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School and Workshop on D-brane Instantons, Wall Crossing and Microstate Counting

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Lectures on Kerr/CFT

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Kerr/CFT and (far) Beyond: Lecture 1

Towards Precision Counting for all String Theory Black Holes

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Theme

Overall theme of school: precision microstate counting for black holes.

Focus on D-brane instantons and wall crossing.

My perspective on the subject: I see hints about the structure of non-BPS string theory in these developments.

My goal for these lectures: a pedagogical review of some ingredients to this story, from several complementary points of view.

Disclaimer: Kerr is one example that we can learn from, but it is not the main focus of the lectures.

Outline of Lecture Series.

- ***Phenomenology of Black Hole Entropy***

  An arena for analysis of general non-extreme solutions.

  Some special limits: BPS, the non-BPS branch, extremal Kerr, and their interrelation.

- ***Non-BPS Extremal Black Holes***

  Much similarity to BPS black holes but some significant differences: flat directions, wall crossing, and more.

  This presents obstacles to precision counting.

- ***Hidden Conformal Symmetry***

  A probe perspective on the geometry $\Rightarrow$ scales and symmetries for a CFT description of general black holes.

- ***Precision Counting for General Black Holes***

  A synthesis: the big picture, as I see it. (Still very much in progress).

No References

I will not be giving systematic references.

Thanks to my recent collaborators:

A. Castro, M. Cvetic, J. Davis, E. Gimon, K.Hanaki, C. Keeler, P. Kraus, J. Simon.

Outline of Lecture 1

- ***The Standard Setting for $D=4$ String Theory Black Holes***

  3M5.

- ***Phenomenology of Black Hole Entropy***

  An arena for analysis of general non-extreme solutions.

- ***The non-BPS Branch (=almost BPS)***

  Black holes with many properties in common with BPS, but also significant differences.

- ***A perspective on Kerr/CFT***

  The relation to the BPS sector and (not!) to the non-BPS branch.

Setting for $D = 4$ Discussion

- Type IIA string theory on $T^6 = T^2 \times T^2 \times T^2$, truncated so the three T^2's do not mix.

- Alternative view point (the STU-model): $N = 2$ SUGRA in $D = 4$ with prepotential:
$$F = \frac{X^1 X^2 X^3}{X^0} \,.$$

- The theory has an obvious embedding in $N = 4$ or $N = 8$ SUGRA so description literally valid there. Other $N = 2$ theories have many similar features.

- There are truncations to Maxwell-gravity theory, and to pure gravity. Those applications are of great interest.

The Configuration Space

There are three ***complex*** scalars in the theory: these are the volumes (and B-fields) on the three compact T^2's.

There are four gauge fields in the theory (the 4th is the graviphoton), so black holes generally have 4 electric and 4 magnetic charges.

Interpretation of electric charges: $D0$, and $D2$'s wrapping each T^2.

Interpretation of magnetic charges: $D6$, and $D4$'s wrapping any two of the T^2's.

The Canonical Black Hole

	0	1	2	3	4	5	6	7	8	9
D4	x			x	x	x	x			
D4	x	x	x			x	x			
D4	x	x	x	x	x					
D0	x									

The coordinates $7, 8, 9$ are three non-compact spatial directions, with radial coordinate r.

Each constituent brane enters the solution through a harmonic function, $H_I = 1 + \frac{Q_I}{r}$ with $I = 0, 1, 2, 3$.

The full solution is constructed by superimposing the harmonic functions for the four constituents.

The brane configuration for the canonical black hole gives the 10D solution:

$$\begin{aligned} ds^2 &= -\frac{1}{\sqrt{H_0H_1H_2H_3}}dt^2 + \sqrt{H_0H_1H_2H_3}(dr^2 + r^2 d\Omega_2^2) \\ &+ \sqrt{\frac{H_0H_1}{H_2H_3}}dz^1 d\bar{z}^1 + \sqrt{\frac{H_0H_2}{H_3H_1}}dz^2 d\bar{z}^2 + \sqrt{\frac{H_0H_3}{H_1H_2}}dz^3 d\bar{z}^3 \, , \\ e^{-2\Phi} &= \sqrt{\frac{H_1H_2H_3}{H_0^3}} \, . \end{aligned}$$

The Solution in $4D$

The 4D dilaton is simply

$$e^{-2\Phi_4} = e^{-2\Phi}\mathrm{Vol}_6 = 1 \, ,$$

so the 4D metric in Einstein frame can be read off immediately

$$ds_4^2 = -\frac{1}{\sqrt{H_0 H_1 H_2 H_3}} dt^2 + \sqrt{H_0 H_1 H_2 H_3}(dr^2 + r^2 d\Omega_2^2) \, .$$

The black hole horizon is at $r = 0$ where $H_I \sim Q_I / r$. The entropy computed from the area is

$$S = \frac{A}{4G_4} = \frac{\pi}{G_4}\sqrt{Q_0 Q_1 Q_2 Q_3} \, .$$

Quantized Charges

The Q_i are "physical" charges that depend on moduli. The quantized charges are related by conversion factors

$$C_0 = \sqrt{\frac{2G_4}{v_6}} \, ,$$
$$C^i = \sqrt{2G_4 v_6} \cdot \frac{1}{v_i} \, ,$$

where v_i are volumes of T^2 measured in string units $v_i = V_i/(2\pi l_s)^2$ and the overall volume is $v_6 = v_1 v_2 v_3$.

The dependence of charges on moduli is such that entropy in fact depends on the quantized charges alone:

$$S = 2\pi \sqrt{n_0 n_1 n_2 n_3} \, .$$

This is one aspect of the attractor mechanism.

BPS and Non-BPS

The solution discussed above makes sense only when $Q_0, Q_1, Q_2, Q_3 > 0$, or else the metric is singular when the harmonic functions vanish.

But: the field strengths appear only quadratically in the action so there are also solutions with $Q_i \longrightarrow -Q_i$ in the field strengths, but $Q_i \longrightarrow Q_i$ in the harmonic functions. Alternatively: take Q_i's of any sign, but insert $|Q_i|$ in the harmonic functions.

Convention: take $Q_1, Q_2, Q_3 > 0$, and consider Q_0 of either sign.

The sign is extremely important: $Q_0 > 0$ is the BPS solution, and $Q_0 < 0$ is the non-BPS solution.

The two branches have many ***qualitative*** differences.

Supersymmetry

Type IIA SUSY has two supersymmetry generators, related by the Dirichlet boundary conditions on the D-branes. The resulting relations between the super-translations become

$$
\begin{aligned}
\tilde{\epsilon} &= \Gamma^{\hat{3}\hat{4}\hat{5}\hat{6}}\epsilon \,, \\
\tilde{\epsilon} &= \Gamma^{\hat{1}\hat{2}\hat{5}\hat{6}}\epsilon \,, \\
\tilde{\epsilon} &= \Gamma^{\hat{1}\hat{2}\hat{3}\hat{4}}\epsilon \,, \\
\tilde{\epsilon} &= \mp\epsilon \,,
\end{aligned}
$$

where the choices in the last relation refers to the sign of Q_0. Consistency of the first three relations give

$$\tilde{\epsilon} = -\epsilon \,,$$

so that only $Q_0 > 0$ is consistent with supersymmetry, as claimed.

M-theory Interpretation

We can lift the $D4 - D4 - D4 - D0$ configuration to M-theory with the result:

	0	1	2	3	4	5	6	7	8	9	10
M5	x			x	x	x	x				x
M5	x	x	x			x	x				x
M5	x	x	x	x	x						x
KK	x										x

In this duality frame there are three $M5$-branes that intersect over a line, denoted x_{10}. The fourth charge is momentum along that line.

The change from BPS to non-BPS is just the sign of the momentum along x_{10}.

However, $M5$-branes are chiral so such a change is not a symmetry.

Aside: MSW Configuration

A more general construction: consider M-theory on $X \times S^1$ where X is a CY 3-fold.

Consider an M5-brane on some divisor $P \in X$. (The divisor is roughly a 4D submanifold.)

We can construct 4D black holes with the magnetic P^I, identified with the M5 projected on to basis four-cycles; and the electric charge Q_0 identified with the KK-momentum along S^1.

The entropy of such black holes is $S = 2\pi\sqrt{Q_0 P^3}$ where the triple intersection number of the divisor, $P^3 = \frac{1}{6} C_{IJK} P^I P^J P^K$.

Some Non-Extremal Solutions

Ultimately we would like to understand the entropy of black holes arbitrarily far from extremality, including Scwarzchild black holes.

The natural generalization of the extremal four charge solutions are

$$ds_4^2 = \frac{-1}{\sqrt{H_0 H_1 H_2 H_3}}(1 - \frac{2\mu}{r})dt^2 + \sqrt{H_0 H_1 H_2 H_3}(\frac{1}{1 - \frac{2\mu}{r}}dr^2 + r^2 d\Omega_2^2)$$

$$H_i = 1 + \frac{2\mu \sinh^2 \delta_i}{r} \, .$$

The gauge fields are essentially the inverse harmonic functions, but there is an overall numerical factor such that the physics charge

$$2Q_i = \mu \sinh 2\delta_i \; , \quad i = 0, 1, 2, 3 \; .$$

Note: the entire solution is in parametric form: it is written in terms of δ_i, μ, which encode the four charges and the total mass.

Physical Parameters

We can extract the physical mass from the solution

$$2G_4M \ = \ \frac{1}{2}\mu \sum_{i=0}^{3} \cosh 2\delta_i \ ,$$

and also find the thermodynamic entropy in parametric variables

$$S = \frac{4\pi\mu^2}{G_4} \prod_{i=0}^{3} \cosh \delta_i \ .$$

The general black hole entropy is a complicated function of the four charges and the mass.

The BPS limit: $\delta_i \to \infty$ for $i = 0, 1, 2, 3$.

The non-BPS extremal limit is $\delta_i \to \infty, i = 1, 2, 3$, $\delta_0 \to -\infty$.

Angular Momentum

The generalization of solutions to include angular momentum is *much* more complicated.

The parametric representation of physical variables with rotation

$$\begin{aligned} 2G_4 M &= \frac{1}{2}\mu \sum_{i=0}^{3} \cosh 2\delta_i \ , \\ 2G_4 Q_i &= \frac{1}{2}\mu \sinh 2\delta_i \ , \\ 2G_4 J &= \frac{1}{2}\mu l \left(\prod_{i=0}^{3} \cosh \delta_i - \prod_{i=0}^{3} \sinh \delta_i \right) \ . \end{aligned}$$

The entropy from the area of these black holes

$$S = 2\pi \left(\sqrt{N_R} + \sqrt{N_L} \right) \ .$$

with

$$\begin{aligned} N_L &= \frac{\mu^4}{G_4^2} \left(\prod_{i=0}^{3} \cosh\delta_i + \prod_{i=0}^{3} \sinh\delta_i \right)^2 \ , \\ N_R &= \frac{\mu^4}{G_4^2} \left(\prod_{i=0}^{3} \cosh\delta_i - \prod_{i=0}^{3} \sinh\delta_i \right)^2 - J^2 \ . \end{aligned}$$

The general entropy is a very complicated function of the physical mass, charges, and angular momentum.

There is one notable simplification:

$$\begin{aligned} N_L - N_R &= \frac{4\mu^4}{G_4^2} \prod_{i=0}^{3} \cosh\delta_i \sinh\delta_i + J^2 \\ &= \frac{1}{4G_4^2} \prod_{i=0}^{3} Q_i + J^2 \\ &= \prod_{i=0}^{3} n_i + J^2 \ . \end{aligned}$$

The final line is the rewriting known from the black hole entropy.

The upshot: the difference $N_L - N_R$ is ***independent of moduli***, and ***it is an integer***. These facts hold for the entire class of black holes considered here.

Extremal Limits

The extremal limit $T \to 0$ corresponds to either $N_R \to 0$ or $N_L \to 0$:

$$N_L \to 0 \ : \quad N_R = |n_0| \prod_{i=1}^{3} n_i - J^2 \qquad (\text{Non} - \text{BPS branch}),$$

$$N_R \to 0 \ : \quad N_L = \prod_{i=0}^{3} n_i + J^2 \qquad (\text{BPS branch})\ .$$

Reminder: the quantization condition

$$N_L - N_R = \prod_{i=0}^{3} n_i + J^2\ .$$

Remark: either case corresponds to AdS_2 near horizon geometry ($\times S^2$ for $J = 0$, but generally some fiber).

The Extremal Kerr Limit

The "BPS branch" is only supersymmetric when $J = 0$.

Angular momentum breaks supersymmetry, but ***the solution is continuously related to the supersymmetric branch***.

Extremal Kerr: BPS branch with no charges and so

$$S = 2\pi\sqrt{N_L} = 2\pi|J| \; .$$

This case has been much studied recently.

It is useful to see it as an excited state of the BPS branch.

A later lecture: propose precision counting for extremal Kerr.

Significance of the Inner Horizon

The division of the entropy in to ***two*** contributions is essential. There is a geometric interpretation in terms of the areas of the ***outer*** and ***inner*** horizons

$$
\begin{aligned}
S_R &= 2\pi\sqrt{N_R} = \frac{1}{2}\left(\frac{A_+}{4G_4} - \frac{A_-}{4G_4}\right) ~, \\
S_L &= 2\pi\sqrt{N_L} = \frac{1}{2}\left(\frac{A_+}{4G_4} + \frac{A_-}{4G_4}\right) ~.
\end{aligned}
$$

The universal quantization rule is

$$
N_R - N_L = \frac{A_+ A_-}{(8\pi G_N)^2} = \text{integer} ~.
$$

This semi-classical quantization rule applies (it seems) to all asymptotically flat black holes and black rings in $D = 4, 5$.

A generalization applies for gauged SUGRA and in higher dimensions.

Black Hole Thermodynamics

The Hawking temperature the general black holes is quite complicated. It is useful to write is as

$$\beta_H = \frac{1}{2}(\beta_R + \beta_L) \ ,$$

where

$$\begin{aligned} \beta_R &= \frac{2\pi\mu^2}{\sqrt{\mu^2 - l^2}} \left(\prod_i \cosh\delta_i + \prod_i \sinh\delta_i \right) \ , \\ \beta_L &= 2\pi\mu \left(\prod_i \cosh\delta_i - \prod_i \sinh\delta_i \right) \ . \end{aligned}$$

The corresponding rotational velocity is

$$\beta_H \Omega = \frac{2\pi l}{\sqrt{\mu^2 - l^2}} \ .$$

Generalized Thermodynamics

There is also a geometrical interpretation of the partial temperatures β_R, β_L in terms of ***two*** surface accelerations of the ***outer*** and ***inner*** horizons

$$
\begin{aligned}
\beta_R &= 2\pi \left(\frac{1}{\kappa_+} + \frac{1}{\kappa_-} \right) \, , \\
\beta_L &= 2\pi \left(\frac{1}{\kappa_+} - \frac{1}{\kappa_-} \right) \, .
\end{aligned}
$$

There is a generalized first law

$$
dM - \Omega dJ = T_R dS_R + T_L dS_L \, .
$$

We will interpret the two temperatures as independent temperatures of Right and Left moving excitations in a dual CFT description of the black holes.

Summary

- Introduced the canonical D4-D4-D4-D0 brane configurations. Emphasis: the ***sign*** of Q_0 determines whether the configuration is BPS or non-BPS.
- Introduced a much more general family of four-charge solutions that includes Schwarzchild, Kerr, and Reissner-Nordström black holes as special cases.
- A generalized quantization condition that holds for the most general black holes.
- One application of the generalized quantization condition: study extremal limits. The BPS and non-BPS branches are qualitatively different, with extremal Kerr part of the BPS branch.
- The ***inner horizon*** is responsible for the chiral (left-right) split. In particular, the product of inner and outer area is independent of moduli, and integer quantized.

Outline of Remaining Lectures

- ***More on the non-BPS Branch***.

- ***Hidden Conformal Symmetry***

  More on the Phenomenology of general black holes, with an AdS/CFT perspective.

- ***Precision Counting for General Black Holes***