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Order flow in financial markets: Origin of persistence and impact of metaorders

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Market order flow

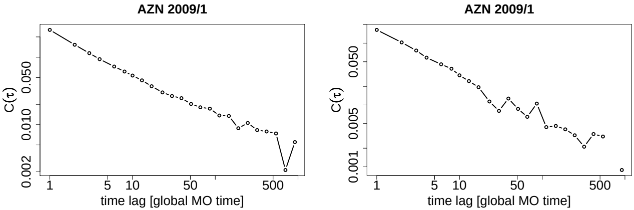
- We focus our attention here on market orders.
- The investigated set is composed by 6 liquid stocks (AZN, BLT, BSY, LLOY, PRU, and VOD) traded at the London Stock Exchange (LSE) in the period 2000-2009. We consider only the electronic market.
- A market order is characterized by a volume v and a sign $\epsilon = +1$ for buy orders and $\epsilon = -1$ for sell orders.
- We consider the time series in market order time, i.e. time advances of one unit when a new market order arrives.
- The unconditional sample autocorrelation function of signs is

$$C(\tau) = \frac{1}{N} \sum_t \epsilon_t \epsilon_{t+\tau} - \left(\frac{1}{N} \sum_t \epsilon_t \right)^2 ,$$

where N is the length of the time series.

Market order flow is a long memory process

It has been shown (Bouchaud *et al.*, 2004, Lillo and Farmer, 2004) that the time series of market order signs is a long memory process.



$C(\tau)$ of market order signs ϵ (left) and signed volumes ϵv (right).
The autocorrelation function decays asymptotically as

$$C(\tau) \sim \tau^{-\gamma} = \tau^{2H-2}$$

where H is the Hurst exponent. For the investigated stocks $H \simeq 0.75$ (i.e. $\gamma \simeq 0.5$).

What is the origin of long-memory in order flow?

Two explanations has been proposed

- Herding among market participants (LeBaron and Yamamoto 2007). Agents herd either because they follow the same signal(s) or because they copy each other trading strategies. Direct vs indirect interaction
- Order splitting (Lillo, Mike, and Farmer 2005). To avoid revealing true intentions, large investors break their trades up into small pieces and trade incrementally (Kyle, 1985). Convert heavy tail of large orders volume distributions in correlated order flow.

Is it possible to quantify **empirically** the contribution of herding and order splitting to the autocorrelation of order flow?
Note that this is part of the question on the origin of *diagonal effect* raised in Biais, Hillion and Spatt (1995).

Decomposing the autocorrelation function

Assume we know the identity of the investor placing any market order.

- For each investor i we define a time series of market order signs ϵ_t^i which is equal to zero if the market order at time t was not placed by investor i and equal to the market order sign otherwise
- The autocorrelation function can be rewritten as

$$C(\tau) = \frac{1}{N}\sum_t \sum_{i,j} \epsilon_t^i \epsilon_{t+\tau}^j - \left(\frac{1}{N}\sum_t \sum_i \epsilon_t^i\right)^2$$

Decomposing the autocorrelation function

We rewrite the acf as $C(\tau) = C_{split}(\tau) + C_{herd}(\tau)$ where

$$C_{split}(\tau) = \sum_i \left(P^{ii}(\tau) \left[\frac{1}{N^{ii}(\tau)} \sum_t \epsilon_t^i \epsilon_{t+\tau}^i \right] - \left[P^i \frac{1}{N^i} \sum_t \epsilon_t^i \right]^2 \right)$$

$$C_{herd}(\tau) = \sum_{i \neq j} \left(P^{ij}(\tau) \left[\frac{1}{N^{ij}(\tau)} \sum_t \epsilon_t^i \epsilon_{t+\tau}^j \right] - P^i P^j \left[\frac{1}{N^i} \sum_t \epsilon_t^i \right] \left[\frac{1}{N^j} \sum_t \epsilon_t^j \right] \right)$$

N^i is the number of market orders placed by agent i, $P^i = N^i/N$, $N^{ij}(\tau)$ is the number of the number of times that an order from investor i at time t is followed by an order from investor j at time $t + \tau$, and $P^{ij}(\tau) = N^{ij}(\tau)/N$

Brokerage data

- We have no data on final investors. We study the relative role of herding and splitting at the broker (or market member) level for which we have data
- At LSE there are typically 250-300 market members trading a stock. Of those roughly 80 are significantly active in a six month period.
- There is a huge heterogeneity in market member activity at LSE. The 15 most active ones are responsible for 80-90% of transactions.
- The activity of market members (independently from their trading direction) is characterized by the persistence

$$\tilde{P}^{ii}(\tau) = P^{ii}(\tau) - (P^{i})^2$$

Market members persistence

Market member activity is highly clustered in (transaction) time. I.e. there is some degree of predictability that a member active now will be active in the near future.

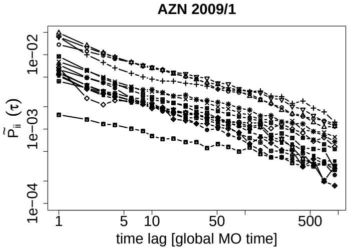


Figure: The diagonal terms of persistence in activity, i.e., $P^{ii}(\tau) - [P^i]^2$ of MO placement for the 15 most active participant codes, the first half of 2009 for AZN. For many codes this quantity is consistently positive, indicating a significant clustering in their activity.

Herding or splitting?

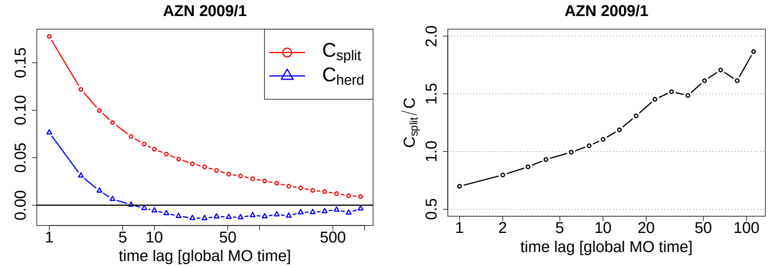


Figure: Left panel. The splitting and the herding term of the correlation of MO signs (the two terms sum up to $C(\tau)$) for the first half year of 2009 for AZN. Right panel. The splitting ratio of MO signs (defined as the ratio of the splitting term in the correlations and the entire correlation) for the first half year of 2009 for AZN.

Splitting dominates herding at the broker level (especially for large lags)

Antiherding?

$C_{herd}(\tau) < 0$ is statistically significant when $10 \lesssim \tau \lesssim 80$. Why?

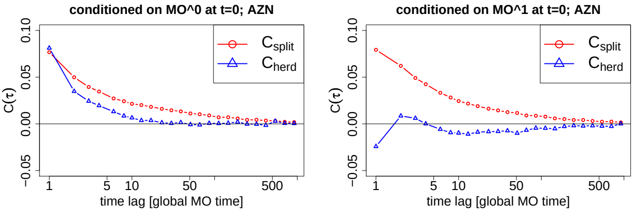
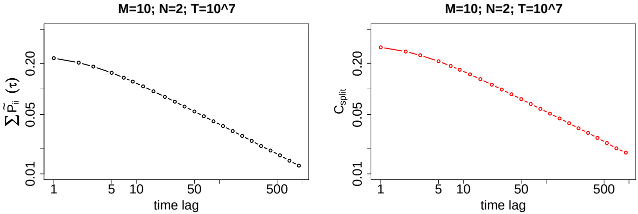


Figure: Splitting and herding component of the MO sign acf conditional to the event at time t, a market order that does not changes the price (left panel) and a market order that does change the price (right panel).

The antiherding phenomenon is due to investors refraining from placing market orders in the same direction of a recent market order of another investor that changed the price.

A splitting model (generalizes Lillo, Mike, and Farmer 2005)

There are M agents, but at any time only N are active. Being active means that they have a large order to trade trough splitting. An agent become inactive (and another becomes active) when the large order is finished. The number of transactions for a large order is Pareto distributed $p(L) \sim L^{-(\alpha+1)}$



The model predicts that $\gamma = \alpha - 1$, i.e. $\alpha \simeq 1.5$.

Brokers vs agents: two stylized models

- Herding and the splitting behavior should be observed at the investor level not at broker level, but our data do not allow to do this
- The relation between investors and brokers is in general complex and not fully explored empirically.
- We develop two stylized agent based models that take into account possible variations of the broker-investor relation and of the mechanism responsible for the long memory.
  - Long memory is generated exogenously by an autocorrelated signal and brokers have a different probability of trading following the signal
  - Long memory is generated endogenously and the apparent splitting at the broker level comes both from the heterogeneity of brokers activity and from the correlated choice of brokers by agents which are close in the network of influence.
- No splitting in both models
- Our point is not to generate a correlated order flow

Herding from broker's heterogeneity and long memory exogenously generated

- At each time step t there is a long memory news signal I_t that can be positive (buy) or negative (sell). Its absolute value gives the strength of the news.
- A subset of $N_t = f(|I_t|)$ investors place a market order with the same sign of I_t. f is an increasing function
- There are K agents and M ($\leq K$) brokers. Each broker i is characterized by a random parameter χ_i (skill) telling how much broker i follows the signal I_t.
- All the agents of broker i have a probability of trading in that day equal to χ_i by placing a market order with the same sign of the signal.

The role of broker's heterogeneity

- $\chi_i = P^i$, i.e. trading activity is proportional to skill. In real data trading activity is very heterogeneous
- It is direct to show that

$$C_{split}^{rand} = C(\tau)\Big(\frac{1}{M} + M\ \mathrm{Var}[P]\Big)$$
$$C_{herd}^{rand} = C(\tau)\Big(\frac{M-1}{M} - M\ \mathrm{Var}[P]\Big). \qquad (1)$$

- The two extreme cases are

$$Var[P] = 0 \rightarrow C_{split}^{rand} = \frac{C(\tau)}{M}; \qquad C_{herd}^{rand} = \frac{M}{M-1}C(\tau)$$
$$Var[P] = \frac{1}{M}\left(1 - \frac{1}{M}\right) \rightarrow C_{split}^{rand} = C(\tau); \qquad C_{herd}^{rand} = 0$$

Results

meas. of inequality		$S(\tau)$				
σ_P/σ_∞	$Var[P]$	$\tau = 1$	$\tau = 10$	$\tau = 50$	$< S(\tau) >$	$\mathrm{std}(S(\tau))$
0.031	0.0000	0.019	0.020	0.021	0.020	0.0004
0.207	0.0008	0.061	0.061	0.061	0.061	0.0002
0.696	0.0095	0.495	0.494	0.496	0.495	0.0002
0.863	0.0146	0.749	0.750	0.750	0.750	0.0002

Table: Results for the news model using $\sigma = 20$. The first column shows σ_P/σ_∞, where σ_P is the standard deviation of P_i and $\sigma_\infty = \sqrt{1/M - 1/M^2}$ (the maximum possible value for σ_P) while the second one shows the variance of P^i. Columns 3-5 show the splitting ratio for lags $\tau = 1, 10, 50$. Columns 6 and 7 show the mean and the standard deviation of the splitting ratio for $\tau \in [1, 50]$.

For real data $\sigma_P/\sigma_\infty = 0.0008$ and $S(\tau) \equiv C_{split}(\tau)/C(\tau) > 0.75$.

Results: shuffling real data

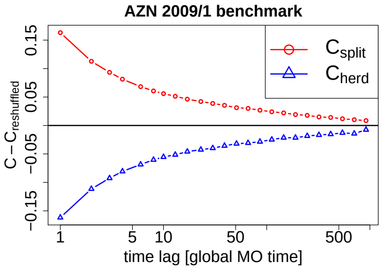


Figure: The difference between the splitting correlation measured on the data and that measured on the reshuffled data ($C_{split} - C_{split}^{rand}$), together with the difference between the herding correlation measured on the data and that measured on the reshuffled data ($C_{herd} - C_{herd}^{rand}$), for MO signs. Note that the two quantities sum to zero by definition.

- Under this model in order to reproduce real data we need to assume an heterogeneity in trading activity (and hence in skills) unrealistically larger than the one observed in real data.

Herding from investors direct interaction and long memory endogenously generated

- K agents (investors) that are linked in an undirected completely connected scale free network of influence. The degree of a node is distributed asymptotically as a power-law function $p(\ell) \sim \ell^{-(1+\eta)}$
- At the beginning each node (agent) is given a random sign ± 1 (buy or sell)
- At each time step t
  - A node i is chosen randomly and a market order with the sign of i is submitted
  - With probability $p \in [0, 1]$ each neighbor of the chosen node places a market order with the same sign of i and changes its state to the state of i, while with probability $1 - p$ it places a market order with the sign given by its state without changing it.
  - A new node is chosen and the process continues
- The model converts the scale free degree distribution of the network of influence into a power law autocorrelation function of the trade signs. In the limit $p \to 1$ the autocorrelation function decays asymptotically as $C(\tau) \sim \tau^{-(\eta-1)}$

Broker mapping

The other ingredient of the model is the investor-broker mapping. We want to map the actions of the agents to the actions of M brokers ($M \leq K$). There are several different possible choices for the mapping between brokers and investors. We considered two alternatives:

- a random mapping
- a correlated mapping, i.e. two agents that are connected in the network choose the same broker with a probability $\Phi + 1/M$ (higher than in the random case). Two possibilities
  - The participation rate of each broker is roughly equal (in number of trades)
  - The participation rate of each broker has a degree of heterogeneity similar to the one observed in real data.

We have considered $K = 10,000$ agents and constructed the network with a preferential attachment mechanism with $m = 1$. We set $p = 0.9$ and we get an Hurst exponent of approximately 0.68, close to the real one, and we observe a very slight dependence on p. We used $M = 50$ brokers

Results

		$S(\tau)$				
Φ	$Var[P]$	$\tau = 1$	$\tau = 10$	$\tau = 50$	$< S(\tau) >$	$\mathrm{std}(S(\tau))$
0.00	0.0000	0.019	0.027	0.037	0.025	0.0067
0.00	0.0008	0.060	0.067	0.088	0.067	0.0102
0.65	0.0012	0.544	0.463	0.476	0.463	0.0326
0.83	0.0059	0.769	0.737	0.746	0.741	0.0181

Table: Results for the network model. The first column shows the parameter Φ. Column 2 shows the variance of the P^i. Columns 3-5 show the splitting ratio for lags $\tau = 1, 10, 50$. Column 6 and 7 show the mean and the standard deviation of the splitting ratio for $\tau \in [1, 50]$. The first two lines correspond to the uncorrelated choice of brokers with homogeneous or heterogeneous broker activity, respectively, while the third and fourth line show the results for correlated choice of brokers.

For real data $\sigma_P/\sigma_\infty = 0.0008$ and $S(\tau) \equiv C_{split}(\tau)/C(\tau) > 0.75$.

Remarks

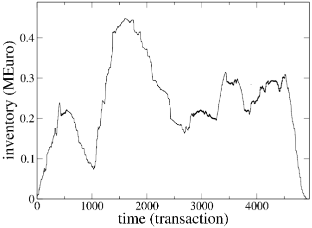
- In order to reproduce the long memory of order flow and the high relative role of splitting vs. herding in the autocorrelation, the network model needs to be calibrated to a very high level of correlation in the choice of brokers.
- Moreover for this value, the heterogeneity of brokers participation rate is much higher than the one observed in real data.
- Even if this possibility cannot be *a priori* excluded, we believe that our modeling poses a strong constraint to the level of correlation that is needed to reproduce real data.

Direct evidence for order splitting

- We have seen that correlated order flow is mostly due to order splitting.
- We want to find direct evidence of splitting, characterize the large trades and the splitting characteristics, and to measure the market impact of these large orders.
- The difficulty is, of course, data.
- Some studies use proprietary data of a large financial institution
- We follow a different approach: **statistical identification of large trades from market member data.**

An example: inventory time series

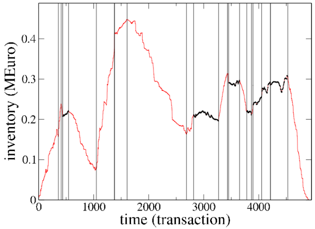
Credit Agricole trading Santander



- Clear trends are visible
- The identification of large trades (*metaorders*) must be statistical: a typical regime switching problem

Segmentation algorithm

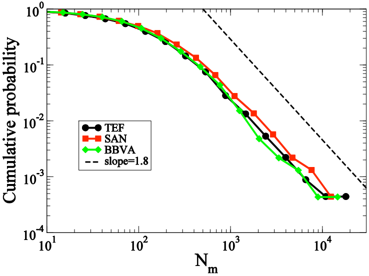
Credit Agricole trading Santander



Different algorithms:

- Modified t-test (G. Vaglica, F. Lillo, E. Moro, and R. N. Mantegna, Physical Review E **77**, 036110 (2008).)
- Hidden Markov Model (G. Vaglica, F. Lillo, and R. N. Mantegna, New Journal of Physics, **12** 075031 (2010)).

Order size distribution and validation of splitting model



- Hidden orders size is asymptotically power law distributed
- The tail exponent is consistent with the splitting model

Impact of metaorders

Here we are interested in measuring the impact of metaorders. We measure the market impact by considering the change in the log price of the stock between time t and time $t + T$, i.e.

$$r_i(t, T) = \log p_{i,t+T} - \log p_{i,t},$$

We define the rescaled market impact as

$$R_i(t, T) = \epsilon_i r_i(t, T)/s_i.$$

where ϵ_i is the sign of the metaorder and s_i is the mean value of the spread of the stock during the year.

Temporary and permanent impact

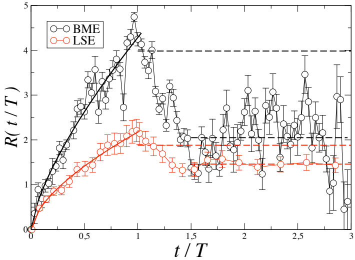


Figure: Market impact versus time. The symbols are the average value of the market impact of the metaorder as a function of the normalized time to completion t/T. The rescaled time $t/T = 0$ corresponds to the starting point of the metaorder, while $t/T = 1$ corresponds to the end of the metaorder.

Impact of metaorders

We find that the market impact of metaorders dominated by market orders is consistent with

$$\langle r|N\rangle = A\epsilon s N^{\gamma} \tag{2}$$

where ϵ is the sign of the order and s is the spread.

Table: Parameters of the fitting of the market impact with Eq. (1)

Market	$A_{f_{mo}>0.8}$	$\gamma_{f_{mo}>0.8}$	$A_{f_{mo}<0.2}$	$\gamma_{f_{mo}<0.2}$
BME	0.63 ± 0.17	0.48 ± 0.07	-0.63 ± 0.22	0.44 ± 0.09
LSE	0.17 ± 0.05	0.72 ± 0.10	-0.16 ± 0.14	0.64 ± 0.30

Relation between temporary and permanent impact: fair pricing condition

A simple argument based on the hypothesis that the price after reversion is equal to the average price paid during execution. If during execution price impact grows like $A \times (t/T)^\beta$ then the average price payed by the agent who executes the order is

$$\langle S \rangle = S_t + A \int_0^1 (t/T)^\beta d(t/T) = S_t + \frac{A}{1+\beta}, \tag{3}$$

i.e. the permanent impact is $1/(\beta+1)$ of the peak impact. In our case and using the exponents β we get $1/(\beta+1) \simeq 0.58 \pm 0.01$ for the BME and $1/(\beta+1) \simeq 0.62 \pm 0.02$ for the LSE which are statistically similar to the ratios R_{perm}/R_{temp} for each market.

Can we build a model on this observation? (and motivate it?)

A model for fair pricing condition of market impact

A portfolio manager makes a long-term decision to either buy or sell a given asset, and then incrementally executes small trades until she has bought or sold the desired quantity. We call this agent a *directional trader*. Following Kyle, the counterparty of the trades is a *market maker* who provides liquidity.
The directional trader's metaorder to either buy or sell is broken up and executed in a series of smaller trades. These trades take place over N time intervals each of length τ, labeled by the index $k = 1, \ldots, N$. The size is chosen from a distribution p_N which is given, and which is public information. Moreover we assume that the distribution p_N has compact support, i.e. that the maximal number of intervals is a known value M.

A model for fair pricing condition of market impact

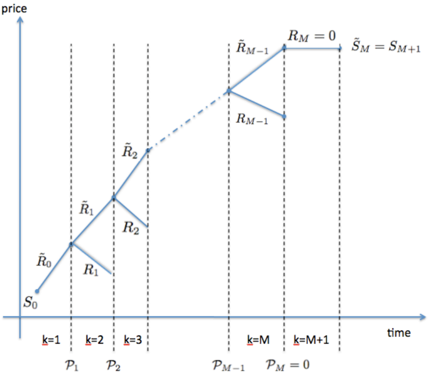


Figure: Sketch of the model

Market efficiency

The only source of uncertainty for the market maker is the size of the metaorder.
The existence of metaorders causes order flow to be correlated.
Let $\mathcal{P}_k$ be the probability that the metaorder will continue based on the knowledge that it is still active at timestep k.
If the market is efficient

$$\mathcal{P}_k \tilde{R}_k - (1 - \mathcal{P}_k) R_k = 0, \qquad k = 1, 2, ..., M - 1 \tag{4}$$

where $\tilde{R}_k = \tilde{S}_{k+1} - \tilde{S}_k$ and $R_k = (\tilde{S}_k - S_{k+1})$. $\tilde{R}_k$ is the expected return if the order continues to $k+1$, and $-R_k$ is the expected return if it stops at k.
Moreover $\mathcal{P}_M = 0$ which implies that $R_M = 0$.

Breakeven when averaged over all sizes

Assume that the market maker sells during N intervals to the directional trader at prices $\tilde{S}_i$ and then buys them back at price S_{N+1}, so that her profit for a trade of size N is

$$\Pi_N \equiv \sum_{i=1}^{N} \tilde{S}_i - N S_{N+1}$$

The condition that the market maker will *breakeven when averaged over all sizes* can be written

$$E_1[\Pi_N] \equiv \sum_{N=1}^{M} p_N \Pi_N = 0$$

This obviously also implies that the directional trader will also break even.
It can be shown that market efficiency implies breakeven when averaged over all sizes if the support of p_N is compact.

Determining the probability continuation

$$\mathcal{P}_k = \frac{\sum_{i=k+1}^{M} p_i}{\sum_{i=k}^{M} p_i}. \tag{5}$$

For Pareto distributed metaorder sizes

$$p_N = \frac{1}{\zeta(\beta)} \frac{1}{N^{\beta+1}}, \qquad N \geq 1 \tag{6}$$

where the normalization constant $\zeta(\beta)$ is the Riemann zeta function it is

$$\mathcal{P}_k = \frac{\zeta(1+\beta, k+1)}{\zeta(1+\beta, k)} \simeq \left(\frac{k}{k+1}\right)^{\beta} \sim 1 - \frac{\beta}{k}. \tag{7}$$

where $\zeta(s, a)$ is the generalized Riemann zeta function.

Fair pricing condition

Efficiency at the end of the first interval implies that $\Pi_1 > 0$, i.e. the market maker makes profit for short orders (as originally suggested by Glosten 1985). Breakeven averaged over all size implies that market maker loses money for other order sizes.
Efficiency at the end of the last interval implies that $\Pi_M < 0$, i.e. the market maker loses money on orders of maximal size.
We postulate that

$$\Pi_N = \sum_{i=1}^{N} \tilde{S}_i - NS_{N+1} = 0 \qquad N = 2, 3, ..., M-1$$

This means that

$$p_1 \Pi_1 + p_M \Pi_M = 0$$

Solution

The system of efficiency (martingale) conditions and fair pricing conditions has solution

$$\tilde{R}_k = \frac{1}{k}\frac{p_k}{\sum_{i=k+1}^{M} p_i}\frac{1-p_1}{\sum_{i=k}^{M} p_i}\tilde{R}_1 \qquad k = 2, 3, ..., M-1$$

$$R_k = \frac{\mathcal{P}_k}{1-\mathcal{P}_k}\tilde{R}_k \qquad k = 1, 2, ..., M-1$$

There are two undetermined parameters:

- $\tilde{R}_0$ is related to market conditions, such as volatility, and to the intensity of the order flow imbalance.
- $\tilde{R}_1$ fixes the scale of the impact.

Pareto distributed size

As a realistic case let us consider the case of Pareto distributed sizes, $p_N \propto N^{-(1+\alpha)}$. The impact $\tilde{S}_k - \tilde{S}_1$ behaves asymptotically for k large but $k \ll M$ as

$$\tilde{S}_k - \tilde{S}_1 \sim \begin{cases} k^{\alpha-1} & \text{for } \alpha \neq 1 \\ \log(k+1) & \text{for } \alpha = 1 \end{cases}$$

In the important case of $\alpha = 1.5$ we get

- A square root law for temporary impact
- Permanent impact is $1/\alpha = 2/3$ of the temporary impact

Empirical results and model predictions in a nutshell

- Order flow is a long memory process
- The origin is order splitting; herding plays a minor role
- The exponent $\gamma \simeq 0.5$ of the autocorrelation function can be related to a tail exponent $\alpha \simeq 1.5$ of the metaorder size (via the splitting model).
- The fair pricing model predicts a square root impact and a 2/3 reversion, similarly to what observed in real (reconstructed) metaorders.
- The model postulates the fair pricing condition, similarly to what observed in real (reconstructed) metaorders.

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