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Spring School on Superstring Theory and Related Topics

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AdS/CFT and Black Holes - Lecture 1

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AdS/CFT & Black Holes I

ICTP

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Castro, Giombi, Lashkari, Lepage-Jutier, Song, Strominger, Witten, Yin, . . .

Overview

The Problem:

Gravity is notoriously difficult to study as a quantum theory...

Classical general relativity doesn't answer many physical questions:

- What happens inside a black hole?
- How can we think about the big bang singularity?

Basic conceptual questions:

- What are the degrees of freedom?
- What is the Hilbert space $\mathcal{H}$?
- What is the Hamiltonian H?
- Are these notions even applicable to gravity?
- Can a Unitary theory describe singularities, inflation, . . . ?

Quantum Gravity

Many of these questions can be addressed, at least in principle, in string theory.

Black Holes:

- Counting of black hole microstates by Strominger & Vafa, ...
- Exact counting only for extremal black holes
- No "geometric" understanding of microstates

Cosmology:

- Construction of inflating vacua by KKLT,...
- Baroque constructions (non-perturbative effects)
- Rococo dynamics (instabilities)

Goal:

To describe recent progress on holographic approaches to black hole entropy.

Lecture I: Core idea of microstate counting: $\mathrm{AdS}_3/\mathrm{CFT}_2$

Lecture II: Realistic (non-extremal) black holes: Kerr/CFT

Lecture III: Geometric interpretation of Microstates

Overview'

Black Holes and AdS/CFT

For an extremally charged ($Q = M$) or rotating ($J^2 = M$) black hole we define the near horizon region by

$$r - r_{hor} << M$$

The geometry of the near horizon region always includes an AdS factor. For many black holes we study in string theory the factor is AdS_3.

So we can understand the dynamics of the black hole by understanding AdS_3.

In fact, much of the important physics can be understand only by thinking about Einstein gravity in AdS_3

$$S(g) = \int_M \sqrt{-g}\left(\frac{1}{G}R + \Lambda + \dots\right)$$

The Plan for Today:

- The Classical Spectrum of AdS_3 Gravity

- Modular Invariance and the Sum over Geometries

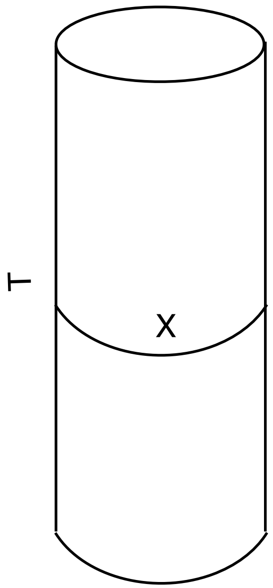
- The Quantum Spectrum of AdS_3 gravity

AdS$_3$ Gravity

Classical Theory:

The prototypical negative curvature metric is AdS_3

$$ds^2 = dr^2 - \cosh^2 r \; dt^2 + \sinh^2 r \; d\phi^2$$



The boundary cylinder $\mathbb{R}^1 \times S^1$ lies at infinite distance. We consider only metrics that look like AdS_3 at infinity.

Spectrum

The two killing vectors

- $H = \partial_t$ generates time translations
- $J = \partial_\phi$ generates rotations

States are labelled by an energy H and angular momentum J. It is convenient to use the CFT language:

$$L_0 = H + J, \qquad \bar{L}_0 = H - J$$

The theory has two types of states:

- Boundary Gravitons
- Black Holes

Boundary Gravitons

Even though there are no *local* gravitons, there are metric perturbations associated with the boundary.

Naively, two metrics describe the same state if they are related by a diffeomorphism. But with AdS boundary conditions, two metrics describe the same state only if they are related by diffeomorphism which vanishes at infinity

- Diffeo's which act on the boundary give new "boundary graviton" states

These infinitesimal diffeomorphisms generate two copies of the Witt algebra

$$[L_m, L_n] = (m - n)L_{m+n}$$

This is reminiscent of the Virasoro algebra in a CFT.

AdS/CFT Correspondence

The isometry group of AdS_3 is $SO(2,2)$. This is the same as the group of rigid conformal transformations in two dimensions. Indeed, these isometries of AdS_3 act as rigid conformal transformations on the boundary cylinder.

The group of "asymptotic symmetries" is generated by the Virasoro algebra. These asymptotic symmetries act as local conformal transformations on the boundary.

The algebra of charges has a central extension

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}(m^3 - m)\delta_{m+n}$$

with $c = \frac{3\ell}{2G}$. So we expect AdS_3 gravity to be dual to a CFT!

Brown & Henneaux, Maldacena, ...

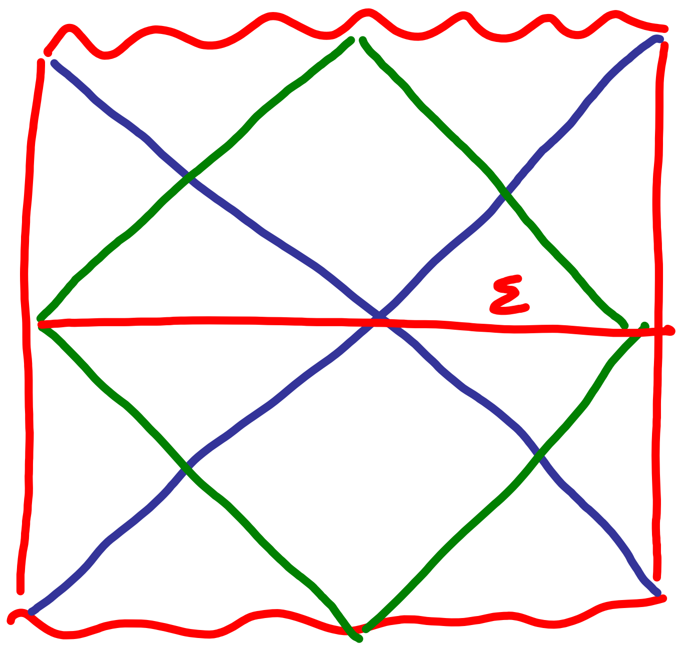
BTZ Black Holes

There are also black holes, which are locally AdS_3 but differ by global identifications.

$$ds^2 = dr^2 - \sinh^2 r\ dt^2 + \cosh^2 r\ d\phi^2$$

This is a quotient $AdS_3/\mathbb{Z}$.

The area L of the black hole horizon is the size of the ϕ circle



The Classical Spectrum

In terms of the L_0 eigenvalue $\Delta = H + J$, the spectrum includes

- A ground state with $\Delta = -c/24$ (this is a choice of normalization)
- A tower of discrete boundary graviton states at integer $\Delta > -c/24$
- A continuum of black holes with $\Delta > 0$

We want to compute the exact quantum spectrum...

Partition Function

Partition Function

We want to compute the number of states $N(E, J)$ of given energy E and angular momentum J.

To do this, we will compute the partition function at finite temperature β^{-1} and angular potential $i\theta$:

$$Z(\beta, \theta) = \mathrm{Tr}_{\mathcal{H}} \left(e^{-\beta H + i\theta J} \right) = \sum_{E,J} N(E, J)\ e^{-\beta E + i\theta J}$$

Write this in CFT language by letting $\tau = \theta + i\beta$

$$Z(\tau) = \mathrm{Tr}_{\mathcal{H}}\ q^{L_0}\ \bar{q}^{\bar{L}_0}, \qquad q = e^{2\pi i \tau}$$

How can we compute this exactly?

Euclidean Path Integral

To compute a thermal partition function we take

$$t \longrightarrow it_E$$

The boundary goes from $\mathbb{R}^1 \times S^1$ to T^2.

The partition function is computed by a *Euclidean* path integral

$$Z(\tau) = \int Dg\ e^{-S(g)}$$

over 3-manifolds which are a torus at infinity.

The geometry of the boundary T^2 depends on $\tau = \theta + i\beta$. The two directions of the torus (t_E, ϕ) are identified

$$z \sim z + 1 \sim z + \tau$$

where $z = \phi + it_E$. The parameter τ determines the *conformal structure* of the boundary T^2.

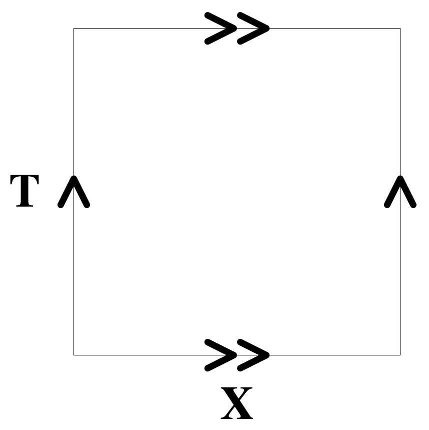
Saddle Points

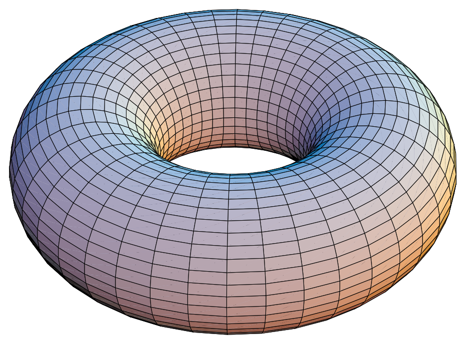
In the saddle point approximation

$$Z(\tau) = \int Dg\ e^{-S(g)} \sim \sum_{M_3} e^{-S(M_3)+\ldots}$$

where M_3 is a constant curvature 3-manifold with T^2 boundary.

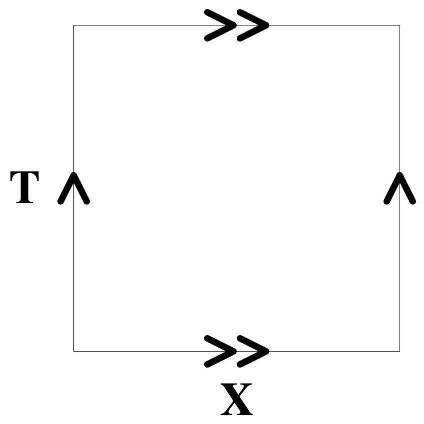
M_3 is a solid torus which "fills in" the boundary T^2:

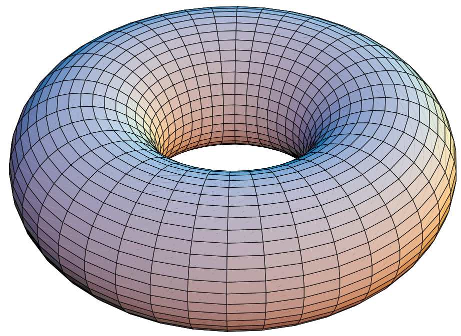




Saddle Points II

There are many such $M_3 = M_{c,d}$. One for each choice of cycle $cT + dX$ which is contractible in the interior:





- When X is contractible, the $M_{0,1}$ is thermal AdS.
- When T is contractible, the $M_{1,0}$ is the Euclidean black hole.

Sum over Geometries

The path integral over Euclidean geometries with boundary T^2

$$Z(\tau) = \int_{\partial M = T^2} Dg \; e^{-S(g)} = \sum_{c,d} e^{-S(M_{c,d})+\dots}$$

is invariant under modular transformations

$$\tau \to \gamma\tau = \frac{a\tau + b}{c\tau + d}, \qquad \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{Z})$$

The geometry $M_{c,d}$ is related to $M_{0,1}$ by the modular transformation γ.

This is the "black hole Farey Tail" of Dijkgraaf, Maldacena, Moore & Verlinde, . . .

Counting Black Holes

Modular Invariance

We have given a gravity argument for the statement that

$$Z(\tau) = Z(\gamma\tau)$$

for any modular transformation $\gamma \in SL(2, \mathbb{Z})$.

This property is obeyed by any CFT partition function, provided the theory is invariant under large Euclidean conformal transformations.

If we assume that the CFT has a normalizable ground state then this *completely* fixes the high energy spectrum of the theory.

$$N(\Delta, \bar{\Delta}) \sim \exp\left\{2\pi\sqrt{\frac{c}{6}\Delta} + 2\pi\sqrt{\frac{c}{6}\bar{\Delta}}\right\}$$

This is *precisely* the Bekenstein-Hawking entropy of the black hole.

Strominger

Conclusions

A generic "extremal" black hole has a near horizon region with an AdS factor. Microstates can be understood using AdS/CFT.

For AdS_3 near horizons the partition function is computed as a sum over geometries.

The result is modular invariant, and with only modest assumptions gives the black hole entropy via Cardy's formula.

Extensions:

- Perturbative corrections $\implies$ subleading corrections.
- All order perturbative corrections $\implies$ exact entropy
- Realistic black holes?
- Physical understanding of black hole microstates?