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Computable dynamic trading equilibria in continuous double auctions or 'optimal trading with linear strategies'

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Computable dynamic trading equilibria in continuous double auctions or "Optimal trading with linear strategies"

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Inspiration

Foucault (99) F, Kadan, Kandel (05) Rosu (10)	Patience Impatience	Flow

- Unit quantity.
- Improving orders.
- Alternance.
- *No cancelation.*

Trading is certain!

- Impatient traders always go market.
  Patient traders never go market.

Inspiration, II

Parlour (98)	Private values/costs	Finite
Goettler, P, Rajan (05)		Flow
Goettler, P, Rajan (09)		Flow + I

- Equilibrium is computable with numerical methods in GPRs.
- Unit quantity and non-improving orders.
- Time constraints.
- (Controlled) no-cancelation
- More “impatience” means more market orders.

This paper

- Limited time to trade $t = 1, .., T$ (few agents).
- For each round, traders with private values/costs are drawn from a large pool and enter the market in random order.
- Two main sources of uncertainty related to $\sigma(\mathcal{A})$.
  1. Time, i.e., position in a random queue, σ.
  2. Trading partners, $\mathcal{A}$.
- Maximize average profit.
- Agents bid/ask using linear functions of the current best bid and best ask.

Results

- “Enough is enough”: impatient traders do not *always* submit market orders.
- “Steal the deal”: patient traders *occasionally* take liquidity.
- In equilibrium, only few states are played. Hence, numerical methods may survive to the curse of dimensionalty of a full blown model.
- (Strategies are simple, description is easier)
- More fundamentally, extreme behavior is due to analytical tractability.

The model

- The market.
  1. Standard CDA with continuous prices in$[0, 1]$.
  2. In one trading sessions, there are $T = 2n$ rounds. Traders enter the market in random order, one for each round.
  3. Unit quantity and no-cancelation.
- The agents.
  1. There are n buyers with known value v_i and n sellers with known cost c_j.
  2. If trading occurs, profits are $v_i - p$ for buyers and $p - c_j$ for sellers.
  3. In each session, nature randomly picks traders from a larger pool of $N + N$ agents.

The agents

- Assume the selected buyers are

$$\mathcal{B} = \{i_1, i_2, \ldots, i_n\} \subset \{1, \ldots, N\},$$

  together with the sellers

$$\mathcal{S} = \{j_1, j_2, \ldots, i_n\} \subset \{1, \ldots, N\}.$$

- Let the (unknown) set of traders be

$$\mathcal{A} = \mathcal{B} \cup \mathcal{S},$$

  and let $\sigma(\mathcal{A})$ be a random permutation of $\mathcal{A}$.
- The i-th buyer solves

$$\max_{B_t \leq v_i} E[\pi_i(B_t | v_i, H_t)],$$

  $E[\cdot]$ is taken over all $\sigma \in \mathcal{P}(\mathcal{A})$ and over all $\binom{n}{N}^2$ choices of $\mathcal{A}$.

Information and strategies

- The position in the queue and $\mathcal{A}$ can be (partially) inferred using information $\mathcal{H}_t$.
- At time t,
$$\mathcal{H}_t = (a_t, b_t).$$
- Large (small) a_t, b_t means there are stronger buyers (sellers).
- The spread $a_t - b_t$ provides information on the position.
- Linear strategies:

$$\begin{aligned} B_t(v_i, H_t) &= v_i - (\alpha_i a_t + \beta_i b_t + \gamma_i) \\ A_t(c_j, H_t) &= c_j + (\alpha_j a_t + \beta_j b_t + \gamma_j) \end{aligned}$$

Parameters

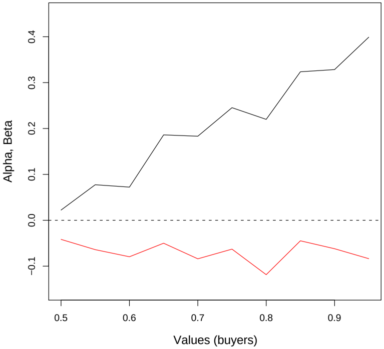
Name	Value	Description
n	10	Number of buyers and sellers
N	380	Total number of buyers and sellers (pool)
V		$\{0.05, 0.10, \ldots, 0.95\} \subset [0, 1]$
C		$\{0.05, 0.10, \ldots, 0.95\} \subset [0, 1]$

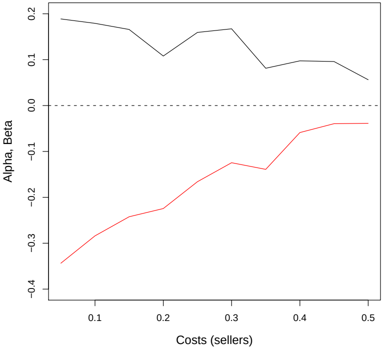
20 agents for each of the 19 types
Ex ante equilibrium price = 0.5
Results are based on 10 runs of 50000 sessions
$2 \times 3 \times 19 = 114$-dimensional problem: ES

Results

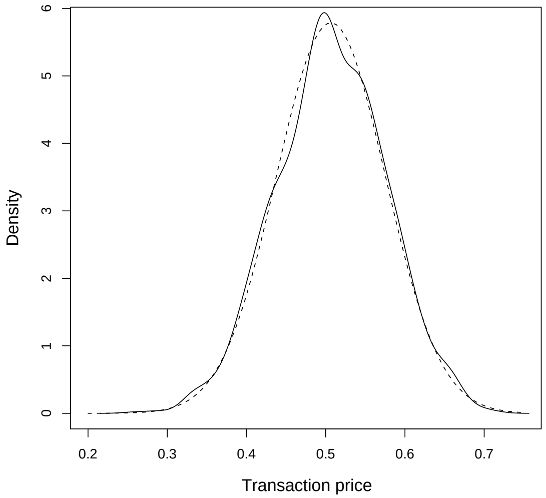
Values of intramarginal buyers						
	0.50	0.60	0.70	0.80	0.90	0.95
α	0.02	0.07	0.18	0.22	0.33	0.40
β	-0.04	-0.08	-0.08	-0.12	-0.06	-0.08
20γ	1.31	1.78	1.06	2.04	1.32	0.99
Costs of intramarginal sellers						
	0.05	0.10	0.20	0.30	0.40	0.50
α	0.19	0.18	0.11	0.17	0.10	0.06
β	-0.34	-0.28	-0.22	-0.12	-0.06	-0.04
20γ	5.97	5.31	4.47	2.03	1.52	1.00

Strategies

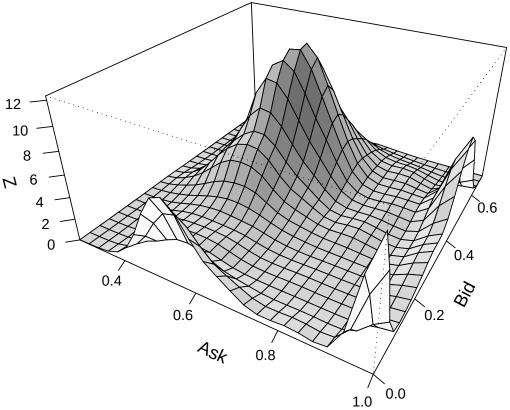

Alpha, Beta
0.4
0.3
0.2
0.1
0.0
−0.1
0.5
0.6
0.7
0.8
0.9
Values (buyers)


Alpha, Beta
0.2
0.1
0.0
−0.1
−0.2
−0.3
−0.4
0.1
0.2
0.3
0.4
0.5
Costs (sellers)

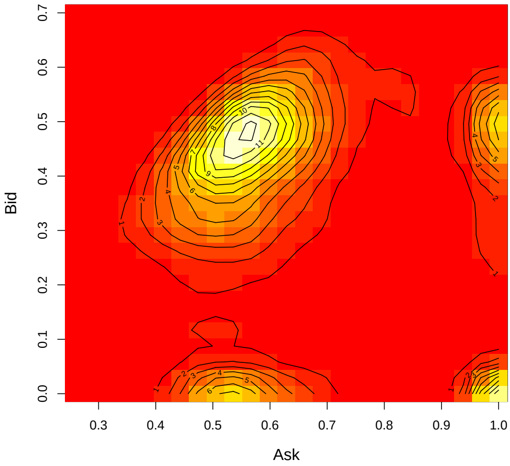
Transaction price



Equilibrium states



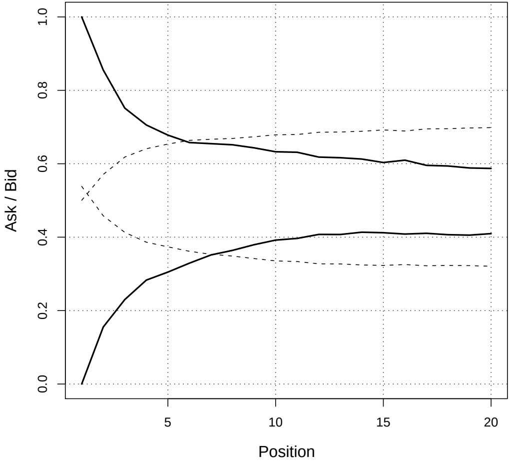
Equilibrium states, II



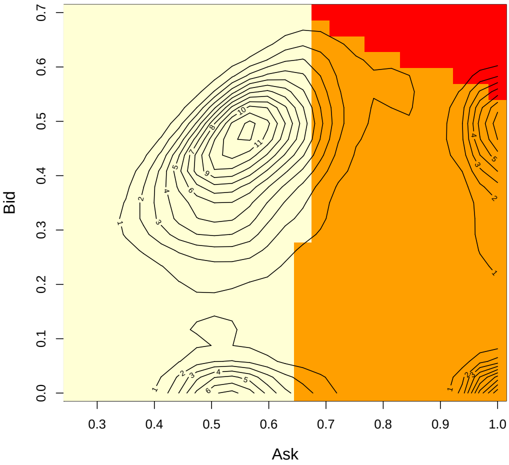
Frequent states

Frequency of states		Bid b_t 0.45-0.55	0.40-0.60	0.35-0.65
Ask a_t	0.45-0.55	0.06	0.12	0.15
	0.40-0.60	0.12	0.23	0.28
	0.35-0.65	0.16	0.29	0.37

Average book

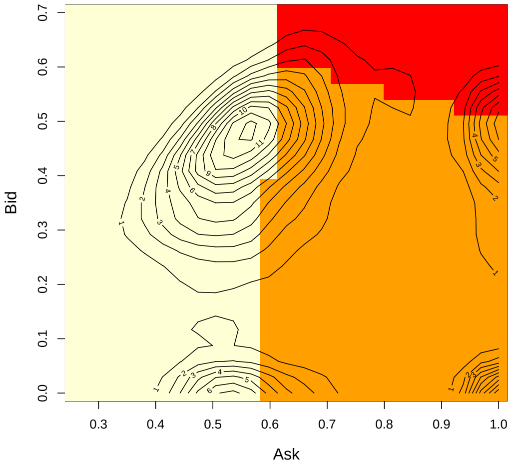


Market, improving and weak orders



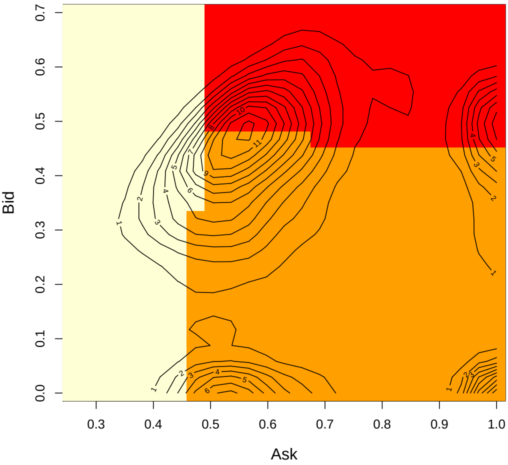
Value = 0.95

Market, improving and weak orders



Value = 0.75

Market, improving and weak orders



Value = 0.55

Evolution strategies - ES

Each population independently maximizes the gain from trade over τ of sessions, given the behavior of the other populations:

$$\max_{\alpha,\beta,\gamma} \sum_{i=0}^{\tau} \pi_t(\alpha, \beta, \gamma | \text{Other types}).$$

1. Set $g = 0$ and initialize the population $\mathcal{P}^{(0)}$ with $y_m^{(0)} = (\alpha_m^{(0)}, \beta_m^{(0)}, \gamma_m^{(0)}, A_m^{(0)}, B_m^{(0)}, C_m^{(0)})$, $m = 1, \ldots, \lambda$;
2. Repeat
    1. sample without replacement $n + n$ agents and trade;
    2. cumulate profit $F_m^{(g)}$ for τ sessions;
    3. select the best μ agents out of λ according to $F_m^{(g)}$. Let the selected agents form the population $\mathcal{Q}^{(g)}$;
    4. s-mutation, y-mutation, $g = g + 1$.

Evolution strategies - ES, II

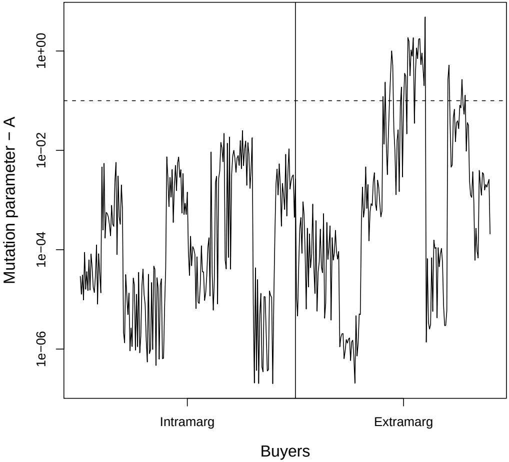
for $l = 1, \ldots, \lambda$ do

1. sample with replacement one agent $(\alpha_k, \beta_k, \gamma_k, A_k, B_k, C_k) \in \mathcal{Q}^{(g)}, k \in \{1, \ldots, \mu\}$
2. let

$$
\begin{aligned}
A_l^{(g+1)} &= \exp(\upsilon \tilde{z}) A_k^{(g)}; & \alpha_l^{(g+1)} &= \alpha_k^{(g)} + \tilde{z} A_l^{(g+1)} \\
B_l^{(g+1)} &= \exp(\upsilon \tilde{z}) B_k^{(g)}; & \beta_l^{(g+1)} &= \beta_k^{(g)} + \tilde{z} B_l^{(g+1)} \\
C_l^{(g+1)} &= \exp(\upsilon \tilde{z}) C_k^{(g)}; & \gamma_l^{(g+1)} &= \gamma_k^{(g)} + \tilde{z} C_l^{(g+1)}
\end{aligned}
$$

let the new individuals $(\alpha_l^{(g+1)}, \beta_l^{(g+1)}, \gamma_l^{(g+1)}, A_l^{(g+1)}, B_l^{(g+1)}, C_l^{(g+1)}), l = 1, \ldots, \lambda$ form the population $\mathcal{P}^{(g+1)}$.

Gauging convergence



A for buyers.

Conclusion

- Heterogeneous agents use simple linear strategies.
- In equilibrium, impatient traders often take liquidity (but not always).
- Patient traders often provide liquidity (but “steal the deal” when possible).
- The set of equilibrium states is small (and numerical methods can be used).
- The on-off behavior of analytical models is no longer present with more heterogeneity (numerical methods are needed).

Thank you

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