Active Matter

Lectures for the 2011 ICTP School on Mathematics and Physics of Soft and Biological Matter

Lecture 1

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Lectures 1-3
SP particles on a substrate & flocking*

Introduction
- What is active matter? Definition and examples on many scales

Flocking transition
- Agent based models: Vicsek model

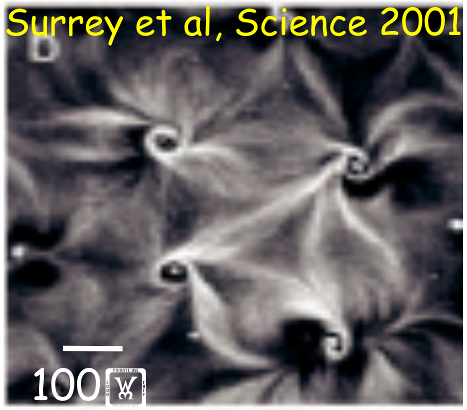
Hydrodynamics as an effective theory
- symmetries, conservation laws and broken symmetries

Two examples of hydrodynamics of flocking:
- Toner and Tu: continuum theory for the Vicsek model
- SP hard rods: an example of deriving hydrodynamics form microdynamics

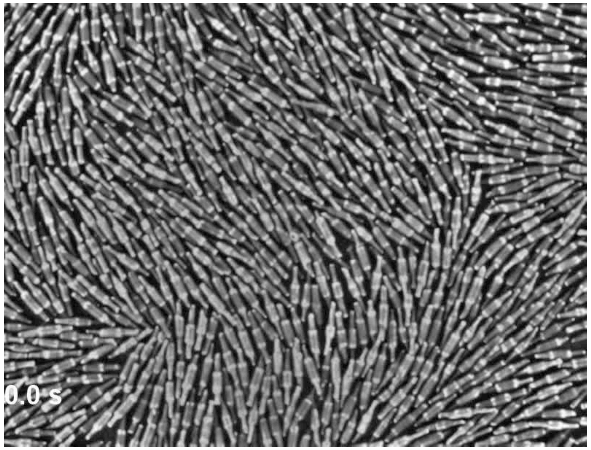
*flocking = onset of collective, coherent motion

What is “active matter”?

- Assembly of interacting “active” (motile, self-driven) particles that collectively generate motion or mechanical stresses
- Drive on each particle, not at boundary as in familiar noneq. systems
- Examples mostly (but not only) from the living world, spanning a huge range of scales.


Surrey et al, *Science* 2001

Narayan et al, *Science* 2007

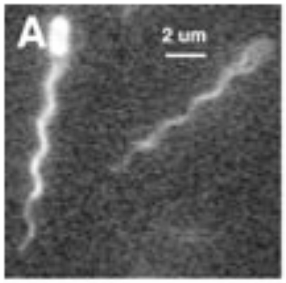


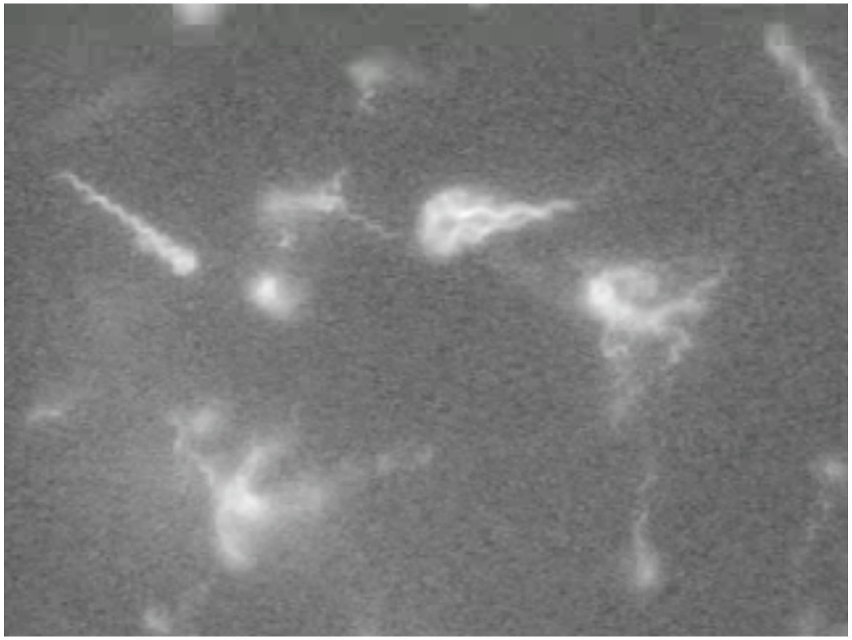
Goal: use ideas and tools of condensed matter physics to describe and *classify* the generic behavior of this diverse group of nonequilibrium systems

Active Particle

An active particle transforms chemical energy into motion or mechanical forces through an internal cyclic transformation

Example: bacteria, such as
E. Coli
Berg Lab
Harvard

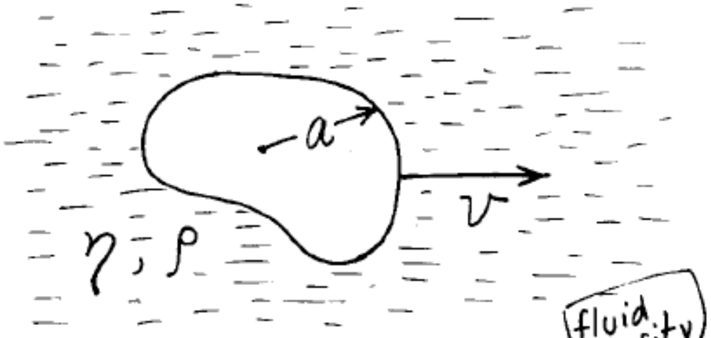




http://webmac.rowland.org/labs/bacteria/movies/fluo_cell_near.mov

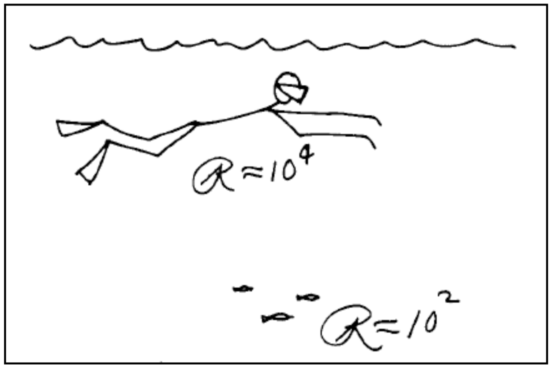
Motion of single bacterium: inertia negligible compared to fluid friction

Eric Purcell, *Life at Low Reynolds Number*, Am. J. Phys. **45**, 3 (1977)



$$\mathrm{Re}=\frac{\text{inertial forces}}{\text{viscous forces}}\approx\frac{a v \overset{\text{fluid density}}{\rho}}{\underset{\text{fluid viscosity}}{\eta}}$$

Fluid turbulence: Re>4000



E. Coli:

L ~ 2-3 microns

v ~ 10 microns/sec

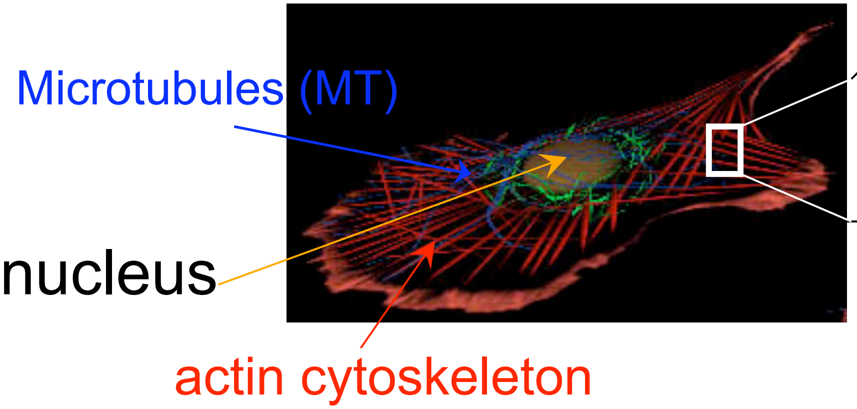
Re~10^{-4}-10^{-5}

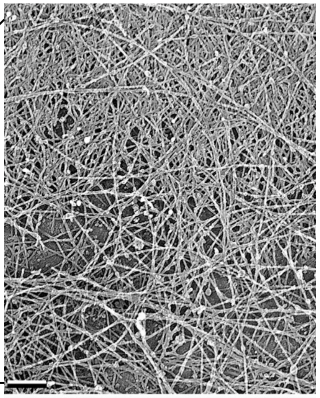
Many bacteria:
collective behavior &
bacterial “turbulence”

A less familiar example: motor-filament complexes

Cell cytskeleton: a polymer network that controls cell motility, shape & mechanical properties → see lectures by MacKintosh & Plastino

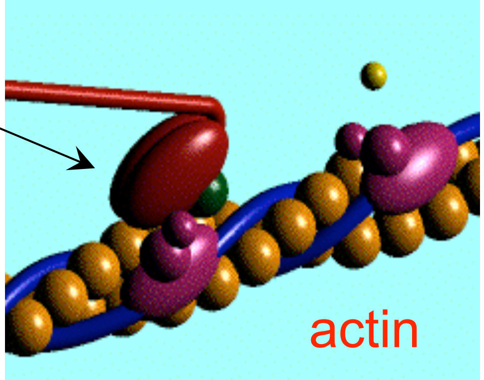
Ingredients:
- Filamentary protein: actin, microtubules
- Motor proteins: myosin, kinesin





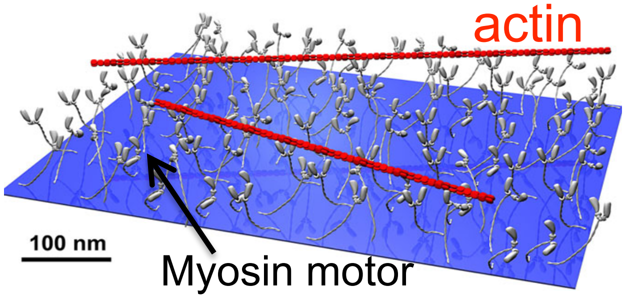
Actin cytoskeleton
of fish keratocyte
(Borisy & Svitkina)

Motor proteins bind to filaments and turn chemical energy from ATP hydrolysis into mechanical work generating forces and torques and remodeling the polymer network



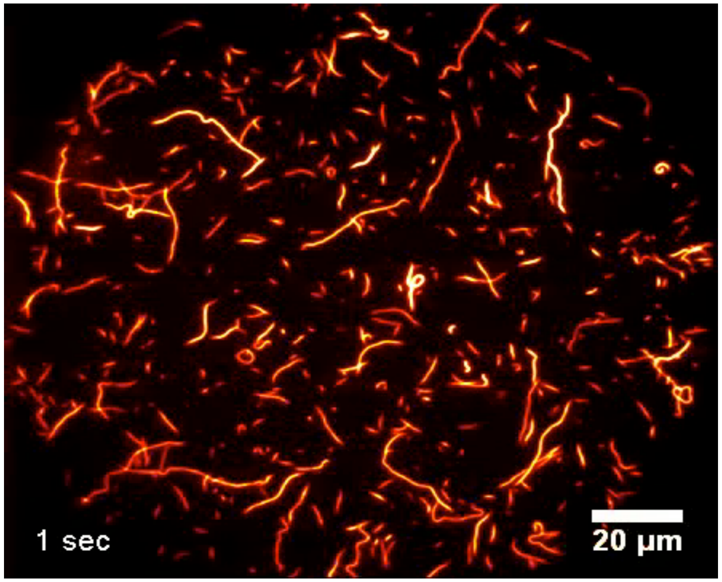
motor-filament complexes are “active particles”

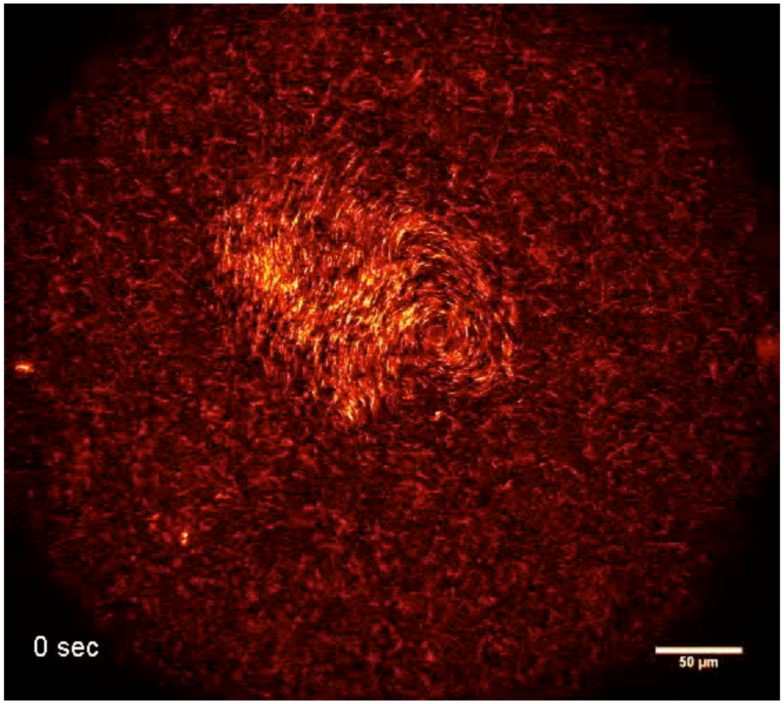
Actin “flocking” in motility assays



Myosin motor proteins

Low density of actin: directed random walk in all directions v_0~4.8 mm/s





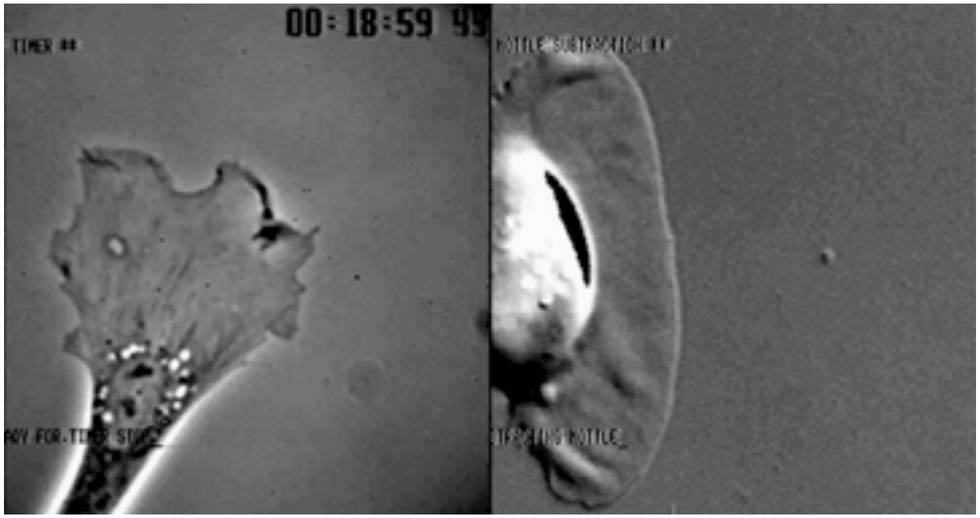
High density of actin: “flocking”

V. Schaller et al, Nature 467, 73 (2010)
(T. Butt et al, J. Biol. Chem. 2010)

http://www.nature.com.libezproxy2.syr.edu/nature/journal/v467/n7311/extref/nature09312-s2.mov

In vivo: active cytoskeleton controls cell motility

Fragment of lamellipodia can move on their own in the absence of cell body. (Verkhovsky et al. 1996)



chick
fibroblasts
(2h)

trout
keratocyte
(4min)
v=15μm/min

V. Small, IMBA, Vienna.

http://cellix.imolbio.oeaw.ac.at/video_tour_2.html

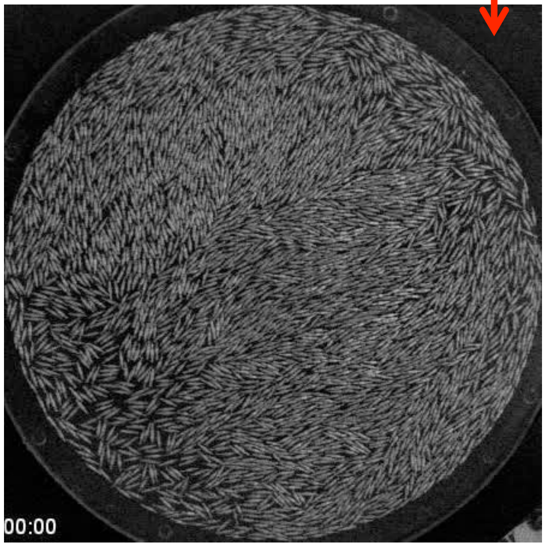
Collective behavior: groups of insects, fish, birds, …

StarFlag
Collaboration

Irene Giardina
et al, Rome

Active Matter is ubiquitous: examples from living & non-living world on many scales

- Inside the cell: cytoskeleton
  - Many cells: bacteria, tissues
    - Schools of fish, flocks of birds
      - Nanoparticles propelled by catalytically self-generated forces
      - Layers of vibrated granular rods
        - Groups of nanobots, hexbugs



Narayan et al
Science **317**
105 (2007)

L. Giomi
Harvard

What do these systems have in common?

- Novel nonequilibrium systems where the drive acts on each unit, not applied at the boundary
- Active particles are elongated, hence can order in states with orientational order → “living liquid crystals”
- “Dynamic self-assembly: onset of coordinated motion at large scales in the presence of noise: no “leaders”, external ordering field, global interactions
  → a nonequilibrium phase transition?

Are there underlying “universal” properties?

Collective behavior of active matter

- Emergent behavior qualitatively different from that of the individual constituents
- Onset of directed large-scale motion (relation to cell motility?)
- Pattern formation on various scales
- Novel correlations (e.g., giant number fluctuations seen in vibrated layers of granular rods)
- Unusual mechanical and rheological properties: cells stiffen when stressed and adapt their mechanical properties to the environment; activity-induced thinning and thickening of bacterial suspensions

Goal of Theoretical Work

Understanding the microscopic mechanisms responsible for collective behavior: why/how do ordered phases and structures form? what are the mechanisms that yield collective motion?

- What is the role of physical interactions vs biochemical or chemotactic ones?

Use the tools and ideas of (soft) condensed matter physics (hydrodynamics and broken symmetries) to develop an effective theory to describe the behavior of active matter:

- Characterization of phases & ordered structures
- Role of noise
- Rheological and mechanical properties
- Can we think of motility as a "material property" and of the onset of motion as a nonequilibrium phase transition?

Theoretical work can be grouped into three classes:

Agent-based models (Vicsek, 1995), studied by numerical simulations (Chate', Ginelli, et al, 2004-2010; Peruani et al, 2006; Yang et al, 2010; …)

Symmetry-based **phenomenological hydrodynamics**

- Toner & Tu (1995, 1998)
- Ramaswamy et al (2002, …)
- Kruse, Joanny, Julicher, Prost & Sekimoto (2004, …)

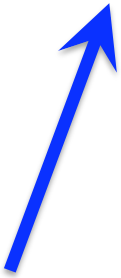
Microscopic derivation of hydrodynamics for specific models

- Liverpool & MCM (2003); Aranson et al. (2005) (motor-filament suspensions)
- Baskaran & MCM (2008 & 2009) (SP rods, swimmers)
- Bertin, Droz & Gregoire (2009), Ihle (2010) (Vicsek model)

Two types of active particles

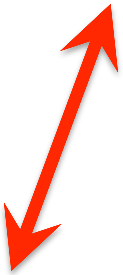
Polar: head$\neq$tail

fish, birds, bacteria,
motor-fils constructs

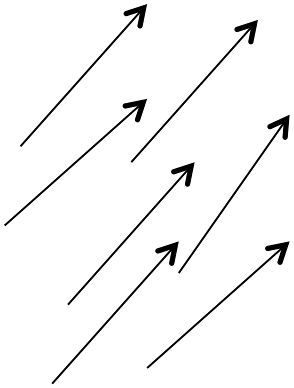


Apolar: head/tail symmetric

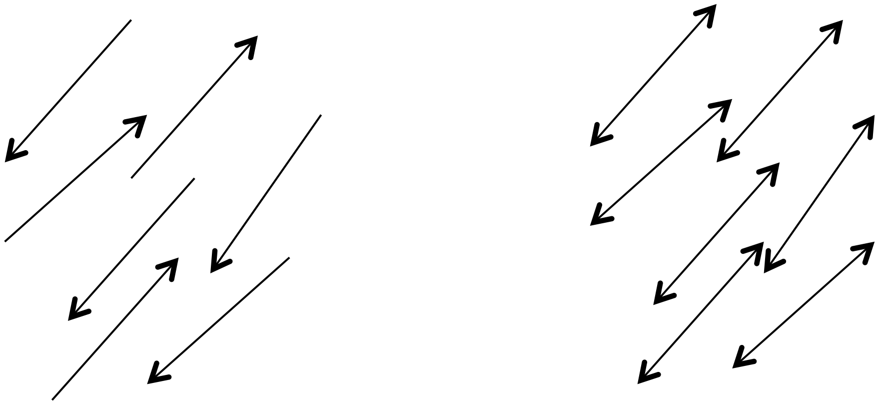
melanocytes, some motor/
fils constructs



Two types of orientational order



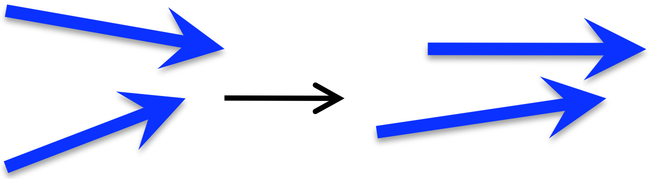
Polar (moving state)



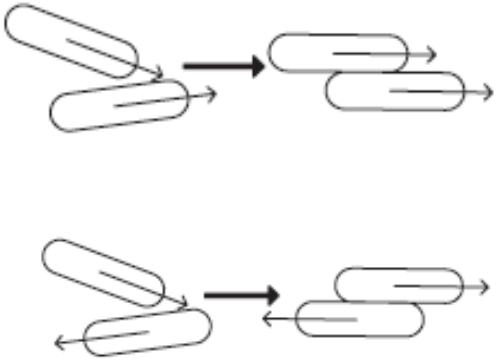
Apolar (nematic)

Two types of interactions

Aligning rule:
polar (e.g., Vicsek model)

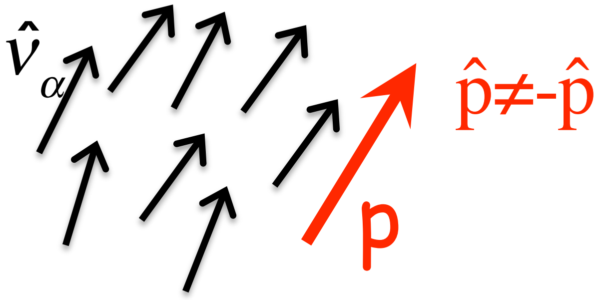


Apolar, e.g., excluded volume/hard core collisions



Polar & Apolar Orientational Order

Polar/Ferroelectric Order

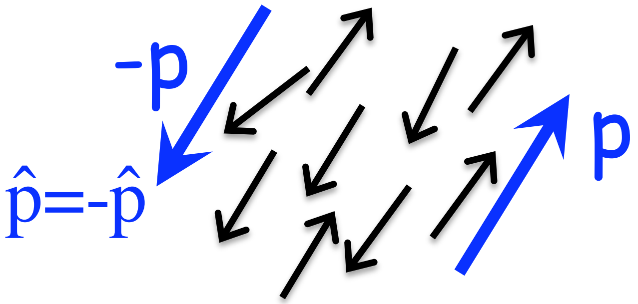


$$OP: \quad \vec{P}(\vec{x},t)=\left\langle \frac{1}{N}\sum_{\alpha}\hat{v}_{\alpha}\delta(\vec{x}-\vec{x}_{\alpha}(t))\right\rangle$$

$$=|P|\;\hat{p}$$

Ordered state is a moving state

Apolar/Nematic Order



$$OP: \quad \overleftrightarrow{Q}=\left\langle \frac{1}{N}\sum_{\alpha}\left(\hat{v}_{\alpha}\hat{v}_{\alpha}-\frac{\overleftrightarrow{1}}{d}\right)\delta(\vec{x}-\vec{x}_{\alpha}(t))\right\rangle$$

$$=S\left(\hat{p}\hat{p}- - \frac{\overleftrightarrow{1}}{d}\right)$$

Ordered state has zero mean motion

Role of medium: suspension vs substrate

Active particles on a substrate (e.g., vibrated rods, myxobacteria, animal herd): the medium only provides passive friction, momentum not conserved

Suspensions (e.g., bacteria): a two-component system → role of hydrodynamic interactions

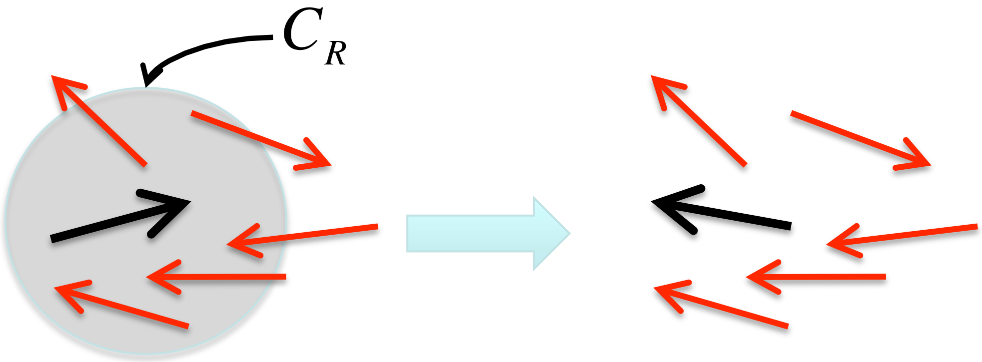
Today lectures:
active particles (mainly polar) on a substrate

A minimal model: Vicsek, 1995

T. Vicsek et al, PRL 75 (1995) 1226; C. Reynolds, SIGGRAPH '87 Conference Proc.

Analogy between flocking and ferromagnetism:

- N point particles
- fixed speed v_0
- noisy aligning rule



$$\theta_i^{t+\Delta t} = \theta_i^t + \left\langle \theta_i^t \right\rangle_{C_R} + noise$$

$$\vec{r}_i^{\,t+\Delta t} = \vec{r}_i^{\,t} + \mathrm{v}_0 \left(\cos\theta_i^t, \sin\theta_i^t \right)$$

$$\left\langle \theta_i^t \right\rangle_{C_R} = \frac{1}{n_R} \sum_{j \in C_R} \theta_i^t$$

Order parameter
Mean velocity:

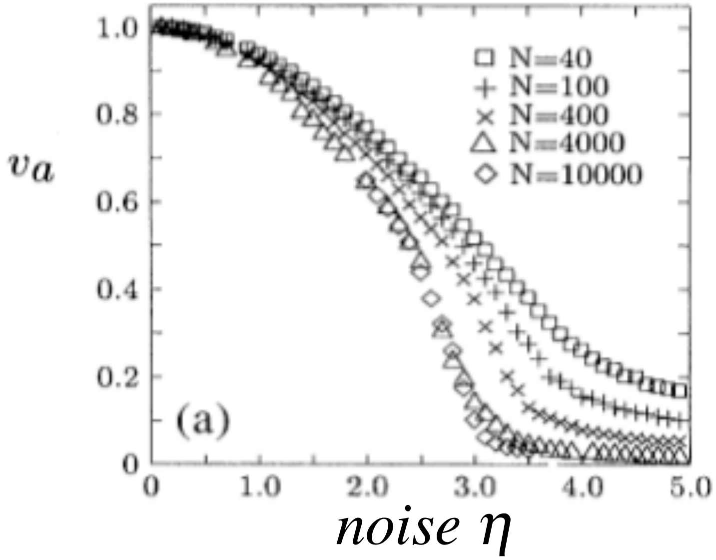
$$\vec{\mathrm{v}} = \frac{1}{N} \sum_{i=1}^{N} \vec{\mathrm{v}}_i(t)$$

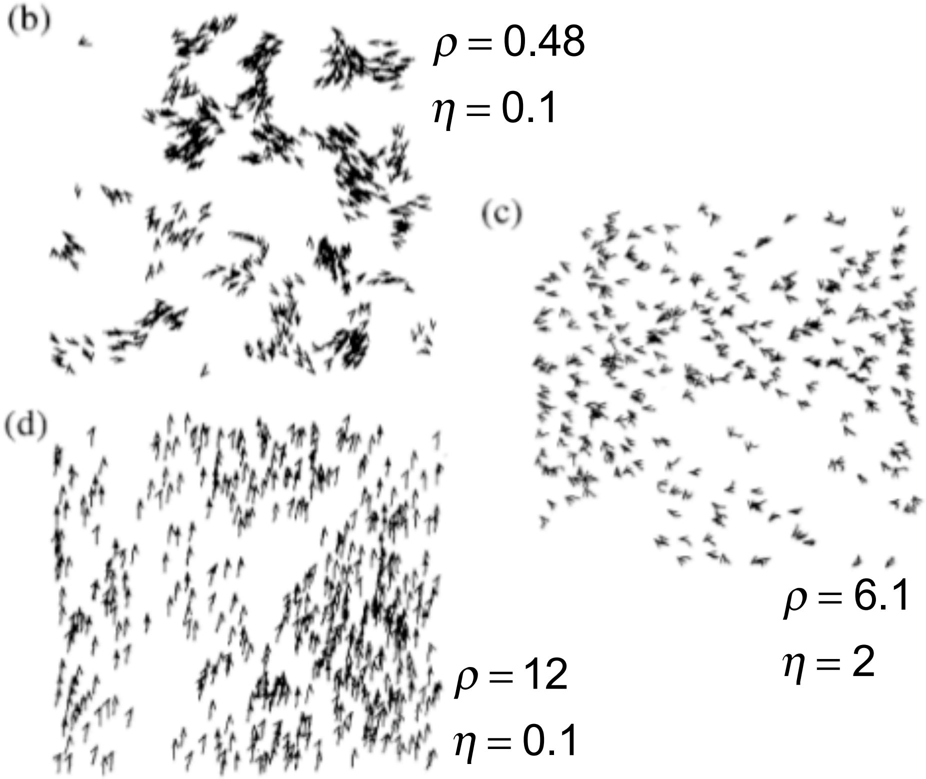
Disorder-order transition in 2d

- Low noise η/high density ρ: ordered flock
- High noise/low density: disordered state

T. Vicsek et al, PRL 75 (1995) 1226

$\rho = N/L^2$=density





Onset of coherent flock with finite v

Spontaneously broken continuous rotational symmetry in 2d

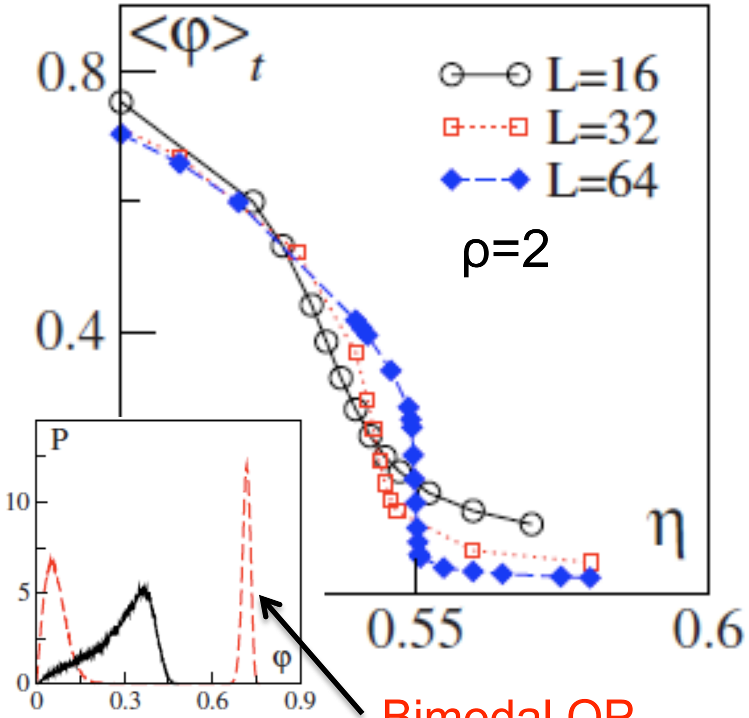
Unlike equilibrium XY: Mermin-Wagner theorem

Recent work: First order transition

Gregoire & Chate, PRL 2004
Chate et al PRE 2008

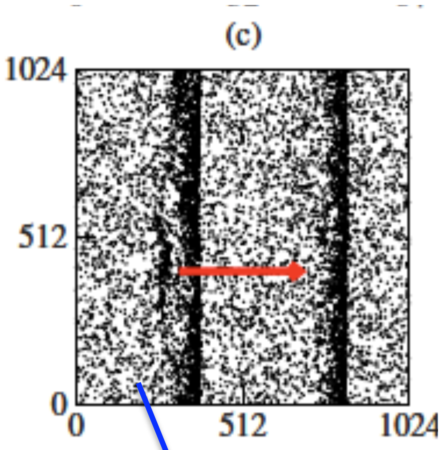
$$\langle \varphi \rangle_t = \frac{\langle v \rangle_t}{v_0}$$

- Discontinuous onset of order
- Coexistence of ordered & disordered states
- Traveling bands

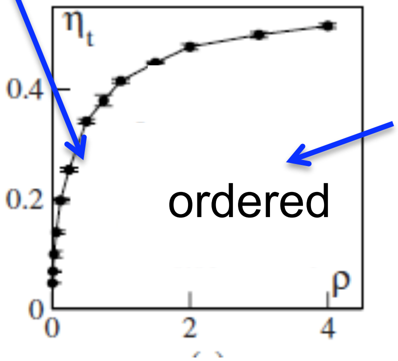


ρ=2

Bimodal OP distribution



ρ=0.5
η=0.3



ordered

large
fluctuations,
intermittency

Summary

- A tour through living and non-living active systems
- Agent-based model and the flocking transition

Next

- Hydrodynamics as an effective field theory: symmetries, conservation laws and broken symmetries
- Two examples of hydrodynamics of active systems on a substrate

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