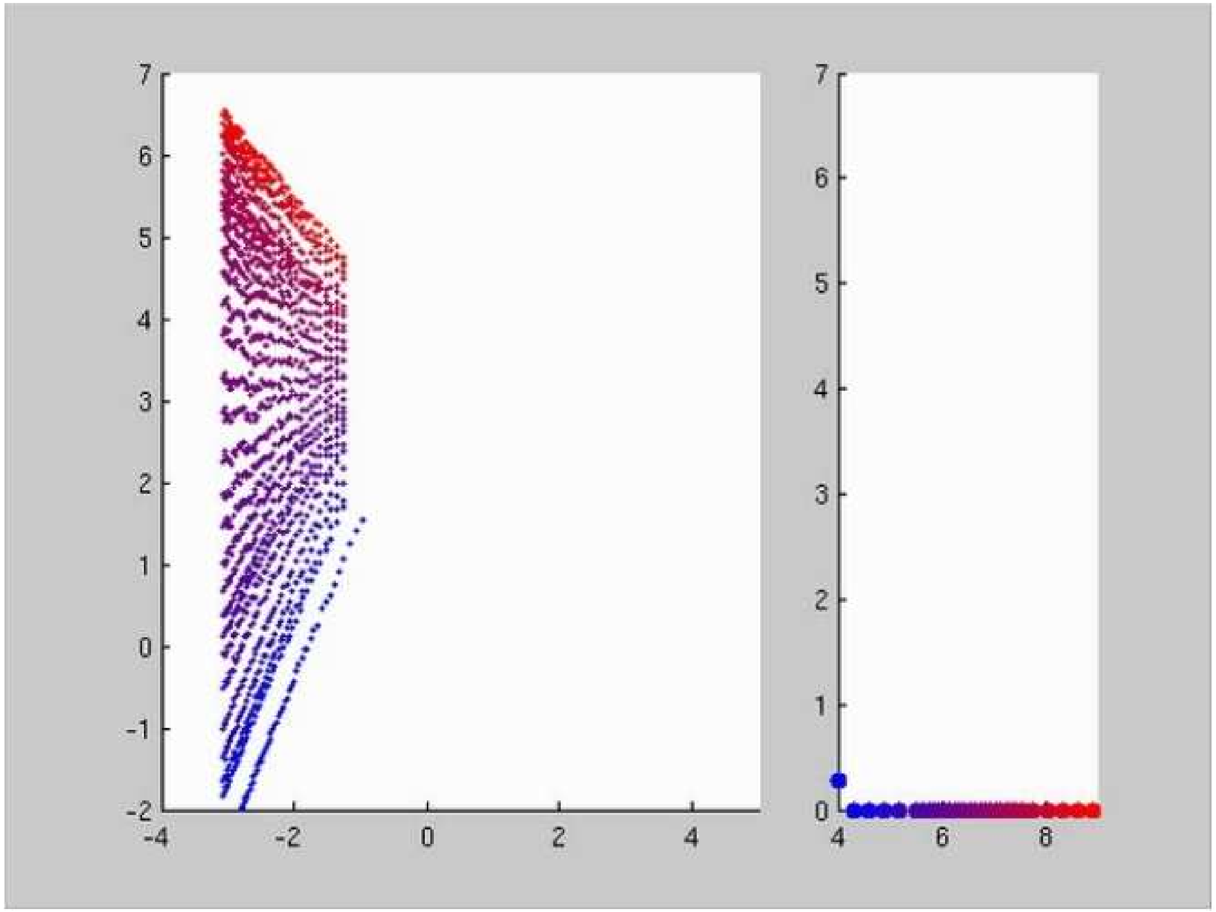
- Existence of $\xi(K)$ and $\mathcal{F}$ such that $\Lambda = \frac{\langle p^2 \rangle}{t^{2/3}} = \mathcal{F}\left[\frac{\xi(K)}{t^{1/3}}\right]$?

$\ln \Lambda = \ln \frac{\langle p^2 \rangle}{t^{2/3}}$

diffusive

critical

localized



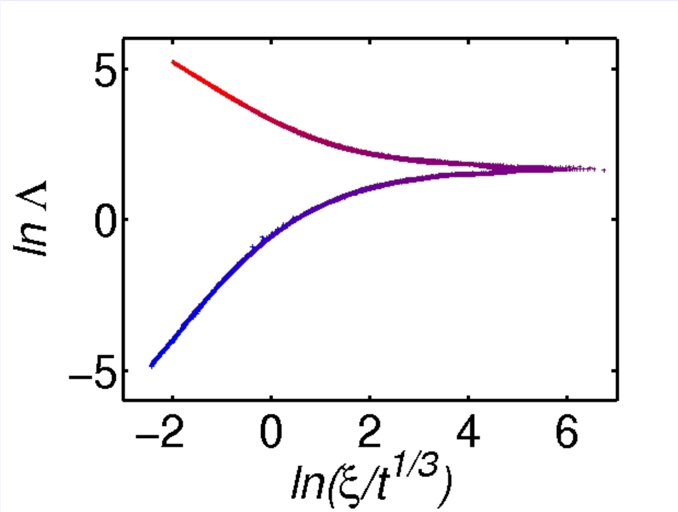
$\ln \xi(K)$

$\ln(1/t^{1/3})$ $\ln\left(\xi(K)/t^{1/3}\right)$ K

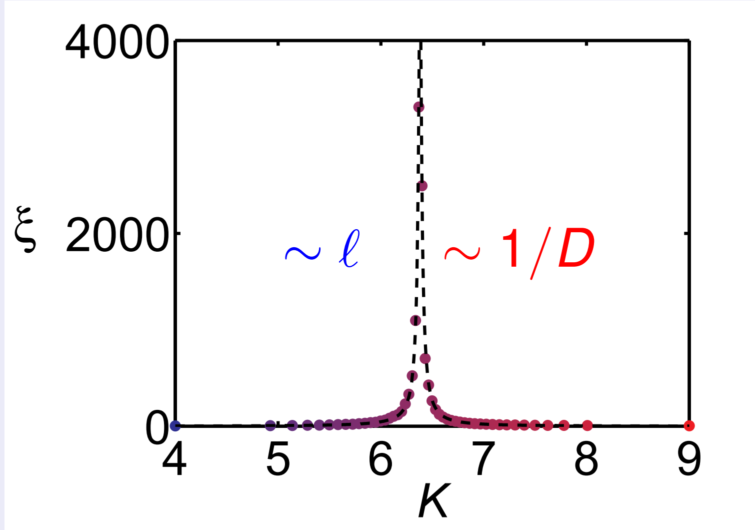
Finite-time scaling analysis of numerical results

Scaling function $\mathcal{F}$

$$\Lambda(K,t) = \frac{\langle p^2 \rangle}{t^{2/3}} \sim \mathcal{F}\left[\frac{\xi(K)}{t^{1/3}}\right]$$



Scaling parameter ξ



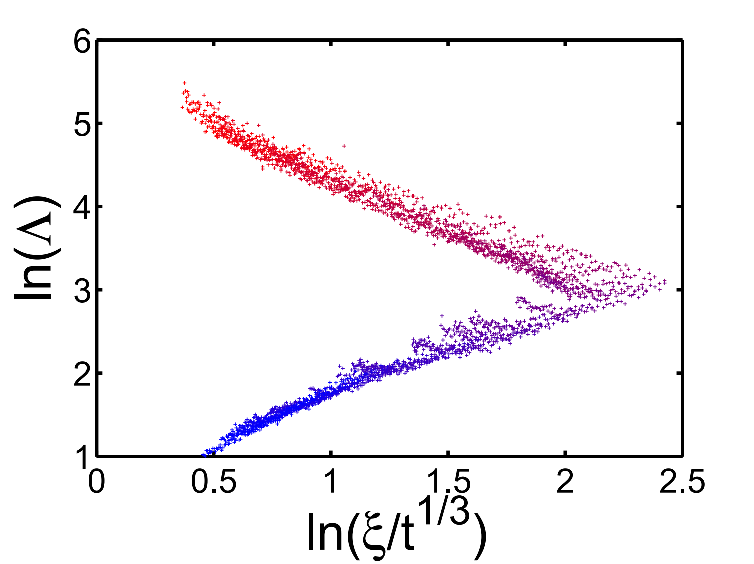
- well fitted by: $\xi \sim |K - K_c|^{-\nu}$
- Critical point: $K_c \simeq 6.4$
- Critical exponent: $\nu \simeq 1.6 \pm 0.05$

in the same Universality Class as for the 3D Anderson model

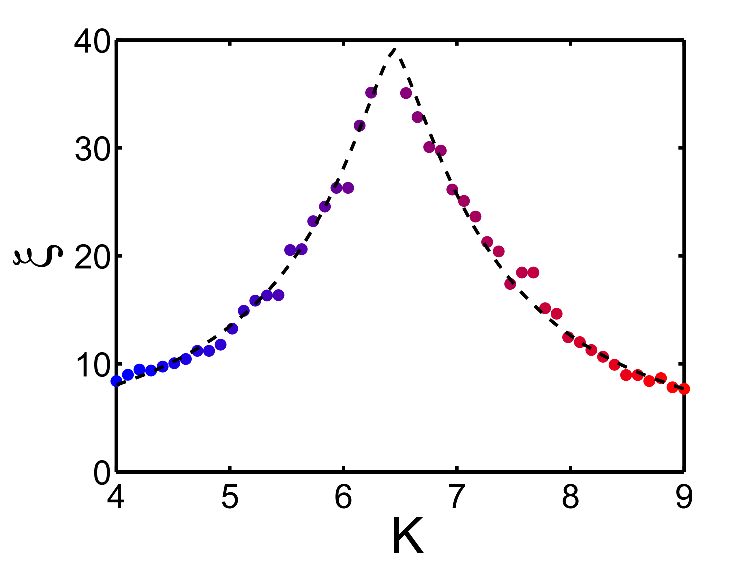
- more refined analysis: $\nu = 1.59 \pm 0.01$ [Lemarié et al., *EPL* **87**, 37007 (2009)]
- Orthogonal: 1.57 ± 0.02 [Slevin et al. (1997)]

Finite-time scaling analysis of experimental results

Scaling function $\mathcal{F}$



Scaling parameter ξ



Experimental determination of the critical exponent ν

- $1/\xi = \alpha|K - K_c|^\nu + \beta$
- β accounts for experimental imperfections
- Critical point: $K_c \simeq 6.4$
- Critical exponent: $\nu \simeq 1.4 \pm 0.3$
- Excellent agreement with numerics (no adjustable parameter)

Critical State of the Anderson Transition: Between a Metal and an insulator

Critical state of the Anderson transition?

Scale invariance? Theoretical description?

At long times, long distance: $P(0, p; t) = |\Psi(p, t)|^2 \sim ?$

???

localized phase	diffusive phase
$t \gg t_\ell$	$t \gg t_D$
$\sim \exp\left[\frac{-2\|p\|}{\ell}\right]$	$\sim \exp\left[-\frac{p^2}{4Dt}\right]$
ℓ localization length	D diffusion coefficient
$\ell \sim (K_c - K)^{-\nu}$	$D \sim (K - K_c)^s$

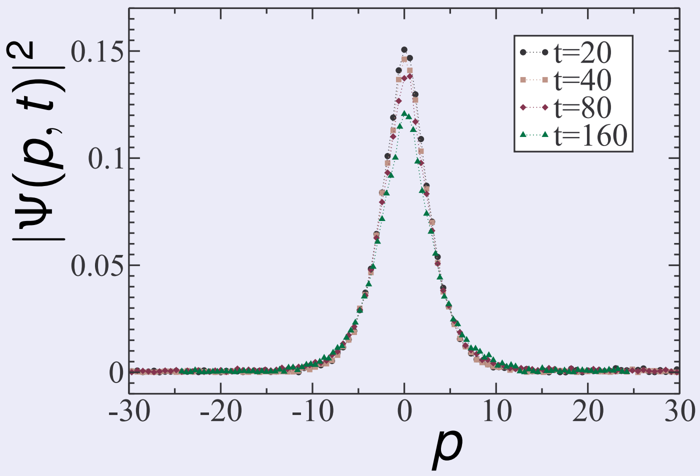
K_c

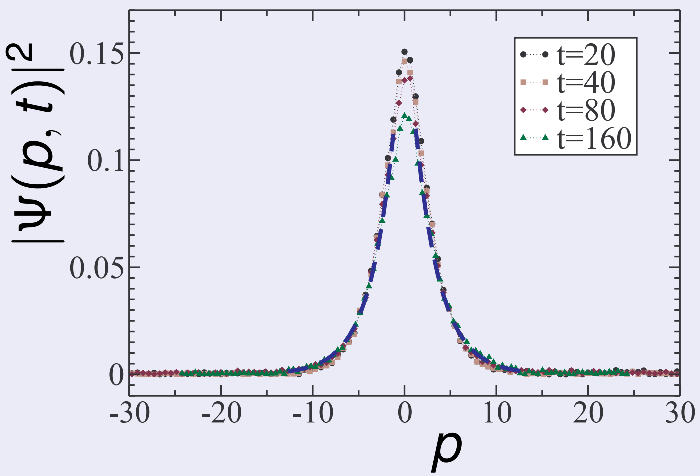
K stochasticity parameter

Standard rescalings in the localized/diffusive regimes

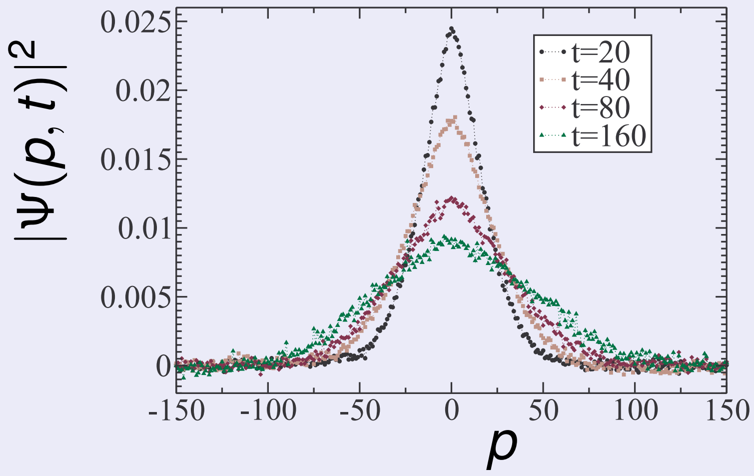
Experimental results

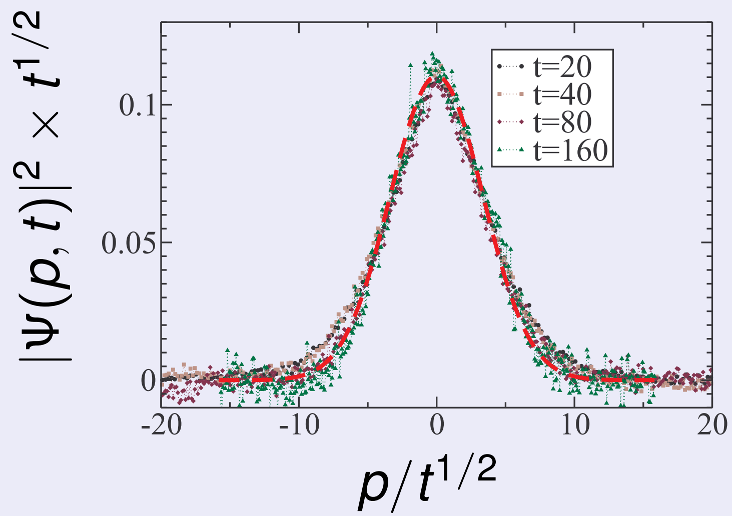
Localized regime, $t \gg t_\ell$:
$|\Psi(p,t)|^2 \sim \exp[-2|p|/\ell]$





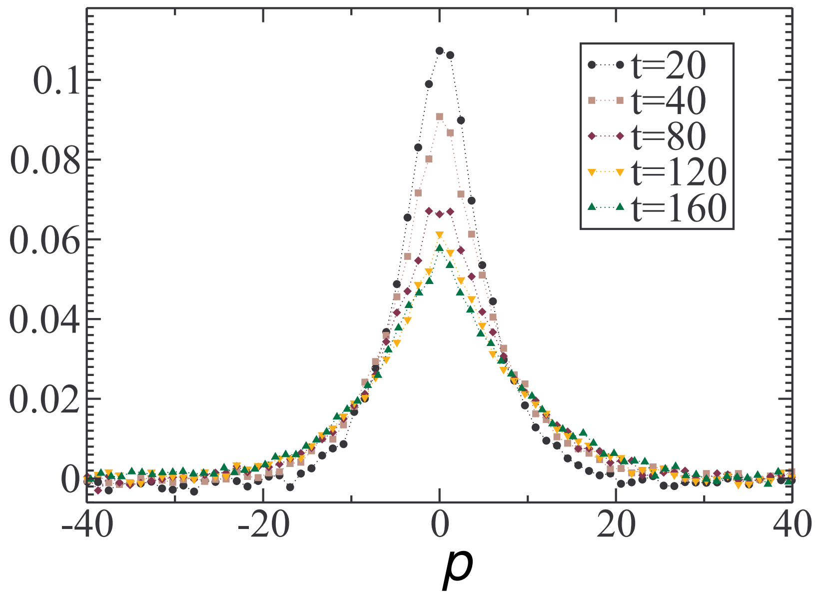
Diffusive regime, $t \gg t_D$:
$|\Psi(p,t)|^2 \sim \exp[-p^2/4Dt]$

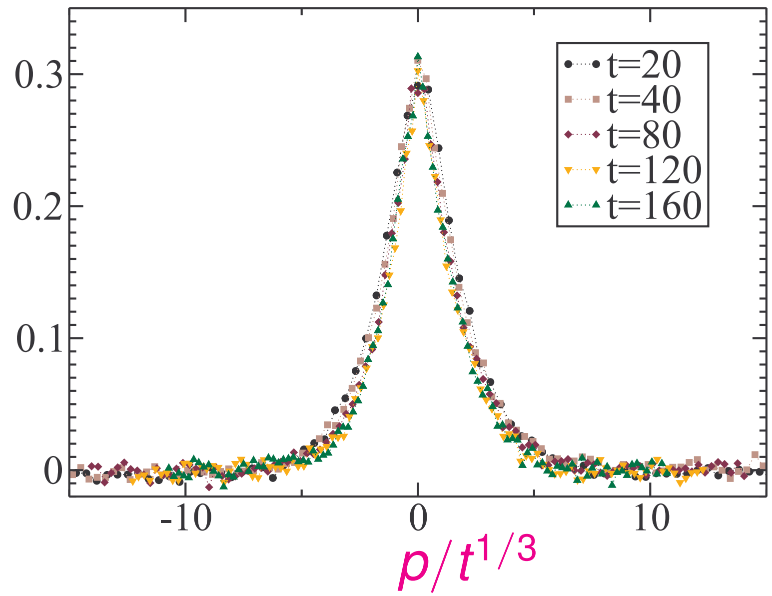




Scale invariance of the critical state
Experimental results

$|\Psi(p,t)|^2$ $\qquad$ $|\Psi(p,t)|^2 \times t^{1/3}$

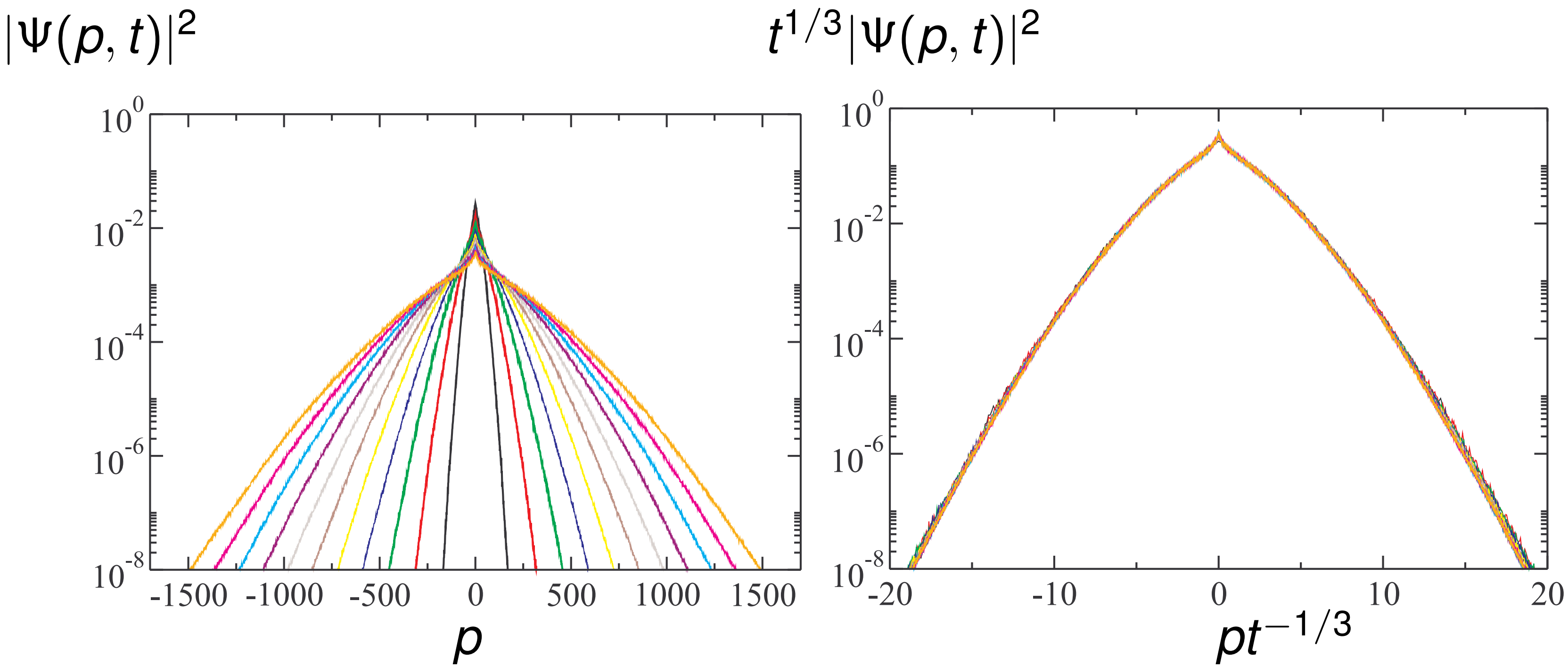




Direct observation of the scale invariance at the threshold

Rescaling of all critical wave functions for t from $t = 20$ to $t = 160$!

Scale invariance of the critical state
Numerical results



Rescaling of all critical wave functions for t from $t = 10^3$ to $t = 10^6$!

Theoretical description of $P(0, p; t)$?

Diffusive transport

- In Fourier space: $P(\boldsymbol{q}, \omega) = \dfrac{1}{-i\omega + Dq^2}$
- Interference effects $\Rightarrow D < D_{\text{cl}}$

Localization

- Generalization: $D \Rightarrow D(\omega)$
- Localized state: $D(\omega) \underset{\omega \to 0}{\sim} -i\omega\ell^2$
- $\dfrac{1}{-i\omega(1 + \ell^2 q^2)} \Rightarrow$ Fourier transform $\Rightarrow \sim \exp[-2|p|/\ell]$

$D(\omega) = ?$

- Perturbative theory to describe weak localization (low disorder $K/\hbar \gg 1$ and not too long times).
- Strong localization $\Rightarrow$ self-consistent theory

Predictions of the self-consistent theory

Self-consistent theory for the quasiperiodic Kicked Rotor

$$D(\omega) = D - 2D(\omega) \int \frac{\mathrm{d}^3\boldsymbol{q}}{(2\pi)^3} \frac{1}{-i\omega + D(\omega)q^2}$$

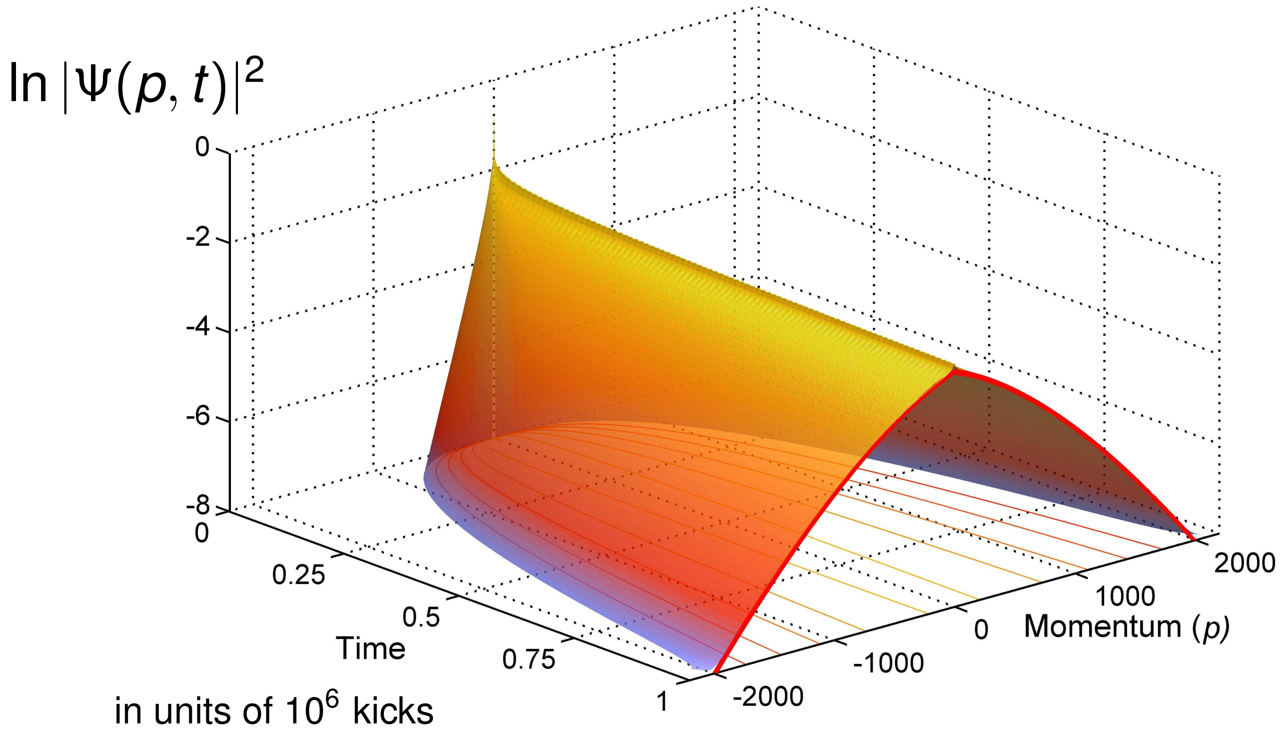
- Anderson transition with $\nu = 1$ (fluctuations are not taken into account by this mean-field theory)
- At criticality: $D(\omega) \underset{\omega\to 0}{\sim} \omega^{1/3}$, thus $\langle p^2 \rangle \sim t^{2/3}$
- Initial condition = plane source = $1_{q_1} \times \delta(q_2) \times \delta(q_3)$

$$|\psi(p,t)|^2 = \int \frac{\mathrm{d}\omega}{2\pi} e^{-i\omega t} \int \frac{\mathrm{d}^3\boldsymbol{q}}{(2\pi)^3} e^{i\boldsymbol{q}\cdot\boldsymbol{p}} \frac{\delta(q_2)\delta(q_3)}{-i\omega + D(\omega)q^2}$$

Analytic prediction for the critical state

$$|\psi(p,t)|^2 = \tfrac{3}{2} \left(3\rho^{3/2} t\right)^{-1/3} \mathrm{Ai}\left[\left(3\rho^{3/2} t\right)^{-1/3} |p|\right]$$

Confrontation with numerics

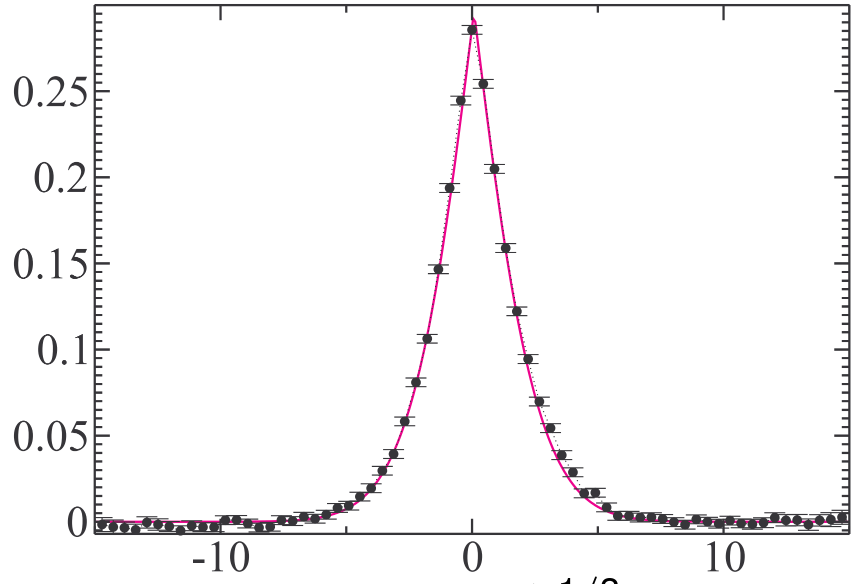


$$|\psi(p,t)|^2 = \frac{3}{2}\left(3\rho^{3/2}t\right)^{-1/3} \mathrm{Ai}\left[\left(3\rho^{3/2}t\right)^{-1/3}|p|\right]?$$

- YES! No adjustable parameter: $\rho = \dfrac{\Gamma(2/3)}{3}\Lambda_c$
- deviations at $p \approx 0 \Rightarrow$ multifractality "$\equiv$ fluctuations" (not taken into account in the self-consistent theory)

Confrontation with the experimental results

$\left\langle |\Psi(p,t)|^2 \times t^{1/3} \right\rangle_t$



$p/t^{1/3}$

$|\psi(p,t)|^2 = \frac{3}{2}\left(3\rho^{3/2}t\right)^{-1/3} \mathrm{Ai}\left[\left(3\rho^{3/2}t\right)^{-1/3}|p|\right]$?

- YES!
- No deviations observed for $t \leq 160$ kicks

Between a metal and an insulator: the critical state of the Anderson transition

At long times, long distance: $P(0, p; t) = |\Psi(p, t)|^2 \sim ?$

No characteristic time

Scale invariance

$$\sim \exp\left[-\frac{\alpha |p|^{3/2}}{t^{1/2}}\right]$$

α critical coefficient

localized phase	diffusive phase
$t \gg t_\ell$	$t \gg t_D$
$\sim \exp\left[\frac{-2|p|}{\ell}\right]$	$\sim \exp\left[-\frac{p^2}{4Dt}\right]$
ℓ localization length	D diffusion coefficient
$\ell \sim (K_c - K)^{-\nu}$	$D \sim (K - K_c)^s$

K_c — K stochasticity parameter

Conclusion

Experimental observation of the Anderson transition with atomic matter waves [*PRL* **101**, 255702 (2008); *PRA* **80**, 043626 (2009); Images de la physique 2009]

- Experimental determination of the critical exponent $\nu \simeq 1.4 \pm 0.3 \simeq \nu_{\text{Anderson}} = 1.57 \pm 0.02$
  New data: $\nu \simeq 1.5 \pm 0.2$

Universality of the Anderson transition with the quasiperiodic Kicked Rotor [*EPL* **87**, 37007 (2009)]

- The quasiperiodic Kicked Rotor belongs to the same universality class as for the Anderson model

Critical State of the Anderson Transition: Between a Metal and an Insulator [*PRL* **105**, 090601 (2010)]

- Direct observation of the scale invariance at the threshold
- Analytical prediction from the self-consistent theory

Perspectives

The Anderson transition in 4D and 2D

- Quasi-periodic modulation with 3 frequencies or 1 frequency
- Critical exponent in 4D? Preliminary result: $\nu \approx 1.2$. Critical dimension?

Symmetries

- Possibility to break the Time-Reversal Symmetry! $\equiv$ effective magnetic field
- $\Rightarrow$ Anderson localization and transition in the Unitary class

Interactions

- Kicked Rotor with BEC
- Interactions controlled by Feshbach resonances