The Abdus Salam
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School on New Trends in Quantum Dynamics and Quantum Entanglement

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ESSENTIAL ENTANGLEMENT

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Essential entanglement

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Nonlinear Dynamics in Quantum Systems

ICTP Trieste, 14-18 February 2011

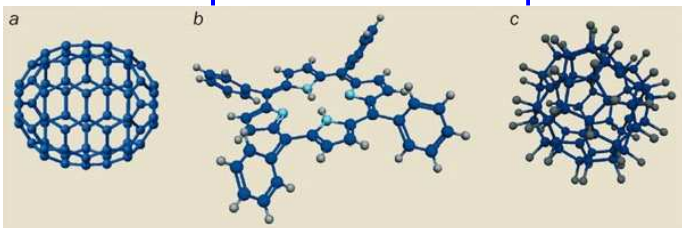
Nonlinear Dynamics in Quantum Systems

Entanglement, a central resource of quantum information processing

Key challenges

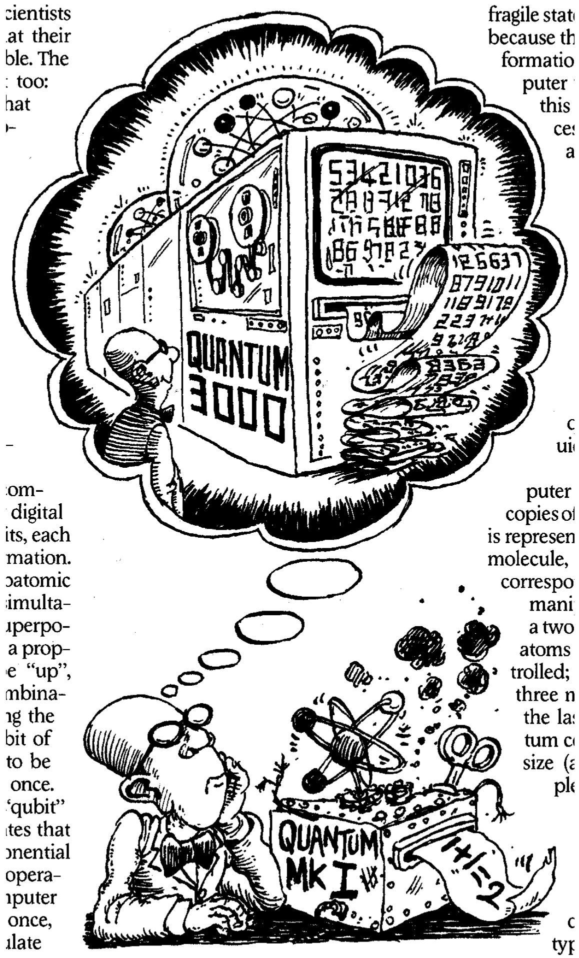
- How long does this special fuel "entanglement" last, under realistic conditions?

- Scalability – how do size and coherence requirements compete?



[Arndt, Hornberger & Zeilinger, Physics World 2005]

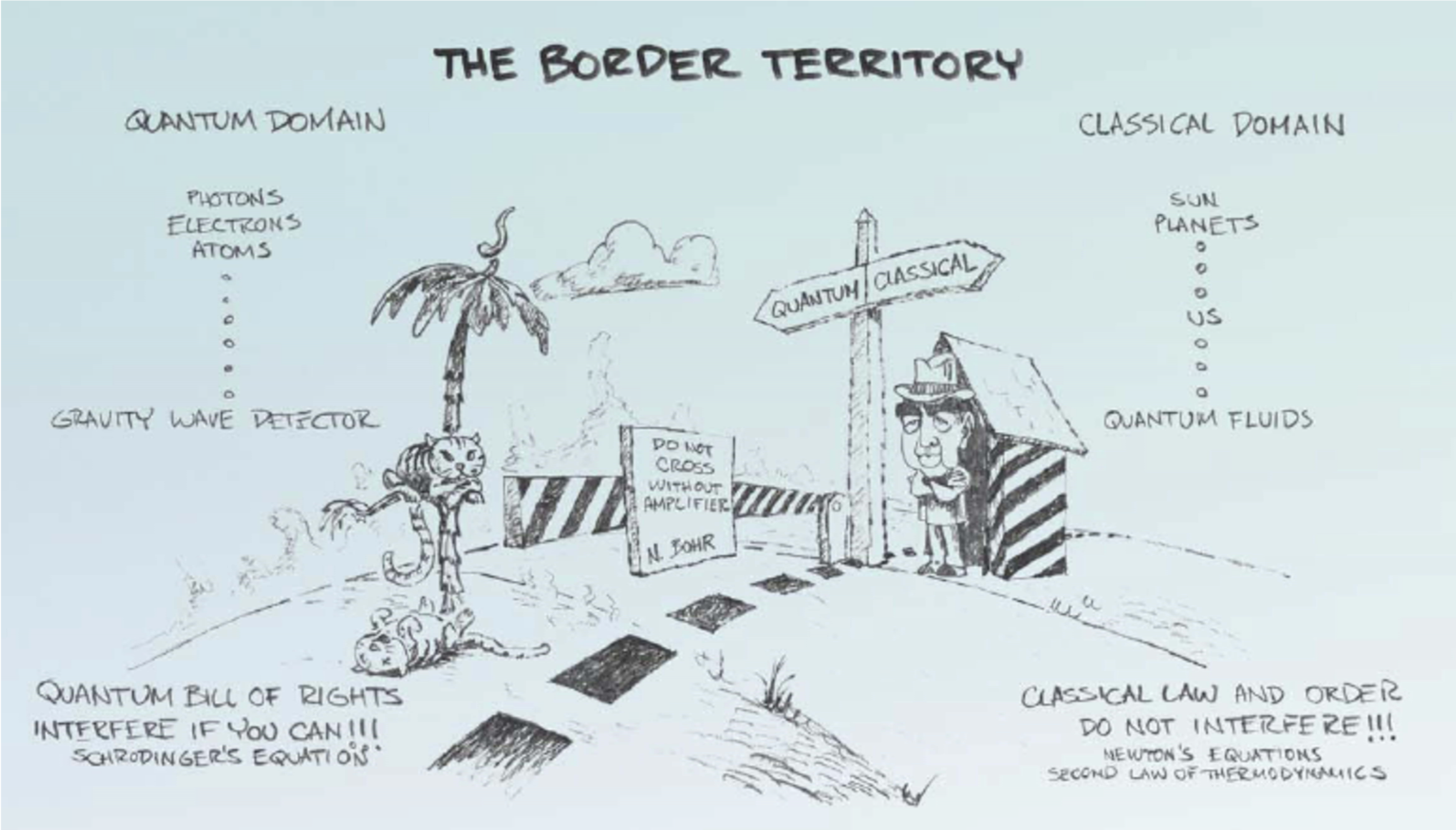
- Which functional rôle?



[The Economist, *Quantum Dreams*, 10/3/2001]

Entanglement, a new perspective for the quantum to classical transition

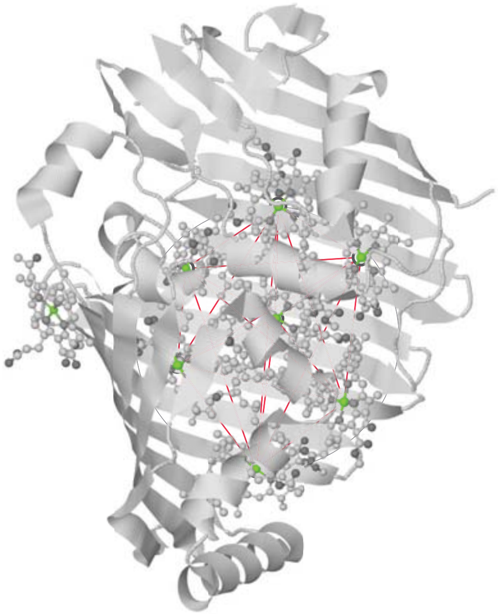
?which size? **?which temperature?**



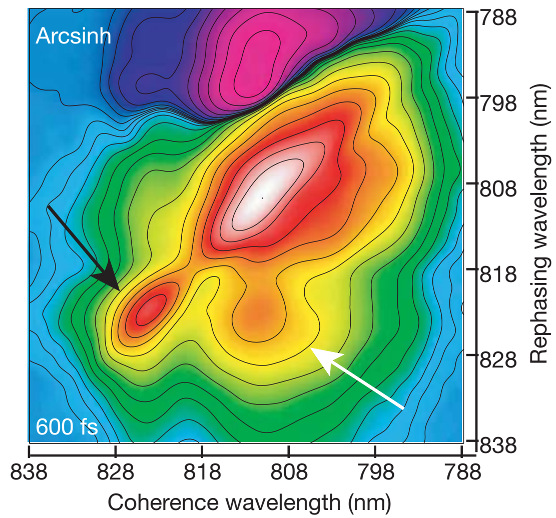
decoherence $\sim$ entanglement with uncontrolled degrees of freedom!?

Quantum coherence in photosynthesis

photosynthetic complex



2D spectroscopy



light harvesting antenna complexes (e.g., "FMO") funnel excitations from receptor to reaction center with ≥ 95 % **quantum efficiency**

at ambient temperature [Engel et al., Nature **446**, 782 (2007); Collini et al., Science **323**, 369 (2009)]

in noisy, multi-hierarchical environment

??? ORIGIN OF THIS EFFICIENCY ???

Wrap-up I-1

Any pure state $|\Psi\rangle$ which *cannot* be written as a product $|\phi\rangle \otimes |\chi\rangle$, i.e.,

$$|\Psi\rangle \neq |\phi\rangle \otimes |\chi\rangle \,, \forall|\phi\rangle \in \mathcal{H}_{\mathrm{A}}, \, \forall|\chi\rangle \in \mathcal{H}_{\mathrm{B}}, \, (|\Psi\rangle \in \mathcal{H}_{\mathrm{A}} \otimes \mathcal{H}_{\mathrm{B}})$$

is **nonseparable** or **entangled**.

Wrap-up I-2

Schmidt decomposition:

Any bipartite state $|\Psi\rangle \in \mathcal{H}_{\mathrm{A}} \otimes \mathcal{H}_{\mathrm{B}}$ can be written as

$$|\Psi\rangle = \sum_j \sqrt{\lambda_j} |a_j^S\rangle \otimes |b_j^S\rangle ,$$

with the (*unique*) **Schmidt coefficients** λ_j, and the **Schmidt basis** $|a_j^S\rangle \otimes |b_j^S\rangle$.

> *A bipartite state* $|\Psi\rangle$ *is separable if and only if it has only one non-vanishing Schmidt coefficient.*

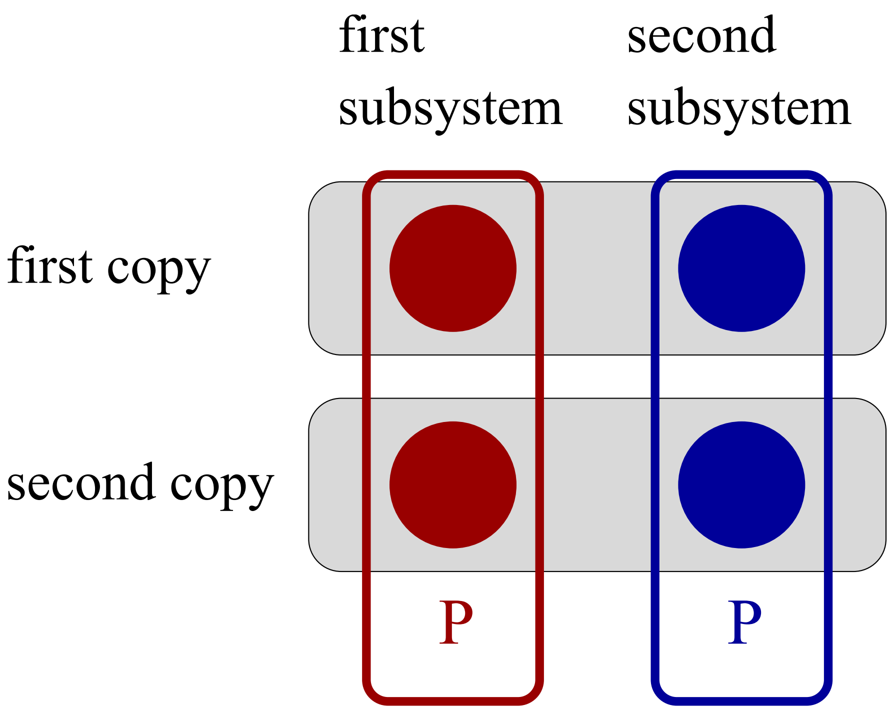
Reformulating concurrence

An efficient quantifier for entanglement is derived by rewriting pure state concurrence as

$$c(\Psi) = \sqrt{\langle\Psi| \otimes \langle\Psi|A|\Psi\rangle \otimes |\Psi\rangle} \ ,$$

where A acts on *two copies* of the given state $|\Psi\rangle$. [Mintert et al, 2004-2008]

$A \sim P_{-}^{(1)} \otimes P_{-}^{(2)}$ *projects on the antisymmetric subspaces of the underlying factor spaces. Thus,* c *vanishes for states which are invariant under exchanges of the individual copies. Possible interpretation in terms of suitable measurement(s) on two copies.*

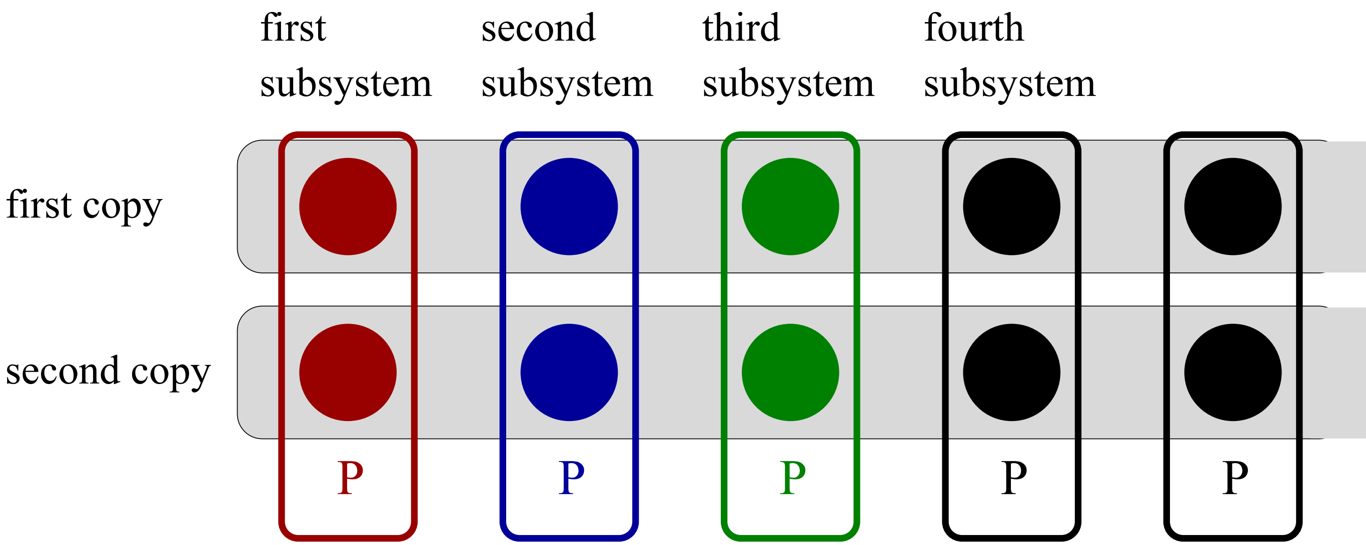


Mixed state concurrence in higher dimensions

For mixed states of bi- or multipartite states of *arbitrary finite dimension* we obtain

$$c(\rho) = \inf_{\{p_j,\Psi_j\}} \sum_j p_j \sqrt{\langle\Psi_j| \otimes \langle\Psi_j|A|\Psi_j\rangle \otimes |\Psi_j\rangle} \ ,$$

with a multipartite generalization of A. [Mintert et al., PRL 2005]



This provides the desired tool for our assessment of the crucial scaling properties!

General structure of A

More explicitely, A in terms of antisymmetric and symmetric operators reads

$$A = \sum_{s_1,\ldots,s_N} p_{s_1,\ldots,s_N} P_{s_1} \otimes \ldots \otimes P_{s_N}\,,\; s_i \in \{-,+\}\,,\; s_1 \cdot \ldots \cdot s_N = +1\,.$$

Special choice of $p_{s_1,\ldots,s_N} = 4$, for all admissible choices of $s_1, \ldots, s_N$, leads to

$$c_N(\Psi) = 2^{1-\frac{N}{2}} \sqrt{(2^N - 2)\langle\Psi|\Psi\rangle^2 - \sum_j \mathrm{tr}\rho_j^2}$$

This has the particular property

$$C_{N+1}(\psi_{1,\ldots,N} \otimes \phi_{N+1}) = C_N(\psi_{1,\ldots,N})\,.$$

Allows to compare the entanglement of states with an increasing number of parties.

[Demkowicz-Dobrzański et al, PRA 2006]

Explicit evaluations

The (numerical) evaluation of the infimum

$$c(\rho) = \inf_{\{p_j, \Psi_j\}} \sum_j p_j \sqrt{\langle \Psi_j | \otimes \langle \Psi_j | A | \Psi_j \rangle \otimes | \Psi_j \rangle} \qquad (1)$$

provides an *upper* bound of $c(\rho)$ . . . we need *lower bounds*!

The algebraic structure of (1) leads to a hierarchy of approximations from below

1. **optimized lower bound** (numerical optimization over lower dimensional – $n_1^2 n_2^2$, instead of $n_1^3 n_2^3$ – optimization space) [Mintert et al., PRL 2004]

2. **algebraic lower bound** (diagonalization of a matrix of dimension equal to the maximal rank – $n_1^2 n_2^2$ – of A)

3. **quasi pure approximation (qpa)** – diagonalization of matrix of dimension of ρ – $n_1 n_2$ [Mintert & –, PRA 2005]

Dynamics under nonvanishing environment coupling

Various types of dynamics

1. **entanglement decay due to coupling of subsystems to "private" baths**

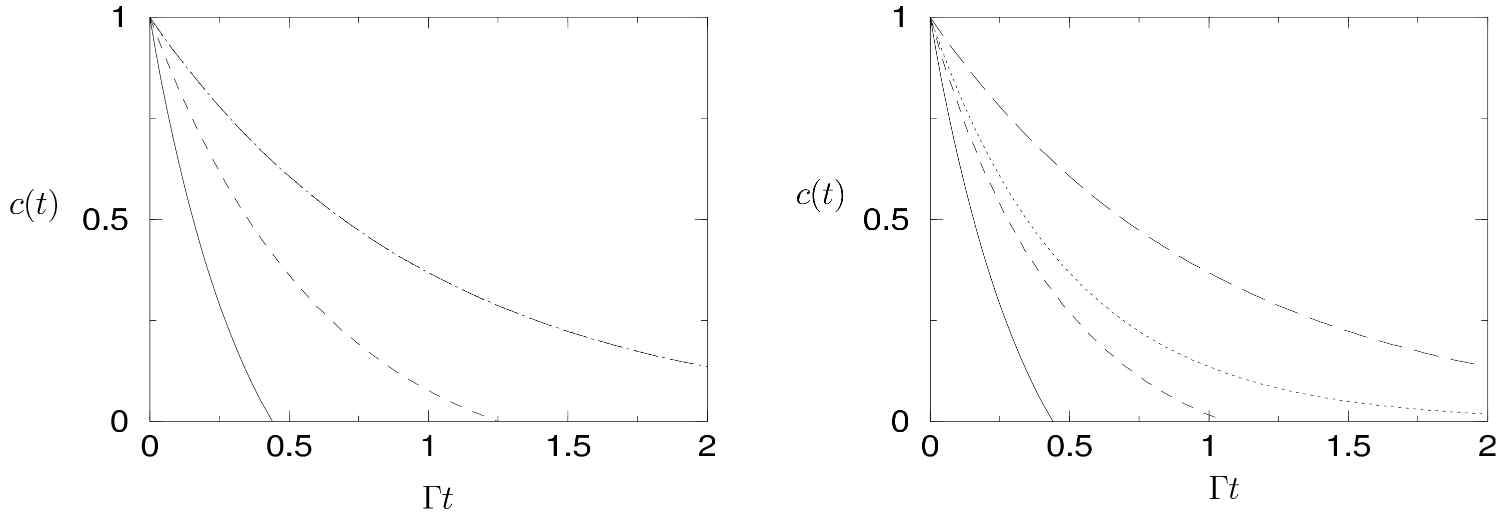
$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H_{\rm sys}, \rho] + \mathcal{L}\rho = -\frac{i}{\hbar}[H_{\rm sys}, \rho] + \sum_j \frac{\Gamma_j}{2} \left(2\, d_j\, \rho\, d_j^\dagger - d_j^\dagger\, d_j\, \rho - \rho\, d_j^\dagger\, d_j\right)$$

2. **random system-environment time evolution with subsequent trace over the "public" environment**

3. **entanglement generation vs. decoherence**

Entanglement decay of bipartite two-level systems

Initial states $|\Psi^+\rangle = (|01\rangle + |10\rangle)/\sqrt{2}$ (left) and $|\Phi^+\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$ (right)



- coupling to **thermal bath** with
  - **zero temperature** (only spontaneous emission; dotted line)
  - **finite temperature** ($\bar{n} = 0.1$ thermal photon in the environment; dashed)
  - **infinite temperature** (noisy environment; solid)

- or to **dephasing reservoir** (only coherence loss; long dashed)

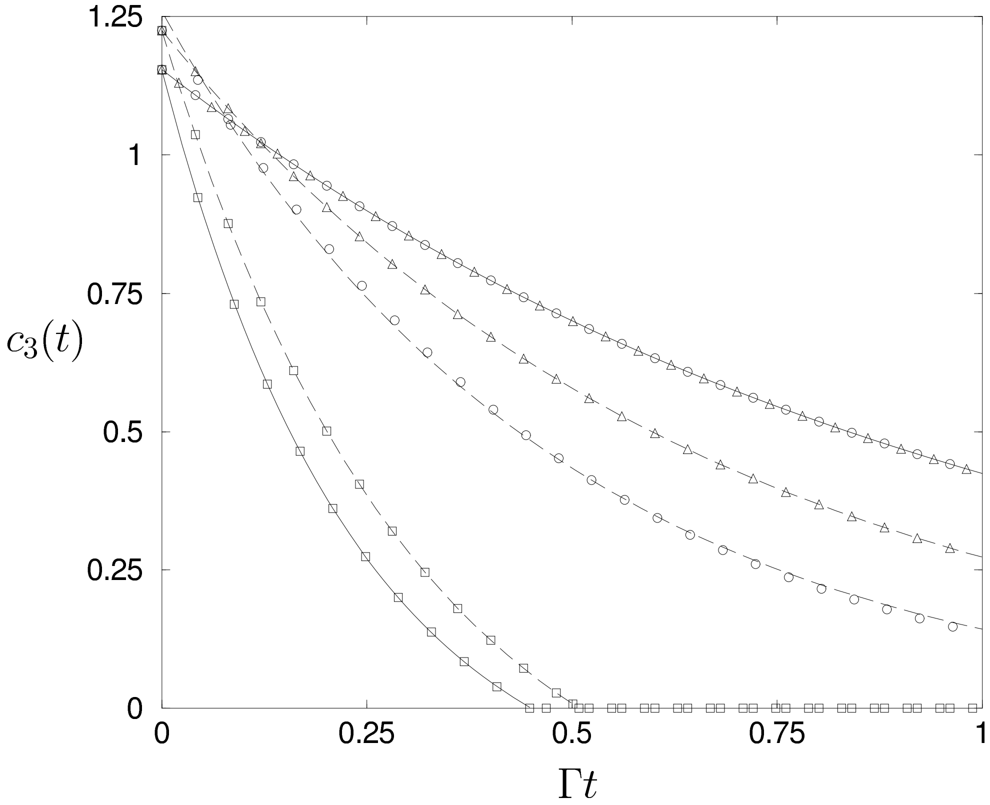
(multi-) exponential decay with finite or infinite separability times

[Mintert, Carvalho, Kuś, -, Phys. Rep. 2005]

N-partite entanglement decay *(private baths)*

We generalize bipartite concurrence $c_2(\Psi) = \sqrt{2(\langle\Psi|\Psi\rangle^2 - \mathrm{tr}\rho_r^2)}$ for N-partite systems (with j counting all possible partitions):

$$c_N(\Psi) = 2^{1-\frac{N}{2}} \sqrt{(2^N - 2)\langle\Psi|\Psi\rangle^2 - \sum_j \mathrm{tr}\rho_j^2}$$



$c_N(t)$ for $N = 3$

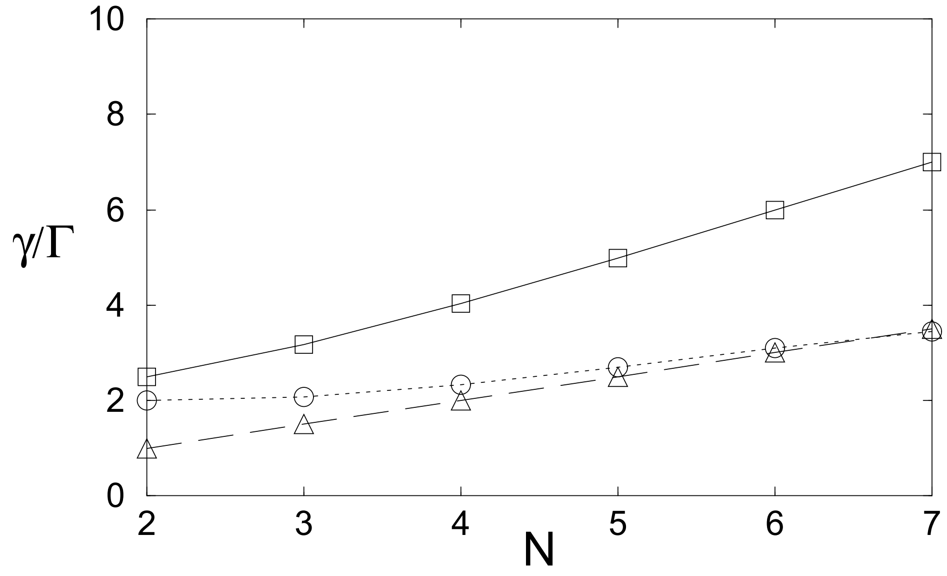
W-states (solid lines)
$|\mathrm{W}_N\rangle = (|00\ldots01\rangle + |00\ldots10\rangle + \ldots + |10\ldots00\rangle)/\sqrt{N}$

GHZ-states (dashed lines)
$|\mathrm{GHZ}_N\rangle = (|00\ldots0\rangle + |11\ldots1\rangle)/\sqrt{2}$

zero temperature (circles), **infinite temperature** (squares), and **dephasing** (triangles) environments.

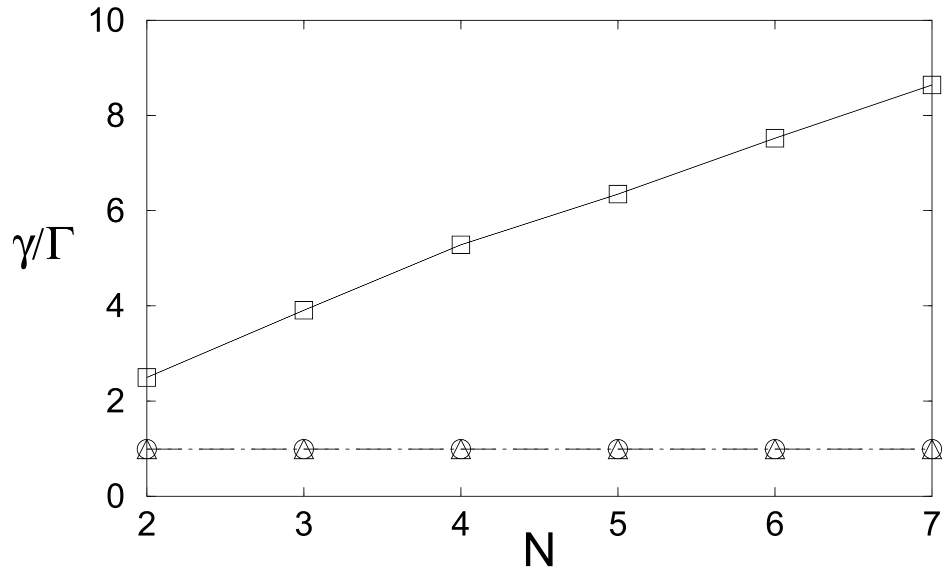
[Carvalho et al., PRL 2004]

Scaling of entanglement decay rates γ (private baths)



top:
$|\mathrm{GHZ}_N\rangle = (|00\ldots0\rangle + |11\ldots1\rangle)/\sqrt{2}$
bottom:
$|\mathrm{W}_N\rangle = (|00\ldots01\rangle + |00\ldots10\rangle + \ldots + |10\ldots00\rangle)/\sqrt{N}$

circles: zero temperature environment
squares: infinite temperature
triangles: dephasing



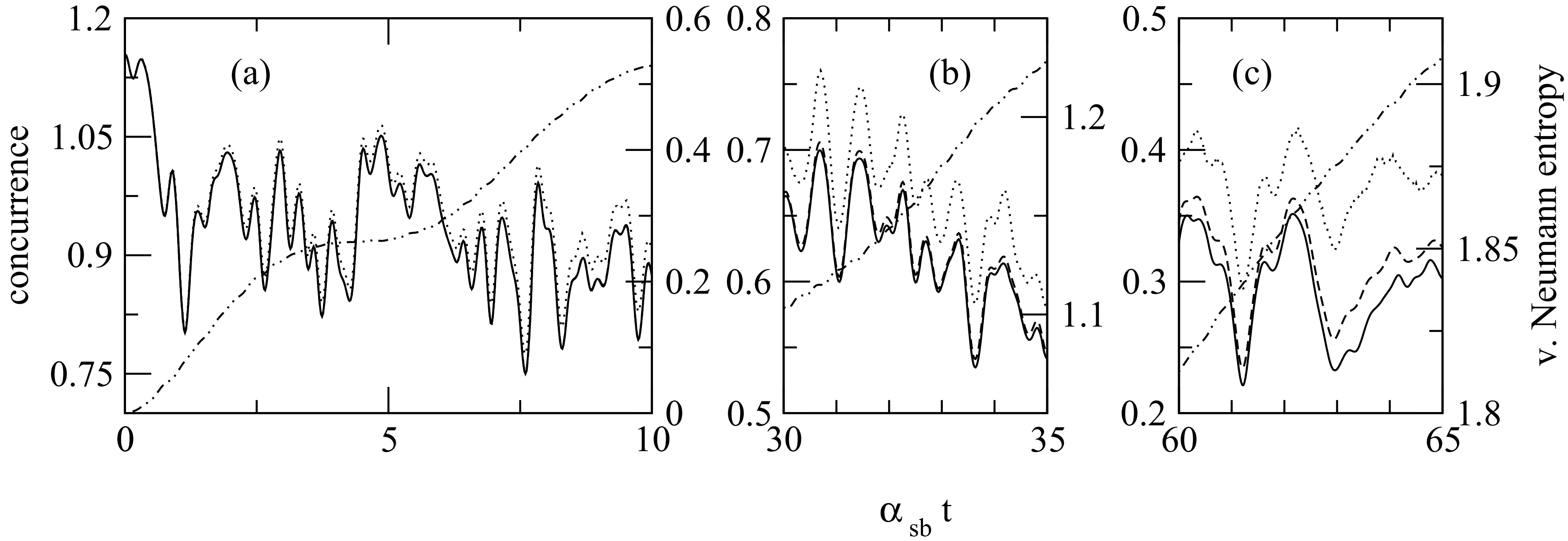
W-states' decay rates *independent* of N for zero temperature and dephasing!

[Carvalho et al., 2004]

similar demarche: [Dür & Briegel, 2004]

(also see [Yu & Eberly, 2004])

Random time evolution *(public bath)*



- Concurrence for an initially pure, maximally entangled 3×5 bipartite state $|\Psi_0\rangle = \sum_{j=1}^{3} |jj\rangle/\sqrt{3}$ under random, non-unitary time evolution; $\alpha_{\rm sb}$ – system-environment coupling strength
- dash-double dotted line: von Neumann entropy $S = -{\rm tr}\rho_{\rm sys} \ln \rho_{\rm sys}$ (measures mixing)

[Mintert, -, PRA 2005]

Wrap-up II-1

- **entanglement monotones as functions of Schmidt coefficients**, for bipartite pure states
- convex roof construction for **mixed states entanglement**
- (multipartite) **concurrence as expectation value of a projection-valued operator** with respect two copies of the state
- efficiently evaluable **lower bounds of multipartite concurrence** which get in general tighter with increasing purity
- examples for **entanglement decay rate scaling** with system size

Wrap-up II-2

BUT: with increasing system size, scaling still unfavourable!

So far: evolve $\rho(t)$, deduce $c[\rho(t)]$

? **Can we assess $c(t)$ directly?**

? **Why does $c(t)$ evolve the way it does?**

! **state space topology** and **reference states**

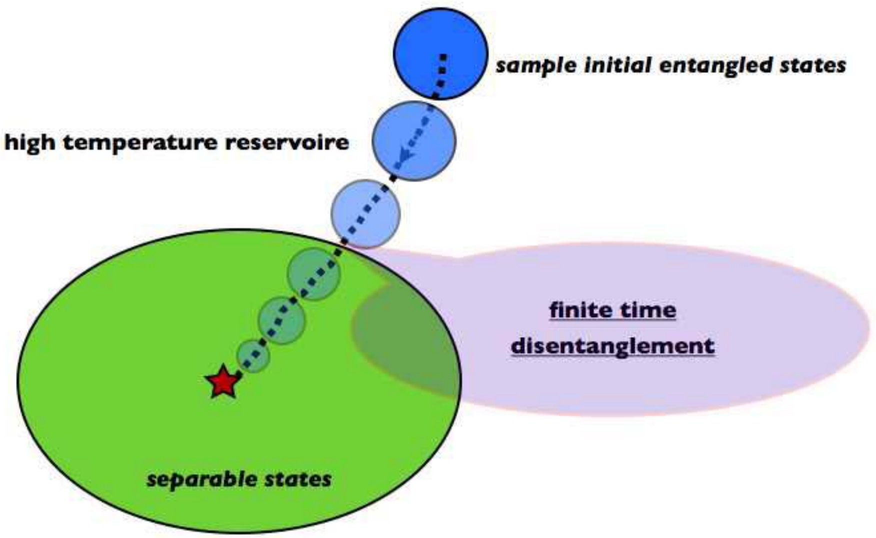
- provide **systematic understanding** of dynamical evolution
- **reduce the complexity** of mixed state entanglement estimation

Statistical-topological approach to entanglement evolution

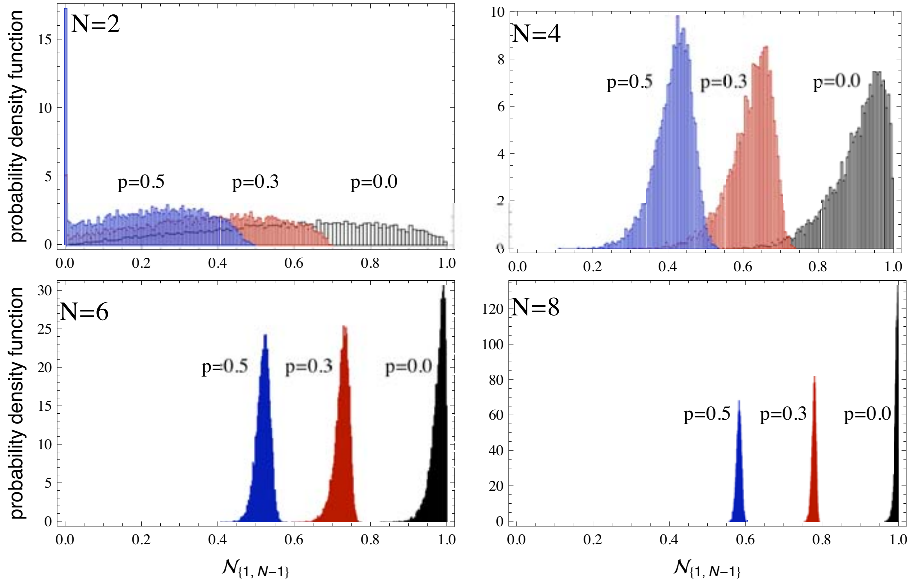
[PhD Markus Tiersch, 2009]

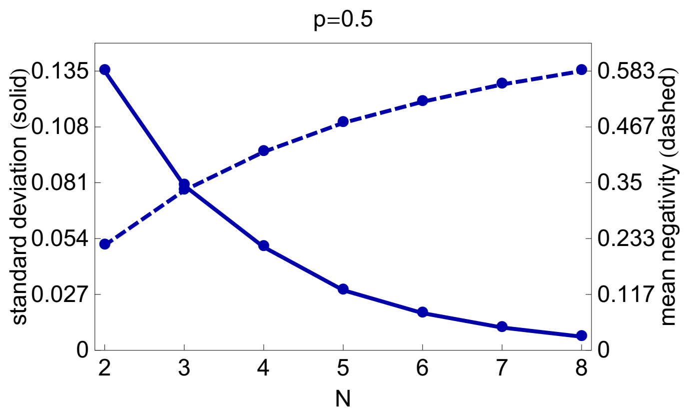
Can we give a **robust, generic** description of $c(t)$?

– monitor evolution of uniform distribution of pure initial states –



Entanglement distribution for increasing system size





$$P\left(|E(\rho)-\overline{E}|>\epsilon\right)\leq 4\exp\left(-\text{const}\times 2^N\epsilon^2\right)$$

– universal dynamics emerge in the macroscopic limit $N\to\infty$ –

[Tiersch et al., arXiv 2008]

Wrap-up III

- **entanglement concentrates** exponentially in high dimensions
- **entanglement** becomes more **robust in high dimensions**
- **entanglement evolution** can be well predicted by **benchmark** state

Take home message, & some open questions

- There are tools to characterize (quantify/estimate/sample) mixed state entanglement in high dimensional, multicomponent, open quantum systems
- Is there a good reason to consider local Hamiltonians as those which abound in nature? (rhetoric question . . .)
- Yet, it is likely that we still need some new ideas to characterize high dimensional entanglement, to aid our intuition (we have some, but any good . . . ?)
- Is it possible to derive a general entanglement evolution equation alike Lindblad?
- Entanglement classification in e.g. atomic/molecular systems with coupled discrete and continuous spectra?

Literature (selection)

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More to read and do . . .

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QE

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