The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific and
Cultural Organization

IAEA
International Atomic Energy Agency

**2177-16**

# ICTP Latin-American Basic Course on FPGA Design for Scientific Instrumentation

*15 - 31 March 2010*

## Digital arithmetic II
## (basic arithmetic operations)

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# Outline

- Digital CMOS design
  - Boolean algebra
  - Basic digital CMOS gates
  - Combinational and sequential circuits
  - Coding - Representation of numbers

# Representing Numbers

How values can be coded ?

In a digital circuit each signal can
take 2 values (0, 1) (Boolean world)

A vector of n bits can represent up to $2^n$ values

# Representing Numbers

How values can be coded ?

What is the meaning of 0100 0110 ?

The character ’F’
The character ’Φ’
The number 46
The number 70
The number 123

Any symbol in a set where the *Card* = 256

# Representing Numbers

How values can be coded ?

by itself a code has no signification

# Representing Numbers

How values can be coded ?

arithmetic : dealing with numbers

How can I represent a number ?

Natural numbers
Relative numbers
Rational numbers
Real numbers

# Representing Numbers

How can I represent a Natural number ?

I need at least n bits for a Natural ranging from 0 to $2^n$-1

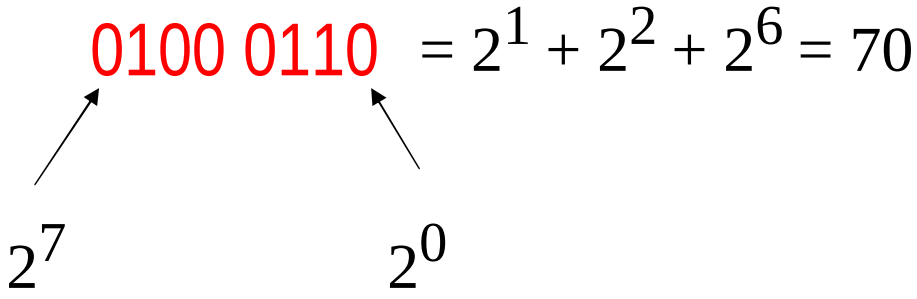
Standards

# Representing Numbers

How can I represent a Natural number ?

Natural Binary Code :

The bits represent the successive powers of 2



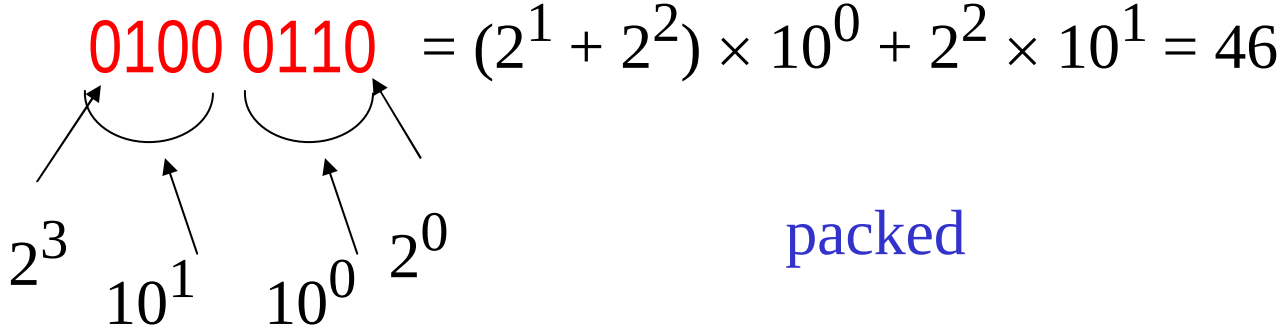
# Representing Numbers

How can I represent a Natural number ?

Binary Coded Decimal :

The bits represent the successive powers of 2

The quartets represent the successive powers of 10




packed

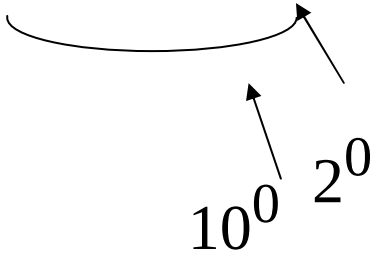
# Representing Numbers

How can I represent a Natural number ?

Binary Coded Decimal - Unpacked :

The bits represent the successive powers of 2
The bytes represent the successive powers of 10
In each byte the 4 Msb are 0

0100 0110 = Illegal  0000 0110 = 6



# Representing Numbers

How can I represent a Relative number ?

Sign + Value

- The bits represent the successive powers of 2
- The Msb represents the sign (1 means negative)

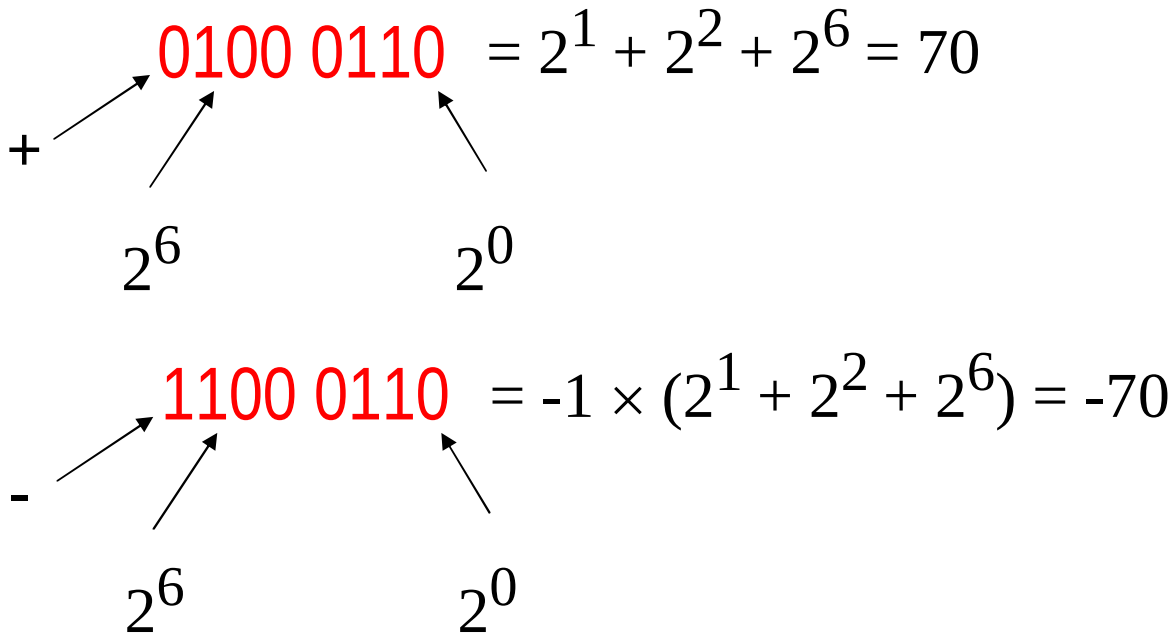
2's complemented

- The bits represent the successive powers of 2
- The Msb represents $-2^n$

# Representing Numbers

How can I represent a Relative number ?

Sign+Value :



# Representing Numbers

How can I represent a Relative number ?

2's complemented :

$0100\ 0110 = 2^1 + 2^2 + 2^6 = 70$

$-2^7$ $2^6$ $2^0$

$1100\ 0110 = 2^1 + 2^2 + 2^6 - 2^7 = -58$

# Representing Numbers

How can I represent a Relative number ?

2's complemented :

$0100\ 0110 = 2^1 + 2^2 + 2^6 = 70$

70

$2^7 = 2^0+2^1+2^2+2^3+2^4+2^5+2^6+1$

$2^7 = 2^0 \quad +2^3+2^4+2^5 \quad +1 + 70$

$1011\ 1010 \quad -70 = 2^0 \quad +2^3+2^4+2^5 \quad +1 - 2^7$

$-70 = \quad 2^1 \quad +2^3+2^4+2^5 \quad - 2^7$

# Representing Numbers

How can I represent a Real number ?

Range

Precision

# Representing Numbers

How can I represent a Real number ?

2's complement Fixed Point :

The bits represents the successive powers of 2

$$0100.0110 = 2^{-3} + 2^{-2} + 2^{2} = 4.325$$

$-2^{3}$ $2^{0}$ $2^{-1}$ $2^{-4}$

# Representing Numbers

How can I represent a Real number ?

Wide range
High precision

Floating Point :
Logarithmic representation

# Representing Numbers

How can I represent a Real number ?

$$R = (-1)^S \times M \times 2^E$$

Normalized scientific representation

S : Sign (1 if negative)

M : Mantissa ( $\in [1, 2[$ )

E : Exponent (Relative number)

# Representing Numbers

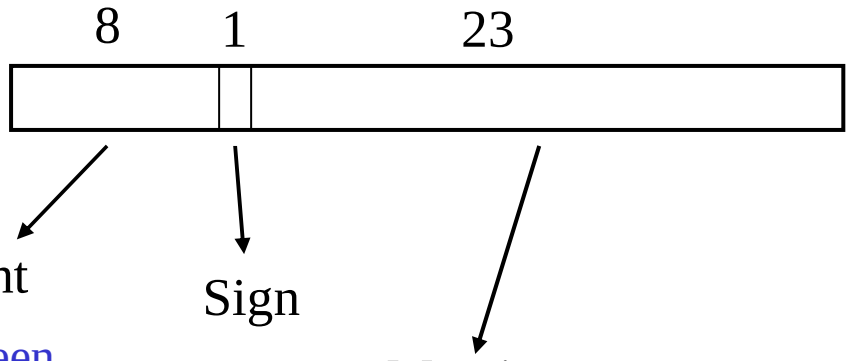
$$R = (-1)^S \times M \times 2^E$$

| | Single Precision 32 bits | Double Precision 64 bits |
|---|---|---|
| S : Sign (1 if negative) | 1 bit | 1 bit |
| M : Mantissa ( $\in [1, 2[$ ) | 23 bits | 52 bits |
| E : Exponent | 8 bits | 11 bits |

# Representing Numbers

$$R = (-1)^S \times M \times 2^E$$

Single precision :




Number between -127 and 127

The code 0000 0000 means -127

Natural Binary Code by Excess of 127

Fixed point positive real number

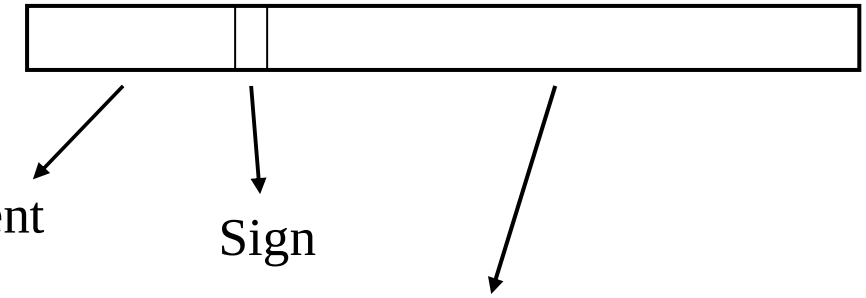
1. • • •

The 1 is not represented !!

# Representing Numbers

$$R = (-1)^S \times M \times 2^E$$

Single precision : Special cases



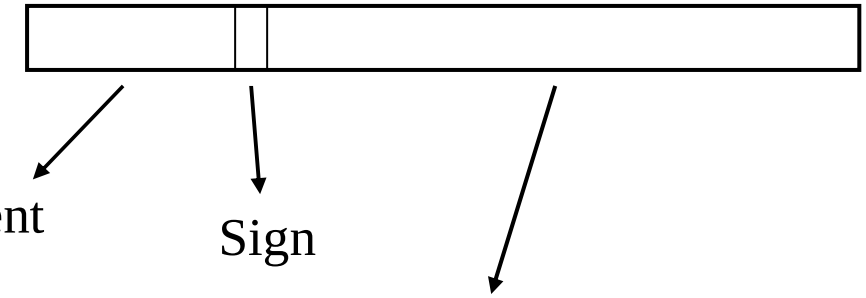

The code 1111 1111 (128) means $\pm\infty$ or an error

.111 … 111 means $\pm\infty$
other values mean error (NaN)

# Representing Numbers

$$R = (-1)^S \times M \times 2^E$$

Single precision : Special cases




The code 0000 0000 (-127) indicates denormalized Mantissa

0.00 … 000 means 0