The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

IAEA International Atomic Energy Agency

2229-13

School and Workshop on Market Microstructure: Design, Efficiency and Statistical Regularities

21 - 25 March 2011

Price impact of metaorders: the latent order book

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Trieste, March 24, 2011

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Outline

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Price impact

What **is** price impact?

- Price impact: Correlation between the sign of an arriving order and the subsequent price change.
- In general, buyer (seller) initiated trades push the price up (down).

Several types of price impact

- Single trade impact vs. impact of several connected trades.
- Immediate impact vs. longer time impact
- Slippage: difference between average **realised** price and **initial** price

Metaorders

A metaorder is a **series** of connected trades in the same direction from the same initiator.
Reason: when trading a large position, typically we have to **split** it up and trade incrementally.

- Not enough liquidity available at any given moment: for a liquid stock the instantaneous volume in the order book is $\approx 10^{-5}$ of market capitalisation, while the total volume traded in a day is $\approx 10^{-3}$
- Not to disseminate too much information

Knowing trading IDs, we can measure it. Evidence for order splitting can be found from trade sign correlations.

Impact of metaorders

Widely observed result

A metaorder of size Q has a price impact: $I \propto \sqrt{Q}$.

More specifically:

$$I(Q) = Y\sigma_T\sqrt{\frac{Q}{V_T}}, \tag{1}$$

where

Q is the volume of the metaorder

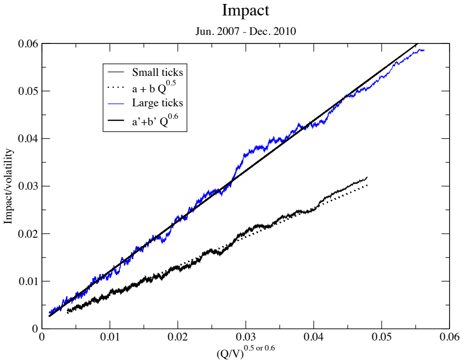
σ_T is the volatility of the market

V_T is the total volume traded in the market

Y is a constant of order unity

$$I(Q) = Y\sigma_T\sqrt{\frac{Q}{V_T}}$$

Results from CFM trades:



Note: this is a huge cost! Trading 1% of daily volume moves the price by 10% of daily volatility!

$$I(Q) = Y\sigma_T\sqrt{\frac{Q}{V_T}}$$

Importance of the square-root

- square-root is not additive ($I(Q_1+Q_2) \neq I(Q_1)+I(Q_2)$): but we are small compared to the market, how can the response be non-linear?
- diverging susceptibility for small volumes: small orders can generate large responses
- Y-ratio is "universal" ($Y \approx 0.5-0.75$ if $\sigma_T := p_{max}-p_{min}$)
- the relation is invariant to execution time: for diffusive prices $\sigma_t \propto \sqrt{N_t}$ and $V_t \propto N_t$ (N_t: number of trades)
- also the increase of impact in volume time is square-root: $I_q \propto I(Q)\sqrt{\frac{q}{Q}}$

What follows?

After having traded $Q/2$, the next $Q/2$ will have less impact
$\Rightarrow$ this means that there is increasing volume available deeper in the book
However the typical order book that one **measures** is not like that.

There has to be a **latent** volume that only appears when we push the price.

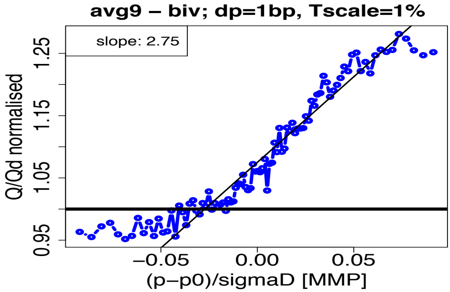
Further: the latent volume has to have a very specific shape.

$$Q = \int_{p_0}^{p_0+I} V_+(p)dp \tag{2}$$

$\Rightarrow$ $V_+(p) \propto (p - p_0)$ the order book should be **linear** (with relatively low volume on the best)

Evidence for latent volume

The volume needed to move the price **up** by an almost infinitesimal value (1 bp) as a function of the deviation of the price from its moving average.



If the price moved up from its average ($(p - p_0) > 0$) it gets harder to further push it upwards!

Outline

The idea of a latent order book

Assumptions

- between time t and $t+dt$, when price is $p(t)$, some investors decide to put a **latent** limit order of size v at a price $p(t)+\Delta$ with probability $\lambda_{-}(\Delta, v)dt$
  $\Rightarrow$ these orders are latent, since they may only actually appear later when the price approaches $p(t)+\Delta$
- investors can also cancel their latent orders: $\delta(\Delta, v)$
- the price, $p(t)$, fluctuates: effect of market orders

$\Rightarrow$ it is this this latent order book that we want to study

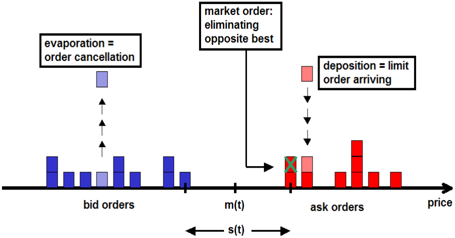
Basics of the model

Simplifications

- the basic model is essentially that of E. Smith et al. (2003)[a]
- latent limit orders arrive with a **constant, flat** rate per unit time per unit price over an infinite support $(\lambda_{\pm}(\Delta, v) \equiv \lambda)$ $\Rightarrow$ no Δ, v dependence
- cancellation: **constant, flat** rate of limit orders being removed per unit time $(\delta(\Delta, v) \equiv \delta)$ $\Rightarrow$ defines a lifetime
- market orders: shares per unit time (μ)
- discretization: tick size and discrete volume

[a] E. Smith, J.D. Farmer, L. Gillemot, S. Krishnamurthy, Quant. Fin. 3(6) (2003)

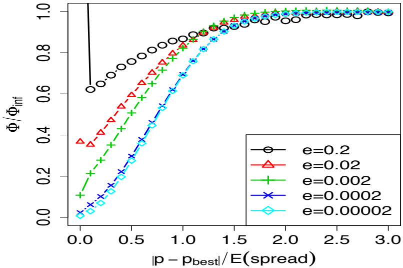
Scheme of the model



- Sell (buy) limit orders arrive at price levels $\geq$ ($\leq$) than the midprice.
- Cancellation rate defines the characteristic lifetime of orders.

Intuitively

The depth of the book far from the mid: λ/δ. Close to the best price MOs remove part of the volume.
The profile of the book will be defined by $\delta/\mu \equiv e$



There is an **increasing volume available** moving away from the current price.

Analytical treatment

Let $\rho(u)$ be the density of the book at a distance u away from the best price. Let's further assume that the price process is **diffusive**. Then in the (moving) reference frame of the best price, we can write the following equation of motion:

$$\frac{\partial \rho}{\partial t} = D\frac{\partial^2 \rho}{\partial u^2} + \lambda - \delta\rho \tag{3}$$

$$\rho = \frac{\lambda}{\delta}\left(1 - e^{-u/u^*}\right) \tag{4}$$

For small distances ($u \rightarrow 0$), we get

$$\rho = \frac{\lambda}{\delta u^*}u. \tag{5}$$

The book is expected to be **linear** around the best.

Analytical treatment

Linearity of the book

The linear regime extends roughly up to a distance u^*, which is of the order of the price variation on the scale of the lifetime of an order.

This is what we are looking for!

But in the these analytic calculations the diffusivity was **assumed**!

The idea again

Latent volume

Not the volume revealed in the real order book. The volume that would reveal itself if the price was to come closer.

What is latent, what is visible?

Clearly trades have to be visible: they appear in the **real** order book.

- But in the model trade signs at this point are not correlated!
  $\Rightarrow$ we need to add autocorrelation in the direction of trades
- Increasing volume away from the best prices: the basic model is mean reverting, prices are subdiffusive.
  $\Rightarrow$ we need to obtain a diffusive price process

Further ingredients

Autocorrelation

Model of Lillo-Mike-Farmer (LMF)[a]:
We generate runs of $+/-$ signs, where the length of runs comes from a Pareto distribution, $P(L) = \alpha L^{-(\alpha+1)}$. (It is like having background metaorders of power law distributed sizes.)
Can be shown that this leads to a power law autocorrelation:

$$C(\ell) \propto \ell^{-\gamma}, \text{ where } \gamma = \alpha - 1 \tag{6}$$

We have long-memory of the signs for $1 < \alpha < 2$. (Notation: LMFα)

Might be thought of as a realistic background.

[a] F. Lillo, Sz. Mike, J.D. Farmer, Phys. Rev. E 71, 066122 (2005)

Further ingredients

Conditioning on volume

For real markets: it is the **fraction** of volume taken from the opposite best, that is (roughly) constant.
To mimic this behaviour we define the following probability of the fraction (f) being taken:

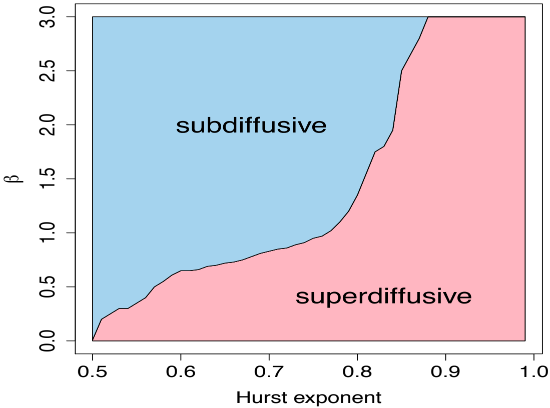
$$P(f) = \beta(1-f)^{\beta-1} \tag{7}$$

- $\beta \to 0$: all volume is taken
- $\beta = 1$: flat distribution of volume taken
- $\beta \to \infty$: minimum (unit) volume is taken

Note: this way the best bid and ask levels are visible!

Diffusion "map"

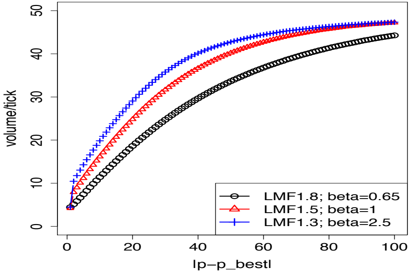
We can construct the map of diffusivity to find the regime that is relevant for us. $(H = 1 - \gamma/2)$



Summary of the model

A model, that aims at describing the latent liquidity. We have:

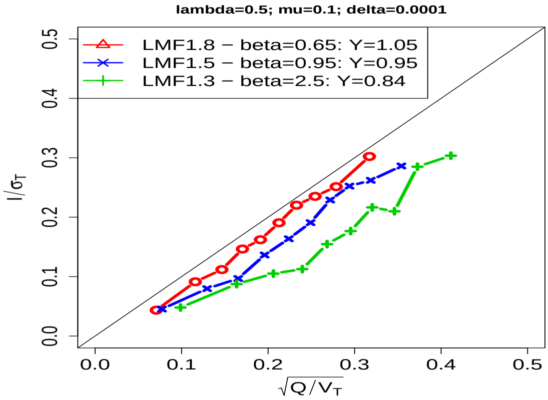
- long-memory of trade signs
- opposing flow of limit orders
- diffusive prices
- on average linear book



Outline

Introducing metaorders

Now we have the background model, we can introduce metaorders, to study their impact.
For now, metaorders trade entirely through MOs.



Introducing metaorders

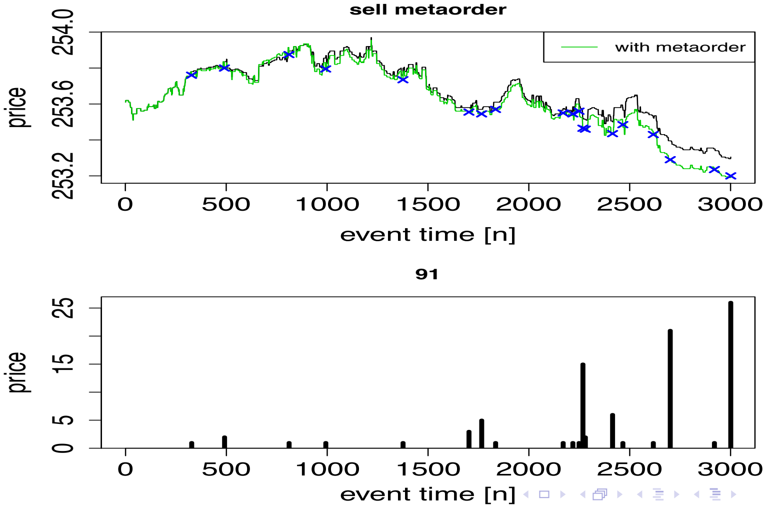
For realistic parameters

- the model reproduces the square root impact

$$I(Q) = Y\sigma_T\sqrt{\frac{Q}{V_T}},$$

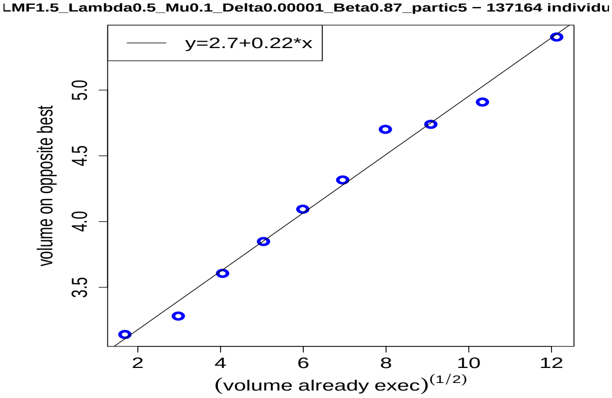
- weak dependence on the autocorrelation exponent
- no dependence on participation rate (went down to 2% participation rate, sqrt is measurable)
- Y-ratio is of order unity (between $0.8 - 1.1$)

Looking into the mechanism – what we cannot do in real life



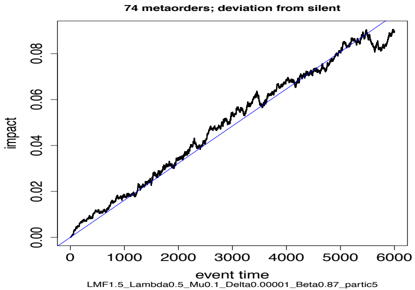
Available volume

The volume on best increases as $Q_{avail} \propto \sqrt{Q} \Rightarrow dQ/dn \propto \sqrt{Q}$
$\Rightarrow \sqrt{Q} \propto n$.



Impact vs real time

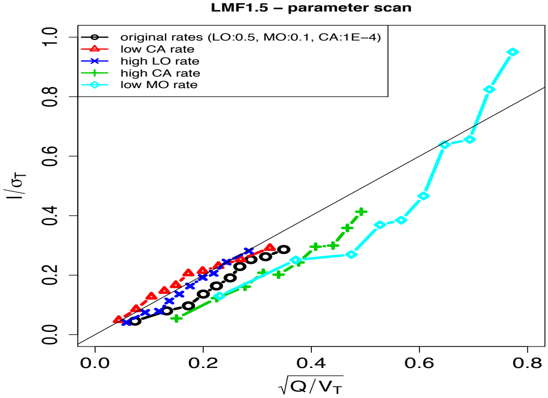
The impact increases linearly in (event) time: $I \propto n$.
This can also be seen in data!



Naïvely: $I \propto n$ and $Q \propto n^2 \Rightarrow I \propto \sqrt{Q}$

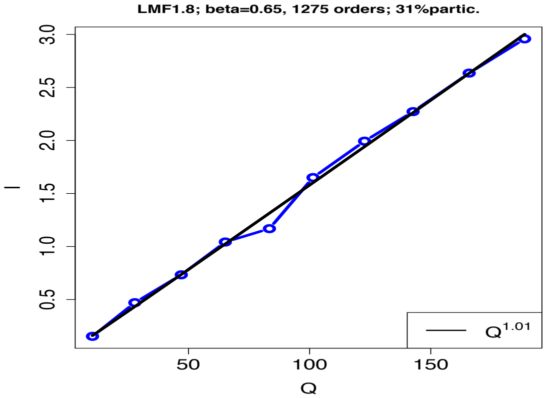
Stability of the results

Varying the parameters λ, μ, and δ.
When diffusive prices can be achieved and $\tau_{execute} << \tau_{life}$, we always find the same form.



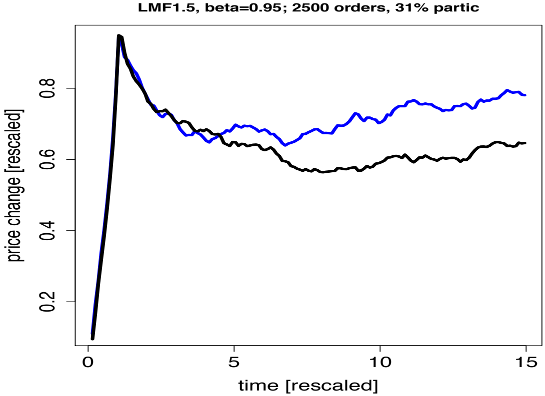
The case of fast relaxation

If $\tau_{execute} > \tau_{life}$ we get linear dependence: $I(Q) \propto Q$, as should expect. (This plot is I vs. Q !!!)



Decay of impact

After the finishing a metaorder, our impact decays. In event time seems to go down to approx $0.6\times$ temporary impact, then it gets noisy. Similar result seen for data by Moro et al. (2010).



Outline

Summary

- Empirical fact: impact of metaorders is sqrt of volume
- Evidence for latent order book

The model

- Latent volume taken into account
- Very few behavioural assumptions
- Diffusive prices
- Analytic calculations tell that on average the book should be linear around the current price

Summary

We find

- Sqrt impact (when diffusive prices and $\tau_{exec} << \tau_{life}$)
- Y-ratio of order 1
- No participation rate dependence
- Stable results against changing parameters
- Zoom into the build-up of impact: $I \propto n$ and $Q \propto n^2$
- More realistic execution: also sqrt, lower Y