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Lectures on Kerr/CFT

Finn LARSEN

Michigan Center for Theoretical Physics
University of Michigan Randall Laboratory
450 Church Street
Ann Arbor, MI 48109-1040
U.S.A.

Kerr/CFT and (far) Beyond: Lecture 2

Aspects of non-BPS Black Holes in D=4

Finn Larsen

Michigan Center for Theoretical Physics

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Outline of Lecture 2

- ***What is the most general black hole solution?***

  Answer using ***duality orbits*** and generating solutions.

- ***Surprising cancellations*** for non-BPS black holes: the mass formula, the attractor behavior, and others.

- Wall crossing: the $D0 - D6$ ***bound state***.

- ***A first order phase transition*** relates the non-BPS branch to a BPS branch.

- Towards a ***microscopic description*** of non-BPS black holes.

  First do no harm: beyond analytical continuation.

Reminder: The STU Model

- The ***STU-model***: $N = 2$ SUGRA with prepotential $F = X^1 X^2 X^3 / X^0$. All considerations generalize to $N = 4, 8$ supergravity.

- ***There are 3 complex scalar fields***: $z^i = X^i / X^0 = x^i - iy^i$ $(i = 1, 2, 3)$.

  Interpretation: this is a consistent truncation of type IIA string theory on $T^6 = T^2 \times T^2 \times T^2$. The scalars are complexified volumes of the three T^2s.

- ***There are 8 charges***: (Q_I, P^I) with $I = 0, 1, 2, 3$.

  Interpretation: Electric charges are $D2$'s (wrapping the T^2s) and the $D0$; magnetic charges are the dual $D4$'s and the $D6$.

The Most General Black Hole

Black holes are usefully parametrized by their asymptotic behavior.

Black hole uniqueness theorems indicate that the most general black hole depends on:

- Gravitational charges: M and J.
- Electromagnetic charges: (Q_I, P^I) with one value of I for each $U(1)$ field. Presently $I = 0, 1, 2, 3$.
- The asymptotic values z^i_∞ of the complex scalar fields . Presently $i = 1, 2, 3$.

Duality Orbits

The classical STU theory has $SL(2,R)^3$ duality symmetry and there is no loss in generality by constructing classical solutions only up to continuous duality.

So: the most general solution (up to duality) has ***8+6-9=5*** parameters (not counting duality invariant M, J).

In detail: the 6 real moduli parametrize the coset $[SL(2)/U(1)]^3$ so we can use $[SL(2)/U(1)]^3 \subset SL(2)^3$ to set the asymptotic moduli equal to some reference value, such as $z^i = -i, i = 1,2,3$.

The remaining $U(1)^3$ duality then simplifies the 8 charges to five parameters, keeping moduli fixed.

The upshot: ***the canonical four charge solutions (discussed in previous lecture) are not sufficiently general***.

Entropy of Extremal Black Holes

The black hole entropy is independent of scalars and invariant under the full $SL(2,R)^3$ duality group. So it must be a function of the unique quartic invariant

$$J_4 = Q_0 P^1 P^2 P^3 - P^0 Q_1 Q_2 Q_3 - \frac{1}{4}(Q_I P^I)^2 + \sum_{i<j} P^i Q_i P^j Q_j \ .$$

Note $J_4 > 0$ for BPS black holes and $J_4 < 0$ for non-BPS black holes.

The entropy fixed by the simplest four parameters solutions must then be general

$$S = \frac{\pi}{G_4} \sqrt{|J_4 + J^2|} \ .$$

BPS and non-BPS differ just by the sign of $J_4 + J^2$ so the ***entropy*** of the two branches is related by analytical continuation.

Analytical continuation is special to the entropy and is due to near horizon enhancement of symmetries.

Non-BPS Generating Solutions

We ***need five parameters*** for the most general solution, up to duality. ***They need not all be charges.***

An instructive duality frame: ***four charges***: $\overline{D0} - D4 - D4 - D4$; ***and one modulus***: a "diagonal" B-field, $z^i = B - i$ with the same B for $i = 1, 2, 3$.

Another instructive duality frame: ***two charges***: $D0 - D6$; ***and three moduli***: independent B-fields on the three T^2's, $z^i = B^i - i$ for $i = 1, 2, 3$.

Explicit solutions can be written down. We will just discuss the physics, in the case of no rotation.

Non-BPS Mass

The $\overline{D0} - D4 - D4 - D4$ solution gives the black hole mass

$$2G_N\, M_{\text{Non-BPS}} = \frac{1}{2}\left(|Q_0| + \sum_{i=1}^{3} P^i\,(1 + B^2)\right) \; .$$

There are no cross-terms depending on several charges: the mass is a ***marginal sum of constituent masses***.

The black hole with $\overline{D0} - D4 - D4 - D4$ charge is interpreted as a bound state of those four kinds of constituents, ***with no binding energy***.

BPS Mass

For comparison, consider a ***BPS*** black hole with $D0 - D4 - D4 - D4$ charge, in the presence of a diagonal background B-field.

The BPS mass follows from the spacetime central charge

$$2G_N\, M_{\mathsf{BPS}} = \frac{1}{2}\left|Q_0 + \sum_{i=1}^{3} P^i\,(1+iB)^2\right| \;.$$

In the presence of a B-field, the spacetime central charge is genuinely complex so ***the mass is not just the sum of constituent masses***.

In the BPS case, the B-field ***binds*** $D0$'s and $D4$'s.

General Duality Frames

Extremal black holes with ***general charge vector and asymptotic moduli*** can be generated from by a $SL(2)^3$ rotation of the canonical solution with $\overline{D0} - D4 - D4 - D4$ charge and diagonal background B-field.

The non-BPS mass formula will always take the form of four underlying BPS constituents, marginally bound.

Example: consider a charge vector with just the charges of D0 branes (Q_0) and D6 branes (P^0).

Then the quartic invariant is negative so the black hole is ***necessarily*** on the non-BPS branch

$$J_4 = Q_0 P^1 P^2 P^3 - P^0 Q_1 Q_2 Q_3 - \frac{1}{4}(Q_I P^I)^2 + \sum_{i<j} P^i Q_i P^j Q_j \ .$$

The D0-D6 configuration has been much studied. The M-theory lift (KK-momentum and KK-monopole) is pure gravity, so this is also a solution in pure 5D gravity, or basic KK theory in 4D.

The non-rotating solution with canonical asymptotic moduli has an unfamiliar mass formula ($Q_0 = Q$, $P = P^0$):

$$M_{D0-D6} = \frac{1}{2G_4}\left[Q^{2/3} + P^{2/3}\right]^{3/2} \ .$$

We can analyze $D0 - D6$ by mapping it to $\overline{D0} - D4 - D4 - D4$ with diagonal B-field.

To remain general (up to duality) we consider $D0 - D6$ with all three B_i-fields, for a total of five parameters.

The $SL(2)^3$ map gives gives in particular the charge vectors $(P^0; P^i; Q_0; Q_i)$ of the four primitive constituents:

$$\begin{aligned}
\Gamma_I &= \frac{1}{4}\Big(P^0; -\frac{P^0}{\Lambda_1}, -\frac{P^0}{\Lambda_2}, -\frac{P^0}{\Lambda_3}; Q_0; \frac{P^0}{\Lambda_2\Lambda_3}, \frac{P^0}{\Lambda_1\Lambda_3}, \frac{P^0}{\Lambda_1\Lambda_2}\Big) \,, \\
\Gamma_{II} &= \frac{1}{4}\Big(P^0; -\frac{P^0}{\Lambda_1}, \frac{P^0}{\Lambda_2}, \frac{P^0}{\Lambda_3}; Q_0; \frac{P^0}{\Lambda_2\Lambda_3}, -\frac{P^0}{\Lambda_1\Lambda_3}, -\frac{P^0}{\Lambda_1\Lambda_2}\Big) \,, \\
\Gamma_{III} &= \frac{1}{4}\Big(P^0; \frac{P^0}{\Lambda_1}, -\frac{P^0}{\Lambda_2}, \frac{P^0}{\Lambda_3}; Q_0; -\frac{P^0}{\Lambda_2\Lambda_3}, \frac{P^0}{\Lambda_1\Lambda_3}, -\frac{P^0}{\Lambda_1\Lambda_2}\Big) \,, \\
\Gamma_{IV} &= \frac{1}{4}\Big(P^0; \frac{P^0}{\Lambda_1}, \frac{P^0}{\Lambda_2}, -\frac{P^0}{\Lambda_3}; Q_0; -\frac{P^0}{\Lambda_2\Lambda_3}, -\frac{P^0}{\Lambda_1\Lambda_3}, \frac{P^0}{\Lambda_1\Lambda_2}\Big) \,.
\end{aligned}$$

The $SL(2)^3$ rotation parameters Λ_i are related to P^0, Q_0 and the B_i:

$$\begin{aligned}
\Lambda_1\Lambda_2\Lambda_3 &= \frac{P^0}{Q_0} \,, \\
\frac{1}{2}[\Lambda_1(1+B_1^2) - \Lambda_1^{-1}] &= \frac{1}{2}[\Lambda_2(1+B_2^2) - \Lambda_2^{-1}] = \frac{1}{2}[\Lambda_3(1+B_3^2) - \Lambda_3^{-1}] \,.
\end{aligned}$$

Interpretation

All four primitive constituents are $D6$-branes, with fluxes:

$$(F_{12},\ F_{34},\ F_{56})^{I} = (f_1,\ f_2,\ f_3)\ ,$$
$$(F_{12},\ F_{34},\ F_{56})^{II} = (f_1,\ -f_2,\ -f_3)\ ,$$
$$(F_{12},\ F_{34},\ F_{56})^{III} = (-f_1,\ f_2,\ -f_3)\ ,$$
$$(F_{12},\ F_{34},\ F_{56})^{IV} = (-f_1,-\ f_2,\ f_3)\ .$$

Fluxes are such that they induce the correct $D0$-charge

$$\begin{aligned} n_0 &= -\frac{n_6}{4}\frac{1}{6(2\pi)^3}\int \mathrm{tr} F\wedge F\wedge F = -\frac{n_6 V_6 f_1 f_2 f_3}{(2\pi)^3}\ , \\ &\Rightarrow\ (2\pi\alpha')^3 f_1 f_2 f_3 = -\frac{Q_0}{P^0}\ . \end{aligned}$$

Complete specification of fluxes require local stability conditions

$$\frac{1}{2}[f_1^{-1}(1+B_1^2) - f_1] = \frac{1}{2}[f_2^{-1}(1+B_2^2) - f_2] = \frac{1}{2}[f_3^{-1}(1+B_3^2) - f_3]\ .$$

The fluxes and $SL(2)^3$ rotation angles (and the constraints they satisfy) are identified as

$$\Lambda_i^{-1} = 2\pi\alpha' f_i \ .$$

The total mass density determined from the DBI action

$$\begin{aligned}\frac{M}{T_6 V_6} &= \mathrm{Tr}\sqrt{\det[G + (2\pi\alpha' F - B)]} \\ = &\ \sqrt{(1+(2\pi\alpha' f_1 - B_1)^2)(1+(2\pi\alpha' f_2 - B_2)^2)(1+(2\pi\alpha' f_3 - B_3)^2)} \\ + &\ \sqrt{(1+(2\pi\alpha' f_1 - B_1)^2)(1+(2\pi\alpha' f_2 + B_2)^2)(1+(2\pi\alpha' f_3 + B_3)^2)} \\ + &\ \sqrt{(1+(2\pi\alpha' f_1 + B_1)^2)(1+(2\pi\alpha' f_2 - B_2)^2)(1+(2\pi\alpha' f_3 + B_3)^2)} \\ + &\ \sqrt{(1+(2\pi\alpha' f_1 + B_1)^2)(1+(2\pi\alpha' f_2 + B_2)^2)(1+(2\pi\alpha' f_3 - B_3)^2)} \ .\end{aligned}$$

Justification: use just abelian part of DBI, for four branes that are each BPS (albeit not mutually BPS).

The total mass depends on the B-fields both ***explicitly*** from the DBI action and ***implicitly*** through the f_i's.

Attractors

So far: lessons from the geometry of non-BPS solutions, specifically the mass and entropy.

Now: consider the scalar fields in the solution.

Qualitative structure: scalar fields flow radially, towards some attractor-value at the horizon of the black hole.

The attractor value of the scalar fields depend on the black hole charges, but not the asymptotic value of the scalars (the moduli).

Thus the moduli decouple from the near horizon behavior.

Example: $D0 - D4 - D4 - D4$ with no B

10D geometry of the canonical solution from lecture 1:

$$ds^2 = -\frac{1}{\sqrt{H_0H_1H_2H_3}}dt^2 + \sqrt{H_0H_1H_2H_3}(dr^2 + r^2 d\Omega_2^2)$$
$$+ \sqrt{\frac{H_0H_1}{H_2H_3}}dz^1 d\bar{z}^1 + \sqrt{\frac{H_0H_2}{H_3H_1}}dz^2 d\bar{z}^2 + \sqrt{\frac{H_0H_3}{H_1H_2}}dz^3 d\bar{z}^3 \,.$$

The volume of the first T^2 near the horizon is independent of the asymptotic volume

$$\frac{V_1}{(2\pi l_s)^2} = v_1\sqrt{\frac{Q_0Q_1}{Q_2Q_3}} = v_1\sqrt{\frac{\frac{n_0}{\sqrt{v_6}}\frac{n_1\sqrt{v_6}}{v_1}}{\frac{n_2\sqrt{v_6}}{v_2}\frac{n_3\sqrt{v_6}}{v_3}}} = \sqrt{\frac{n_0n_1}{n_2n_3}} \,.$$

Interpretation: attractor behavior is an equilibrium between branes squeezing the cycles they wrap, and blowing up transverse cycles.

Flat Directions

Consider BPS black holes in $N = 2$ SUGRA with vector- and hyper-multiplets.

In this case the attractor mechanism applies to all scalars in vector multiplets.

The attractor mechanism does not apply to scalars in hyper multiplets. Those decouple from the flow and so keep their (arbitrary) asymptotic value.

The hyper-multiplets parametrize flat directions of the effective potential for scalars in the black hole background.

For non-BPS black holes there are also flat directions among the vector-multiplets!

Interpretation of Flat Directions

The origin of new flat directions can be understood generally from group theory. But the clearest is to just consider the $D0 - D6$ duality frame.

Attractor behavior is due to branes squeezing the cycles they wrap, expanding transverse cycles.

$D6$ on $T^6 = T^2 \times T^2 \times T^2$ squeeze the overall T^6, and $D0$ blows it up. ***Both are indifferent to the volumes of each T^2 component*** by themselves.

The two flat directions in the $D0 - D6$ duality frame are the ratios of T^2 volumes!

The two flat directions in other duality frames (like the $\overline{D0} - D4 - D4 - D4$) are generally much more complicated, but they are determined by the duality transformation from $D0 - D6$.

$D0 - D6$ Supersymmetry?

The SUSY-projections due to Dirichlet conditions on the $D0$ and the $D6$ branes are

$$\begin{aligned}\tilde{\epsilon} &= \Gamma^{\hat{1}}\Gamma^{\hat{2}}\Gamma^{\hat{3}}\Gamma^{\hat{4}}\Gamma^{\hat{5}}\Gamma^{\hat{6}}\epsilon\,, \\ \tilde{\epsilon} &= \pm\epsilon\,.\end{aligned}$$

There are no solutions because $(\Gamma^{\hat{1}}\Gamma^{\hat{2}}\Gamma^{\hat{3}}\Gamma^{\hat{4}}\Gamma^{\hat{5}}\Gamma^{\hat{6}})^2 = -1$.

Background B-fields rotate the $D6$ condition by a factor

$$\prod_{i=1}^{3} \frac{1 + iB_i}{1 - iB_i}\,.$$

Corrollary: $D0 - D6$ is SUSY in the presence of B-fields if

$$\sum_{i<j} B^i B^j = 1\,.$$

$D0 - D6$ Bound States

It is simple to compute the spectrum of ***open strings stretching between the $D0$ and the $D6$***.

The spectrum is ***supersymmetric*** if

$$\sum_{i<j} B^i B^j = 1\,.$$

For

$$\sum_{i<j} B^i B^j > 1\,.$$

there is ***a tachyon*** in the spectrum.

The tachyon may condense into a ***supersymmetric ground state***, interpreted as ***a genuine bound state*** of the $D0 - D6$-system (the Higgs-branch).

The Multi-Center BPS solutions

There are no ***single center*** BPS black holes with $D0 - D6$ charges but there are ***BPS multicenter solutions***. Some of their properties:

- The simplest multi-center configuration: two centers, one $D0$, the other $D6$.

  (Single-center non-BPS $D0 - D6$: four $1/2$-BPS constituents, all $D6$'s with fluxes.)

- The charge vectors of the 1/2-BPS ***constituents are mutually non-local***, *i.e.* they have non-zero intersection number.

  (The four constituents of the non-BPS black holes are mutually local.)

The Wall of Marginal Stability

- BPS configurations of $D0 - D6$ branes exist only for

$$\sum_{i<j} B^i B^j \geq 1 \,.$$

  This is a co-dimension one wall of marginal stability in moduli space

  (The non-BPS black holes exist everywhere in moduli space.)

- BPS multicenter solutions exist in the same range, with ***a specified separation scale***

$$R = |\vec{x}_1 - \vec{x}_2| = \frac{|Q_0 + iP^0 \prod_{i=1}^{3}(1 + iB^i)|}{\sum_{i<j} B^i B^j - 1} \,.$$

  (Constituents of the non-BPS black holes can move freely in the supergravity approximation.)

The $D0 - D6$ constituents move apart as the wall of marginal stability is approached; ***they are removed from the spectrum***.

First Order Phase Transition

- The BPS solutions ***cannot be continuously connected*** to the non-BPS solution through the wall of marginal stability.
- There ***can be decay*** from the non-BPS branch to the BPS branch on the part of moduli space where BPS solutions exist.
- The BPS mass is always ***strictly smaller*** than the non-BPS mass with otherwise identical quantum numbers.
- So the ***transition will release energy***, entropy and generally also angular momentum.
- This indicates ***a first order transition*** between the two branches.

Is $D0 - D6$ Unstable?

What is the faith of $D0 - D6$ on the part of moduli space where BPS states cannot exist?

The non-rotating solution with canonical asymptotic moduli has mass formula ($Q_0 = Q$, $P = P^0$):

$$M_{D0-D6} = \frac{1}{2G_4}\left[Q^{2/3} + P^{2/3}\right]^{3/2} \ .$$

This formula applies even when there is angular momentum, as long as $J < PQ/2$ (recall $S = 2\pi\sqrt{-J_4 - J^2}$ on the non-BPS branch).

The energetics allows spontaneous decay into widely separated $D0$'s and $D6$'s:

$$M_{D0-D6} > \frac{1}{2G_4}\left[Q + P\right] = M_{D0} + M_{D6} \ .$$

But: ***this process is forbidden*** by angular momentum conservation: widely separated $D0$'s and $D6$'s have $J \geq PQ/2$!

Candidate final states consistent with conservation laws must have at least three bodies.

The decay we know is important for the system is the marginal one: spontaneous separation into four constituents. No energy is released in this process.

It is not known what the dominant decay mode of $D0 - D6$ is.

Non-BPS Microscopics

- Classically, the entropy of BPS and non-BPS black holes are related by analytical continuation: $S_{\rm BPS} = 2\pi\sqrt{|J_4|}$.
- This suggests that their microscopic origins are virtually identical, *i.e* related by analytical continuation.
- The problem: ***there are significant differences between the two branches***.
- For example, ***the classical moduli spaces are completely different***: they have different dimension.
- Also, ***the mass formulae*** on the two branches are ***not related by analytical continuation***.

- These distinctions are relevant: ***precise counting of BPS states involves choosing a favorable point in moduli space***.

- Specifically, one may want to turn on a small B-field to avoid bound states at threshold. ***This is not possible for the non-BPS states***.

- Conclusion: ***the corresponding microstates are not be related by analytical continuation***.

Presumably there is a simple understanding of extremal non-BPS entropy anyway. But the significant differences between the two branches must be addressed by a more detailed understanding of the microscopics.

Indeed, these differences may give guidance towards a microscopic description.

Summary

The non-BPS extremal black holes exhibit some surprising properties:

- The most general black hole solutions in $D = 4$ can be generated from a ***seed solution with 5 parameters***.
- The ***mass formula*** of the black hole suggests an interpretation as ***a marginal bound state of four primitive constituents***.
- ***Flat Directions:*** some of the scalars in the theory experience a flat potential when all forces are taken into account.
- ***Phase Transition:*** the non-BPS mass is strictly greater than the BPS bound, even in regions of moduli space where BPS multi-center solutions exist. The branches are related by a first order phase transition.