The Abdus Salam International Centre for Theoretical Physics

2162-14

Advanced Workshop on Anderson Localization, Nonlinearity and Turbulence: a Cross-Fertilization

23 August - 3 September, 2010

Co-localization of Indistinguishable Photons in Disordered Waveguide Arrays (Photons in Lattices: Localization and Quantum Correlations in Waveguide Arrays)

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Photons in Lattices:

Localization and Quantum Correlations in Waveguide Arrays

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ICTP Trieste, August 2010

Thanks...

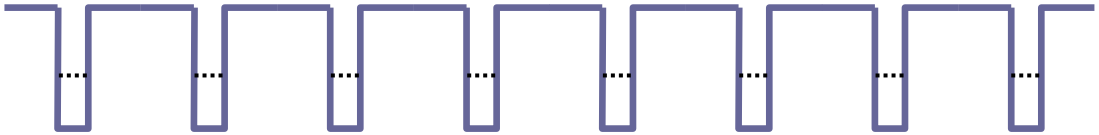
Outline

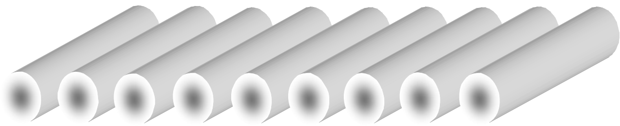
- AL in a photonic lattice
- Quantum correlations in a lattice
- Quantum correlations & disorder

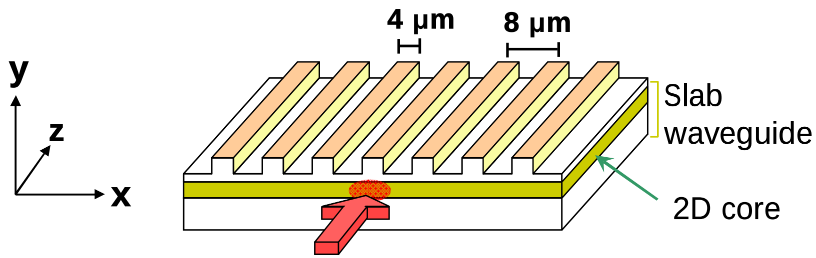
The 1d waveguide lattice

- The Tight Binding Model (Discrete Schrödinger Equation)

$$-i\frac{\partial\psi_n}{\partial t} = E\psi_n + T\left[\psi_{n+1} + \psi_{n-1}\right]$$



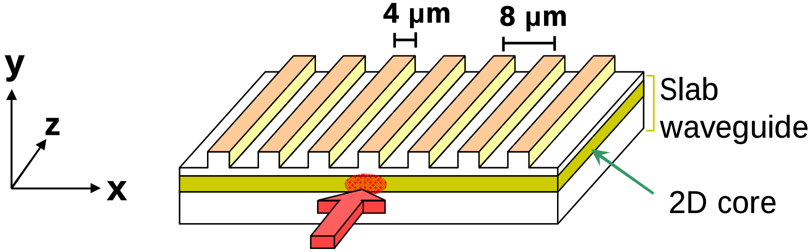




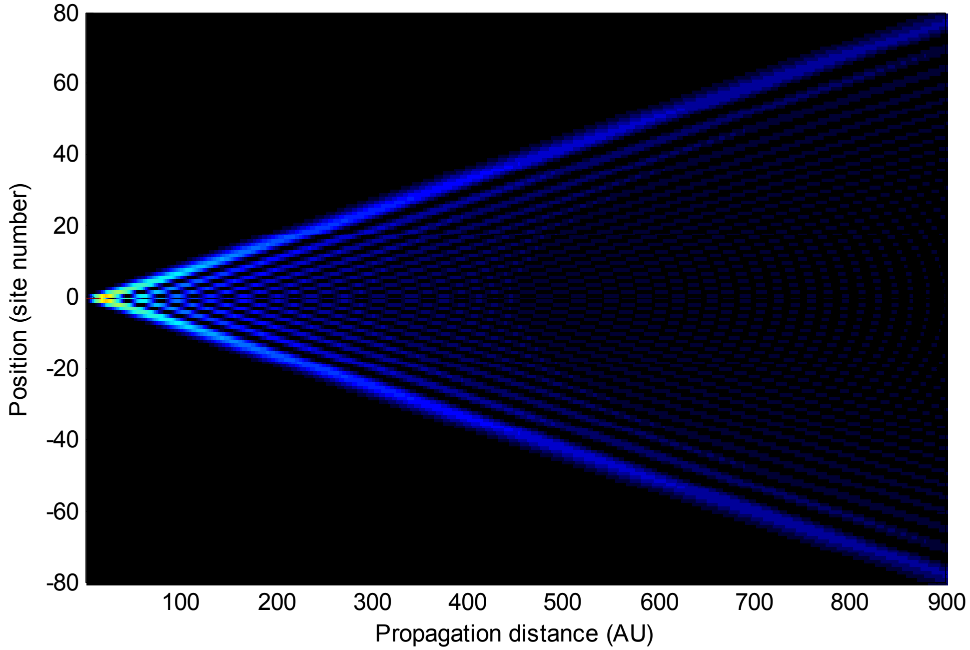
- The discrete nonlinear Schrödinger equation (DNLSE)

$$i\frac{\partial U_n}{\partial z} = \beta U_n + C\left[U_{n+1} + U_{n-1}\right] + \gamma\left|U_n\right|^2 U_n$$

Discrete Diffraction



$$i\frac{\partial U_n}{\partial z} = \beta U_n + C\left[U_{n+1} + U_{n-1}\right]$$

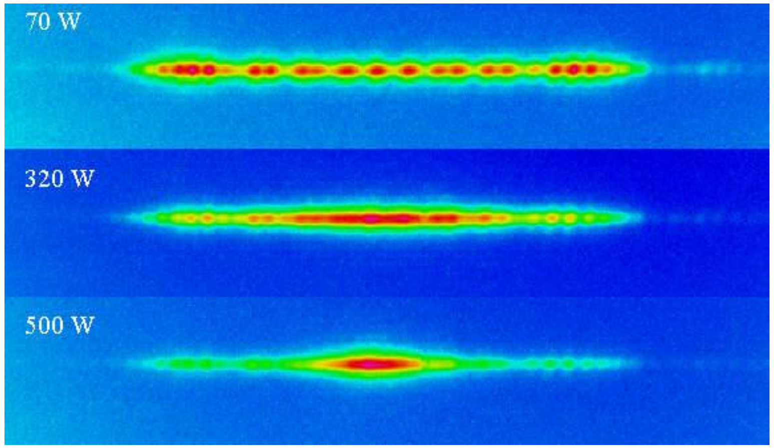


$$a_n(z) = i^n J_n(2Cz)a_0$$

Nonlinear localization in a periodic lattice

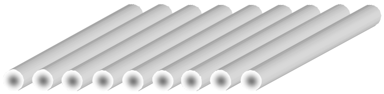
$$i\frac{\partial U_n}{\partial z} = \beta U_n + C\left[U_{n+1} + U_{n-1}\right] + \gamma\left|U_n\right|^2 U_n$$

Solitons of the discrete nonlinear Schrödinger equation (DNLSE)

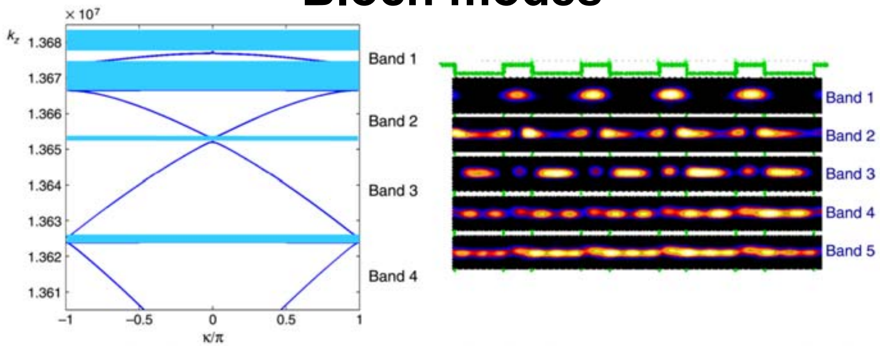


Eisenberg *et al*, 1998

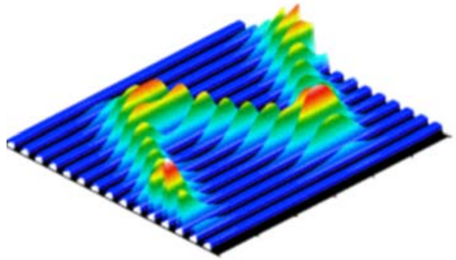
Optical analogues of quantum effects



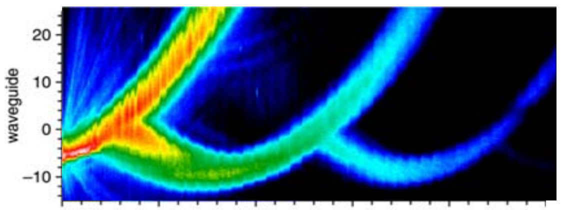
Bloch modes



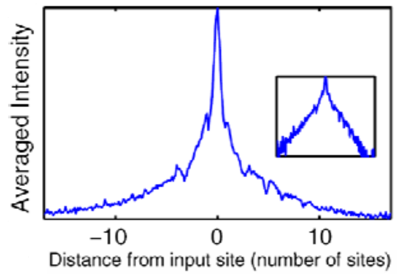
Bloch oscillations

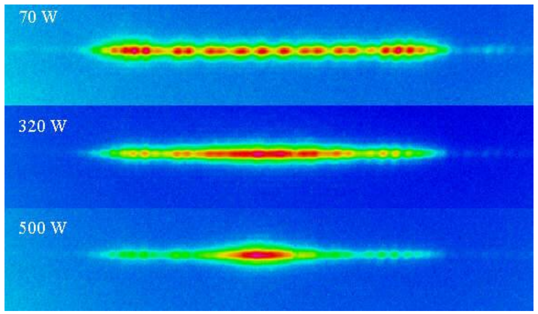


Zenner tunneling



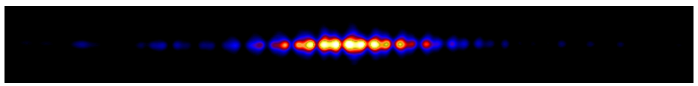
Anderson localization





Gap Solitons

Schwartz et. al. Nature 446 53, (2007); Lahini et. al. Phys. Rev. Lett. 100, 013906 (2008).

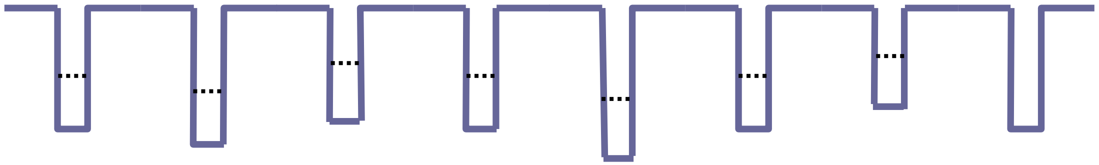


Lederer *et al.*, Phys. Rep. **463** (2008)
Christodoulides *et. al.*, *Nature*, **424** (2003)

The Disordered Lattice

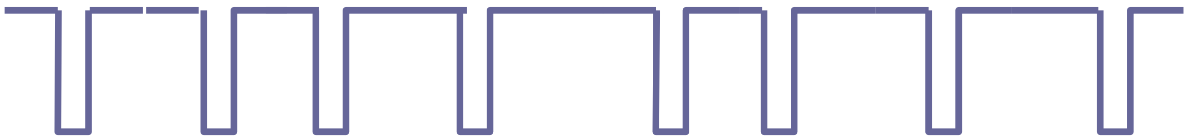
- Diagonal Disorder (Anderson, 1958)

$$-i\frac{\partial \psi_n}{\partial t} = E_n \psi_n + T\left[\psi_{n+1} + \psi_{n-1}\right]$$

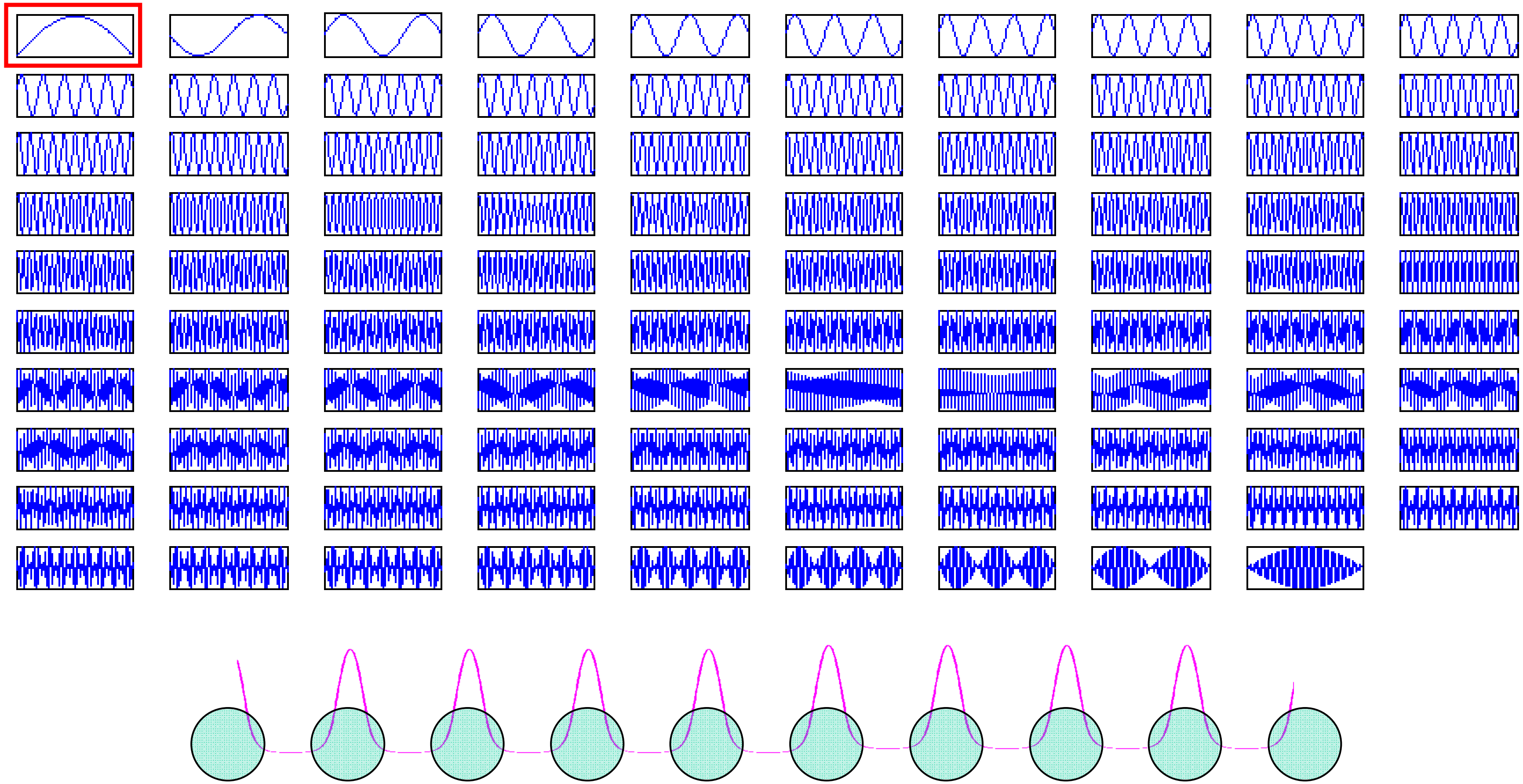


- Off-Diagonal Disorder

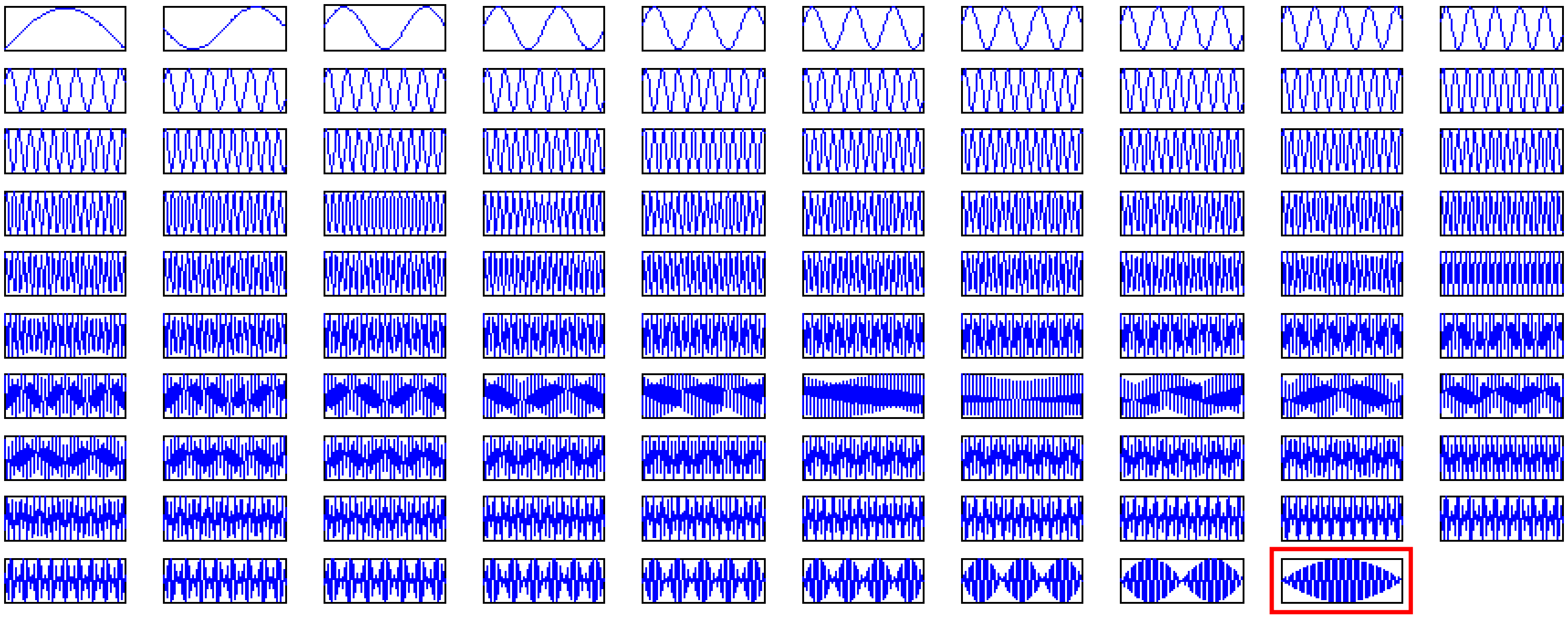
$$-i\frac{\partial \psi_n}{\partial t} = E \psi_n + T_{n,n\pm 1}\left[\psi_{n+1} + \psi_{n-1}\right]$$

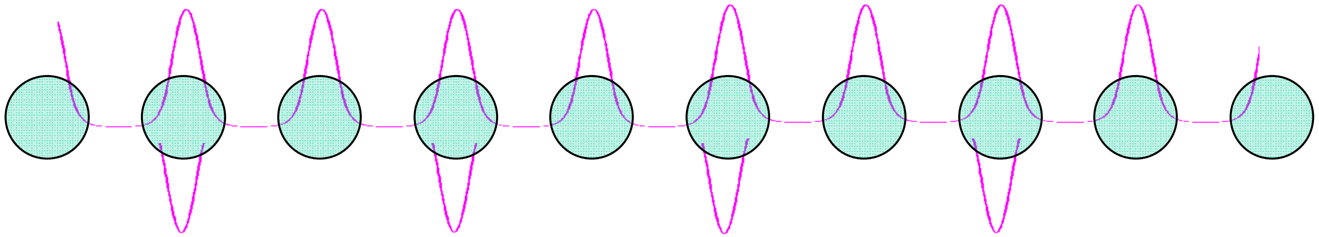


Eigenmodes of a periodic lattice,   N=99

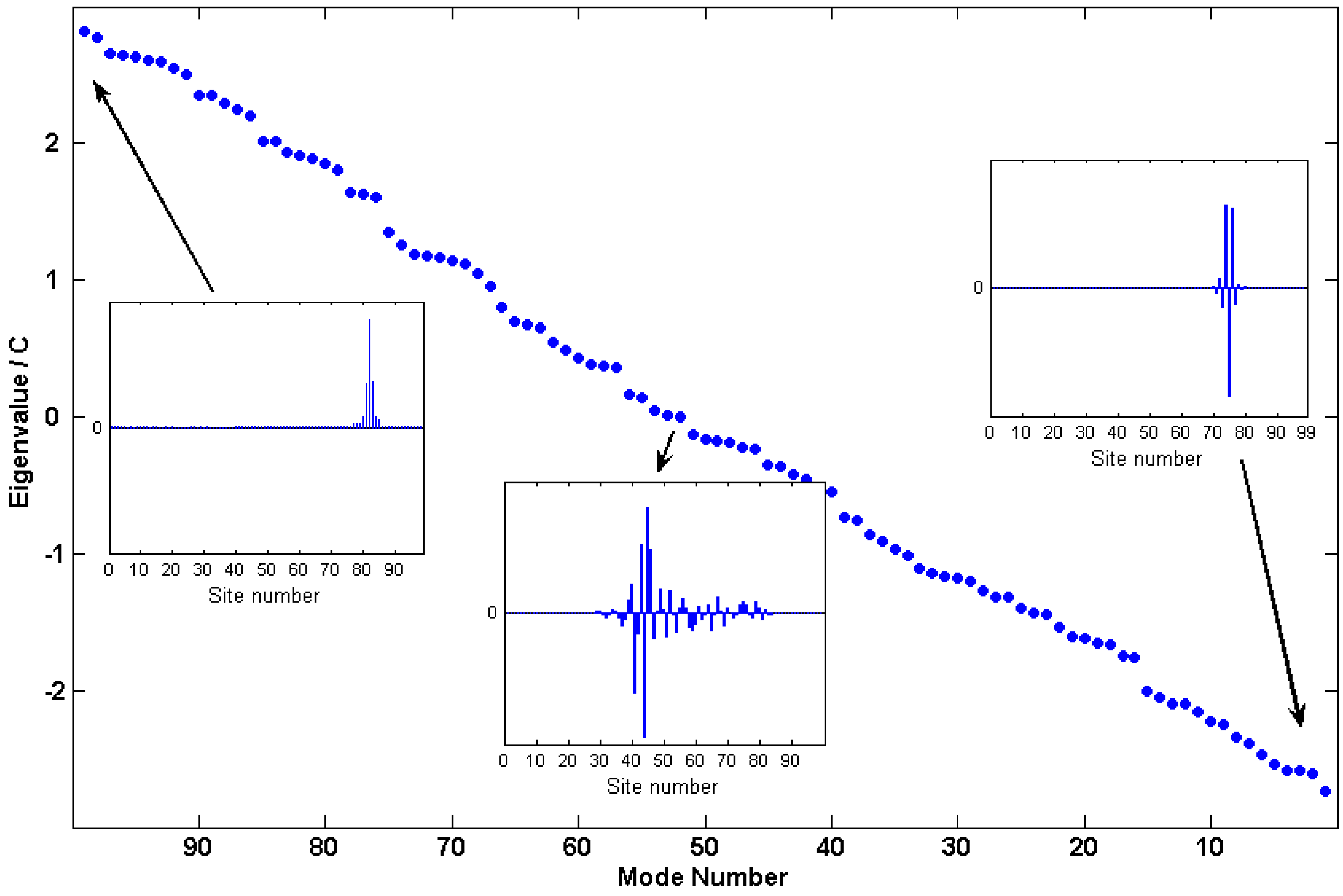


Eigenmodes of a periodic lattice, N=99

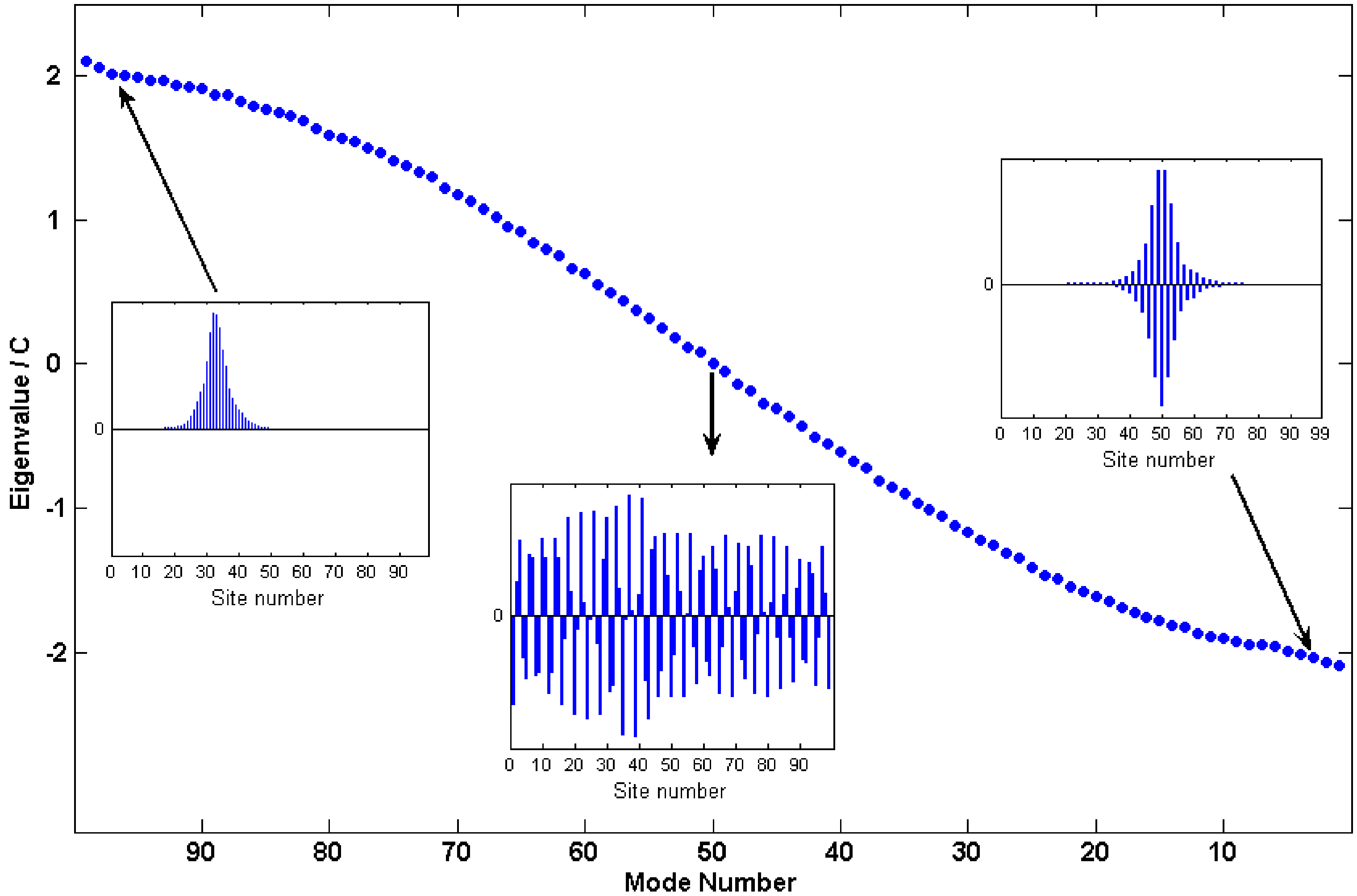




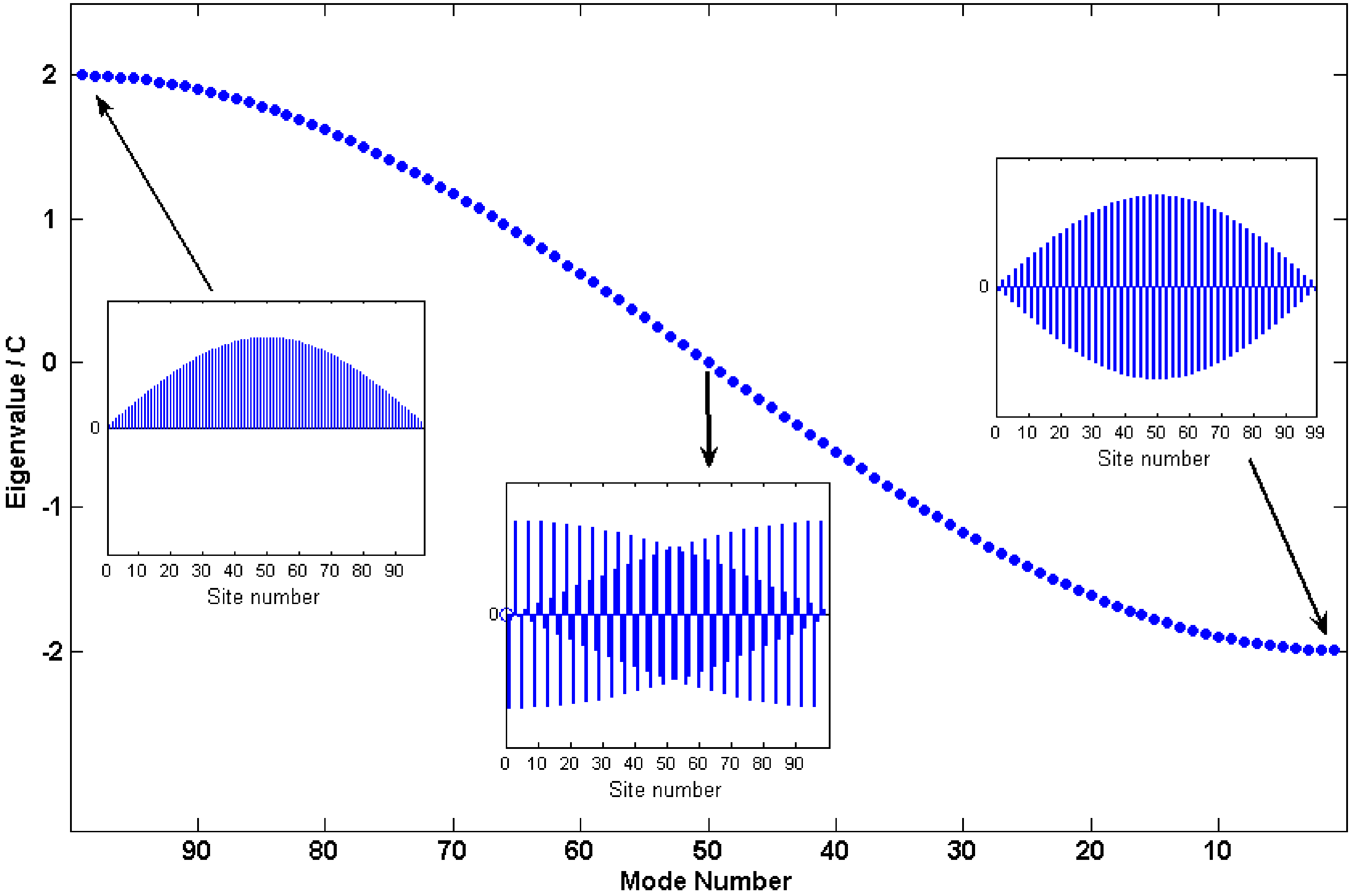
Eigenvalues and eigenmodes for N=99, Δ/C=0



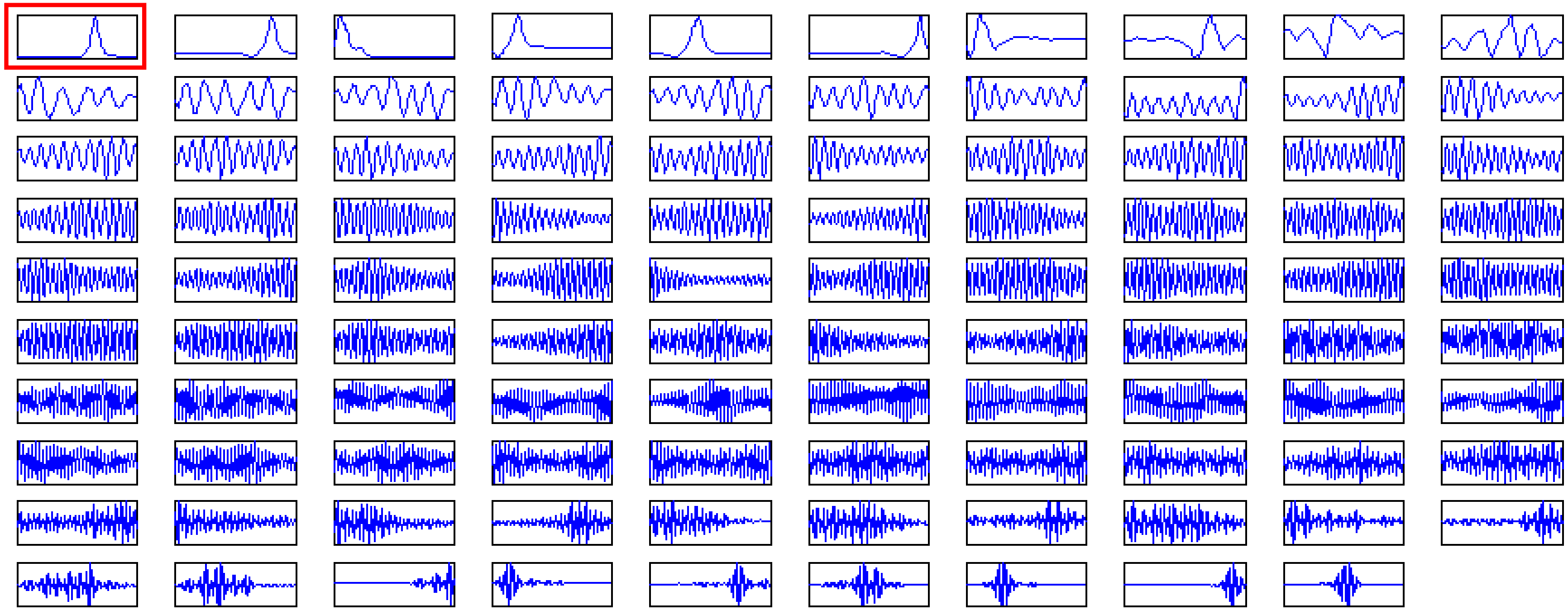
Eigenvalues and eigenmodes for N=99, Δ/C=1

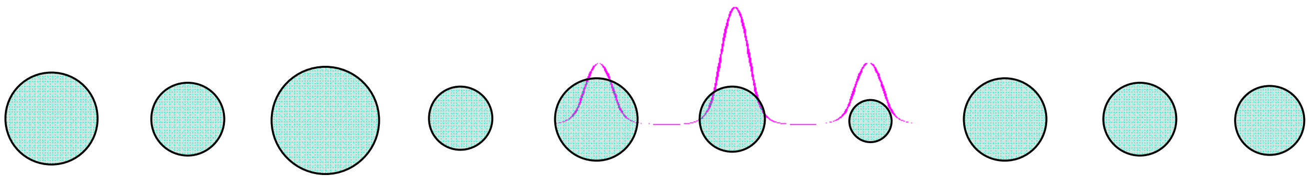


Eigenvalues and eigenmodes for N=99, Δ/C=3

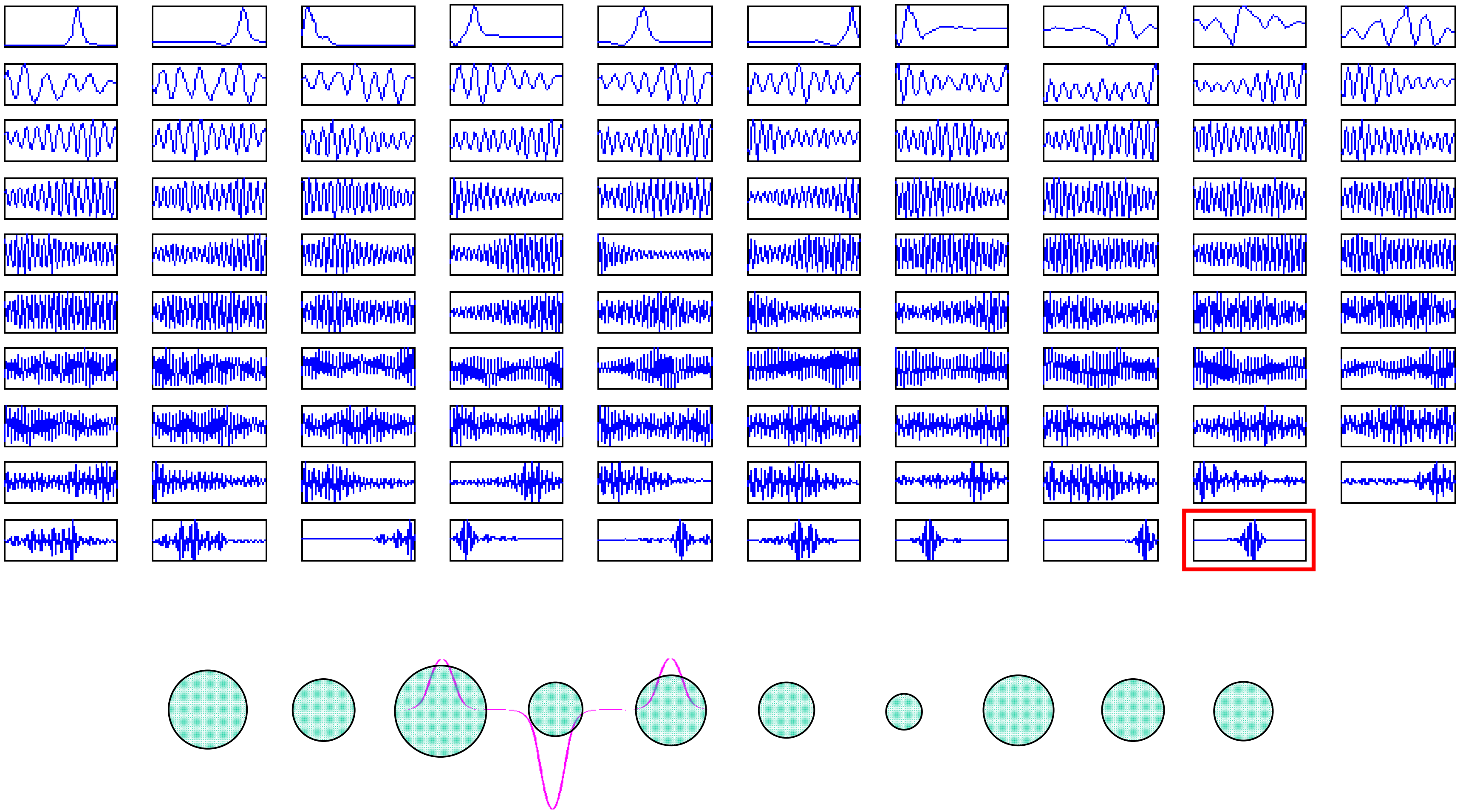


Eigenmodes of a disordered lattice, $\Delta/C=1$

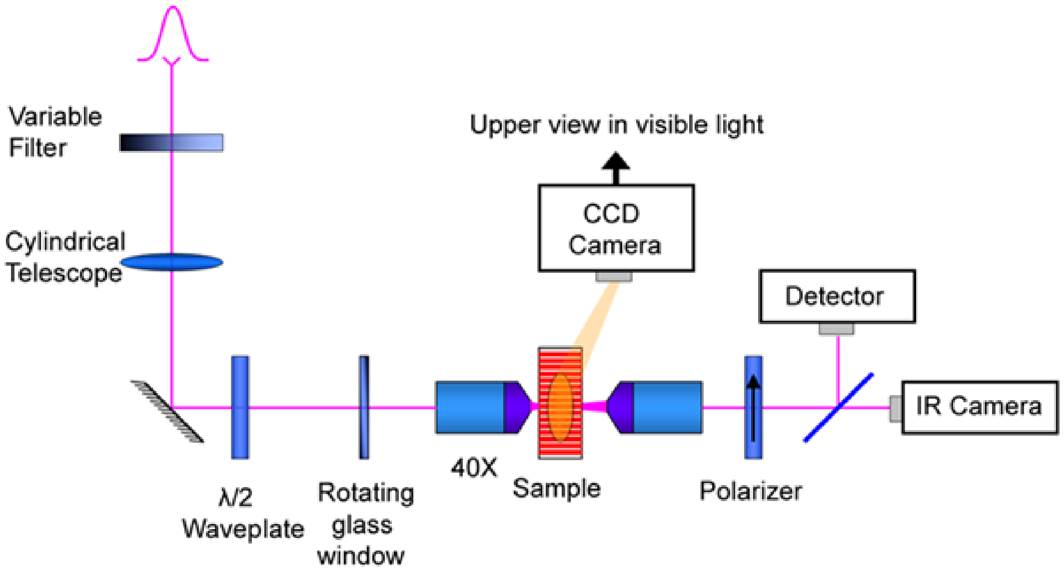




Eigenmodes of a disordered lattice, $\Delta/C=1$

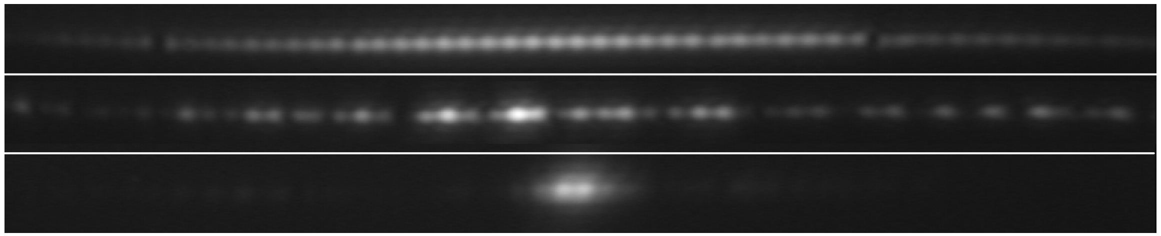


Experimental setup



- Injecting a narrow beam (~3 sites) at different locations across the lattice

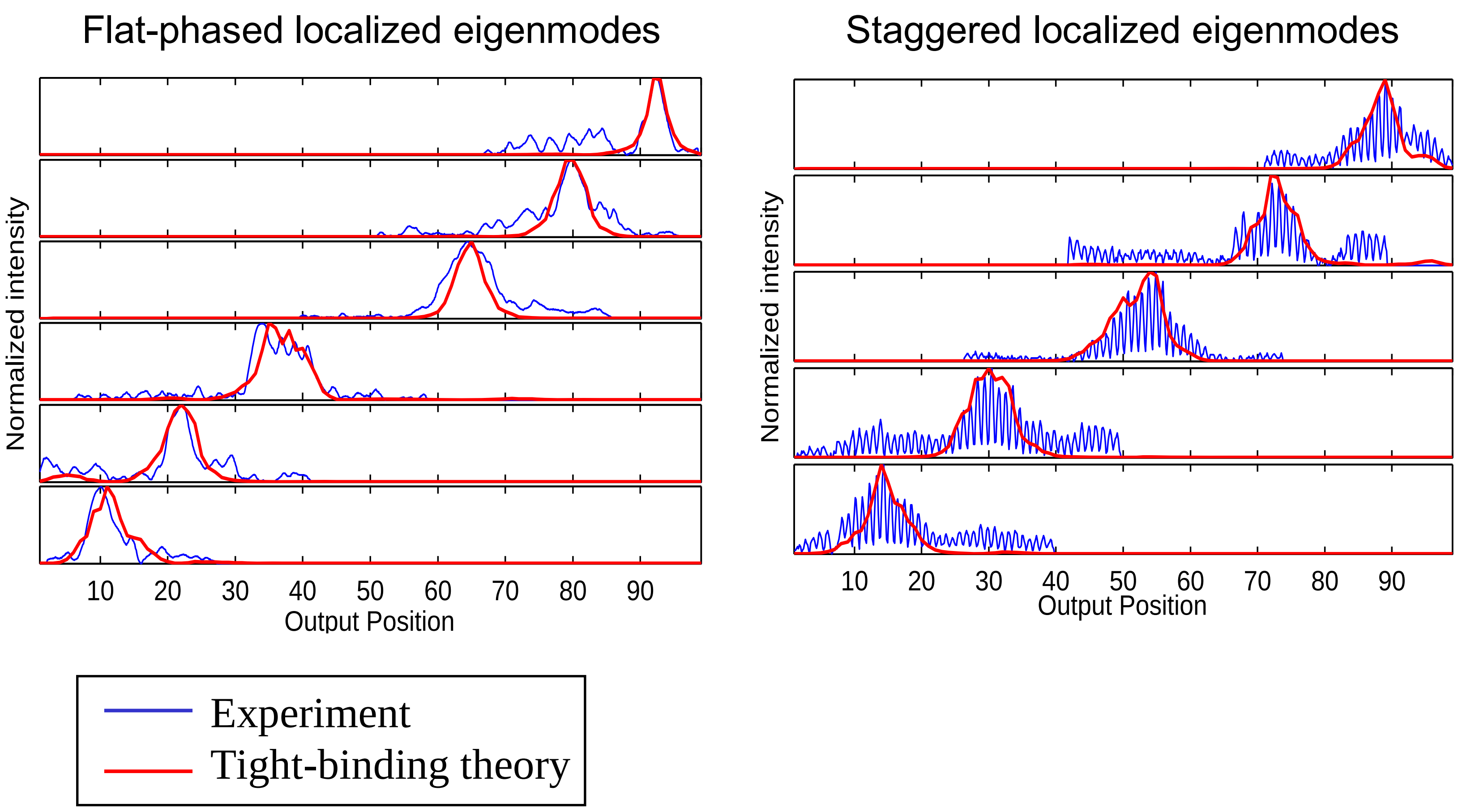
(a)
(b)
(c)



(a) Periodic array – *expansion*
(b) Disordered array - *expansion*
(c) Disordered array - *localization*

Exciting Pure localized eigenmodes

- Using a wide input beam (~8 sites) for low mode content.



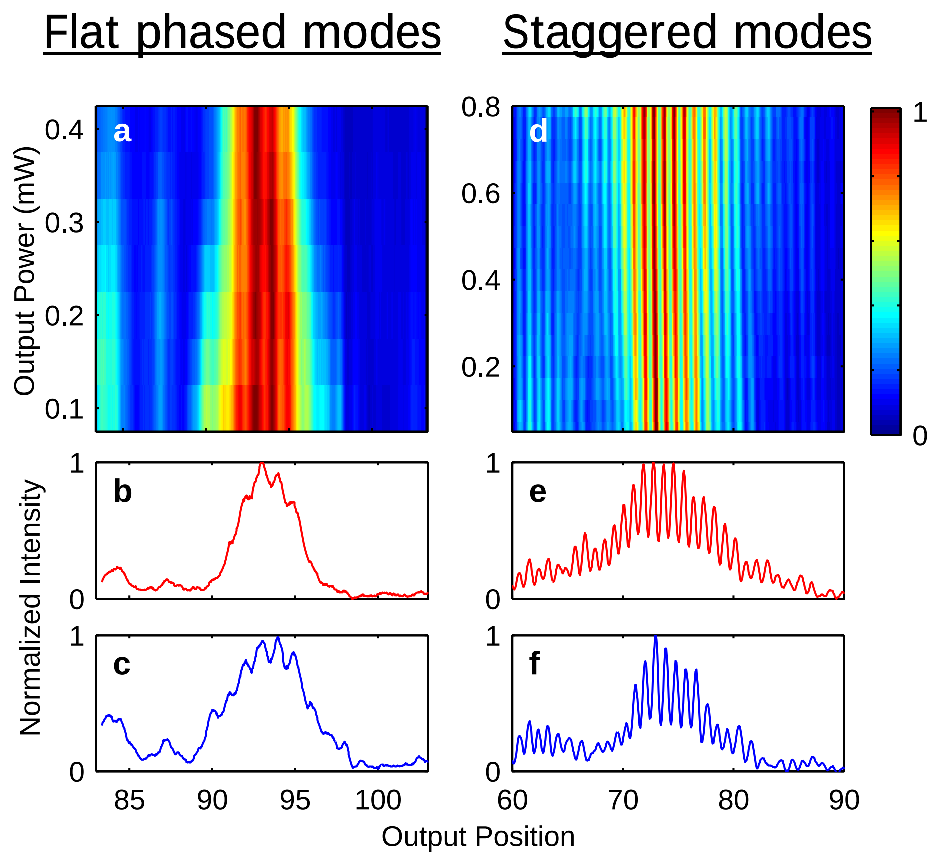
Nonlinearity and Localization

Nonlinear Localization – Discrete Solitons

Disorder-induced localization – Anderson Localization

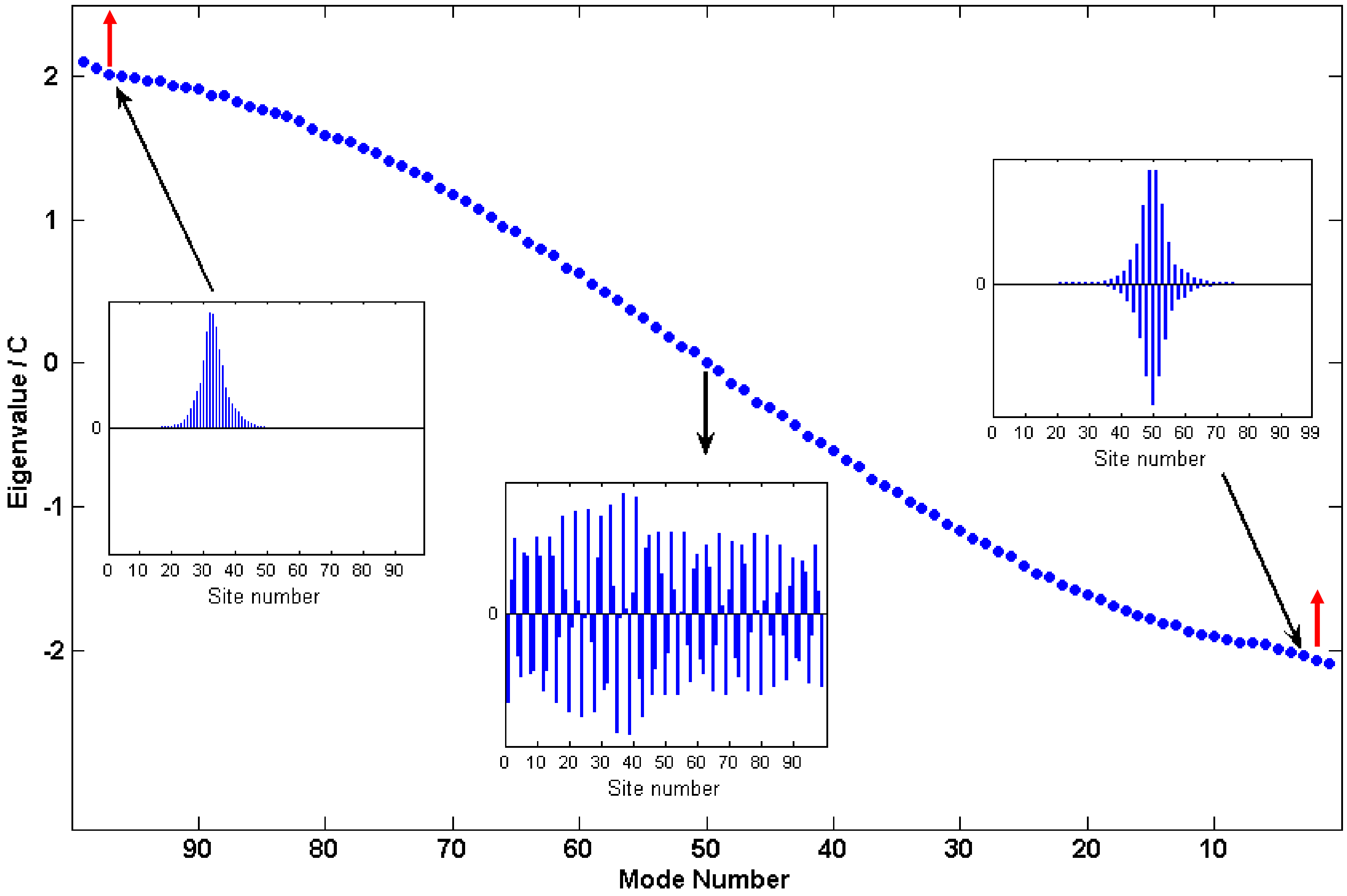
Disorder + Nonlinearity ?

The effect of nonlinearity on localized modes – weak disorder



- Two families of eigenmodes, with opposite response to nonlinearity
- Delocalization through resonance with the 'extended' modes

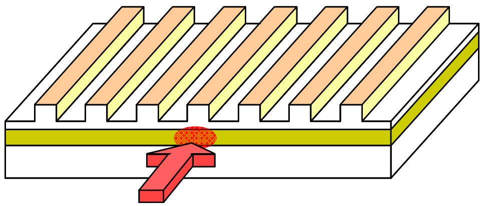
The effect of nonlinearity on localized modes – weak disorder



Wavepacket expansion in disordered lattices

The effect of nonlinearity on wavepacket expansion

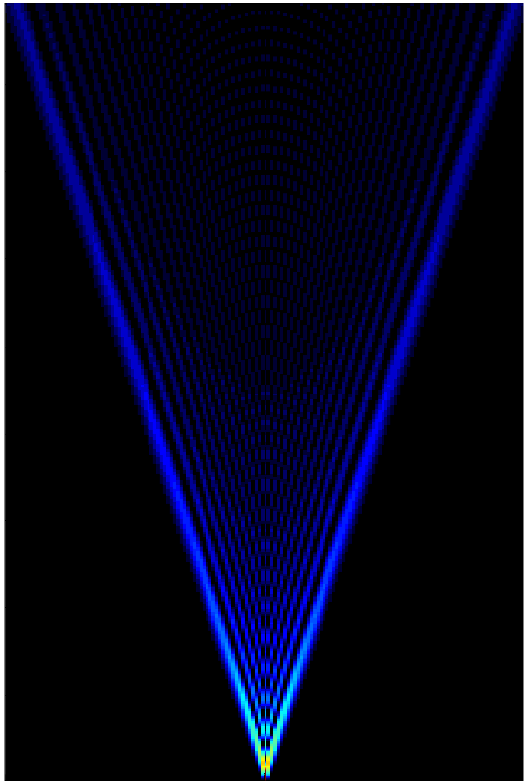
- Single-site excitation
- Short time behavior – from ballistic expansion to localization



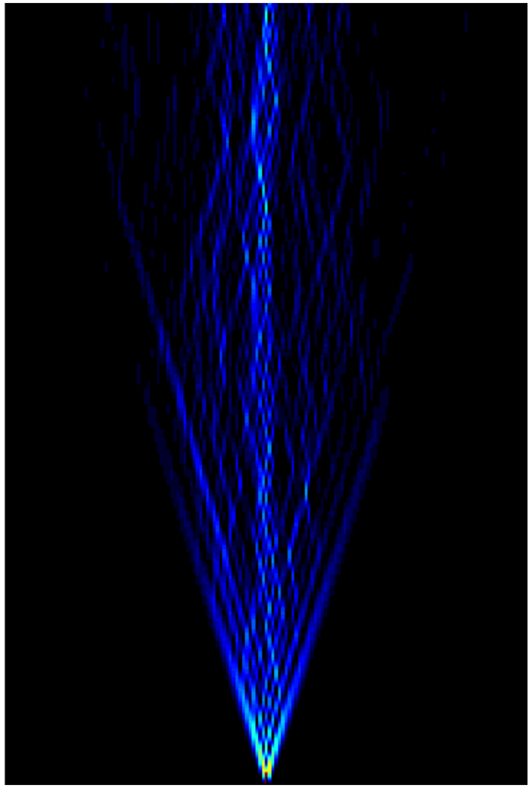
Wave packet expansion in disordered arrays

- Exciting a *single* site as an initial condition

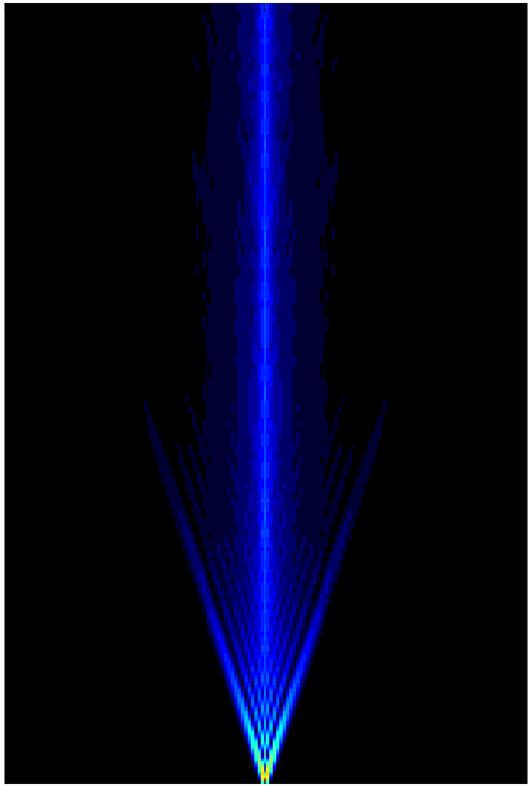
Ordered lattice



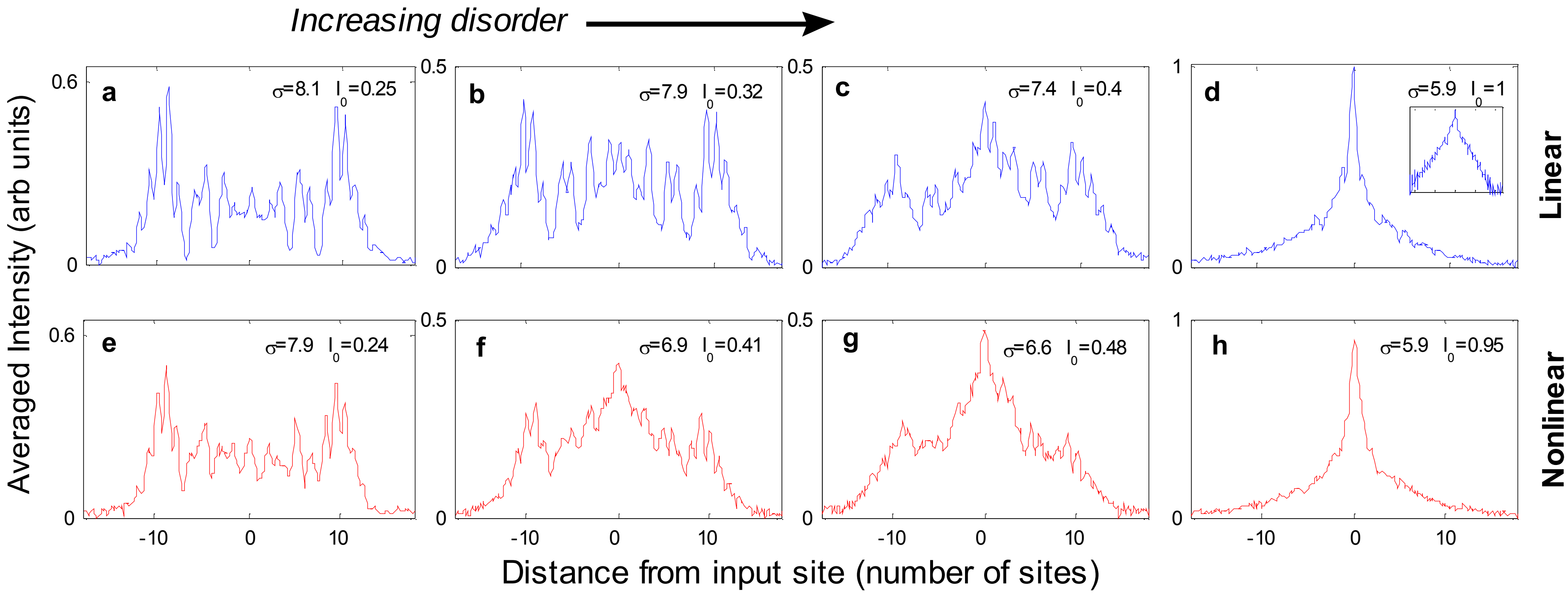
Disordered lattice



Disordered lattice - averaged



Wave packet expansion in disordered arrays: experiments



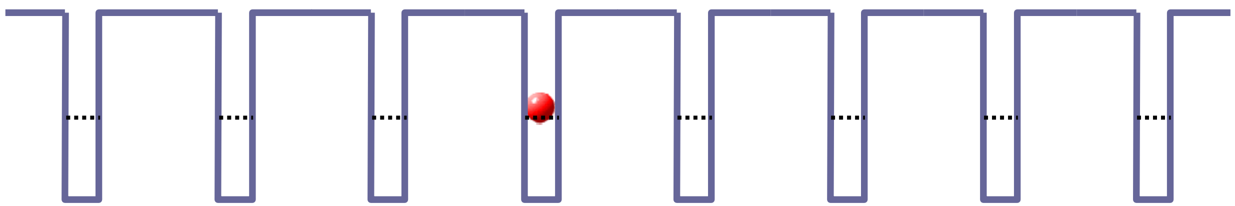
- **A crossover from ballistic expansion to exponential (Anderson) localization**
- **Coexistence of a ballistic and a localized component at intermediate times**
- **The effect of nonlinearity – accelerated crossover**

Schwartz et. al. Nature 446 53, (2007); Lahini et. al. Phys. Rev. Lett. 100, 013906 (2008).

Outline

- AL in a photonic lattice
- Quantum correlations in a lattice
- Quantum correlations & disorder

Single Photons in Waveguides

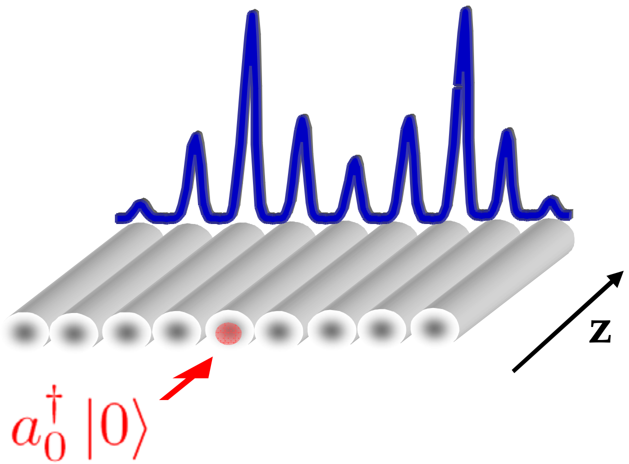


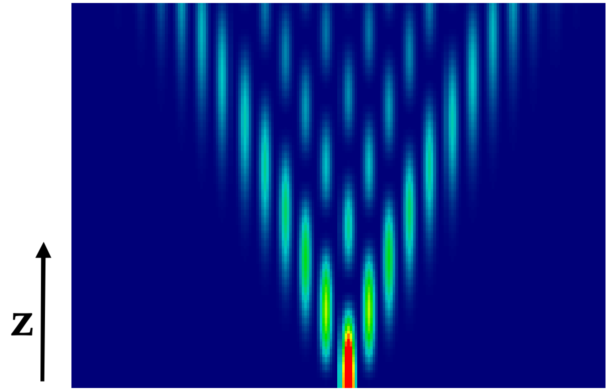
- What new features are added when single \ few photons are used?

Quantum walk of a single photon in a lattices

The Heisenberg equations for the **creation/annihilation** operators:

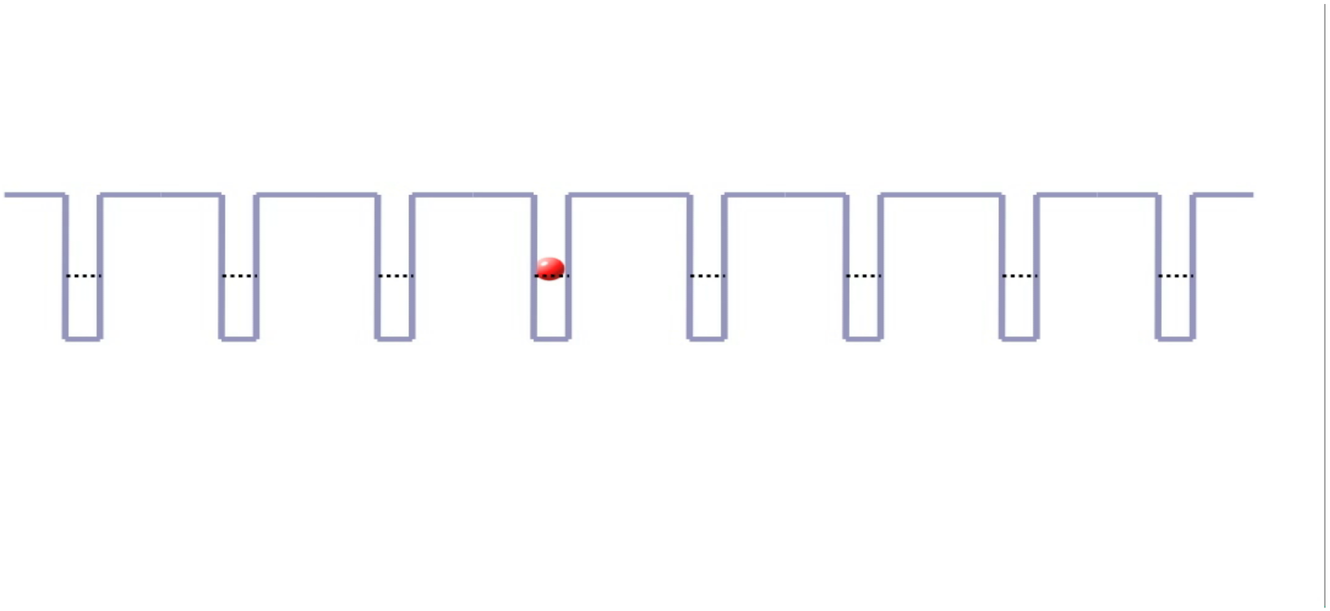
$$i\frac{\partial a_n^\dagger}{\partial z} = \beta a_n^\dagger + C\left(a_{n+1}^\dagger + a_{n-1}^\dagger\right) \qquad [a_m, a_n^\dagger] = \delta_{m,n}$$





- The mean photon number $\langle a^\dagger_n a_n \rangle$ propagates exactly like a classical wave
- **Quantum** features are revealed only when considering **correlations**.

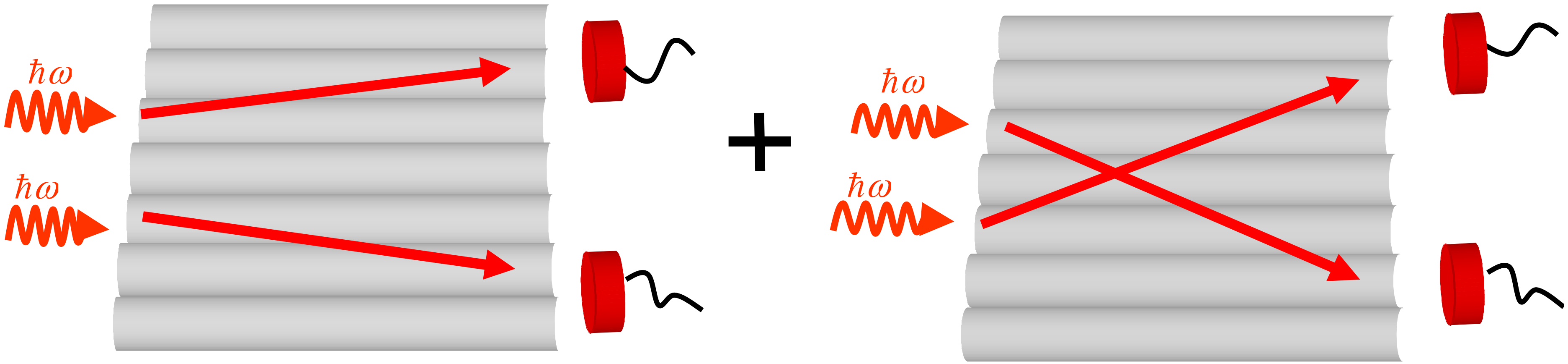
Two-particle interference



movie

Two-photon (HBT) interference

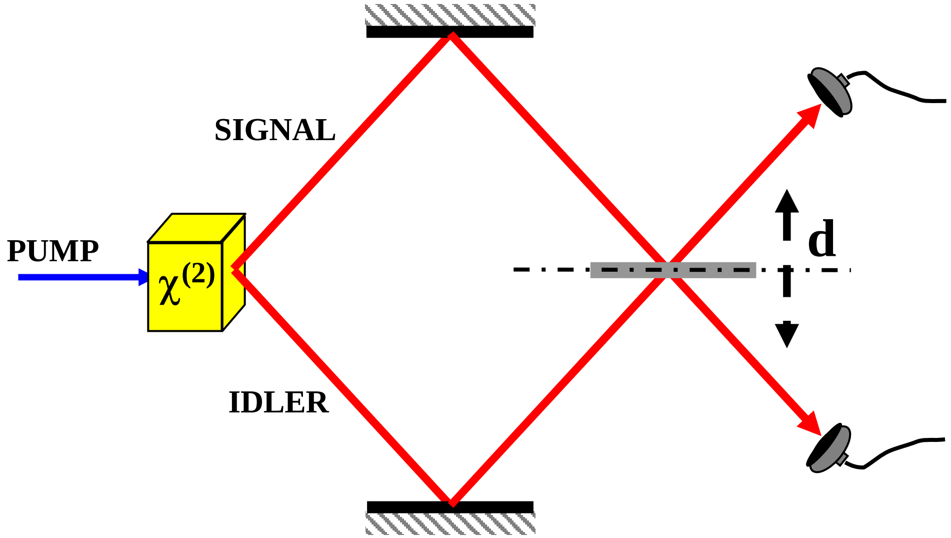
Two indistinguishable paths yield a coincidence event ⇨ interference

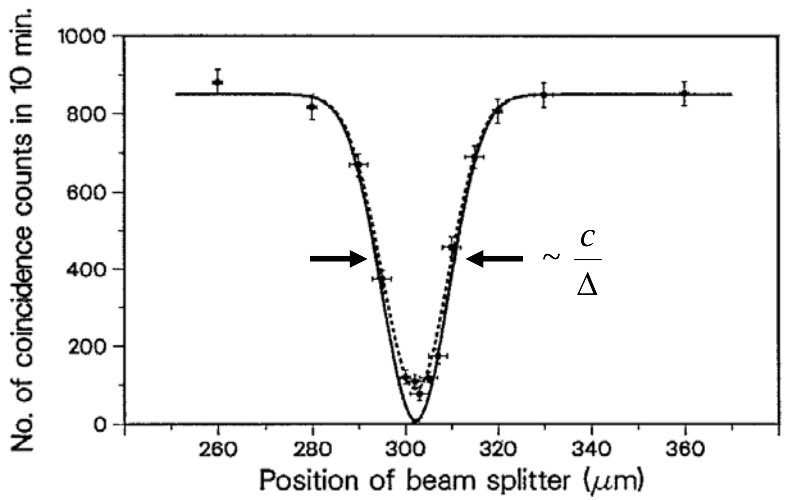


Two-Photon Coincidence Interference :
Hong-Ou-Mandel Dip

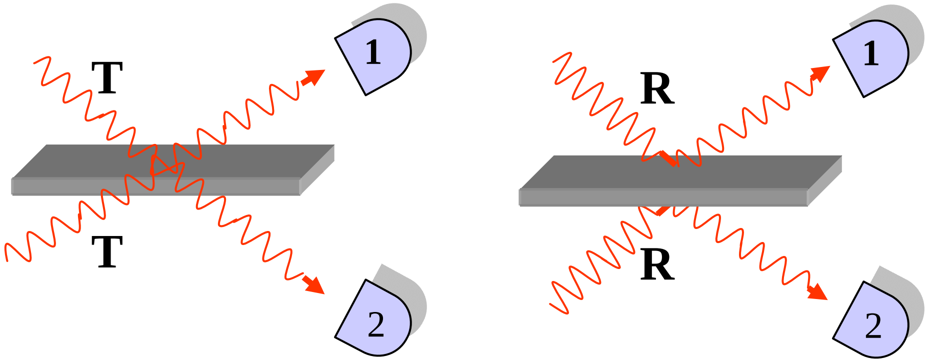
"Measurement of Subpicosecond Time Intervals between Two Photons by Interference"

C.K. Hong, Z.Y. Ou and L. Mandel, PRL 59 (1987)





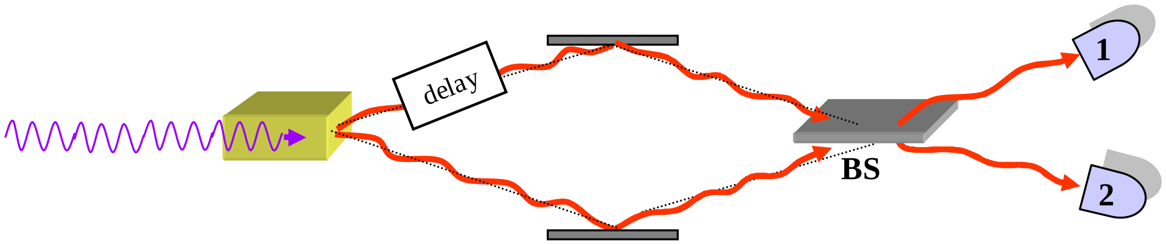
Indistinguishable Paths



$$\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\cdot\frac{i}{\sqrt{2}} = 0$$

Indistinguishable paths which lead to the same event interfere

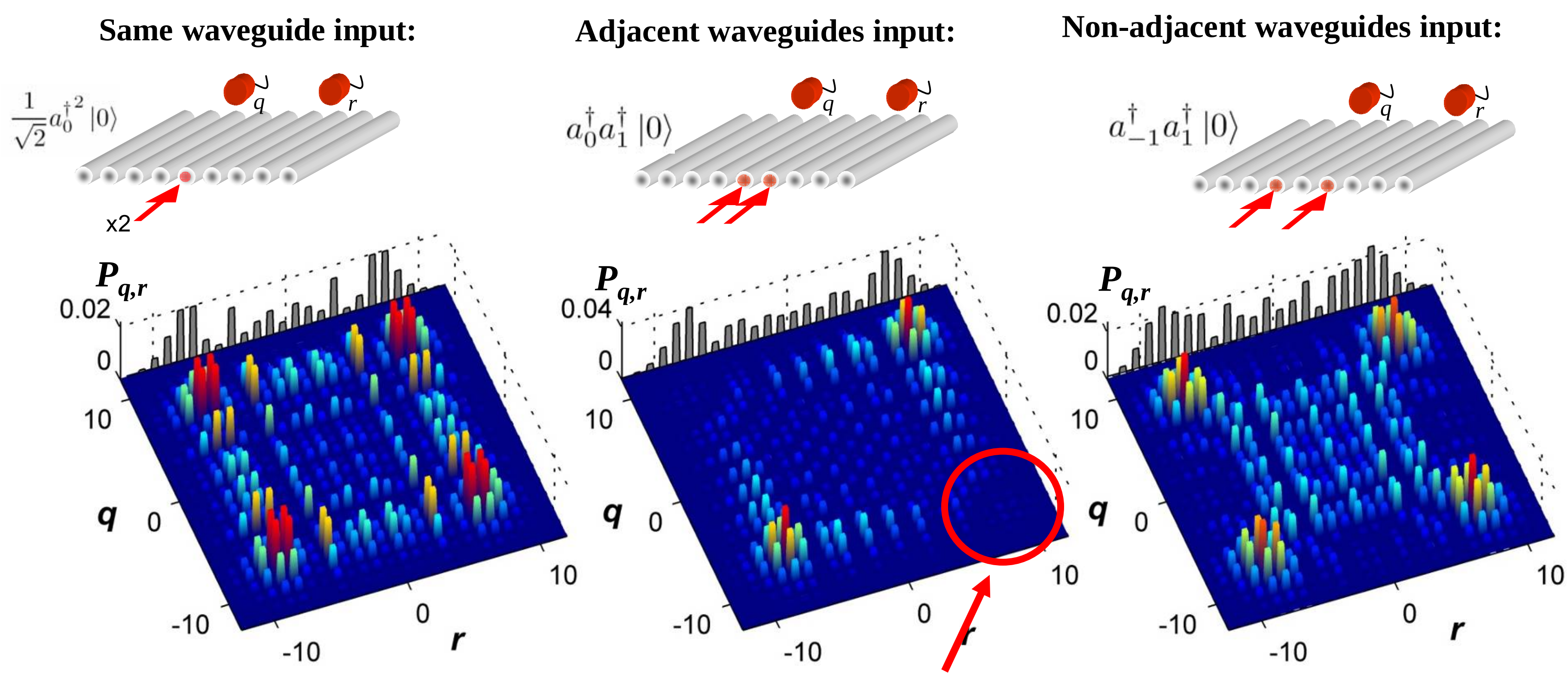
→ destructive interference → no coincidence



delay → distinguishable paths → no interference

Quantum correlations between photon-pairs

- The probability to detect one photon at waveguide *q* and its twin photon at waveguide *r* is given by $P_{q,r} = \langle \psi_{in} | \hat{a}_q^\dagger \hat{a}_r^\dagger \hat{a}_q \hat{a}_r | \psi_{in} \rangle$



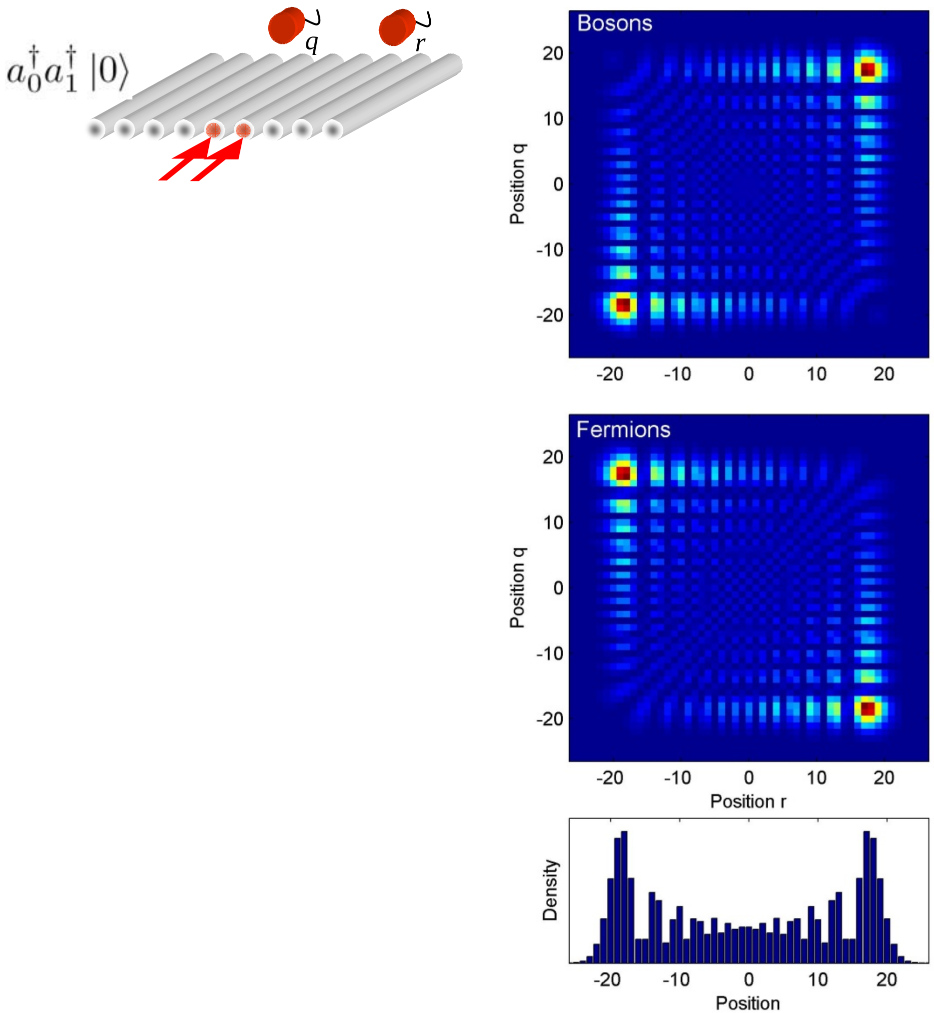
$$P_{q,r} = \langle \hat{a}_q^\dagger \hat{a}_q \rangle \langle \hat{a}_r^\dagger \hat{a}_r \rangle$$

"photon bunching"

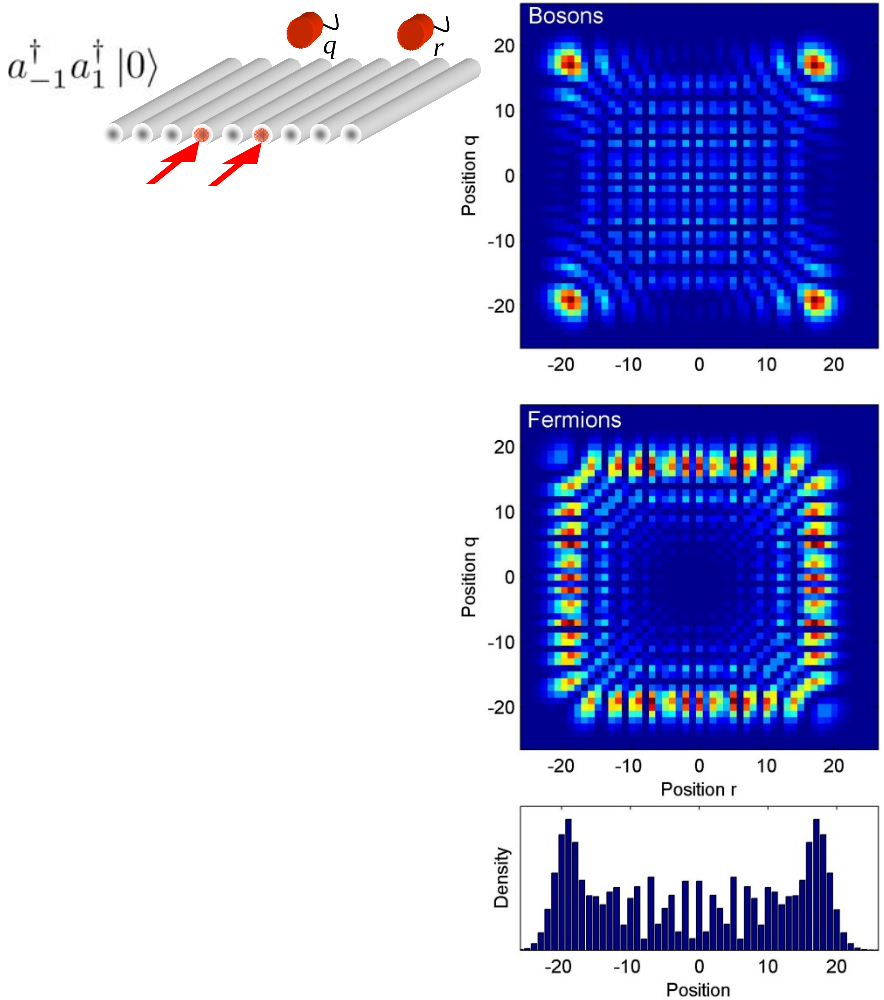
Zero probability to detect each photon at opposite lobes.

Bosons vs Fermions

Adjacent waveguides input:

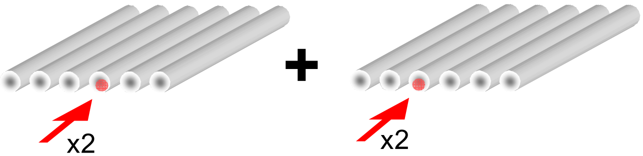


Non-adjacent waveguides input:

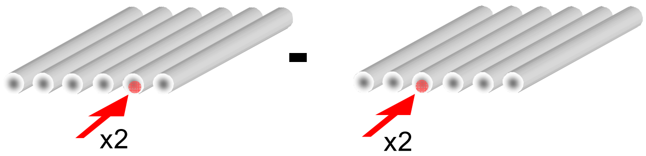


States without a classical analogue

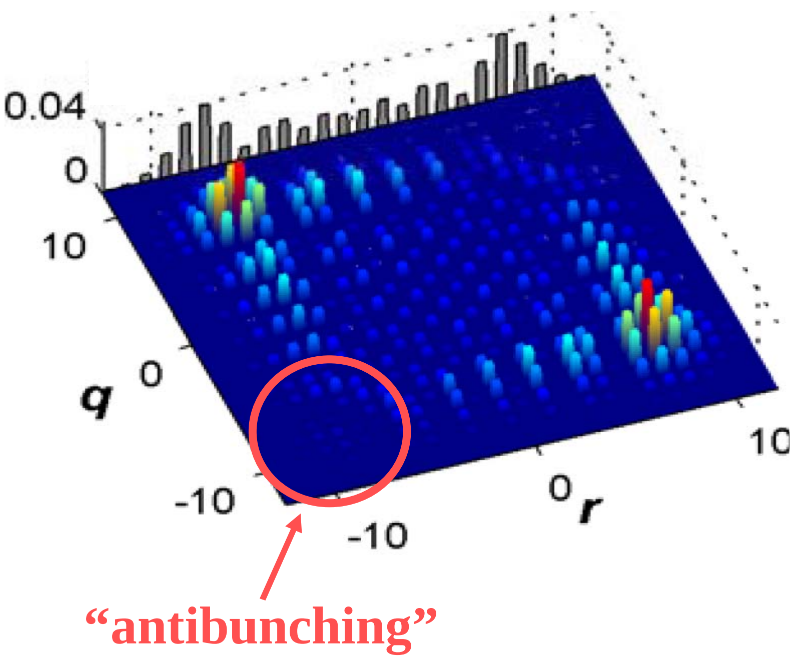
- Entangled input states

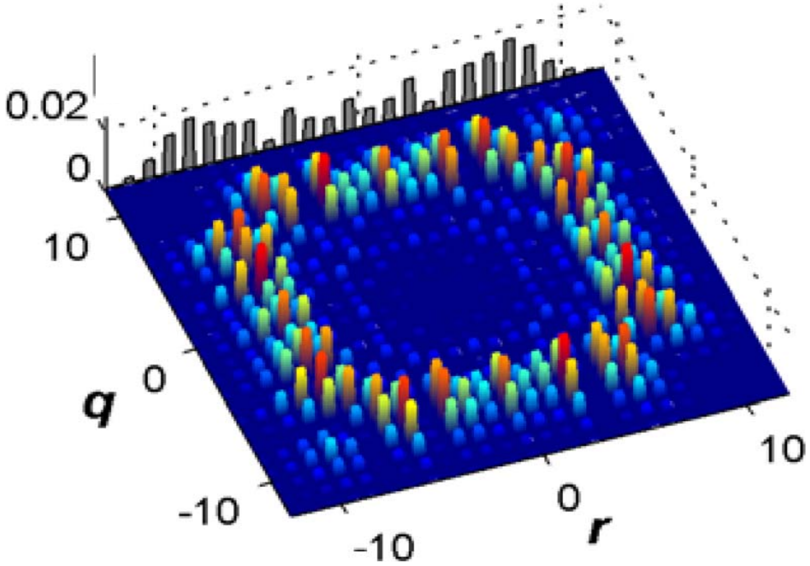


$$|\psi_{in}\rangle = \frac{1}{2}\left(a_0^{\dagger 2} + a_1^{\dagger 2}\right)|0\rangle$$



$$|\psi_{in}\rangle = \frac{1}{2}\left({a_{-1}^{\dagger}}^{2} - a_1^{\dagger 2}\right)|0\rangle$$

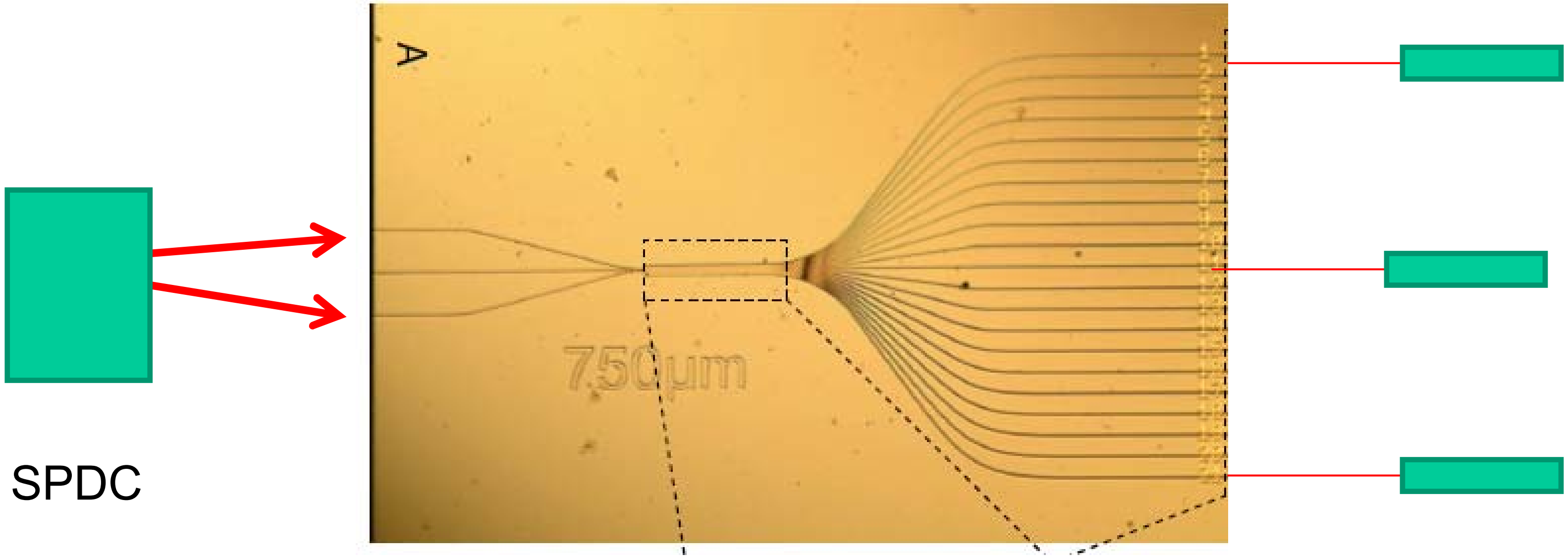




- Violate Cauchy-Schwartz inequality

$$P_{q,r} \leqslant \sqrt{P_{q,q}P_{r,r}}$$

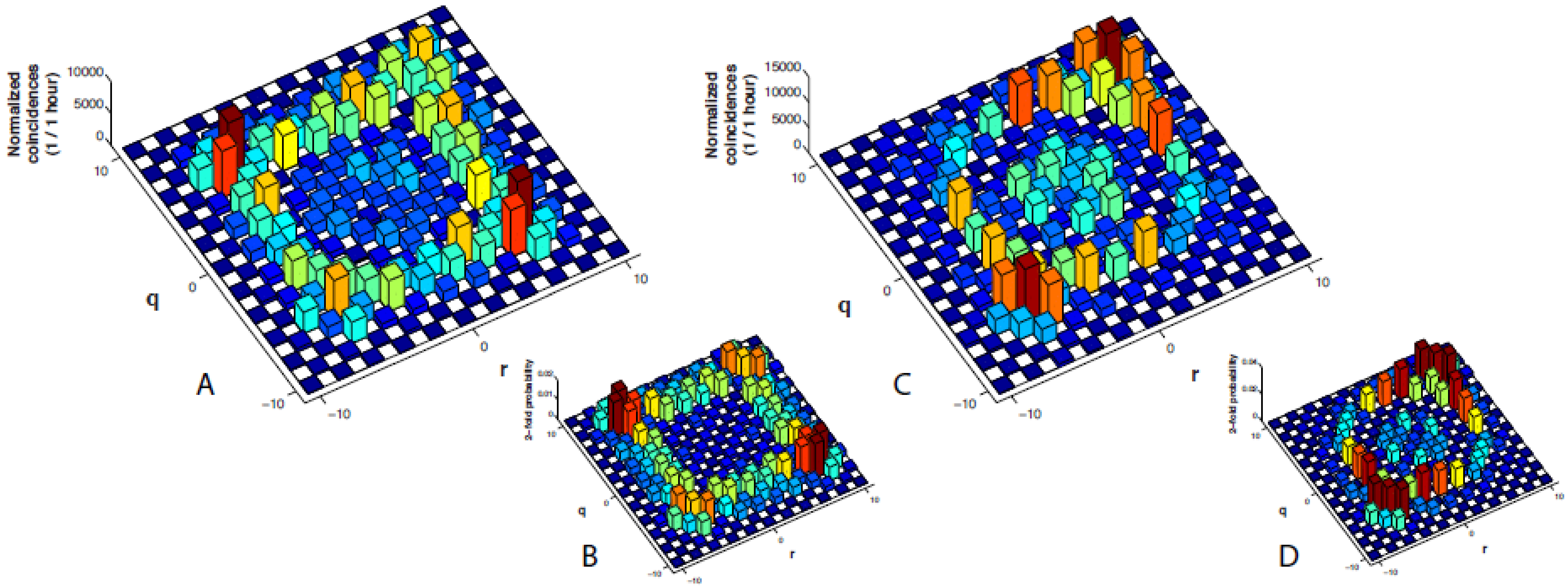
Quantum correlations experiments



With Jeremy O'Brien group, Bristol

Quantum correlations experiments – (0,1) input

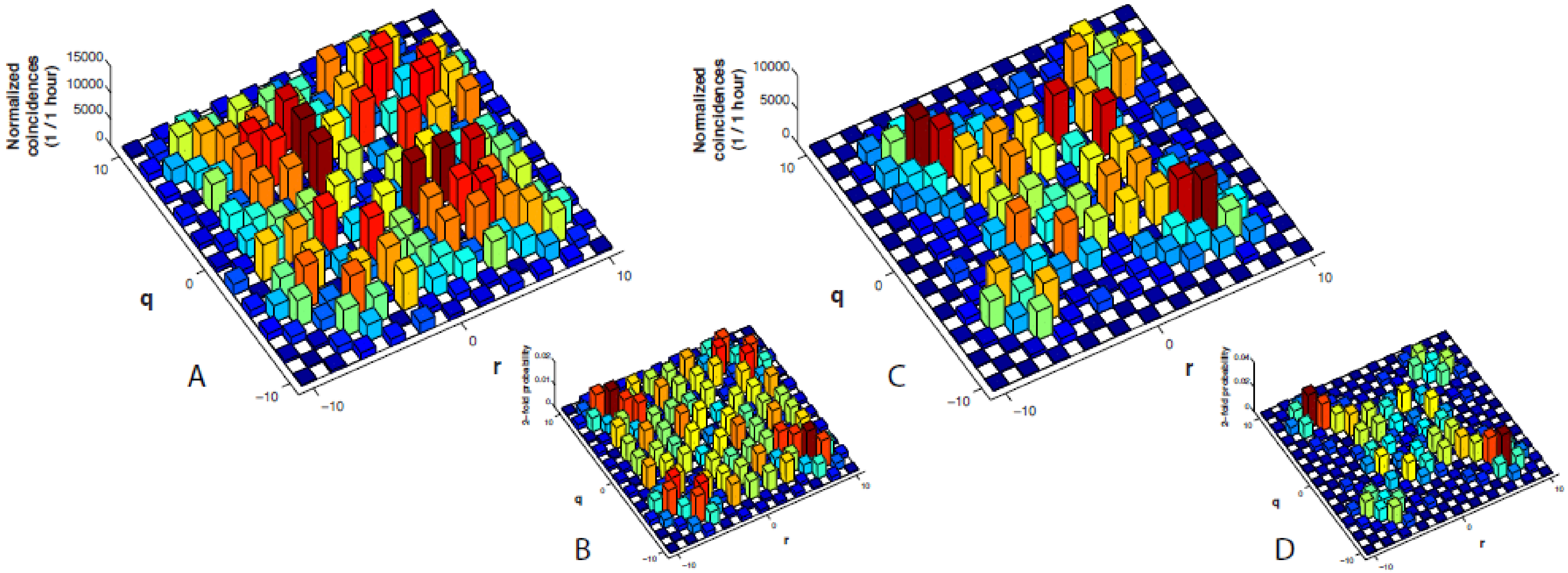
Distinguishable | Indistinguishable



Quantum correlations experiments – (-1,1) input

Distinguishable

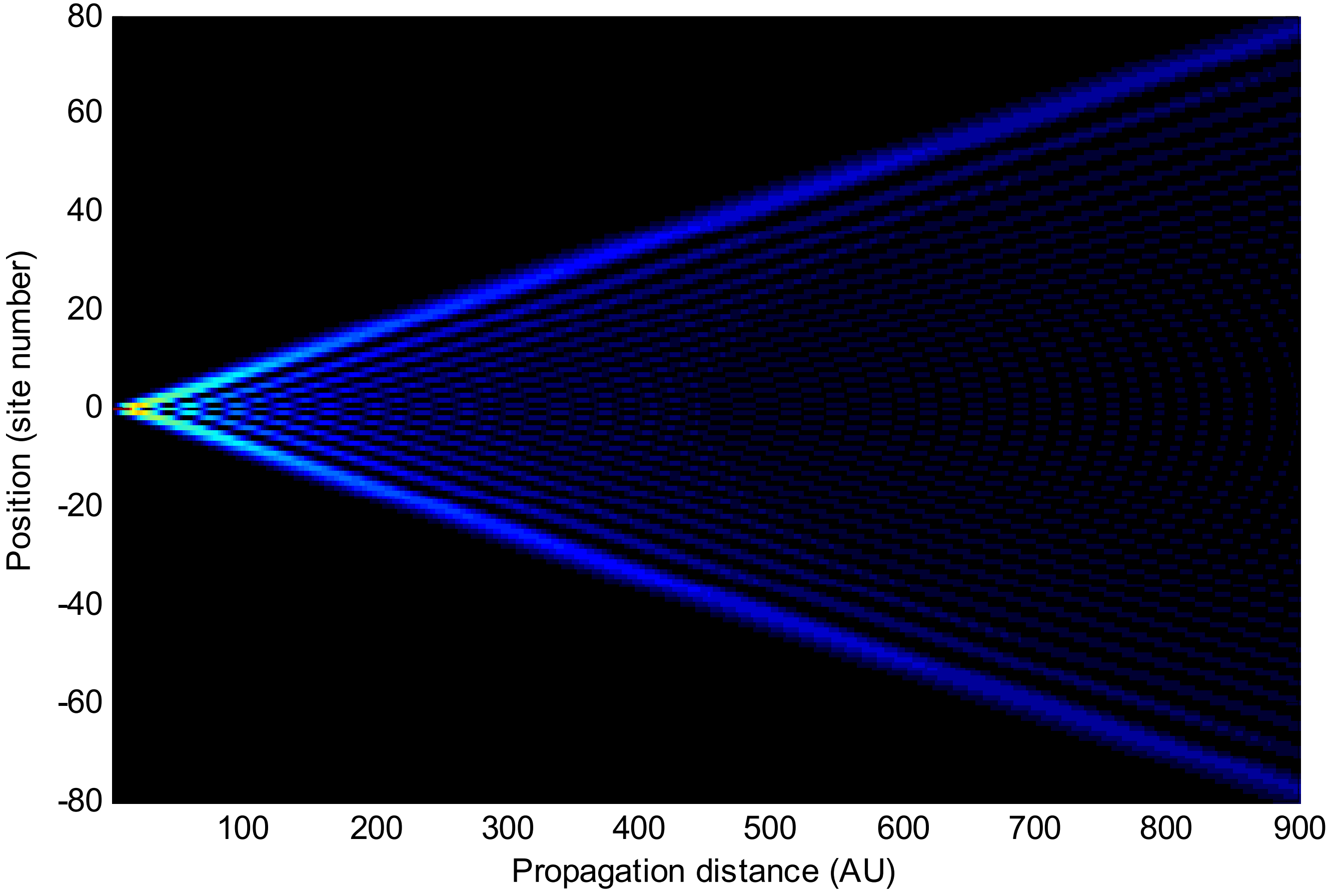
Indistinguishable



Outline

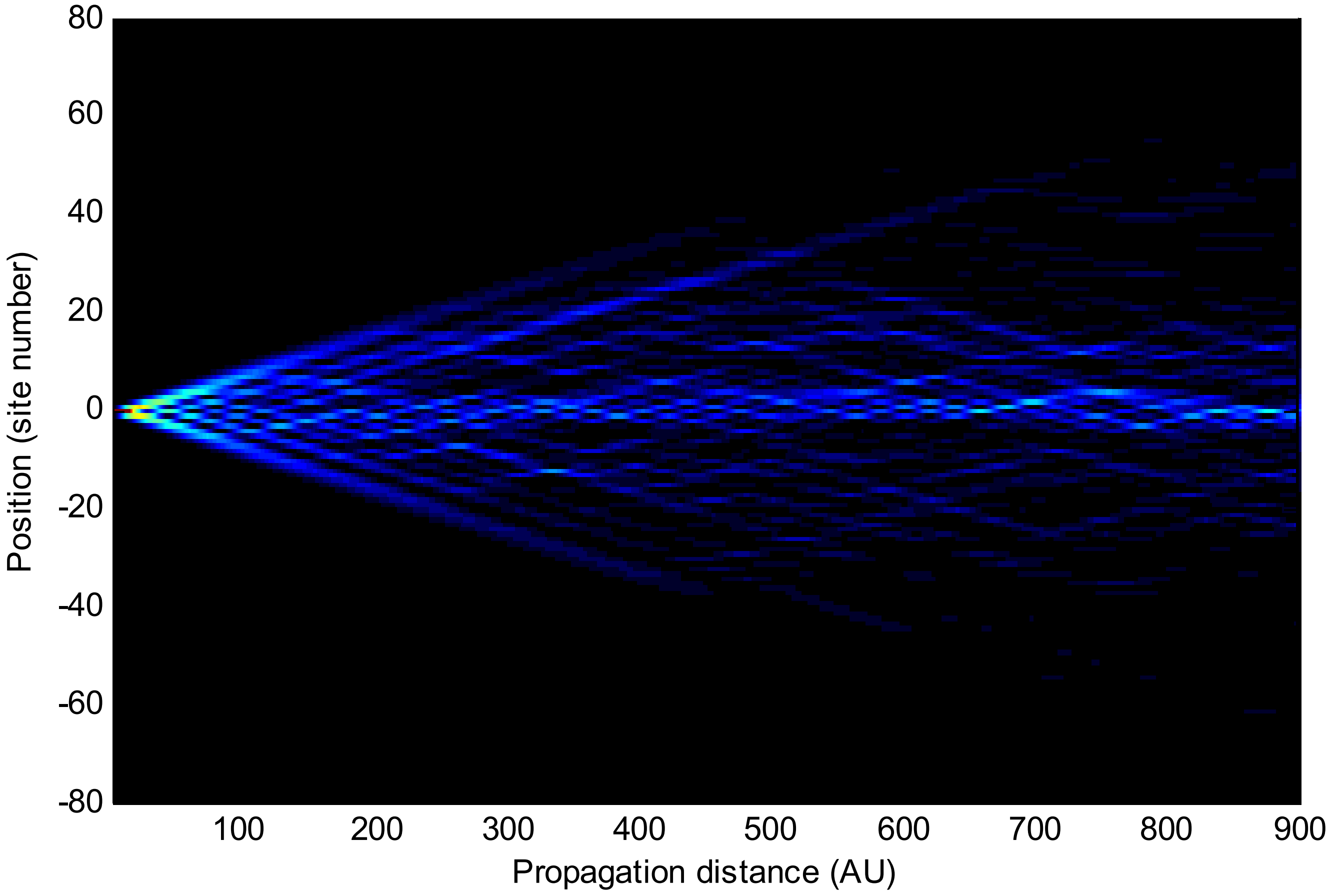
- AL in a photonic lattice
- Quantum correlations in a lattice
- Quantum correlations & disorder

Wavepacket expansion in a 1D disordered lattice

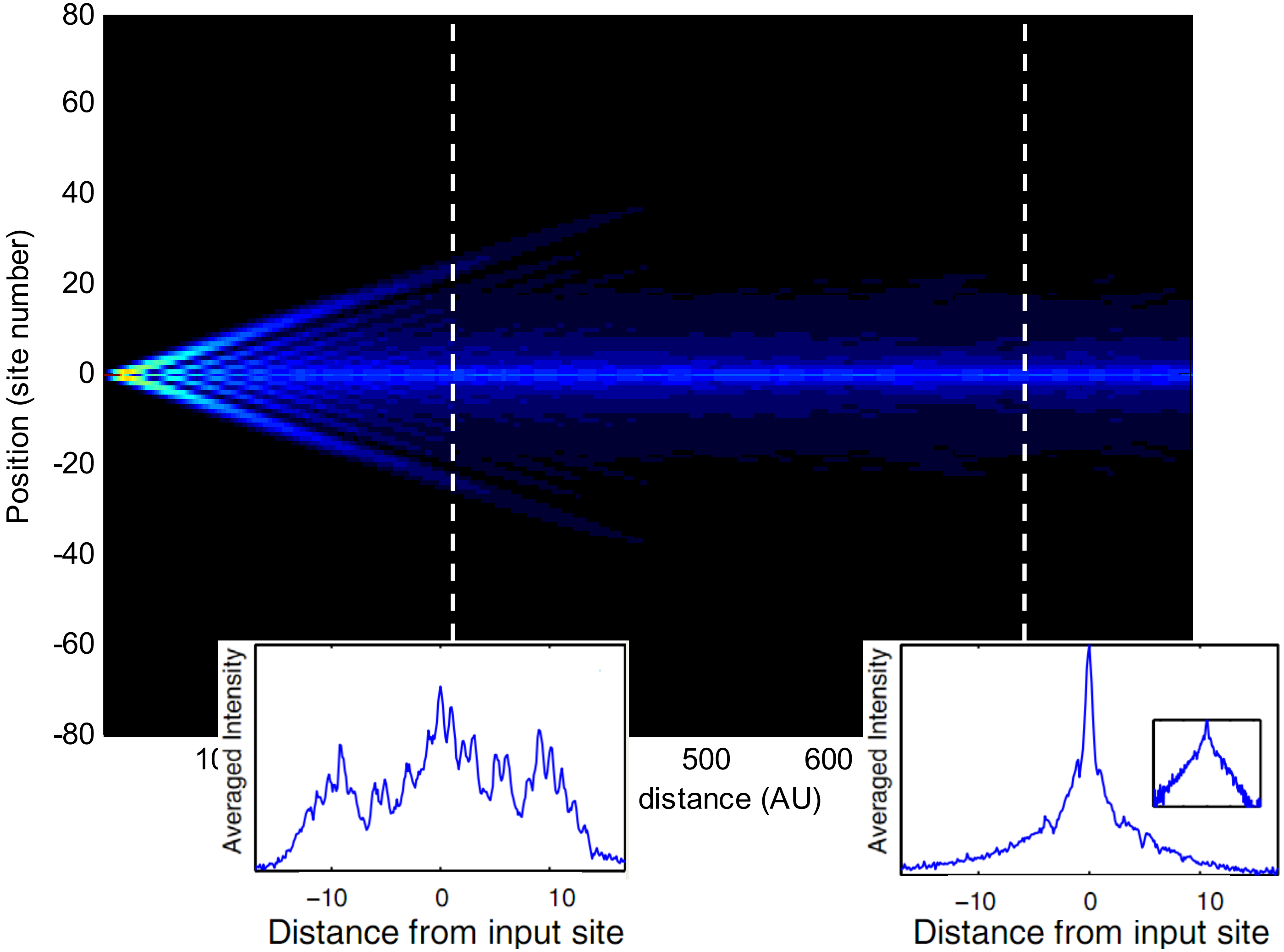


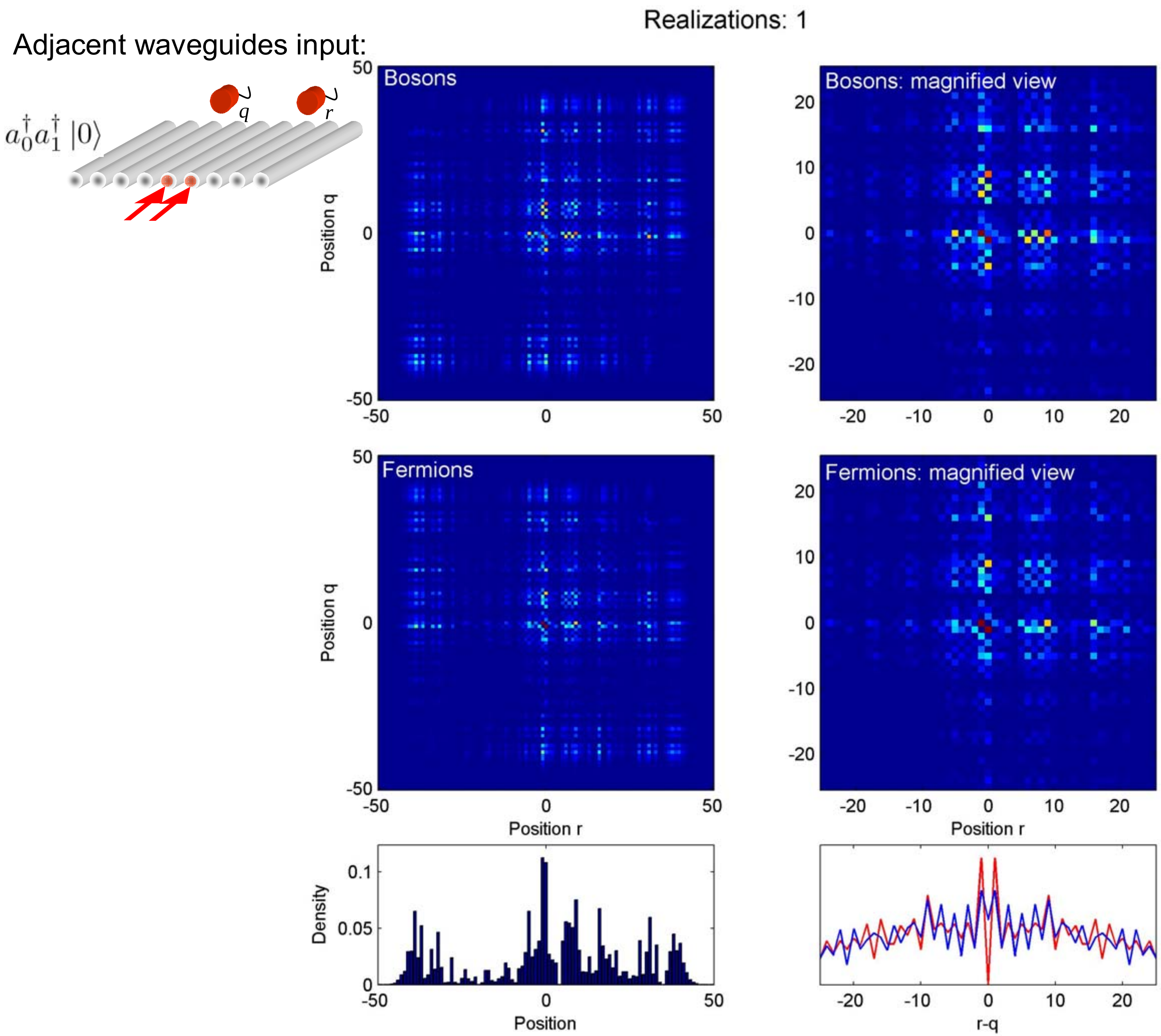
$$a_n(t) = i^n J_n(2Cz)a_0$$

Wavepacket expansion in a 1D disordered lattice

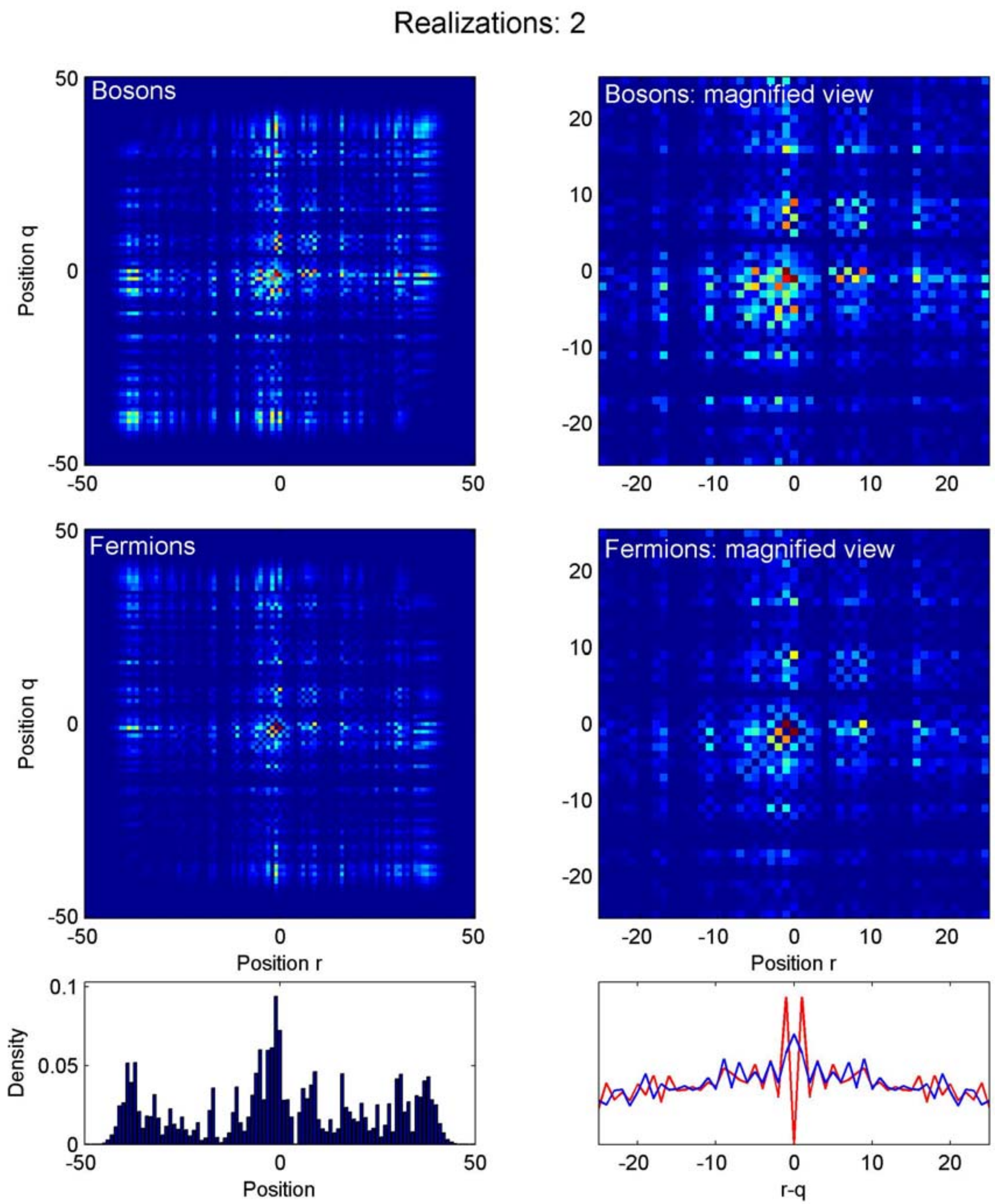


Wavepacket expansion in a 1D disordered lattice



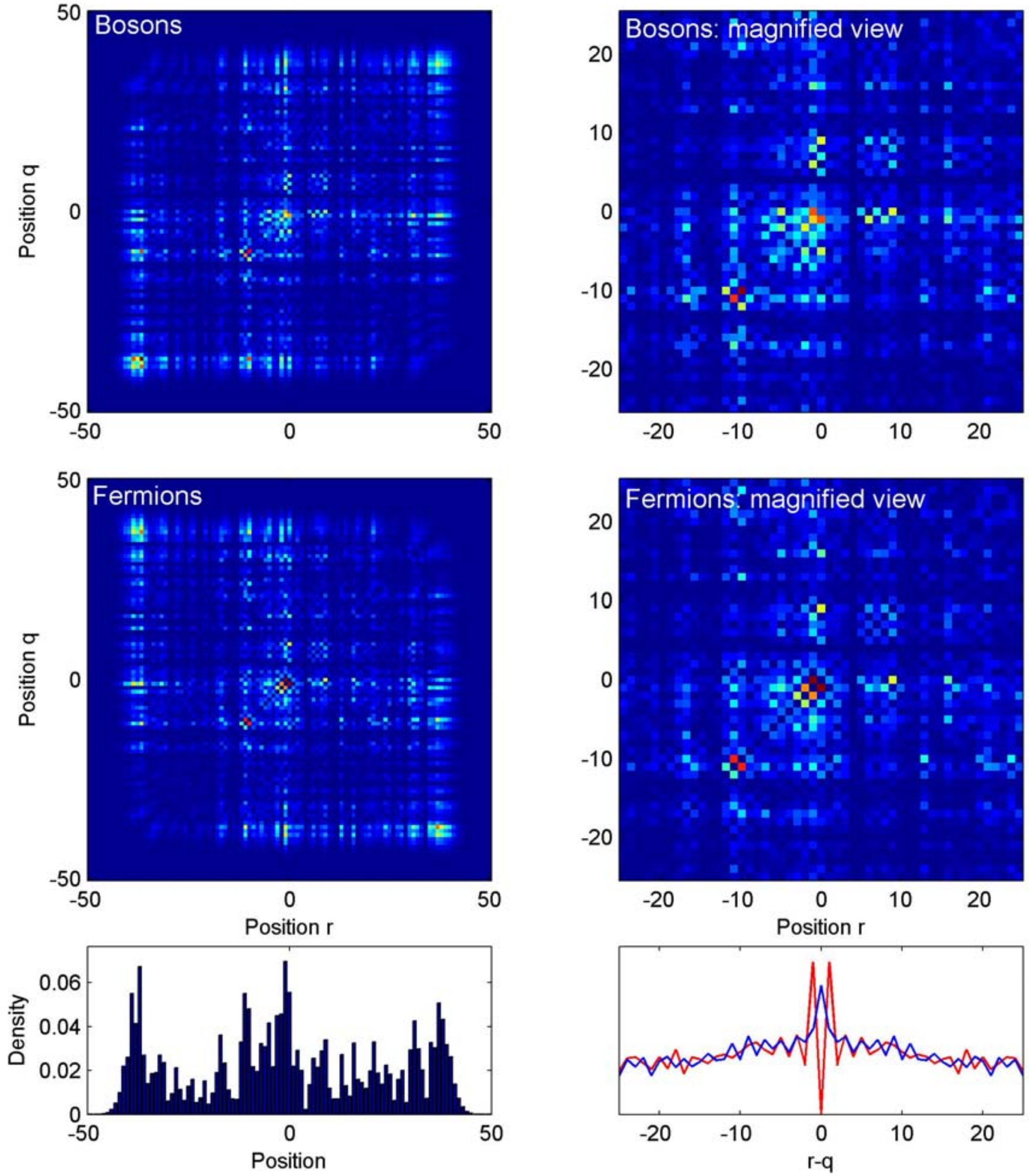


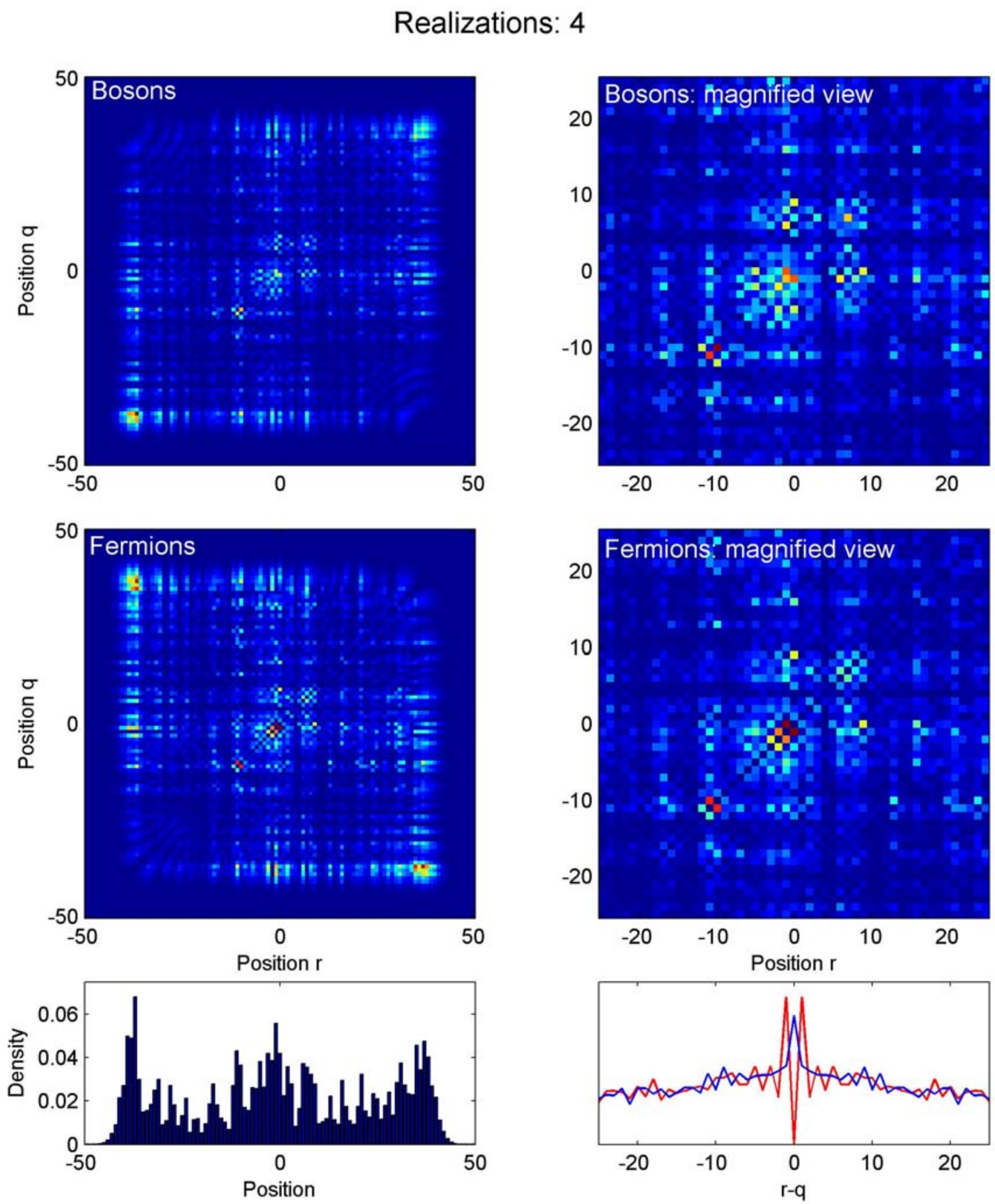
Realizations: 1
Adjacent waveguides input:
$a_0^\dagger a_1^\dagger |0\rangle$
q
r
Bosons
Bosons: magnified view
Fermions
Fermions: magnified view
Position q
Position r
Density
Position
r-q

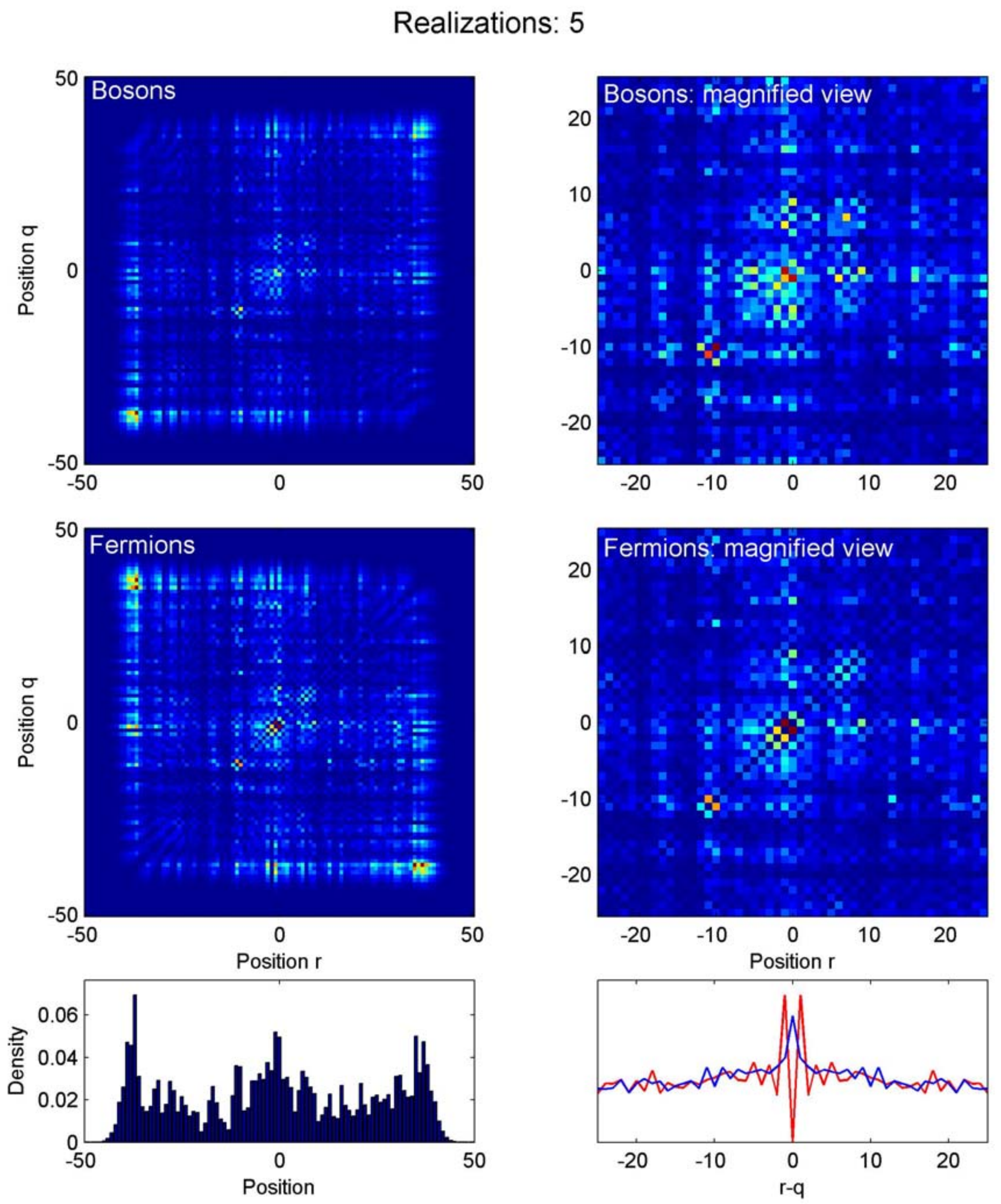


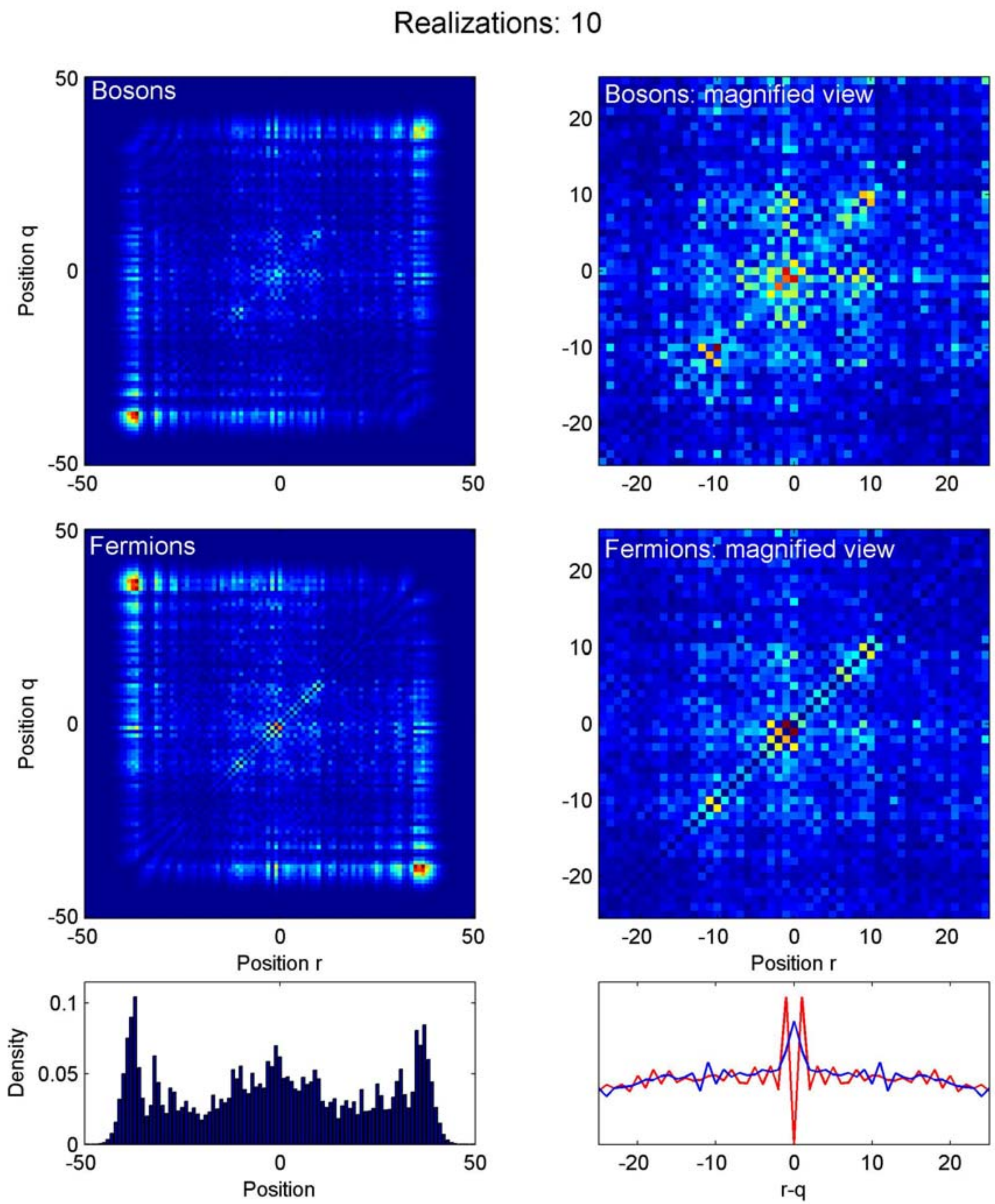
Realizations: 2
Bosons
Bosons: magnified view
Fermions
Fermions: magnified view
Position q
Position r
Density
Position
r-q
50
0
-50
20
10
-10
-20
0.1
0.05

Realizations: 3

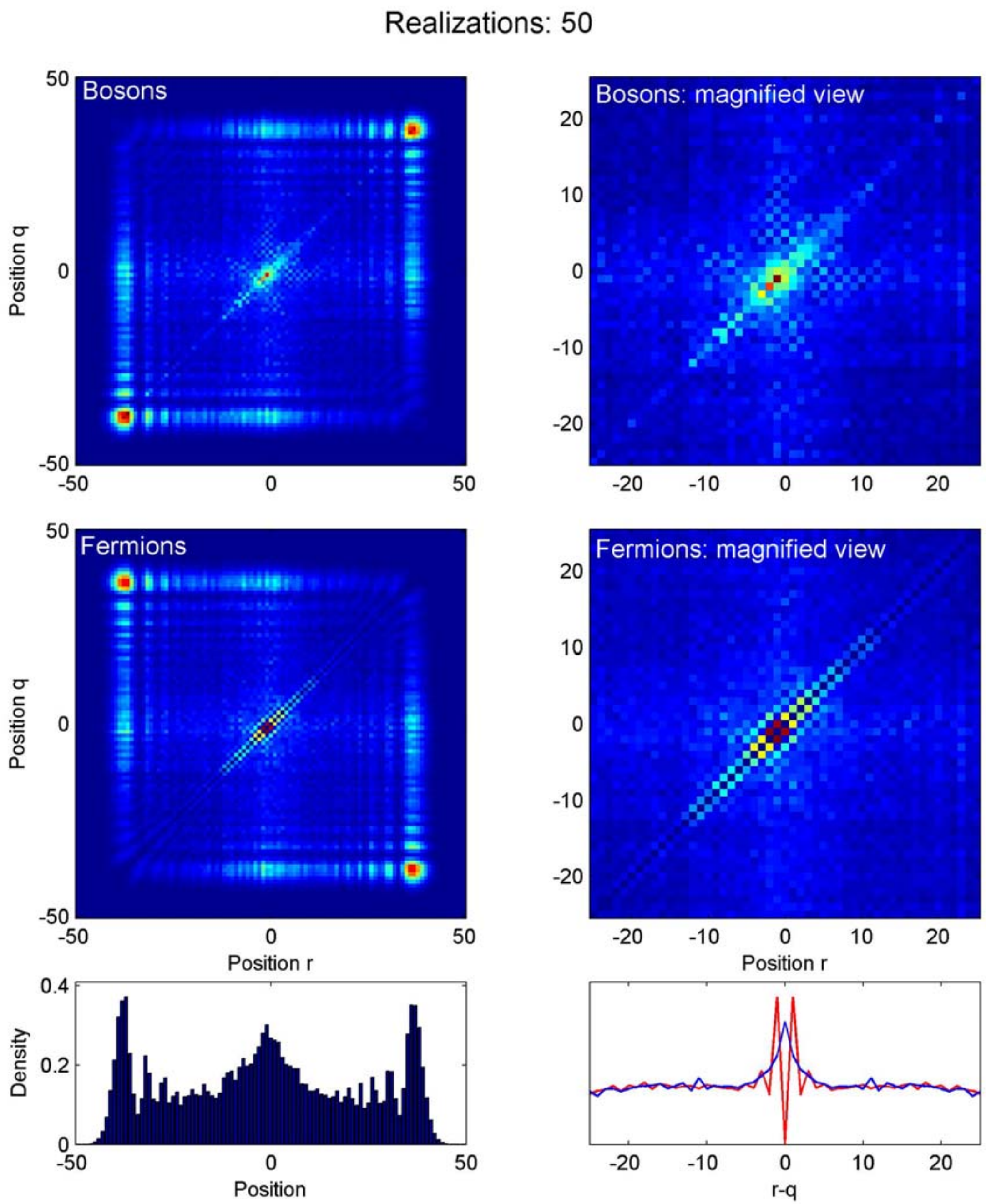



Realizations: 4
Bosons
Bosons: magnified view
Fermions
Fermions: magnified view
Position q
Position r
Density
Position
r-q

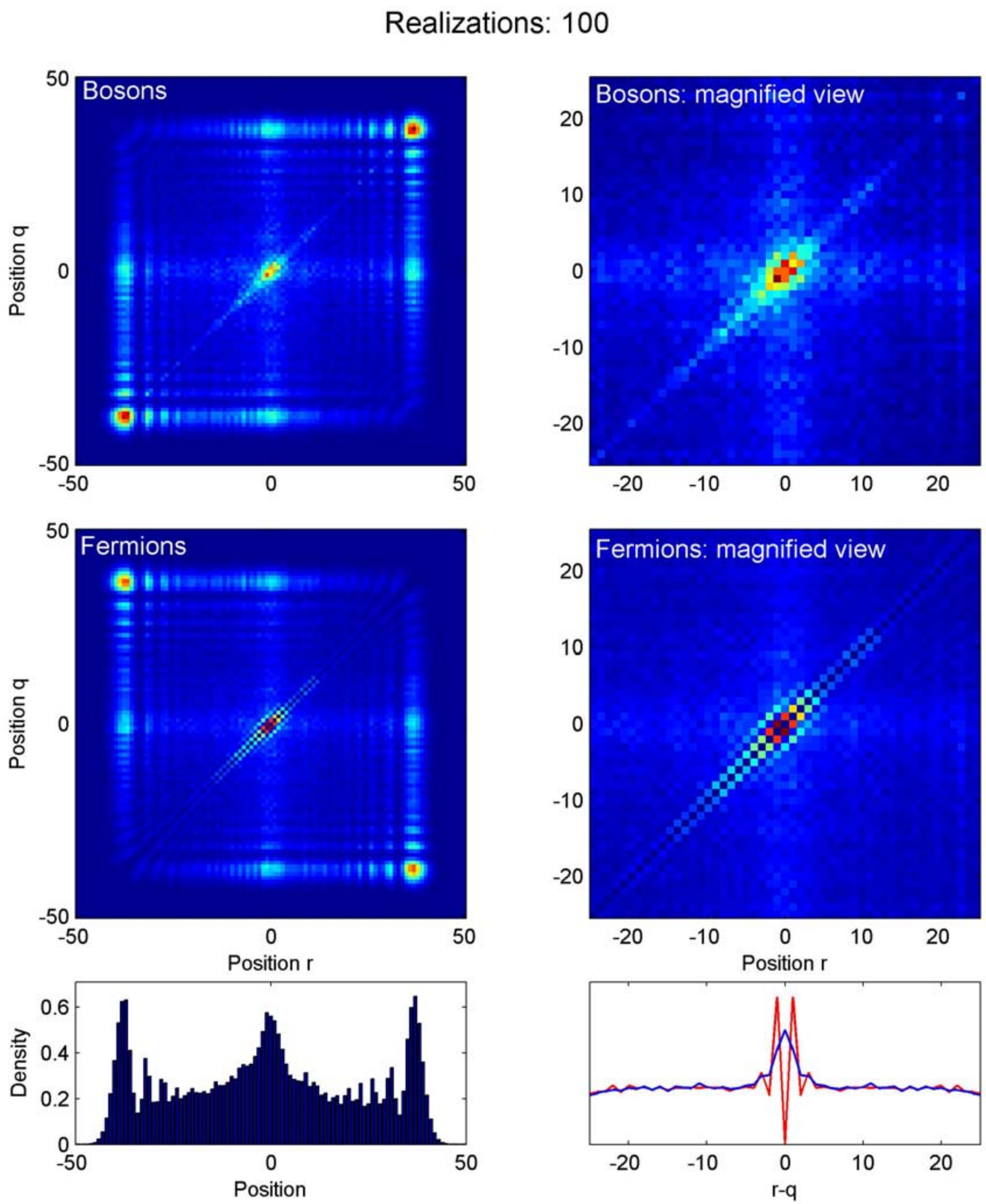

Realizations: 5
Bosons
Bosons: magnified view
Fermions
Fermions: magnified view
Position q
Position r
Density
Position
r-q



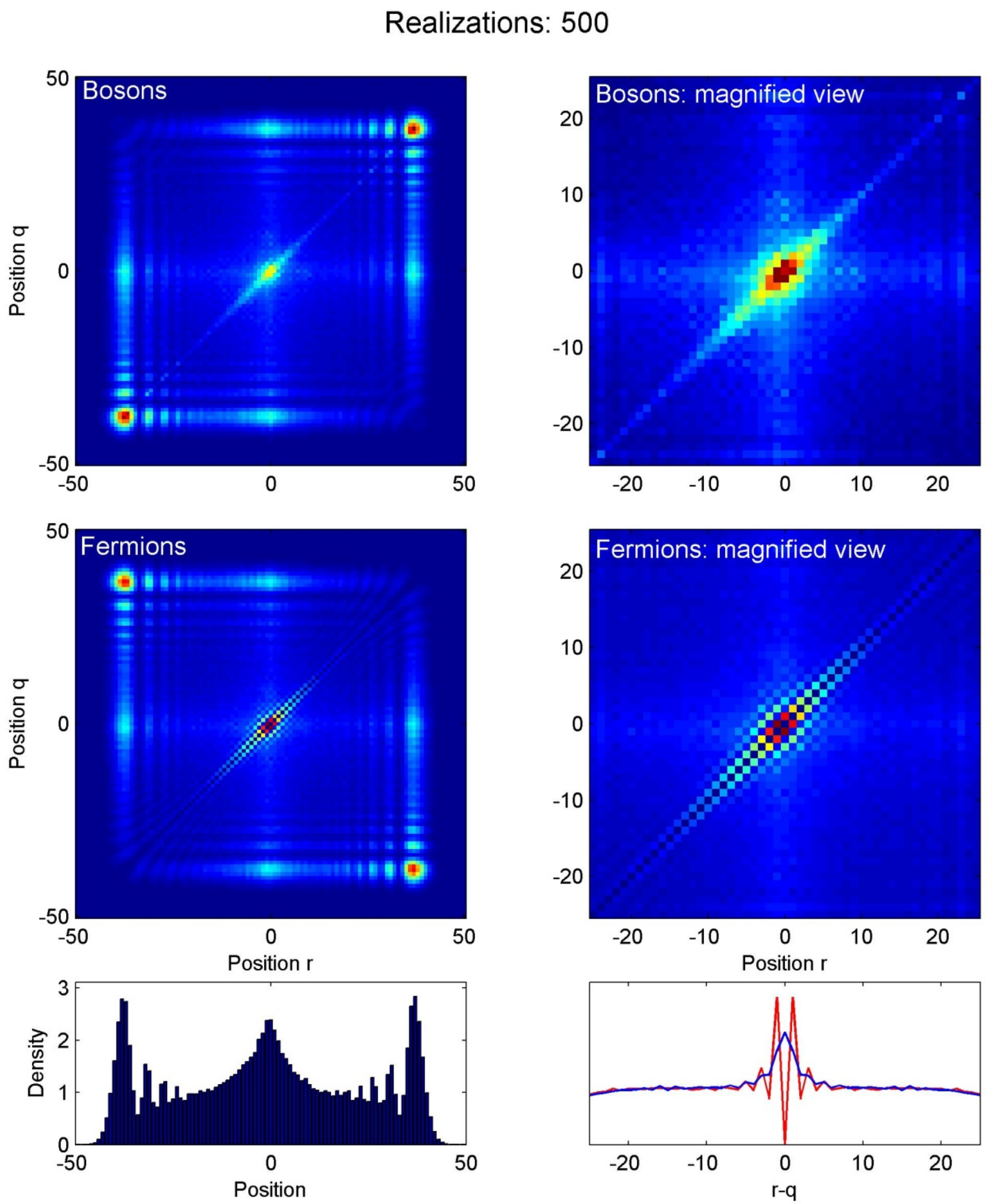
Realizations: 10
Bosons
Bosons: magnified view
Fermions
Fermions: magnified view
Position q
Position r
Density
Position
r-q



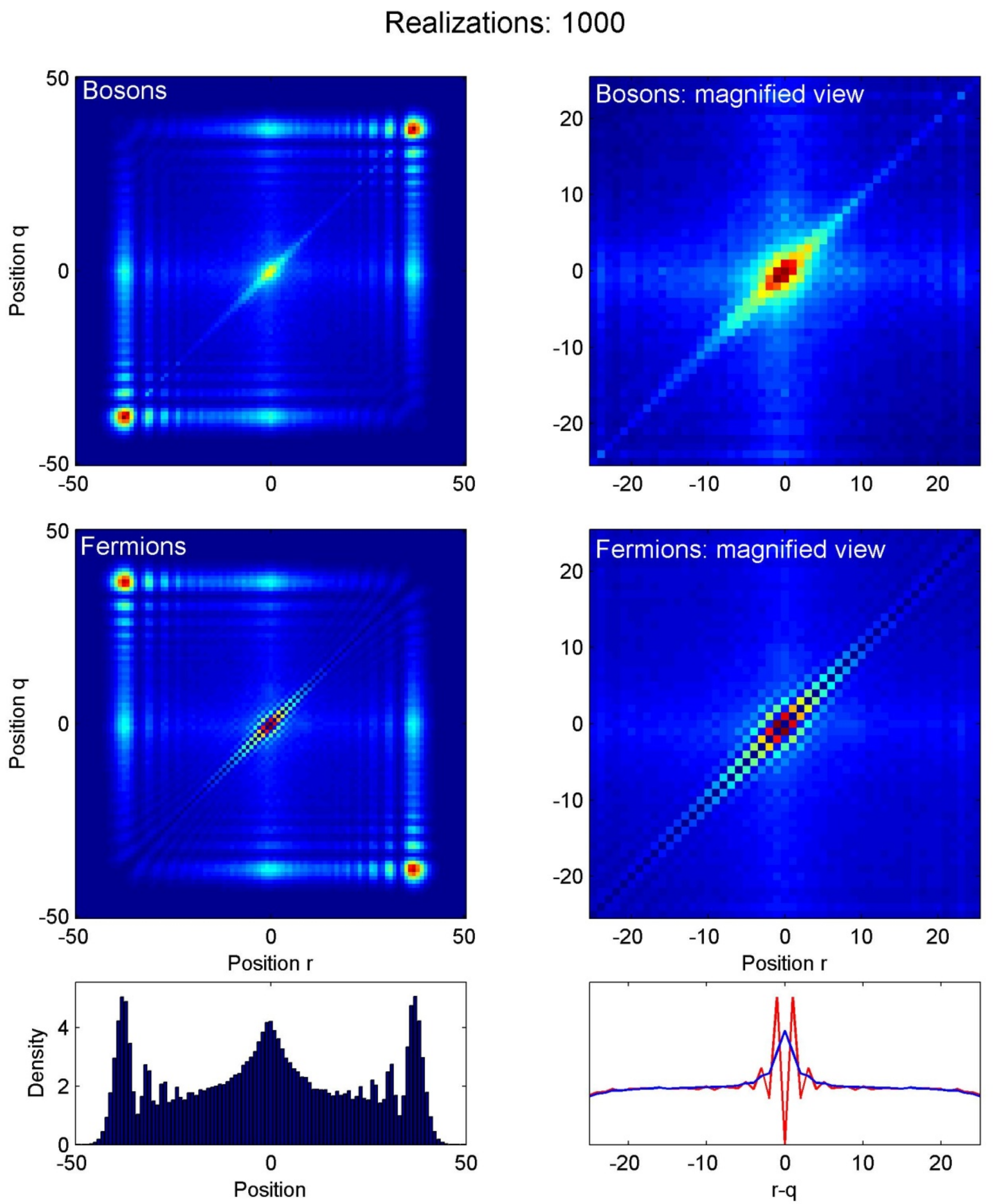
Realizations: 50
Bosons
Bosons: magnified view
Fermions
Fermions: magnified view
Position q
Position r
Density
Position
r-q



Realizations: 100
Bosons
Bosons: magnified view
Fermions
Fermions: magnified view
Position q
Position r
Density
Position
r-q

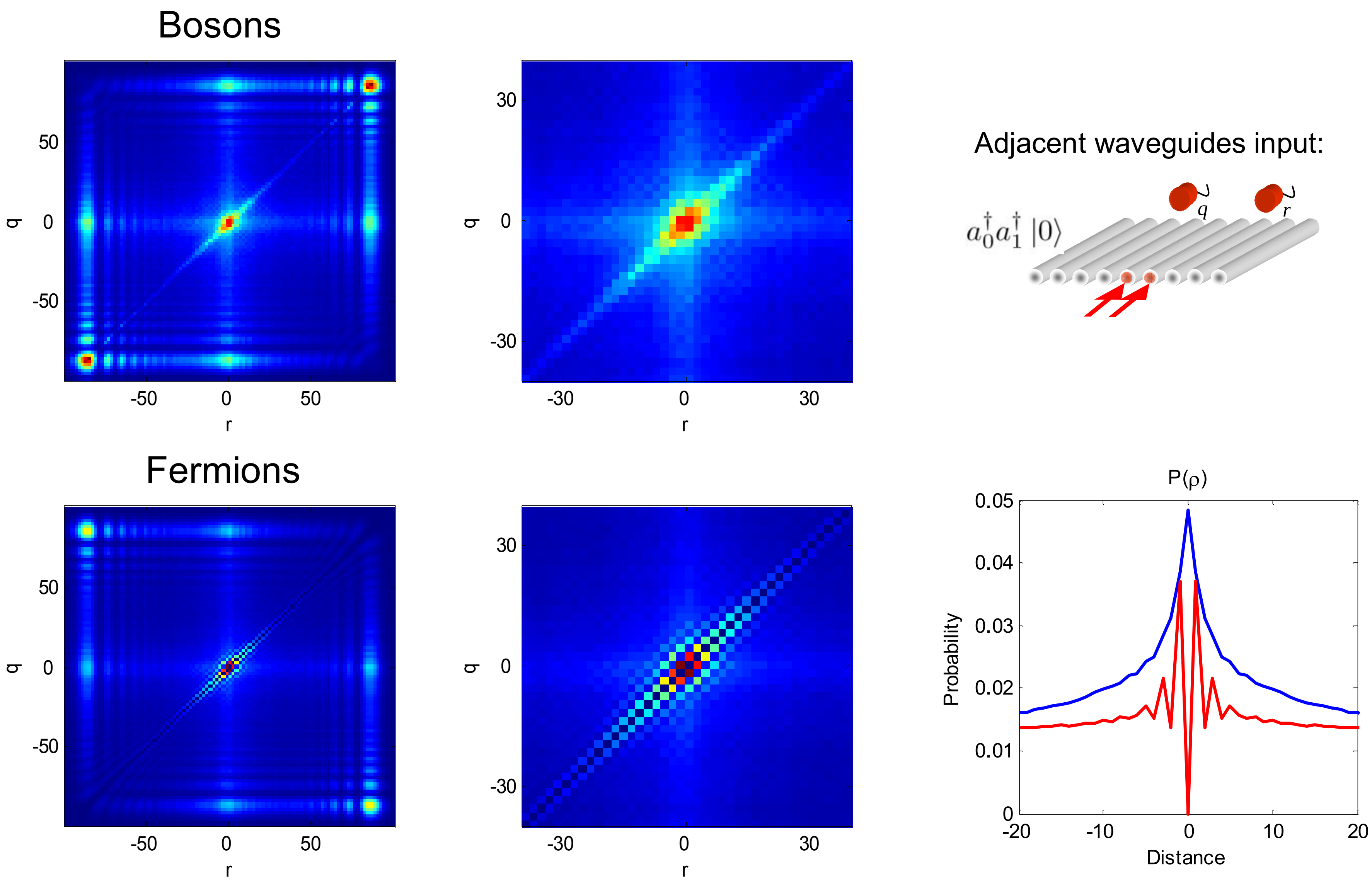


Realizations: 500
Bosons
Bosons: magnified view
Fermions
Fermions: magnified view
Position q
Position r
Density
Position
r-q
50
0
-50
20
10
-10
-20
3
2
1



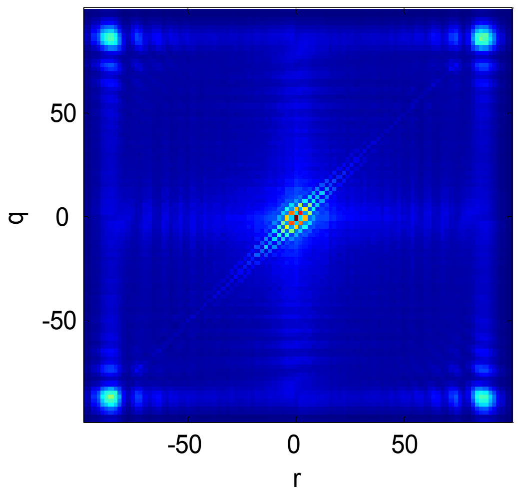
Realizations: 1000
Bosons
Bosons: magnified view
Fermions
Fermions: magnified view
Position q
Position r
Density
Position
r-q

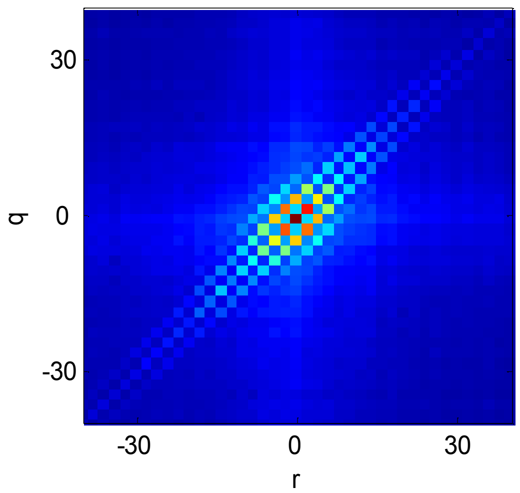
(0,1) input, off-diagonal disorder



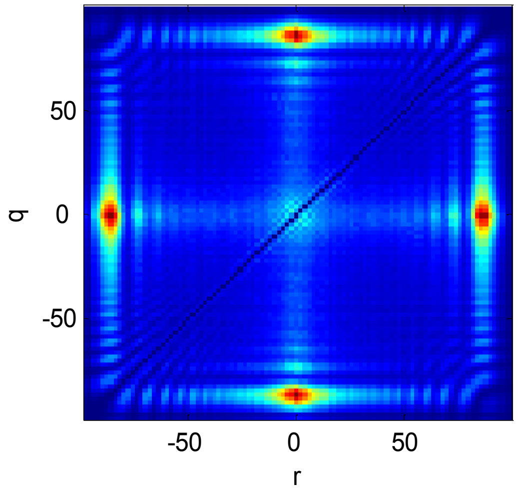
(-1,1) input, off-diagonal disorder

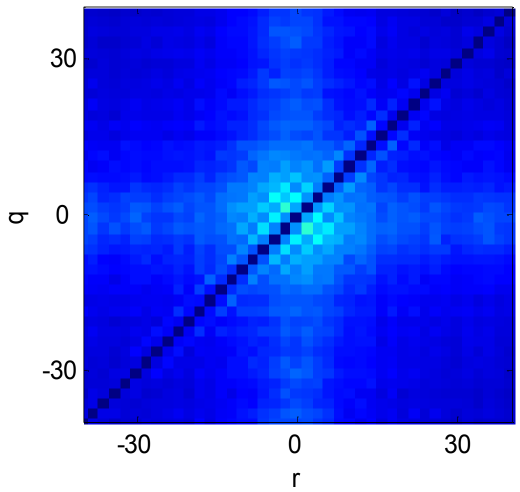
Bosons



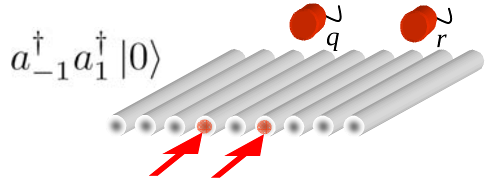


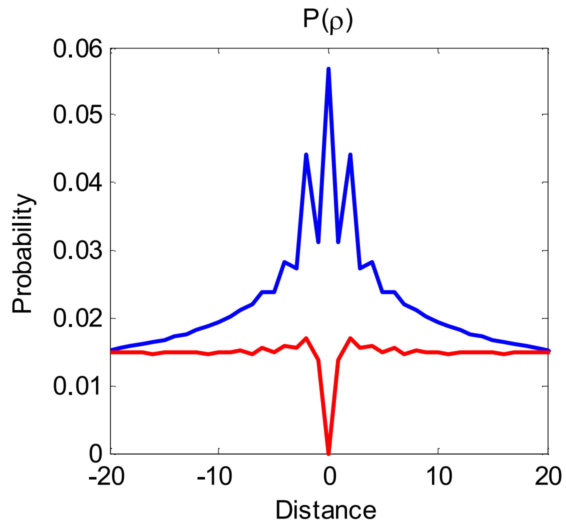
Fermions

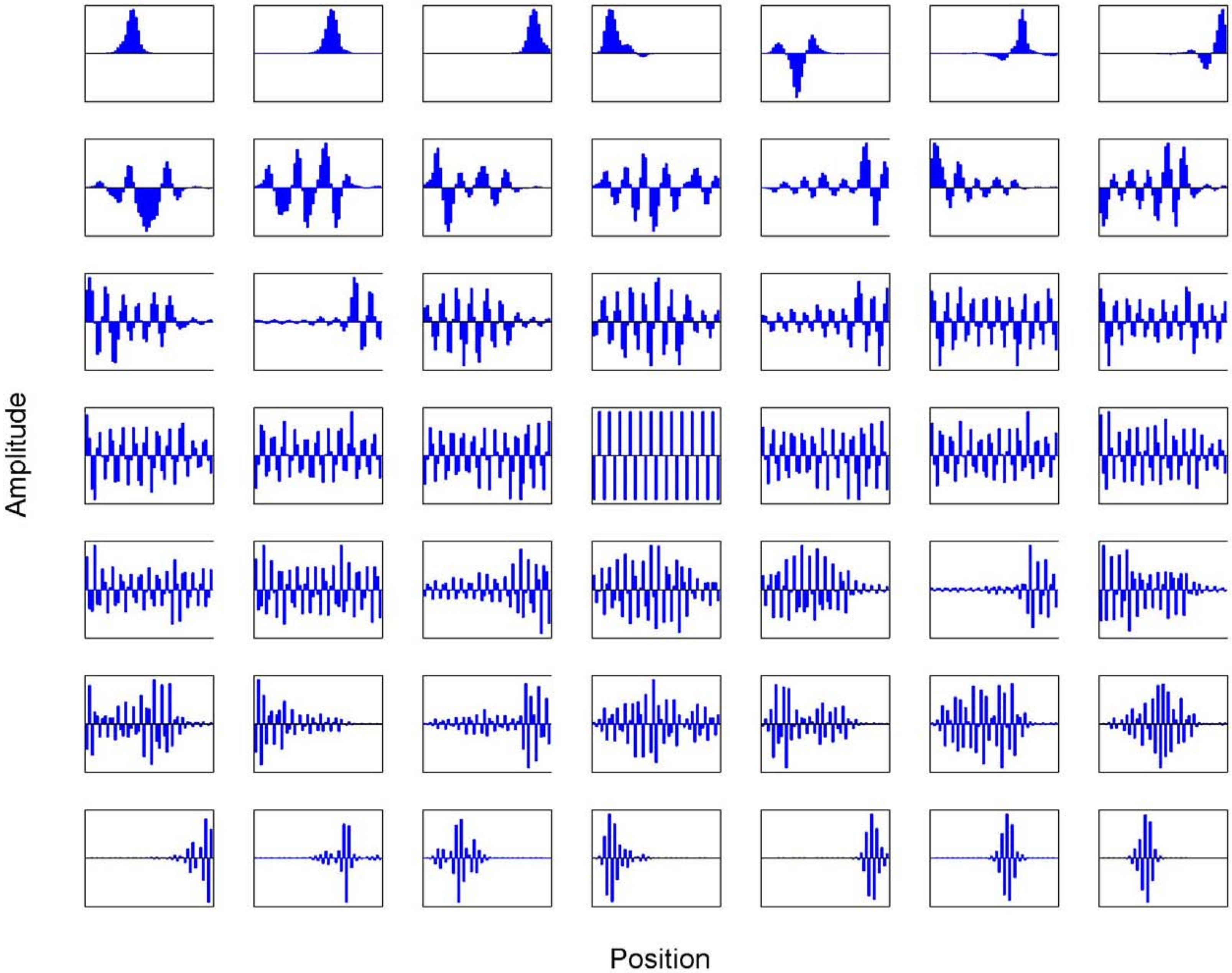




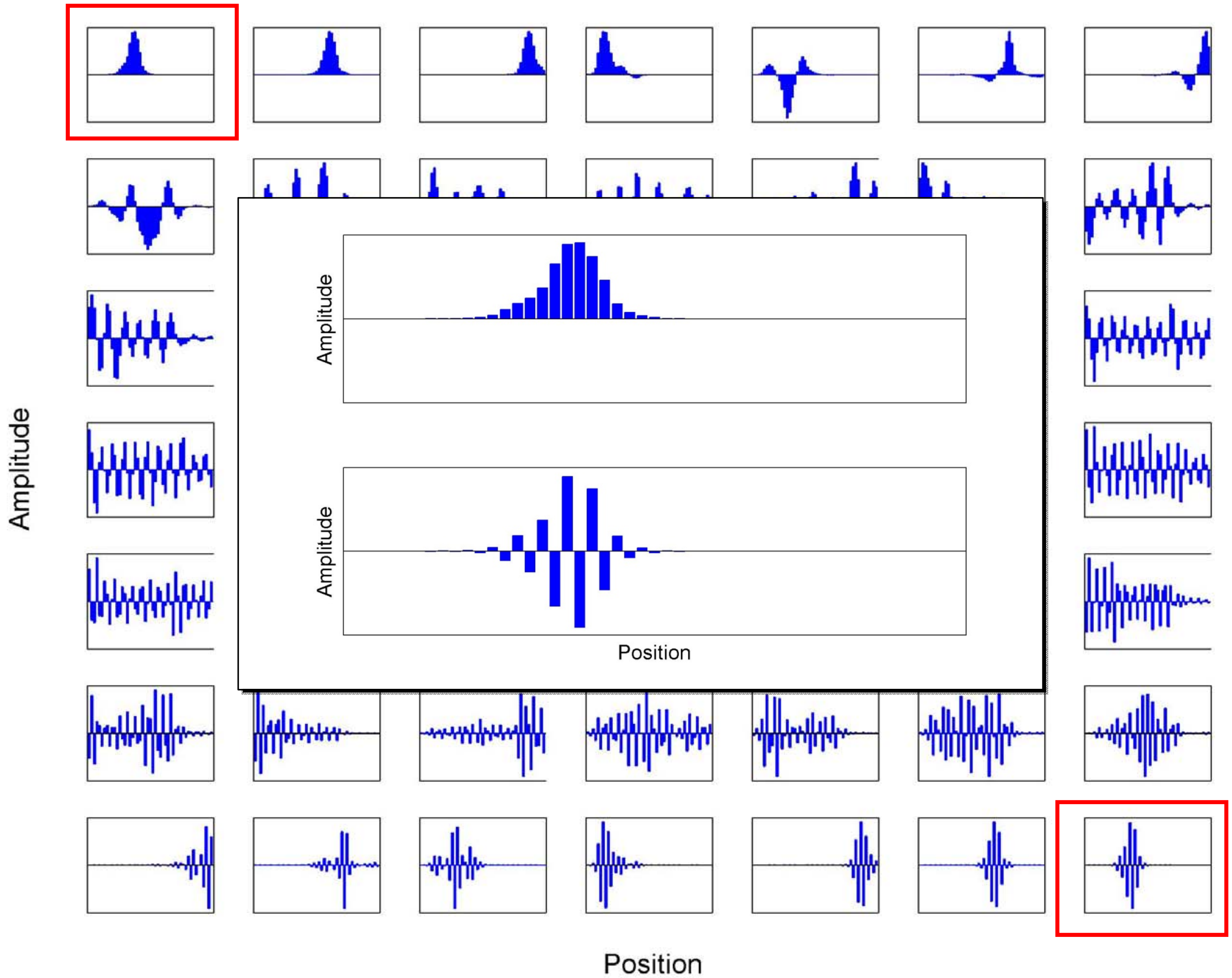
Non-adjacent waveguides input:

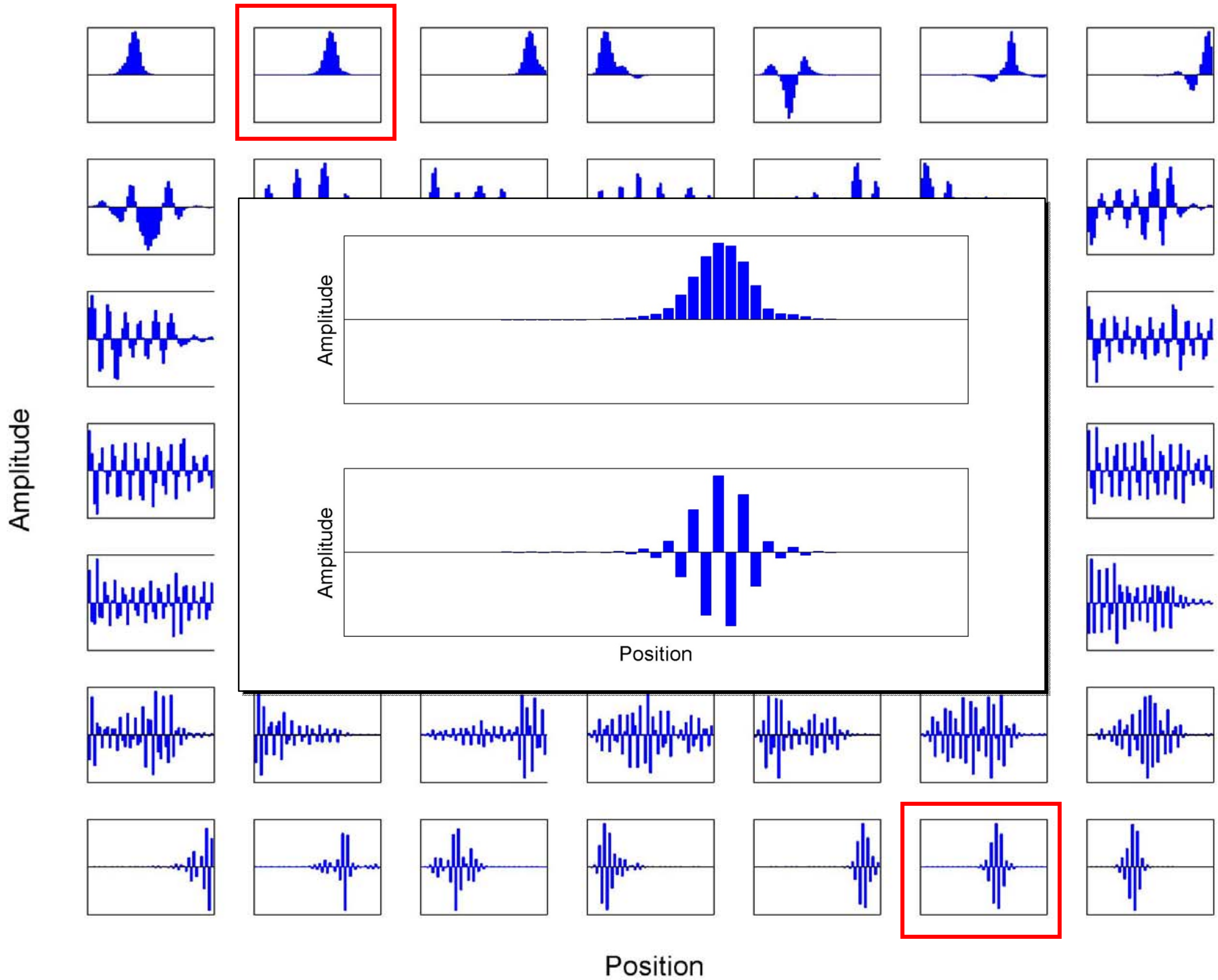


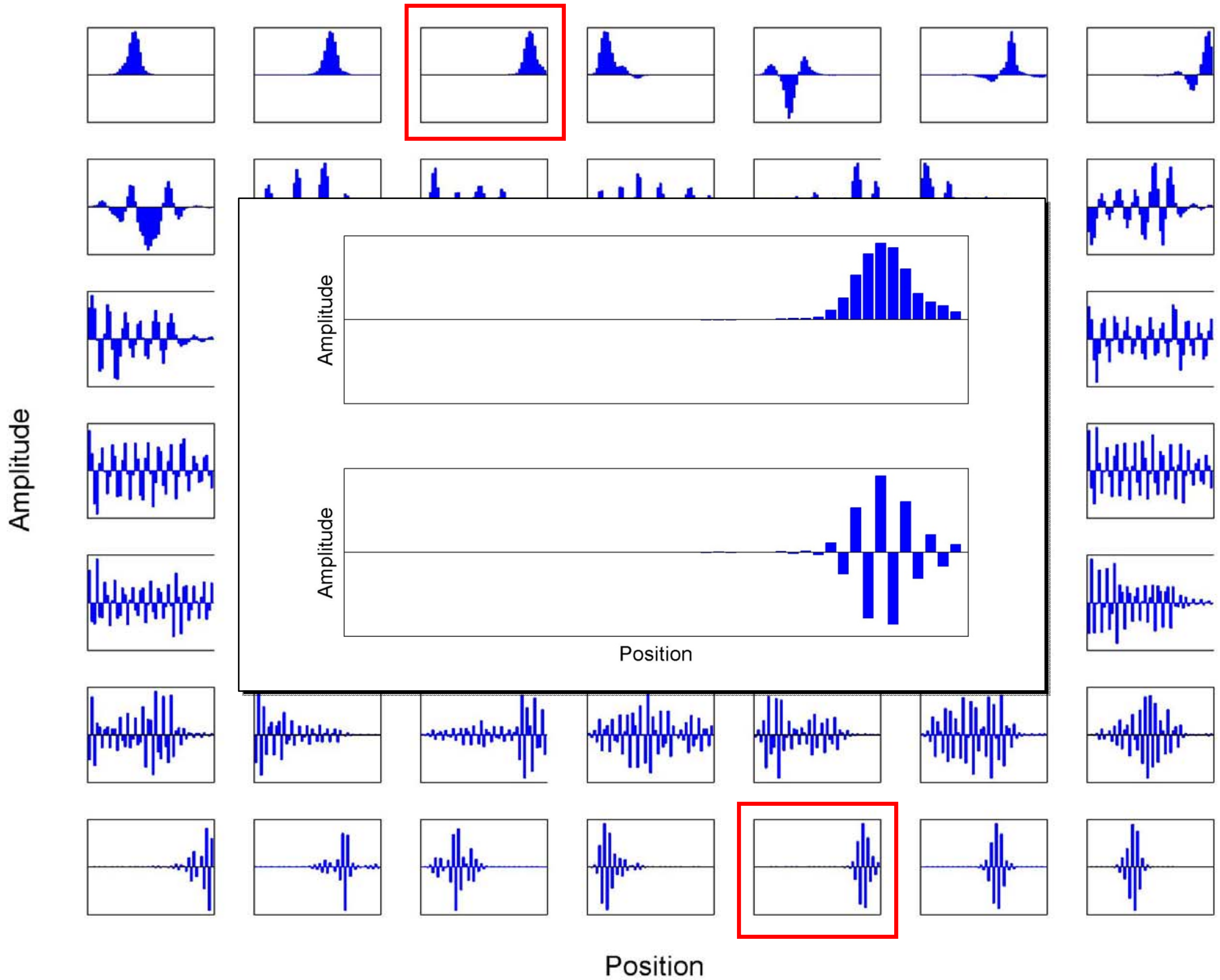




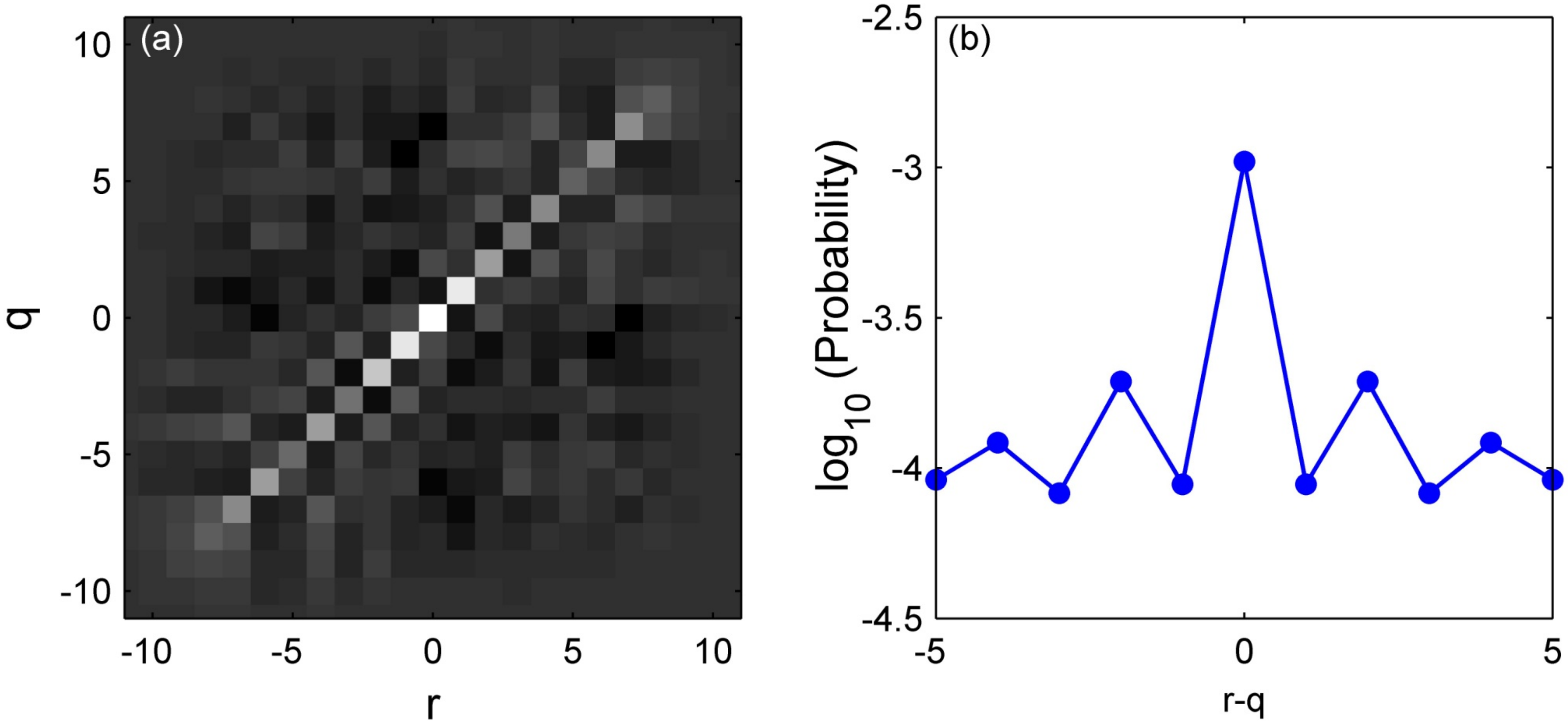
Amplitude
Position


Amplitude
Amplitude
Amplitude
Position
Position

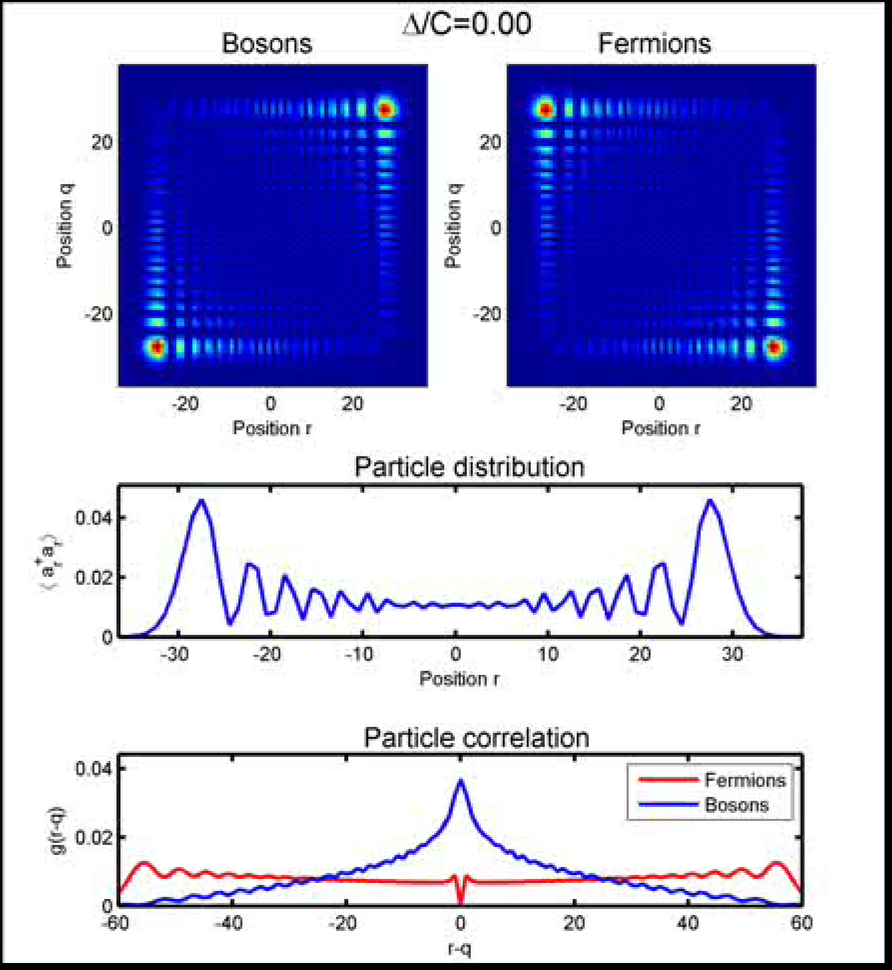

Amplitude
Amplitude
Amplitude
Position
Position


Amplitude
Amplitude
Amplitude
Position
Position

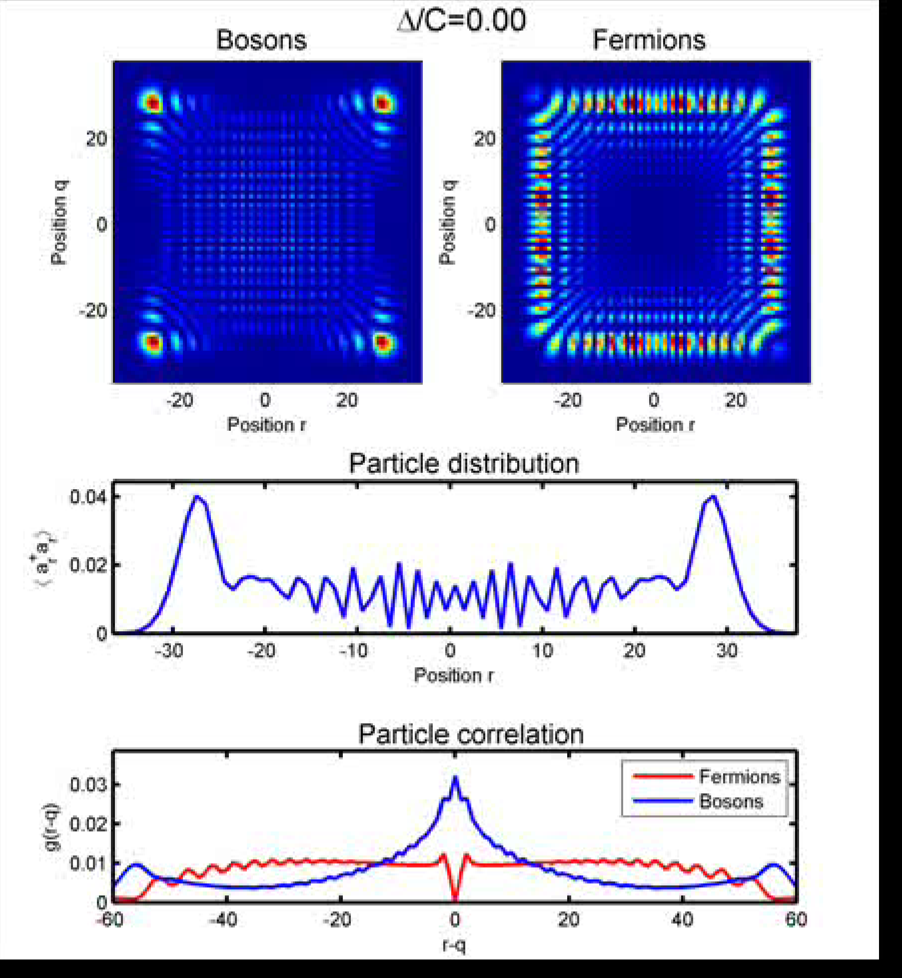
Correlations in disordered sample – single input

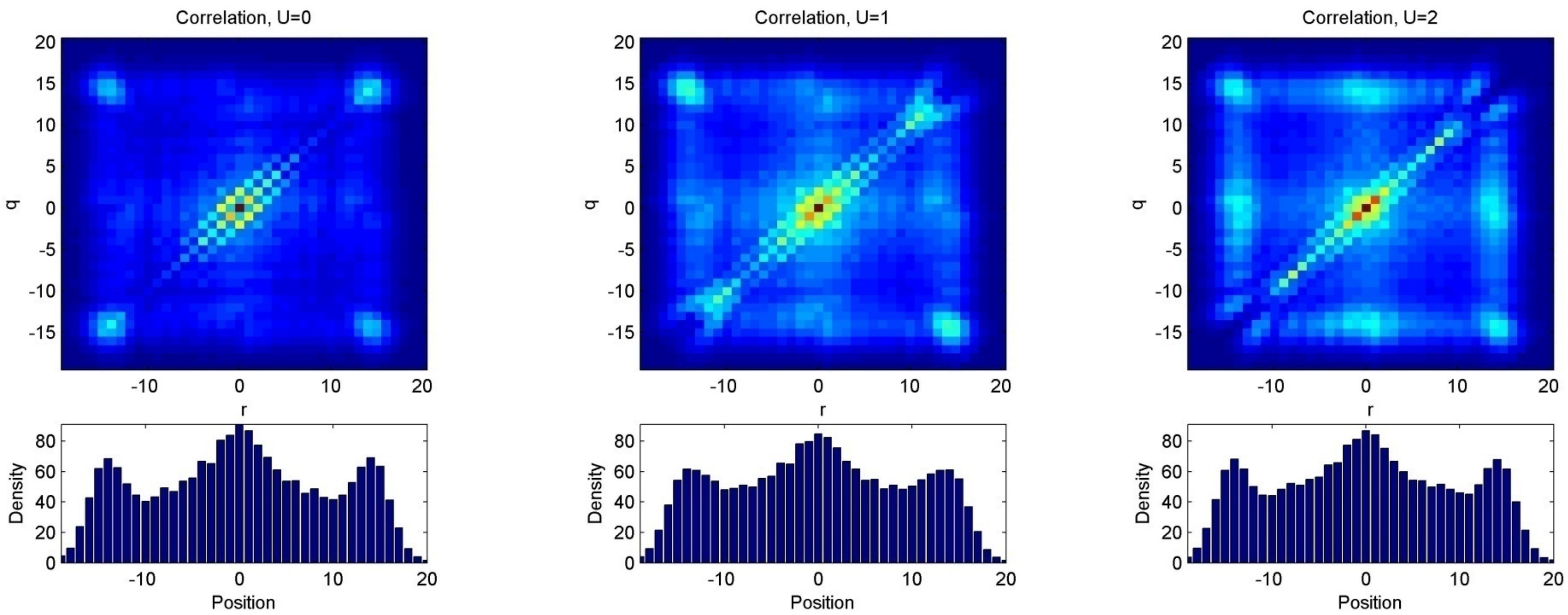


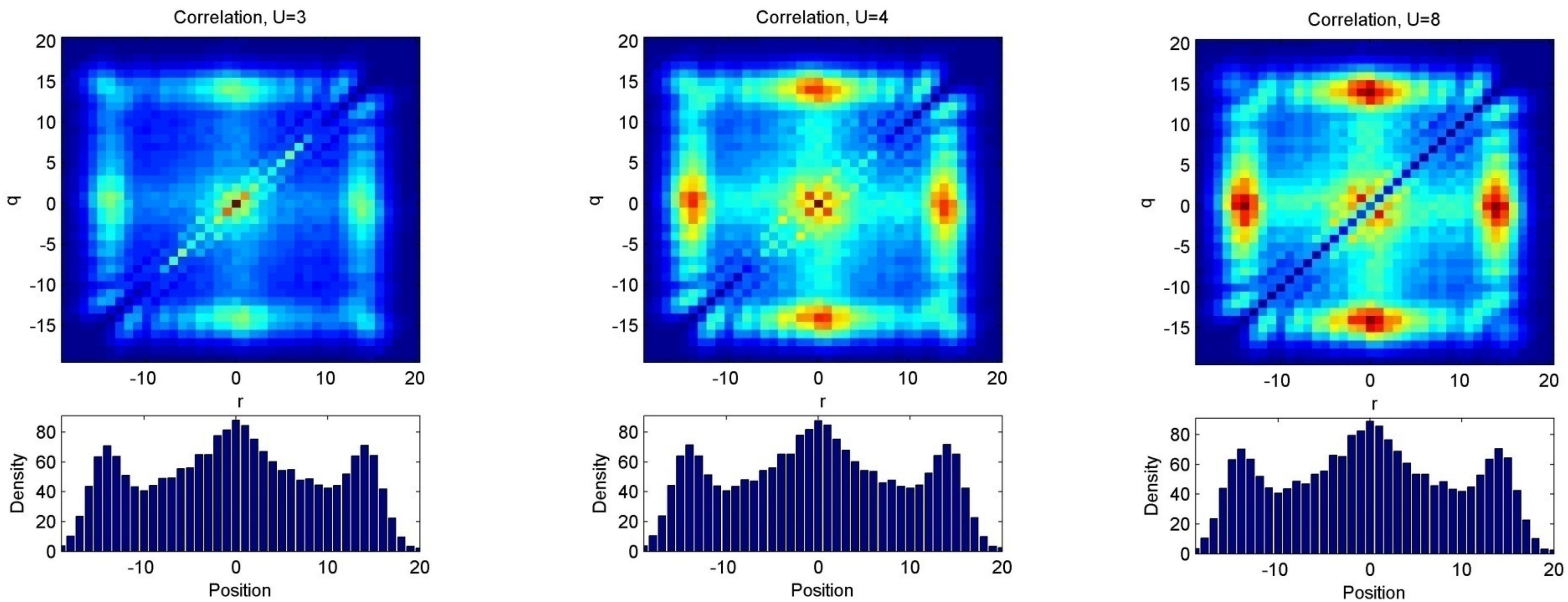
0,1 input – Increasing Disordrer

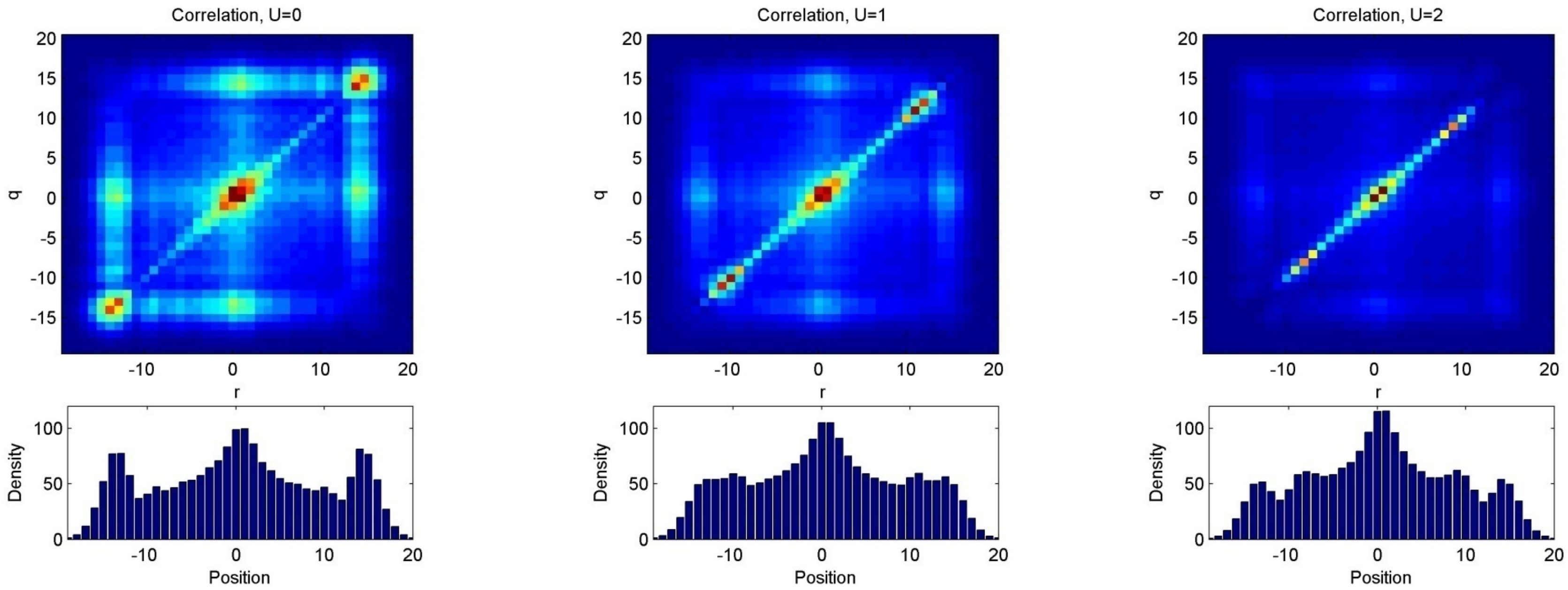


+1,-1 input - Increasing disorder

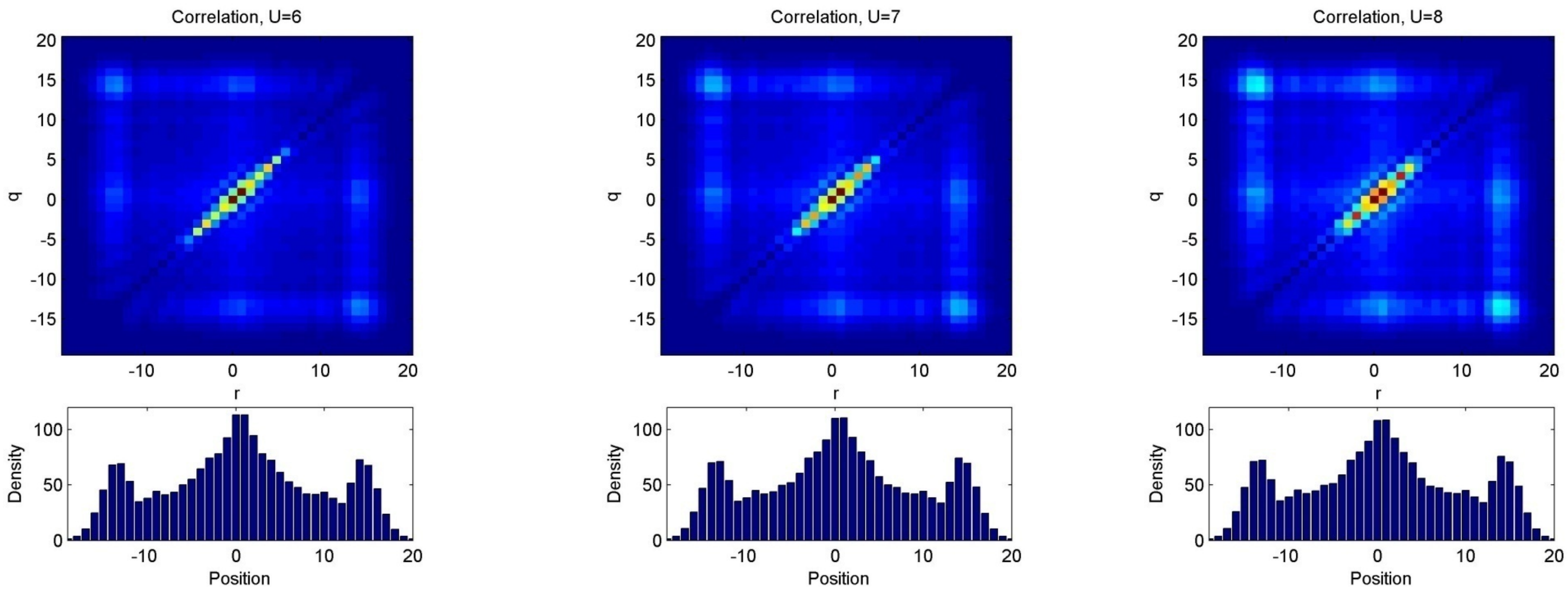



Correlation, U=0
Correlation, U=1
Correlation, U=2
q
r
Density
Position


Correlation, U=3
Correlation, U=4
Correlation, U=8
q
r
Density
Position



Correlation, U=0
Correlation, U=1
Correlation, U=2
q
r
Density
Position



Correlation, U=6
Correlation, U=7
Correlation, U=8
q
r
Density
Position

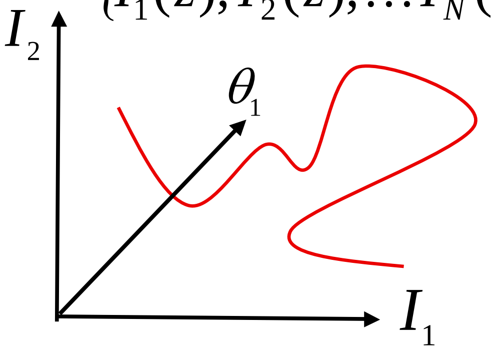
What we study now...

- Quantum & classical correlations patterns in disordered lattices in higher dimensions
- The interplay between interactions and correlations- how correlations are affected by attractive or repulsive interactions
- Correlations in non-stationary (z-dependent) potentials
- Experimental system for quantum correlation measurements

Statistical Mechanics Approach

$$i\frac{d\,u_n}{d\,z} = (u_{n-1} + u_{n+1}) + \Gamma\,|\,u_n\,|^2\,u_n$$

$$u_n = \sqrt{I_n}\,e^{i\varphi_n}$$



- The probability $p(I_1, I_2, \ldots I_N, \varphi_1, \ldots, \varphi_N)$ **maximizes the entropy:**

$$S[p] = -\int p(I_1, \ldots, \varphi_N)\ln\big(p(I_1, \ldots, \varphi_N)\big)\,dI_1 \ldots dI_N\,d\varphi_1 \ldots d\varphi_N$$

- Two conserved quantities (**constraints**)

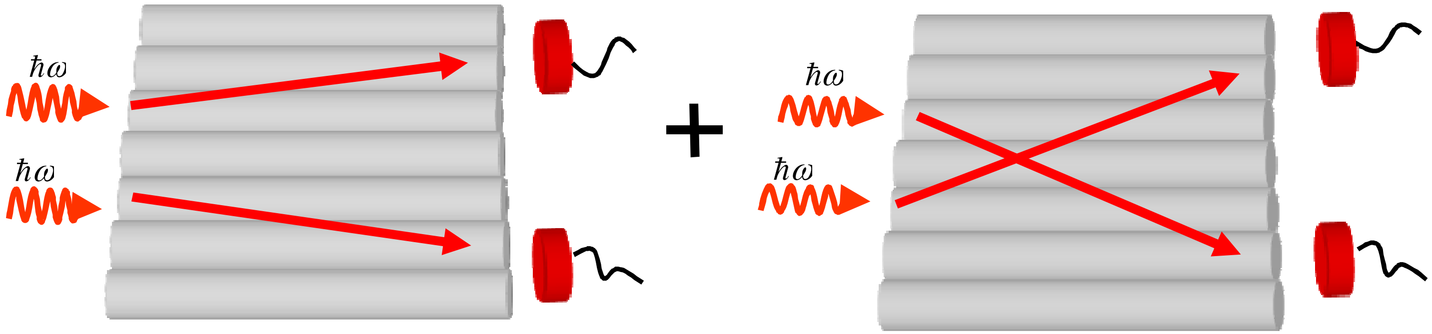
***i)* Hamiltonian** $\quad H \equiv \frac{1}{2}\Gamma\sum_n I_n^{\,2} + 2\sum_n \sqrt{I_n I_{n+1}}\cos(\varphi_{n+1} - \varphi_n)$

***ii)* Total intensity** $\quad A \equiv \sum_n I_n$

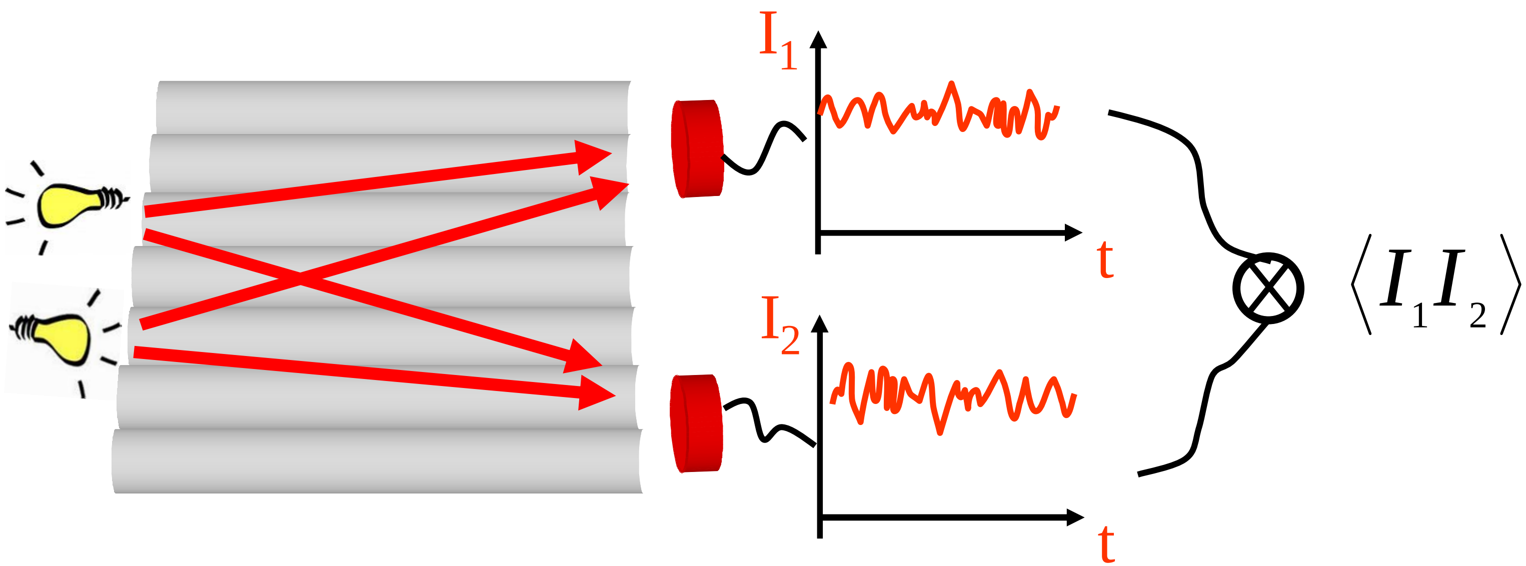
Rasmussen *et al.* PRL **84,** 3740 (2000); Rumpf PRE **77** 036606 (2008)

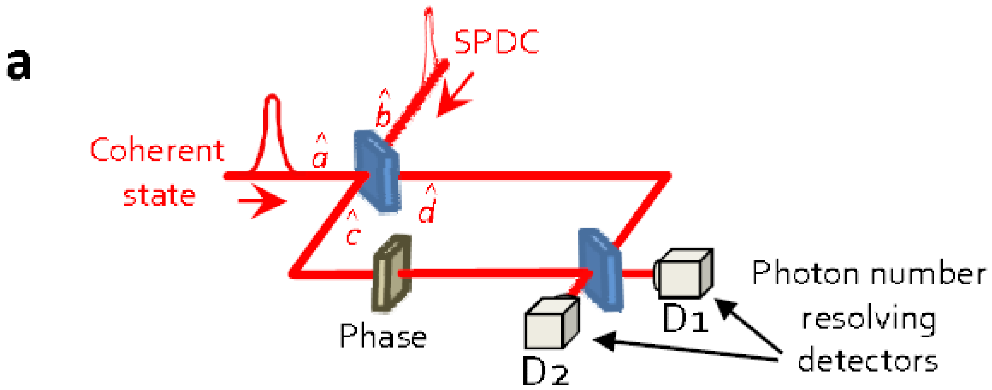
Classical intensity correlations

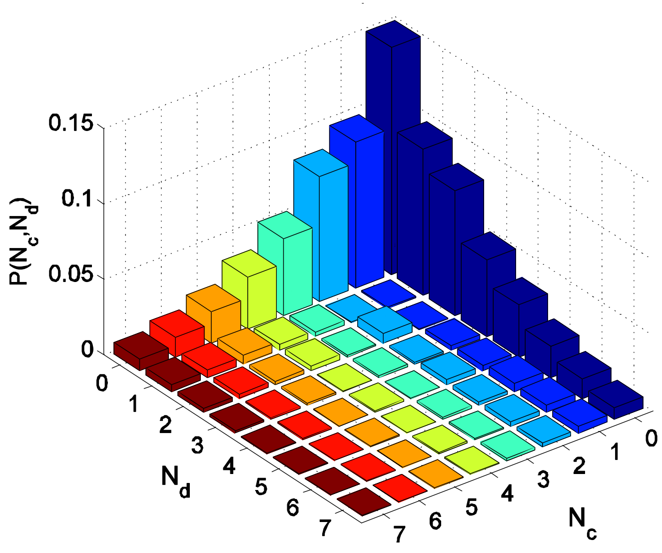
Quantum interference

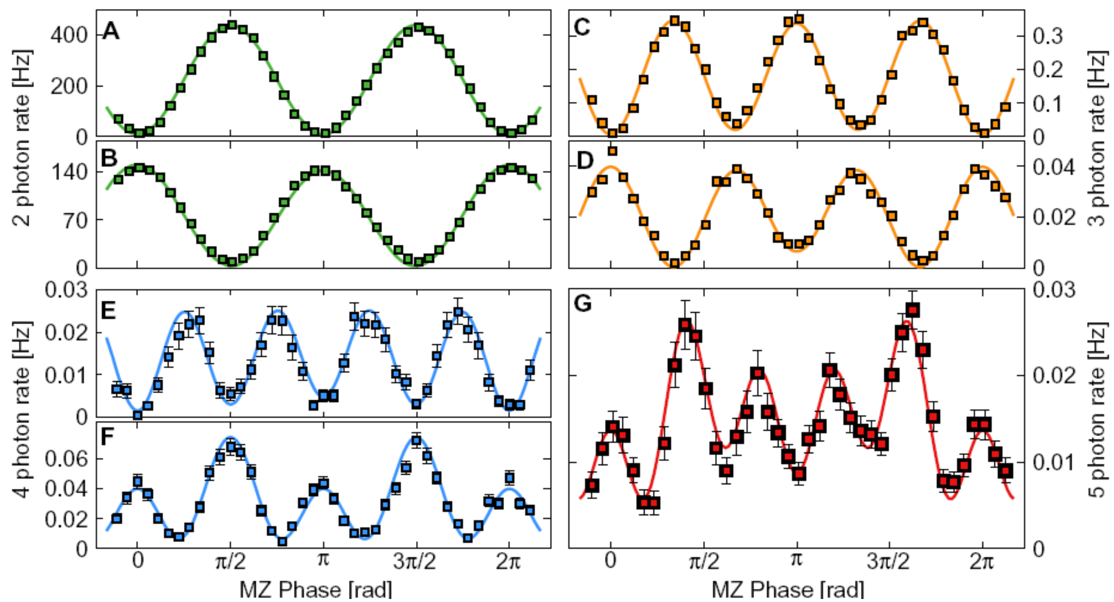


Classical analogue – intensity correlations







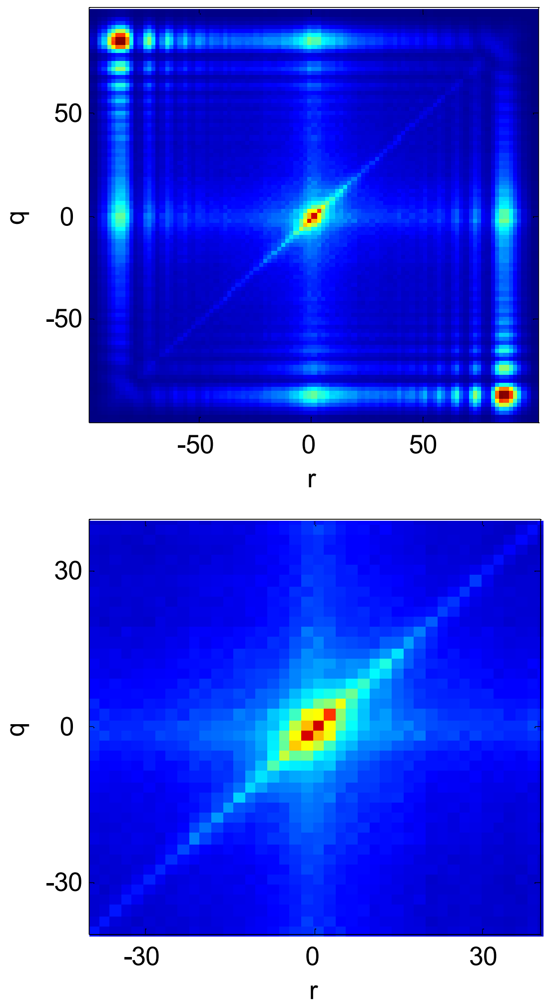


Afek, Ambar, YS, *Science* **328,** 879 – 881 (2010)

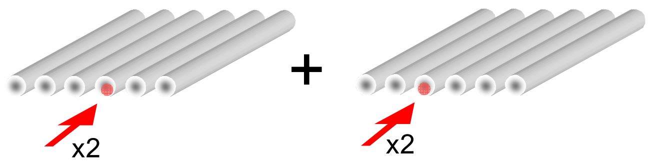
PRL **104**, 123602 (2010)

Adjacent waveguides input, off-diagonal disorder, entangled input state

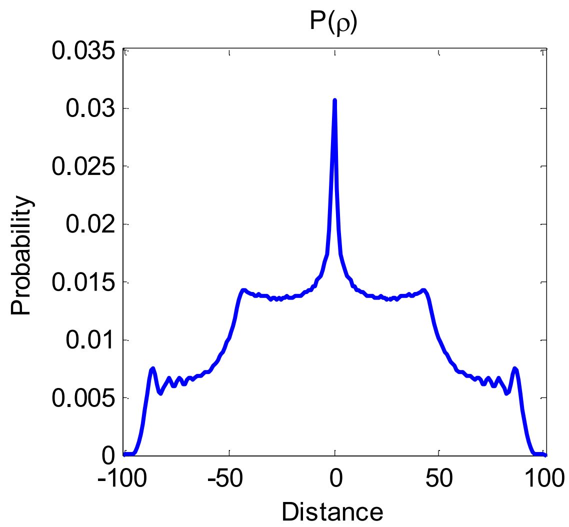
Bosons



Entangled input states



$$|\psi_{in}\rangle = \frac{1}{2}\left(a_0^{\dagger\,2} + a_1^{\dagger\,2}\right)|0\rangle$$



Eigenmodes of a disordered lattice N=99, $\Delta/C=1$:
Intensity distributions