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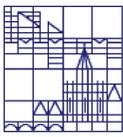
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and Statistical Regularities**

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**On the Evolution of Market Institutions  
The Platform Design Paradox**

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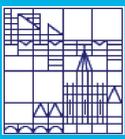
# On the Evolution of Market Institutions

## The Platform Design Paradox

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## On the Evolution of Market Institutions

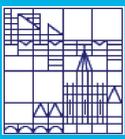
**Introduction**

**Platforms**

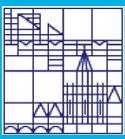
**Learning Process**

**Platform Design**

**Conclusion**



# Introduction



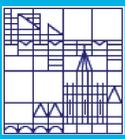
# Introduction

Introduction Platforms Learning Process Platform Design Conclusion

▶ Alós-Ferrer and Kirchsteiger (2010, 2011):

If several alternative market institutions are available for the same good (some of them biased, hence involving rationing) traders follow myopic behavioral rules, they will learn to coordinate on market-clearing ones (if available) at least part of the time.

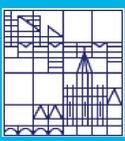
▶ **This paper:** What happens when rational market designers anticipate the behavior of market traders and actively design market platforms?



# Motivation

Introduction Platforms Learning Process Platform Design Conclusion

- ▶ About 95% of world e-commerce is actually B2B (Business-To-Business), trade of relatively standardized products.
- ▶ Estimated 2004 transaction volume  $\sim$  \$ 1 trillion.
- ▶ About one third of B2B platforms are run by third parties (neither sellers nor buyers), who actively design the platform characteristics.  
The rest are run by one market side, but there is still active design.



# Asymmetric Rationality

Introduction Platforms Learning Process Platform Design Conclusion

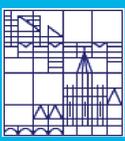
- ▶ Economic agents are boundedly rational... (*yawn!*)
- ▶ ...but maybe some agents are more rational than others?

## Asymmetric Rationality models.

- ◇ Fully rational **firms** face boundedly rational **consumers**.

Gabaix and Laibson (2003), Severinov and Deneckere (2004), Hopkins (2005).

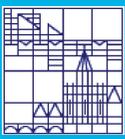
- ◇ Market designers vs. traders: **this paper**.
- ◇ Social planners vs. citizens: Fei Shi, 2011 (migration model).



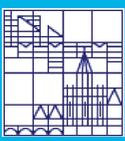
# Competing Platforms

Introduction Platforms Learning Process Platform Design Conclusion

- ▶ The paper is an exploratory, extended example.
- ▶ The following is general:
  - ◇ Boundedly rational, myopic **buyers and sellers** as in yesterday's model.
  - ◇ General dynamics.
  - ◇ Buyers can be heterogeneous.
  - ◇ (Relatively) Rational, long-lived **Market Designers** actively design the platforms.
- ▶ The following is particularized:
  - ◇ **Two** competing platforms
  - ◇ Platforms characterized by **constant price bias  $\beta$**  and **fee  $f$** .
  - ◇ Sellers are producers endowed with a **particular production technology**.



# Platforms



# Sequence of Actions

Introduction Platforms Learning Process Platform Design Conclusion

## ▶ Stage 1: Design

Market designers introduce trading platforms and decide about

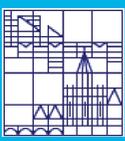
- ◇ the trading fees demanded from the traders, and
- ◇ market clearing properties of the platform (bias).

## ▶ Stage 2: Competition between platforms

Boundedly rational traders dynamically learn which platform to use.

- ◇ Traders simultaneously choose platform on the basis of last outcomes.
- ◇ Market outcomes realized.
- ◇ Iterate.

## ▶ Q: Will this process lead to the introduction of market clearing institutions?



# Platforms

- ▶ Two platforms,  $i = 1, 2$

Platform characteristics:

$$s_i = (\beta_i, f_i)$$

- ▶ Fee:  $f_i$

Market Designer's Profits:  $\Pi_{D,i} = f_i \cdot ER_i$

$ER_i$  = Expected per-round revenue on  $i$ .

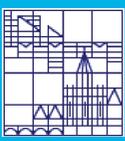
Assume  $f_i \in F = \{f_{\min}, f_{\min} + \gamma, \dots, f_{\max}\}$  with  $0 < f_{\min} < f_{\max} < 1$ .

- ▶ Bias:  $\beta_i > 0$

Price at platform  $i$ :  $p_i = \beta_i \cdot p_i^*$ .

$p_i^*$  = Market-Clearing price.

$\beta_i \in B = \{\beta_{\min}, \beta_{\min} + \delta, \dots, 1, \dots, \beta_{\max}\}$  with  $0 < \beta_{\min} < 1 < \beta_{\max}$ .



- ▶ Finite set of sellers  $M$ .

Sellers  $m$  are profit-maximizing firms.

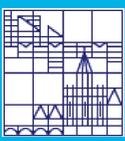
Constant marginal costs  $c > 0$ .

This is restrictive; allows to obtain results for general dynamics.

- ▶ Finite set of buyers  $N$ .

Buyer  $n$  endowed with demand functions  $d_n(p)$ , strictly decreasing in  $p$ ,  $d_n(p) > 0$  for all  $p$ .

Buyers can be heterogeneous.



# Price Formation

- ▶ A platform is **active** if both sellers and buyers are present and strictly positive quantities are traded; **inactive** if not.
- ▶ A buyer pays  $p_i$ ; the seller receives  $(1 - f_i)p_i$ , the market designer  $f_i p_i$ .
- ▶ Due to constant returns, in active platforms the market-clearing price is

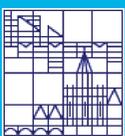
$$p_i^*(s_i) = \frac{c}{1 - f_i}$$

and the realized price is

$$p_i(s_i) = \beta_i \frac{c}{1 - f_i}$$

- ▶ If  $\beta_i < 1$ , supply is zero and the platform is inactive.
- ▶ Let  $N_i, M_i$  be the subsets of buyers and sellers who choose platform  $i$ .  
 $i$  is active if and only if  $N_i \neq \emptyset \neq M_i$  and  $\beta_i \geq 1$ .
- ▶ Define

$$D_{N_i}(p) = \sum_{i \in N_i} d_n(p)$$



# Platform Outcomes

- ▶ A buyer  $n$  at  $i$  trades

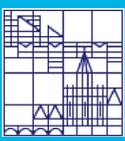
$$q_{n,i}(N_i, M_i, s_i) = \begin{cases} d_n(\beta_i \frac{c}{1-f_i}) & \text{if } i \text{ is active,} \\ 0 & \text{otherwise,} \end{cases}$$

Buyers are never rationed if they get to trade.

- ▶ A seller  $m$  at  $i$  trades

$$q_{m,i}(N_i, M_i, s_i) = \begin{cases} \frac{1}{|M_i|} D_{N_i} \left( \beta_i \frac{c}{1-f_i} \right) & \text{if } i \text{ is active,} \\ 0 & \text{otherwise.} \end{cases}$$

(equal rationing)



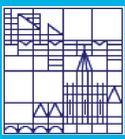
# Evaluation of Outcomes

Introduction Platforms Learning Process Platform Design Conclusion

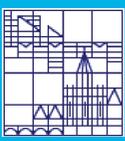
- ▶ **Behavioral assumption:** Buyers prefer platforms with lower prices; inactive platforms evaluated as worse than active ones.
- ▶ Sellers evaluate platforms by profits.

$$\pi_{m,i}(N_i, M_i, s_i) = \begin{cases} q_{m,i}(N_i, M_i, s_i)(\beta_i - 1)c & \text{if } i \text{ is active,} \\ 0 & \text{otherwise.} \end{cases}$$

Note sellers always prefer active non-market clearing platforms to market-clearing ones.



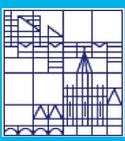
# Learning Process



# Learning Process

Introduction Platforms Learning Process Platform Design Conclusion

- ▶  $t = 0$ : Market designers select  $s_i, i = 1, 2$ .
- ▶ At the end of a period  $t = 1, 2, \dots$ , traders observe outcome of both platforms.
- ▶ **Behavior**: If a trader is allowed to revise his choice of platform, he switches to the platform with the outcome at  $t$ , which is best for him (myopia!). This determines distribution of traders over institutions for period  $t + 1$ .  
In case of indifference, choice is randomized, with both platforms chosen with strictly positive probability.
- ▶ State of the (Markov) process:  $\omega_t \in \Omega = \{1, 2\}^N \times \{1, 2\}^M$ .
- ▶ **Monomorphic states**:  $\omega_i^*$  ( $i = 1, 2$ ) such that  $\omega(n) = i$  for all  $n \in N$  and  $\omega(m) = i$  for all  $m \in M$ .



# Revision opportunities

- ▶ Random revision opportunities.

$E(k, \omega)$  event that trader  $k$  receives revision opportunity at state  $\omega$ .

$E^*(k, \omega) \subseteq E(k, \omega)$  event that  $k$  is the only trader of his type with revision opportunity at  $\omega$ .

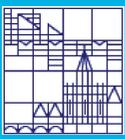
- ▶ For every trader  $k$  and state  $\omega$ ,

(D1)  $Pr(E^*(k, \omega)) > 0$ .

(D2) either  $Pr(E^*(k, \omega) \cap E^*(k', \omega)) > 0$  for any trader  $k'$  of the other type, or  $Pr(E^*(k, \omega) \cap E(k', \omega)) = 0$  for all such  $k'$ .

- ▶ Encompasses many standard learning models, like those with

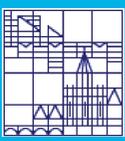
- ◇ **independent inertia**: Exogenous, independent, strictly positive probability that an agent cannot revise.
- ◇ **asynchronous learning**: only one agent per period has positive probability of revision.



# Stochastic Stability

Introduction Platforms Learning Process Platform Design Conclusion

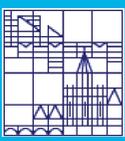
- ▶ Small experimentation probability  $\varepsilon > 0$ : platform chosen at random, both with positive probability.
- ▶ **Stochastically stable states**: those in the support of the limit invariant distribution  $\mu^*$  as  $\varepsilon \rightarrow 0$
- ▶ By the Ergodic Theorem: only those observed a significant proportion of the time in the long run (for  $\varepsilon$  small).



# Stochastic Stability

Introduction Platforms Learning Process Platform Design Conclusion

- ▶ Given two states  $X$  and  $Y$ , let the **transition cost** from  $X$  to  $Y$ ,  $c(X, Y) > 0$ , denote the minimal number of experiments necessary for a direct transition from  $X$  to  $Y$ .
  
- ▶ **Theorem 1:**
  - (a) If  $\beta_i > 1$  and  $\beta_j \leq 1$ , the only stochastically stable state is  $\omega_i^*$ .
  - (b) If  $\beta_1 \leq 1$  and  $\beta_2 \leq 1$ , all states in  $\Omega$  are stochastically stable.
  - (c) If  $\beta_1 > 1$  and  $\beta_2 > 1$ , only  $\omega_1^*, \omega_2^*$  can be stochastically stable.  
Further,  $\omega_i^*$  is stoch.stable if and only if  $c(\omega_i^*, \omega_j^*) \geq c(\omega_j^*, \omega_i^*)$ .
  
- ▶ **Proposition:** Suppose  $\beta_i, \beta_j > 1$  and  $p_i = \frac{\beta_i c}{1-f_i} \leq \frac{\beta_j c}{1-f_j} = p_j$ . Then,  $c(\omega_i^*, \omega_j^*) \geq 2 = c(\omega_j^*, \omega_i^*)$  and hence  $\omega_i^*$  is stochastically stable.



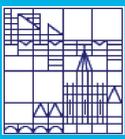
# Further Results?

- ▶ These results are independent of the dynamics, elasticities, population sizes, etc. Further results would not. For example...
- ▶ **Proposition:** Suppose  $\beta_i, \beta_j > 1$ ,  $p_i < p_j$ , so that  $\omega_i^*$  is stochastically stable.
  - (a) in a dynamics with independent inertia,  $\omega_j^*$  is also stoch. stable if and only if there is at least one buyer  $\tilde{n} \in N$  such that

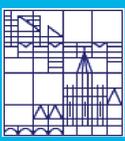
$$d_{\tilde{n}}(p_j)(\beta_j - 1) \geq \frac{1}{|M| - 1} D_{N \setminus \{\tilde{n}\}}(p_i)(\beta_i - 1).$$

- (b) in a dynamics with asynchronous learning,  $\omega_j^*$  is also stoch. stable if and only if there is at least one buyer  $\tilde{n} \in N$  such that

$$\frac{1}{|M| - 1} d_{\tilde{n}}(p_j)(\beta_j - 1) \geq D_{N \setminus \{\tilde{n}\}}(p_i)(\beta_i - 1).$$



# Platform Design



# Platform Design

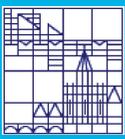
Introduction Platforms Learning Process Platform Design Conclusion

- ▶ Market Designers select  $s_i$  at  $t = 0$ .
- ▶ Objective is to maximize long-run profits: hence, focus on the limit invariant distribution.
- ▶ Benchmark: **Monopolistic market design**

A monopolist would set  $\beta^* = 1$ .

*Intuition: maximize trade volume, he lives off the fees.*

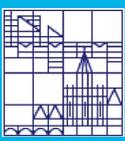
For monopolistic market design, market-clearing leads to maximum designer's revenues.



# The Platform Design Game

Introduction Platforms Learning Process Platform Design Conclusion

- ▶ Market Designers play a (finite) 2-player, non-cooperative, normal-form game.
- ▶ To define the payoffs in this game, we need the invariant distribution  $\mu^*$  ... which can not be computed (in practice) except in specific cases.
- ▶ But we can get enough information to enable equilibrium analysis.



# The Platform Design Game

Introduction Platforms Learning Process Platform Design Conclusion

**Lemma 2:** Let  $s_i = (\beta_i, f_i)$ ,  $s_j = (\beta_j, f_j)$ .

Let the prices be  $p_i = \beta_i \frac{c}{1-f_i}$ ,  $p_j = \beta_j \frac{c}{1-f_j}$ .

(a) If  $s_i = s_j$ , then,  $\Pi_{D,k}(s) = \frac{1}{2} f_i p_i D_N(p_i) > 0$  ( $k = 1, 2$ ).

(b) If  $\beta_i = \beta_j = 1$  and  $f_i < f_j$ , then  $f_k p_k D_N(p_k) > \Pi_{D,k}(s_i, s_j) > 0$  ( $k = 1, 2$ ).

(c) If  $\beta_i > 1$  and  $\beta_j \leq 1$ , then  $\Pi_{D,i}(s_i, s_j) = f_i p_i D_N(p_i)$  and  $\Pi_{D,j}(s_j, s_i) = 0$ .

(d) If  $\beta_i, \beta_j > 1$ ,  $p_i \leq p_j$ , and  $c(\omega_i^*, \omega_j^*) > 2$ , then

$$\Pi_{D,i}(s_i, s_j) = f_i p_i D_N(p_i) \quad \text{and} \quad \Pi_{D,j}(s_i, s_j) = 0$$

(e) If  $\beta_i, \beta_j > 1$ ,  $p_i \leq p_j$ , and  $c(\omega_i^*, \omega_j^*) = 2$ , then

$$\Pi_{D,k}(s_i, s_j) = \mu^*(\omega_k^*) f_k p_k D_N(p_k) > 0 \quad k = 1, 2$$



# Main Result

- ▶ The Market Designers game (allow for mixed strategies).
- ▶ **Theorem 2:** Let  $(\sigma_i^*, \sigma_j^*)$  be a Nash equilibrium (possibly in mixed strategies). If the grid  $B$  is fine enough, for any pure strategy  $s_i = (\beta_i, f_i)$  of player  $i$  such that  $\sigma_i^*(s_i) > 0$ , we have that  $\beta_i > 1$ .

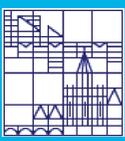
- ▶ In other words...

Although a monopolist would favor market clearing platforms, competing market designers never allow for them; hence traders necessarily coordinate on platforms with prices biased above market-clearing level.

- ▶ Thus **competition among market designers induces** them to select biased platforms...

...hence inducing **non-competitive market outcomes**.

- ▶ Note that: If  $\beta_i = 1$ , sellers make zero profits and it is very easy to attract them away from platform  $i$ .



# Sketch of proof

- ▶ No platforms with  $\beta_k < 1$  are introduced in equilibrium (inactive!).
- ▶  $C(\sigma_j^*) = \left\{ s_j = (\beta_j, f_j) \in S \mid \sigma_j^*(s_j) > 0 \right\}$ .
- ▶ By contradiction, suppose  $\exists \bar{s}_i = (1, \bar{f}_i)$  with  $\sigma_i^*(s_i) > 0$ . Let

$$\bar{p} = \frac{c}{1 - \bar{f}_i}.$$

- ▶ Suppose  $\beta_j > 1$  for all  $s_j \in C(\sigma_j^*)$ .

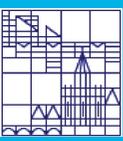
By Lemma 2(c),

$$\pi_{D,i}(\sigma_i^*, \sigma_j^*) = \pi_{D,i}(\bar{s}_i, \sigma_j^*) = 0.$$

For  $s'_i \in C(\sigma_j^*)$ , by Lemma 2(a),

$$\pi_{D,i}(s'_i, \sigma_j^*) \geq \sigma_j^*(s'_i) \pi_{D,i}(s'_i, s'_i) > 0.$$

Hence, player  $i$  has an incentive to deviate, contradiction.



# Sketch of proof

- ▶ Thus  $\exists s_j \in C(\sigma_j^*)$  with  $\beta_j = 1$ .

Let  $C_1(\sigma_j^*) = \left\{ s_j = (\beta_j, f_j) \in C(\sigma_j^*) \mid \beta_j = 1 \right\}$ .

By Lemma 2(c),  $\pi_{D,i}(\bar{s}_i, s_j) = 0$  for all  $s_j \in C(\sigma_j^*)$  with  $\beta_j > 1$ .

By Lemma 2(a,b),

$$\pi_{D,i}(\bar{s}_i, \sigma_j^*) < \sum \left\{ \sigma_j^*(s_j) \bar{f}_i \bar{p} D_N(\bar{p}) \mid s_j \in C_1(\sigma_j^*) \right\}$$

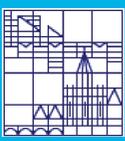
- ▶ For  $s'_i$  with  $\beta'_i > 1$ ,  $f'_i = \bar{f}_i$ , by Lemma 2(c)

$$\pi_{D,i}(s'_i, \sigma_j^*) \geq \sum \left\{ \sigma_j^*(s_j) \bar{f}_i \beta'_i \bar{p} D_N(\beta'_i \bar{p}) \mid s_j \in C_1(\sigma_j^*) \right\}$$

- ▶ By continuity in  $\beta'_i$ ; For  $\beta'_i \rightarrow 1+$ ,

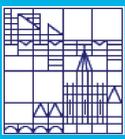
$$\pi_{D,i}(s'_i, \sigma_j^*) > \pi_{D,i}(\bar{s}_i, \sigma_j^*) = \pi_{D,i}(\sigma_i^*, \sigma_j^*).$$

Hence, if the grid is fine enough, player  $i$  has an incentive to deviate, contradiction.

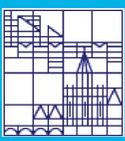


# Comments

- ▶ Nothing more can be said about the form of the Nash Equilibria.  
A full characterization depends on details of the dynamics, elasticity of demand, etc. For example:
- ▶ **Proposition:** Assume independent inertia, identical buyers and  $|M| = |N|$ . If  $B$  and  $F$  fine enough and  $\epsilon_p(p) = -\frac{pd'(p)}{d(p)}$  sufficiently small, there exists no Nash equilibrium where both designers introduce only platforms with  $\beta_i = \beta_j = 1 + \delta$ .
- ▶ We assumed constant returns to scale.  
If  $\beta_i = 1$ , sellers make zero profits and it is easy to attract them away from platform  $i$ .
- ▶ Under decreasing returns to scale, results are not so sharp.  
However, for certain dynamics we can find examples with decreasing returns to scale where the paradox appears again – it is not a phenomenon of constant returns to scale only.

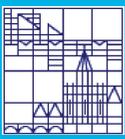


# Conclusion



# Conclusion

- ▶ The evolutionary analysis of which institutions survive in the long run is important; but it is only one aspect.
- ▶ **Asymmetric Rationality** approach: Market Designers are rational enough to anticipate the bounded rationality of traders.
- ▶ As a consequence, not all institutions (not even all “natural” institutions) will be introduced!
- ▶ **The Platform Design Paradox:**  
Competition among market designers might lead to a predominance of biased institutions, resulting in non-competitive market outcomes.



**Thank You for Your Attention (again!)**