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AdS/CFT and Black Holes - Lecture 2

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AdS/CFT & Black Holes II

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Castro, Strominger & A. M., . . .

Overview

Black Holes:

Can we understand the quantum states of a black hole?

String theory provides a precise accounting of the microstates of certain extremal black holes. They have zero temperature and differ qualitatively from astrophysical black holes.

Can we use stringy techniques to study non-extremal black holes?

Today I will focus on Kerr black holes with mass M and angular momentum J. These black holes have finite Hawking temperature and have been (indirectly) observed.

These are the most interesting black holes, but in a sense I will have the least to say.

For less interesting black holes the constructions are much more explicit so one can make more detailed statements.

Plan for Today:

- Black Holes and Conformal Symmetry

- Conformal Structure of Kerr

- Counting States

Extremal Black Holes (A Caricature)

For an extremally charged ($M = Q$) or rotating ($M = J^2$) black hole we define the near horizon region by

$$r - r_{hor} << M$$

The geometry of the near horizon region includes an AdS factor.

For example, the near horizon limit of the D1-D5 system includes an AdS_3 factor

The isometry group of AdS is the same as the conformal group in one less dimension; this symmetry group acts as conformal transformations on the asymptotic boundary of the near horizon geometry.

So the Hilbert space of states is that of a conformal theory. This allows us to understand black hole entropy.

Brown & Henneaux, Maldacena, Strominger, . . .

The Idea:

The near-horizon region of a realistic astrophysical black hole is Rindler space, not AdS.

Nevertheless, the states of quantum gravity still organize themselves into representations of the conformal group. The difference is that the conformal symmetry is not geometrically realized.

Aside from this, the computation proceeds in the exact same way as for extremal black holes.

But many features of this CFT remain mysterious.

Non-Extremal Black Holes

A non-extremal black hole is unstable, but this does not preclude a CFT description. It just means that the CFT must be coupled to external degrees of freedom.

What is the analog of the "near-horizon" region?

For an extremal black hole the near-horizon region

$$r - r_{hor} << M$$

is the part of the geometry probed by low energy modes

$$\omega << M^{-1}$$

For non-extremal black holes these two definitions do not coincide. The first definition gives Rindler space.

What about the second?

The Near Region

Probe a non-extremal black hole by low energy modes

$$\omega << M^{-1}$$

These modes do not live near the horizon. But we can define the "near" region by

$$r << \omega^{-1}$$

This definition is probe-dependent, so is not a limit of the geometry.

When ω is small it includes

- The inner and outer horizons at $r = r_\pm$
- The ergosphere
- Regions outside the black hole

Matching Surface

Consider a field in a Kerr background. Since $\omega << M^{-1}$ the two regions

- Near: $r << \omega^{-1}$
- Far: $r >> M$

overlap. So we can study physics in the near and far regions and match together along a *matching surface* at

$$M << r_{match} << \omega^{-1}$$

This surface plays the same role as the boundary of AdS (or NHEK) in the extremal case.

Claim: Near region physics has a conformal symmetry, realized as conformal transformations of the matching surface.

Conformal Symmetry of Kerr

Kerr Metric

A Kerr black hole with mass M and angular momentum J has inner and outer horizons at $r = r_\pm$ given by

$$M = \frac{r_+ + r_-}{2}, \qquad \frac{J}{M} = \sqrt{r_+ r_-} \equiv a$$

In Boyer-Lindquist coordinates the metric is

$$ds^2 = \frac{\rho^2}{\Delta} dr^2 - \frac{\Delta}{\rho^2}\left(dt - a\sin^2\theta d\phi\right)^2 + \rho^2 d\theta^2 + \frac{\sin^2\theta}{\rho^2}\left((r^2 + a^2)d\phi - adt\right)^2$$

$$\Delta = (r - r_+)(r - r_-), \qquad \rho = \sqrt{r^2 + a^2\cos^2\theta}$$

The Ergosphere is at $\rho = 0$.

The Wave Equation

Consider a field Φ in the Kerr background.

$$\Phi = e^{i\omega t} f(r, \theta, \phi)$$

For $r << \omega^{-1}$ the Kerr Laplacian is

$$\nabla^2 = H^2 + L(L+1) = \bar{H}^2 + L(L+1)$$

where

$$H^2 = -H_0^2 + \{H_1, H_{-1}\}$$

is the Casimir of $SL(2, \mathbb{R})$ and $L(L+1)$ is an eigenvalue of the S^2 Laplacian.

Thus states organize into representations of $SL(2, \mathbb{R})_L \times SL(2, \mathbb{R})_R$, the rigid conformal group in $1+1$ dimensions.

This is true for any free field, including spin 2.

Conformal Action

The generators act as conformal transformations on the matching surface. If we let

$$w^+ = \sqrt{\frac{r - r_+}{r - r_-}} e^{2\pi T_R \phi}, \qquad w^- = \sqrt{\frac{r - r_+}{r - r_-}} e^{2\pi T_L \phi - t/2M}$$

then the $SL(2, \mathbb{R})$ generators are

$$\begin{aligned} H_1 &= i\partial_+ \\ H_0 &= i(w^+\partial_+ + \frac{1}{2} y\partial_y) \\ H_{-1} &= i(w^{+2}\partial_+ + w^+ y\partial_y - y^2\partial_-) \end{aligned}$$

where y is a “radial” coordinate

$$y = \sqrt{\frac{r_+ - r_-}{r - r_-}} e^{\pi(T_L + T_R)\phi - \frac{t}{4M}}$$

Black Hole States

This extends to local conformal symmetries $Vir_L \times Vir_R$.

Thus states of low energy probes of the near region organize into representations of the local conformal group, realized as conformal transformations of the matching surface.

This is just like in normal AdS/CFT, except that now conformal transformations act on the phase space of the theory rather than on the geometry itself.

In AdS/CFT this is a signature of the conformal invariance of the quantum gravity in asymptotically AdS space.

We conjecture that the same is true here.

The Kerr Entropy

The CFT dual to Kerr

The quantum states of Kerr organize into representations of the conformal group.

They are states in a CFT.

To describe the black hole microstates we need to know

- which states describe black holes
- the central charge of the CFT

We will also assume that the CFT satisfies a few reasonable properties.

Finite Temperature

To find the state, note that because

$$\phi \sim \phi + 2\pi$$

the conformal generators are not globally defined. We must identify

$$w^+ \sim e^{4\pi^2 T_R} w^+, \qquad w^- \sim e^{4\pi^2 T_L} w^-$$

The $w^\pm$ coordinates are accelerating (Rindler) coordinates.

So we have a state a finite temperature

$$T_R = \frac{r_+ - r_-}{4\pi\sqrt{r_+ r_-}}, \qquad T_L = \frac{r_+ + r_-}{4\pi\sqrt{r_+ r_-}}$$

The state breaks $SL(2,\mathbb{R}) \times SL(2,\mathbb{R})$ down to $U(1) \times U(1)$.

The theory is conformally invariant but the state is not.

Decoupling the Far Region

The $T_{L,R}$ temperatures are conjugate to $\frac{\partial}{\partial w^{\pm}}$, not $\frac{\partial}{\partial t}$ and $\frac{\partial}{\partial \phi}$. So they differ from the usual Hawking temperatures.

With respect to these generators, the Kerr black hole has positive specific heat and can be put in thermal equilibrium. It makes sense to talk about a microstate.

This is related to the fact that we decoupled the black hole from the far region.

To describe the *decay* of Kerr we would need to couple this CFT to additional degrees of freedom.

Central Charge

We need to find a vacuum state invariant under the symmetries.

If we set $T_R = 0$ the black hole is extremally rotating $M = J^2$. Then

- The "near" region is the "near-horizon extremal Kerr" geometry of Bardeen & Horowitz
- the $SL(2, \mathbb{R})_R$ is unbroken and geometrically realized.

We compute the central charge by studying the diffeomorphisms which generate the asymptotic symmetry group, a la Brown & Henneaux:

$$c_R = 12J$$

This is the "Kerr/CFT" used to describe extremal black holes of Guica, Hartman, Song & Strominger.

Density of States

Assume that the CFT

- is parity invariant so $c_L = c_R$
- is modular invariant
- possesses a ground state with a gap.

Then we can compute the density of states using Cardy's formula

$$N \sim \exp\left\{\frac{\pi^2}{3}\left(c_L T_L + c_R T_R\right)\right\}$$

This is a good approximation when T_L and T_R are large, which happens when the mass M is large.

Entropy

This reproduces the Bekenstein-Hawking entropy

$$S = \frac{\pi^2}{3}(c_L T_L + c_R T_R) = \frac{\mathrm{Area}}{4}$$

including the numerical coefficient!

We have reproduced the entropy of a *realistic* black hole – not just an extremal one – using a dual CFT.

Moreover,

- Scattering amplitudes reproduce CFT correlation functions.
- This works for a variety of other non-extremal black holes.

Conclusions

We are closing in on a microscopic description of realistic, astrophysical black holes.

Using holographic techniques,

- States live in a dual CFT; conformal symmetries act on the phase space but not on the geometry.
- This reproduces the Bekenstein-Hawking entropy, including the numerical coefficient.

But several puzzles remain:

- We don't know much about the dual CFT.
- We don't even know its vacuum state!
- How can we understand black hole microstates geometrically?