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Models of Continuous Double Auctions

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Models of continuous double auctions

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Outline

- What's a Continuous Double Auction (CDA)?
- See one in action (from `batstrading.com`)
- A model is model (is a model, is a model, is a model...)
- Things we want to explain and motivations to trade.
- Parlour (1998) and Goettler et al. (2005, 2009).
- Foucault (1999), Foucault, Kadan and Kandel (2005), Rosu (2010).
- Wrap-up.

CDA

- Many buyers and sellers can electronically submit offers at any time.
- Offers are binding proposals to buy (bid) or sell (ask) a specified quantity for a limit price, i.e, they are couples (q, p).
- Offers are immediately executed if they are marketable; otherwise, they are stored for future use in *limit order books*.
- Traders can change or cancel their offers at any time (if they are in the book).
- Much more can be done: *splitting, stop-loss orders, all-or-nothing. . .*
  Let's see what's going on right now!

CDA (2)

- We say that you can *offer liquidity*, placing orders in the book or...
- *Consume liquidity*, submitting market orders.
- Both orders are “risky”:
  1. Limit orders can be picked-off.
  2. Market orders *always* trade at the worst possible price.

Fundamental trade-off

Immediacy *versus* efficacy:

1. Market orders are certainly executed, but they are costly.
2. Limit orders are more favorable, but execution is uncertain.
3. Clash! Should I stay or should I go?

A model is a model

- Trading in a CDA is difficult:
  1. Quantity and (limit) price.
  2. History (trades, bids, asks. . .)
  3. Fundamental value, beliefs, asymmetric information.
  4. Cancellation, resubmission and timing.
- The order (q_τ, p_τ) is

$$(q_\tau, p_\tau) = f(a_\tau, b_\tau | \mathcal{H}_\tau, \mathcal{B}_\tau, \mathcal{I}_\tau, \ldots)$$

- Simplification is needed, so:
  1. Information is neglected (uh?)
  2. Cancellation is forbidden (30% wrong).
  3. Quantity is ignored (unit trading).
  4. Timing is tampered (Poisson or one-shot chance).

  Still, understanding the CDA is hard.

A model is a model: MDP

- The framework is a Markov Decision Process (MDP): outcomes are random and only partly under control.
  1. There is a set of actions $a \in \mathcal{A}$ to be taken by traders and there is a set of states $s \in \mathcal{S}$ of the book (world).
  2. There is a stochastic reward (immediate or "delayed") for any action in any state: $\tilde{\pi}_i(a_t, s_t)$.
  3. The state of the book is possibly changing after a_t, hence
  $$\tilde{s}_{t+1} = g(a_t, s_t).$$
  4. Traders independently maximize the reward:
  $$\max_{a_1, a_2, \ldots, a_T} E\left[\sum_{t=1}^{T} \pi_i(a_t, s_t)\right]$$
- Non-cooperative game (with Nash equilibria), a dynamical programming problem (solved by backward induction, in blessed cases), a stochastic optimization problem. . .

A model is a model: equilibrium

- We look for actions that cannot be improved (otherwise, competitive traders will behave differently!)
- Equilibrium: a sequence of functions $\{r_i, i = 1, \ldots, N\}$ of the state $a_{it} = r_i(s_{it})$ such that

$$\pi(r_i, r_{-i}) \geq \pi(r'_i, r_{-i}), \forall i$$

- A strategy can be thought also as a look-up table dictating what to bid/ask in any possible state $\rightarrow$ computationally heavy.
- Typical sources of randomness are entry times, values, (cancellation times).

Facts and questions

- Diagonal effect, Biais et al (95): orders of the same type form streaks.

	Order at t							
	Buy				Sell			
Order at $t-1$	1	2	3	4	1	2	3	4
Buy 1	**22.01**	**11.86**	5.96	5.87	10.29	**25.25**	12.02	6.74
Buy 2	14.66	6.80	**9.99**	**13.32**	**20.86**	**16.37**	10.04	7.96
Buy 3	13.38	9.62	**21.57**	7.17	13.79	13.17	11.33	9.97
Buy 4	15.05	8.70	8.02	**18.08**	**16.57**	15.28	9.53	8.77
Sell 1	15.93	**20.66**	6.36	3.81	**17.65**	**16.73**	10.87	7.99
Sell 2	**24.67**	**11.85**	5.16	5.09	10.19	8.66	**14.21**	**20.17**
Sell 3	20.38	8.25	6.51	4.37	10.58	10.36	**29.49**	10.06
Sell 4	20.89	10.70	6.65	5.96	7.87	11.56	**14.20**	**22.17**
Unconditional	19.26	11.34	7.88	7.11	12.91	15.16	14.53	11.80

- Who and why use market orders? Who takes and who provides liquidity?
- Why is the book so sparse?
- (Why do we have fat tails in daily stock returns?)

Why do you trade?

- Private values / costs:
  1. Buyers have a privately known redemption value v_i. Their profit is

$$\pi_i = \begin{cases} v_i - p & \text{if they trade;} \\ 0 & \text{otherwise.} \end{cases}$$

  2. Sellers have a privately known cost c_i and their profit is

$$\pi_i = \begin{cases} p - c_i & \text{if they trade;} \\ 0 & \text{otherwise.} \end{cases}$$

- Alternatively, let the valuation of the asset be β_i: if $\beta_i \leq E[\beta]$, the agent is a seller; if $\beta_i \geq E[\beta]$, the agent should buy.

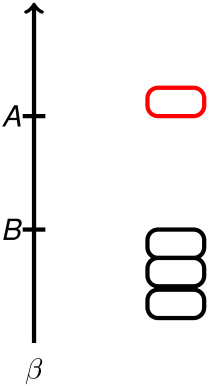
Why do you trade? I'm in a hurry!

- Patience / impatience: traders pay a cost that depends on the time spent in the book (waiting for execution).
  1. Patient buyers have a low cost of waiting γ
     $$E[\pi_{it}] = E[\tilde{p}_\tau - \gamma(\tilde{\tau} - t)],$$
     they can afford to submit a limit order and wait for a trading opportunity.
  2. Impatient traders have higher waiting cost $\gamma' >> \gamma$:
     $$E[\pi_{it}] = E[\tilde{p}_\tau - \gamma'(\tilde{\tau} - t)]$$
- "For simplicity, it is assumed that γ' is much larger than γ, which implies that impatient traders always submit market orders".

Parlour (1998)

- MO or LO under time constraints. Only bids at B and asks at A are allowed
- Time to trade $t = 1, \ldots, T$ is limited and time priority is extremely important.
- Value $\beta \sim U[0, 2]]$: buyers have $\beta > 1$, sellers $\beta < 1$.
- Traders enter sequentially and are aware of time.
- Four choices:
  1. MOS: market to sell (cash B).
  2. MOB: market to buy (pay A).
  3. LOS: limit order to sell (queue at A).
  4. LOB: limit order to buy (queue at B).

Parlour (1998), II



1. At T only market orders. . .
2. At $T-1$, the probability of execution enters the scene.

Parlour (1998), III

- Agents act differently based on β: patient / impatient.
- Both sides of the markets count: increasing P^B (probability of a MOB), increases the probability of a LOS.
  P^B depends on the number of LOB queued at B.
- Example at $T-2$. Pretend you are a seller.
  Assume no order at the bid.
  1. Will increase the probability of a LOB at $T-1$.
  2. Hence, decrease the probability of execution of a LOS at $T-2$.

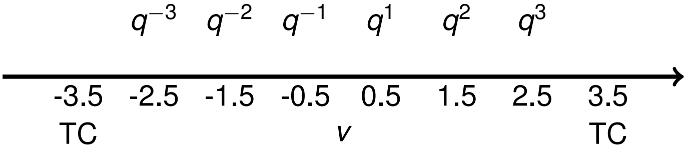
  Conversely, assume plenty of bids at $T-2$
  1. Will increase the probability of a MOB at $T-1$.
  2. Hence, increase the probability of execution of a LOS at $T-2$.

Parlour (1998), diagonal effect

- The depth at quotes is reduced by MOB and MOS and increased by LOB and LOS.
- After a MOB, there is less depth at ask A.
- Hence, after a MOB, traders know that
  1. Sellers will issue more LOS than MOS.
  2. Consequently, buyers know that their LOB is less likely to be filled and they use a MOB.
- All in all, after a MOB buyers issue more MOB than MOS.
- Same can be said for other orders.

Goettler, P, Rajan (2005)

- The equilibrium is found numerically (!), 8 ticks relative to zero consensus fundamental value, cancellation is a mechanical Poisson process.



- The state of the book is $s = (q^{-3}, q^{-2}, q^{-1}, q^{1}, q^{2}, q^{3})$.
- Traders have to figure out price and quantity as a function of β, given s (and fundamental price v).

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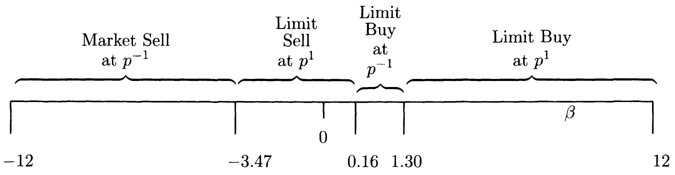
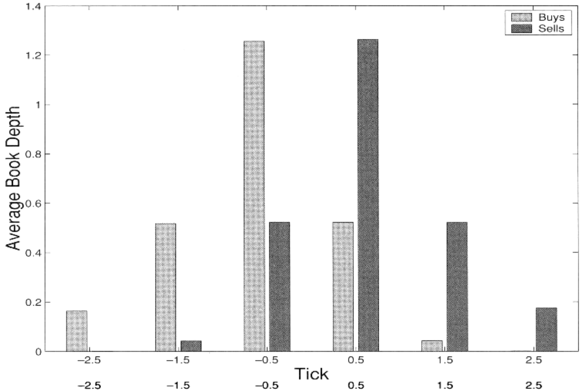
Goettler, P, Rajan (2005), II



Two limit buys at p^{-1}, i.e., $q^{-1} = 2$.

Goettler, P, Rajan (2005), III



Average depth at the ticks.

Foucault, Kadan and Kandel (2005)

- Patient (θ_P) and impatient ($\theta_I = 1 - \theta_P$) traders, with different waiting costs $\delta_I \geq \delta_P$.
- Trading crowd outside of $[B, A]$. Inside the spread, traders arrive at Poisson rate λ.
  1. One MO or LO per trader, no cancellation.
  2. LO must be improving.
  3. Buyers and sellers alternate with certainty.

Main results

1. Impatient agents *always* go market.

   Patient traders *always* go limit.
2. The book is sparse.

Rosu (2010)

- There is a flow of traders that experience waiting costs: patient and (very) impatient agents + trading crowd.
- Poisson processes with rates λ_P and λ_I.
- Continuous-time model: orders can be canceled or changed *at any time*, i.e., never!
- There are price-wars fought in infinitesimal time and other technicalities.
- Traders know that they will trade with certainty (sooner or later).

Main idea

In equilibrium, all orders must provide the same utility.

1. A more competitive LO gains less but is executed sooner.
2. A less competitive LO gains more but waiting costs are bigger.

A simple version

- For simplicity, all sellers are patient ($\lambda_{PS} = \lambda_1$) and all buyers are impatient ($\lambda_{IB} = \lambda_2$).
  1. The first seller places an ask $a_1 = A$.
  2. The second seller undercuts with $a_1 - \delta$, then the first undercuts,...
  3. ...till the second place $a_2 < a_1$ in such a way that they get the same utility.
- Denote the number of sellers in the book as m and let the expected utility of the m-th seller be f_m. At most M limit orders can be in the book and $f_M = B$.
  If the market is in state $m = 1, \ldots, M - 1$, it can go to $m + 1$ sellers (another ask) or to $m - 1$ (a market buy).
- Utilities must be the same in different states...

The full Rosu

- Now, there are patient buyers/sellers and impatient buyers sellers.
- Utilities and levels now depend on two indexes, $f_{m,n}, a_{m,n}, b_{m,n}$.

1.000	0.965					
1.000	0.905	0.824				
1.000	0.828	0.726	•			
1.000	0.770	0.616	0.500	•		
1.000	0.726	0.526	0.384	0.274	0.176	
1.000	0.697	0.468	0.300	0.177	0.095	0.035
1.000	0.682	0.440	0.260	0.131	0.045	0.000

- The book is sparse (only few levels are used).
- Let *activity* $\lambda = \lambda_1 + \lambda_2$ and *competition* $C = \lambda_1/\lambda_2$. Then
  1. $C > 1 \rightarrow$ resilient book.
  2. Average spread $\overline{S}$ is smaller when sellers are more patient and activity is high.
- Patient go limit, impatient go market

In a nutshell

	Analytical		Numerical
	Flows	Finite T	
Patient vs Impatient	Foucault (99) FKK (05) Rosu (10)		
Private Values		Parlour (98)	GoettlerPR (05) GoettlerPR (09)

Thanks (`paolop@unive.it`)