Active Matter

Lectures for the 2011 ICTP School on Mathematics and Physics of Soft and Biological Matter

Lecture 5: Rheology of Active Fluids

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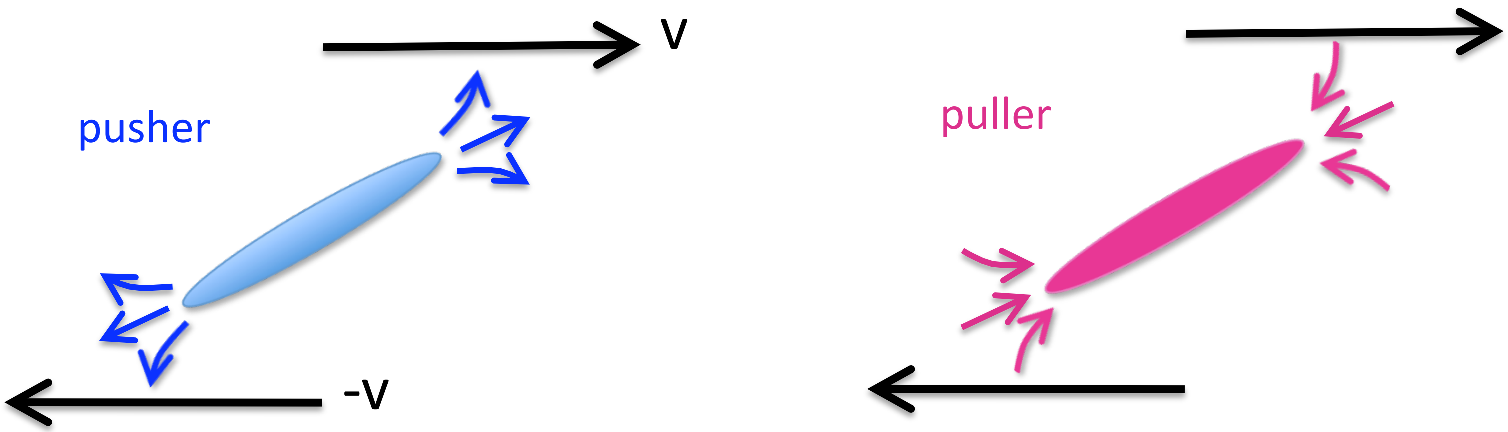
Rheology of active fluids

Theory: **rod-like** pushers decrease the viscosity of the suspension, **rod-like** pullers increase it.

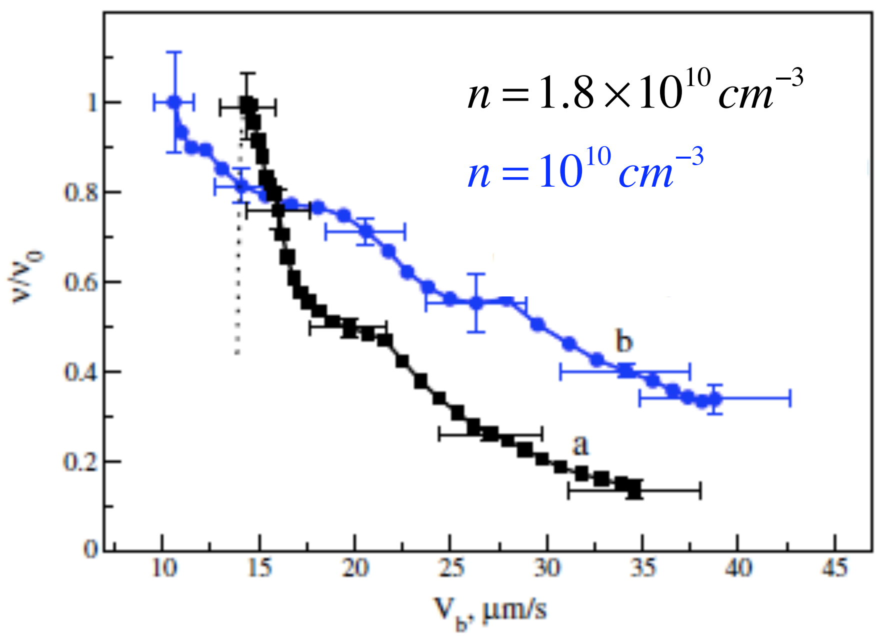
- Hatwalne, Ramaswamy, Rao & Simha, PRL 92, 118101 (2004)
- Liverpool & MCM, PRL 97, 268101 (2006)

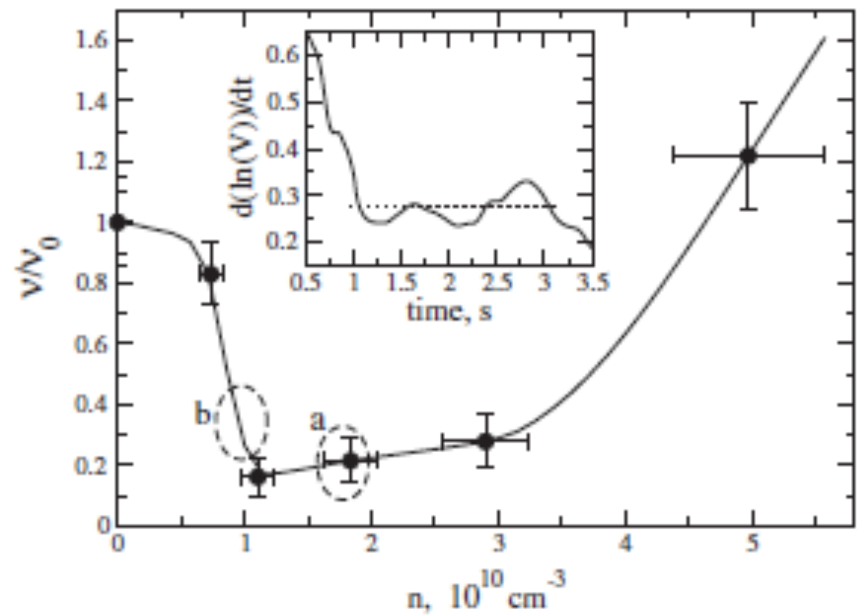


The rheology of active suspensions is controlled by the interplay of **flow alignment** l and **active tensile/contractile stresses** a>0/a<0

Activity-induced "thinning"

Sokolov & Aranson, Phys. Rev. Lett. 103, 148101 (2009):
Bacillus Subtilis swimming in freely suspended films **decrease** the viscosity of the suspension



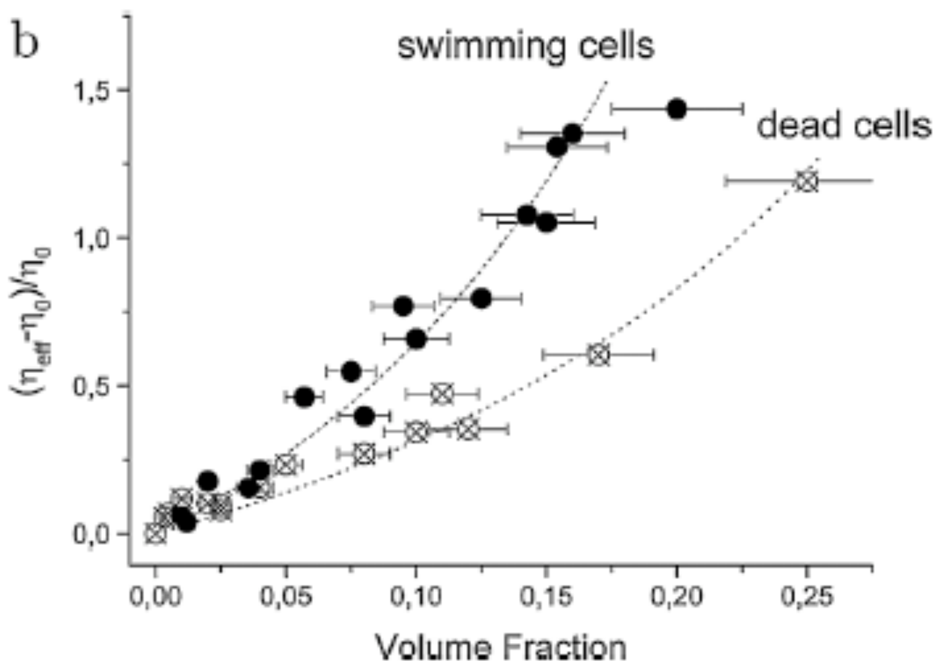


Viscosity from vortex decay. n_0 is viscosity with non-motile bacteria.

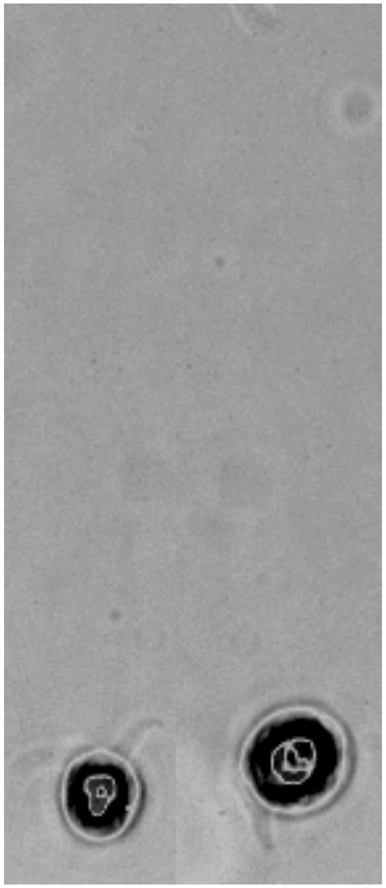
Viscosity from torque on rotating magnetic particle. n_0 is the viscosity of the medium.

Activity-induced "thickening"

Rafai, Jibuti, Pelya, Phys. Rev. Lett. 104, 098102 (2010)
Chlamydomonas swimming in a suspension **increase** the effective viscosity relative to that of a suspension with the same concentration of dead cells.

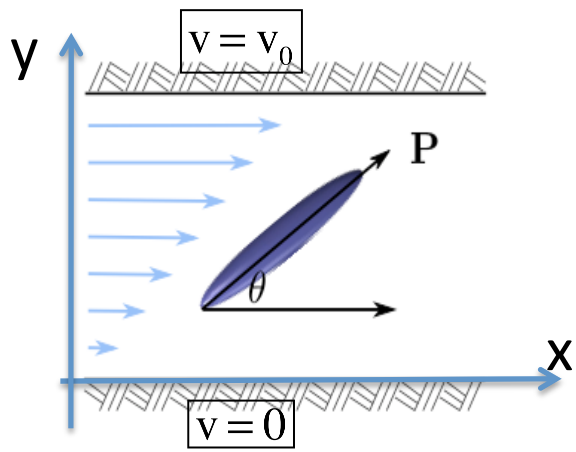


h_0 is the viscosity of the medium



Rheology of active suspensions

[Giomi, Liverpool & MCM, PRE 81, 051908 (2010)]



$\theta(0)=\theta(L)=0$

strain rate $\dot{\gamma}=\mathrm{v}_0/L$

stress $\sigma(\dot{\gamma})$

apparent viscosity $\eta_{app}(\dot{\gamma})=\sigma/\dot{\gamma}$

Tensile/contractile stress in polar & nematic a>0 pullers, a<0 pushers

$$\rho\partial_t \mathrm{v}=\partial_y\sigma=\partial_y\left[\eta\partial_y \mathrm{v}+\alpha P_x P_y+\beta\ \partial_y P_x\right]$$

$$\partial_t c=-\partial_y\left[c\beta P_y-D\partial_y c\right]$$

Active polar currents

$$\left(\partial_t+\beta P_y\partial_y\right)P_i=\frac{\lambda-1}{2}\left(\partial_y \mathrm{v}\right)P_x-w\partial_y c+\Gamma\frac{\delta F}{\delta P_i}$$

Also: Ishikawa & Pedley, J Fuid. Mech. (2007); Haines et al., Phys. Biol. 2008 & PRE 2009; Saintillan, PRE 2010

$\alpha,\beta\propto$ Activity
ATP consumption rate
Forces exerted by swimmers

Fix strain rate $\dot{\gamma} = \int_0^L \partial_y \mathrm{v} = \mathrm{v}_0 / L \quad \rightarrow$ calculate stress $\sigma(\dot{\gamma})$

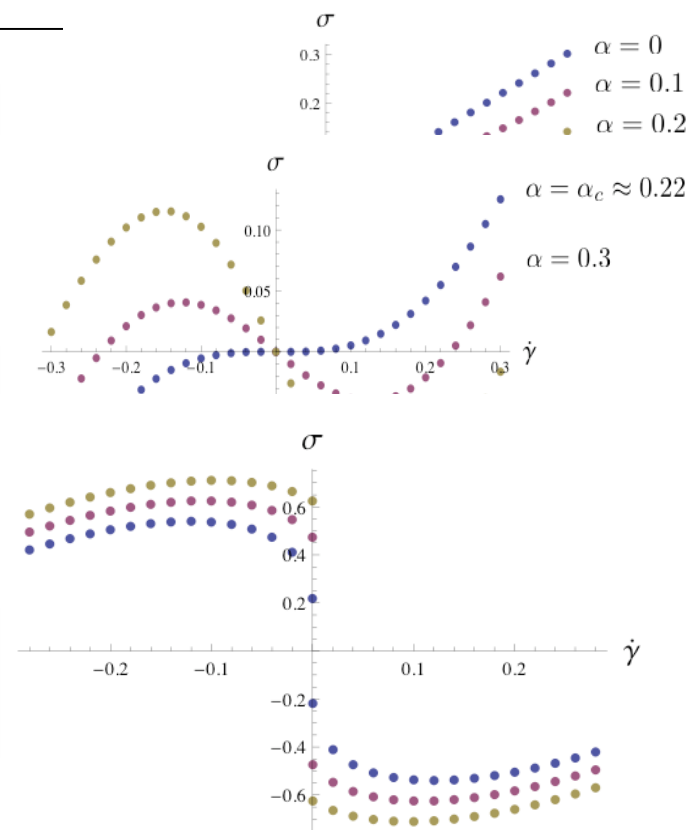
apparent viscosity $\eta_{app}(\dot{\gamma}) = \dfrac{\sigma(\dot{\gamma})}{\dot{\gamma}}$

- $|\alpha|<\alpha_{c1}$ (onset of spontaneous flow in film with one no-slip and one free boundary): stress/strain curve monotonic and **linear** for a broad range of shear rates

- $\alpha_{c1}<|\alpha|<\alpha_{c2}$ (onset of spontaneous flow in film between two no-slip planes): stress/strain curve **nonmonotonic** and nonlinear

- $|\alpha|>\alpha_{c2}$: stress/strain curve has jump

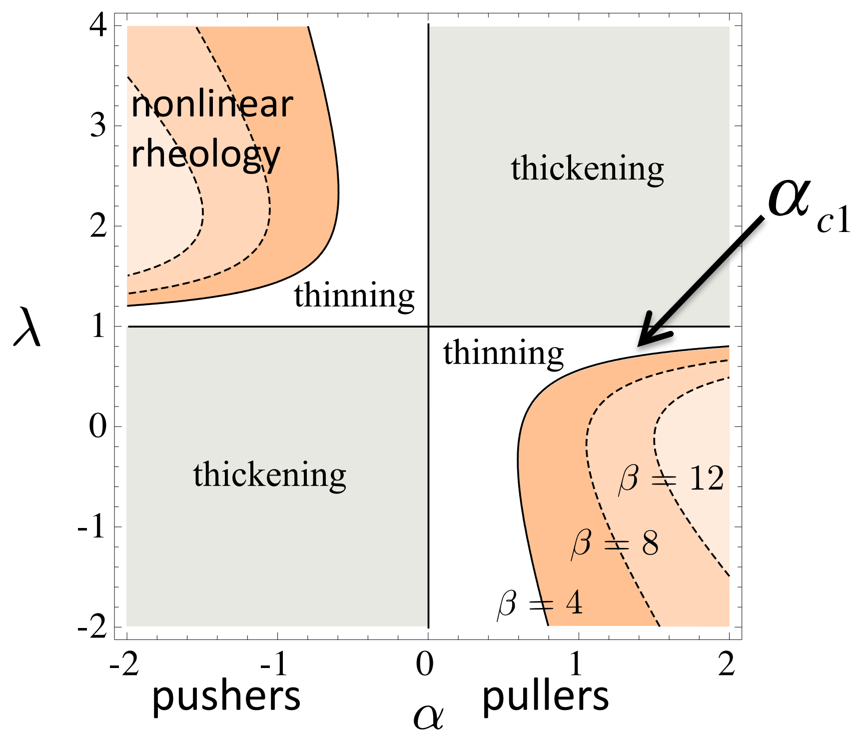
$$\sigma_c \equiv \sigma(\dot{\gamma}=0) \neq 0$$

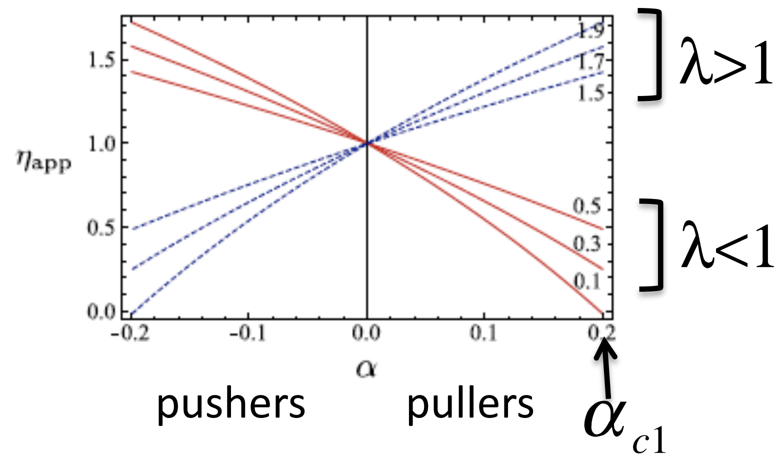


Linear region: rheological "duality" of active fluids

At small shear stresses and activity, flow-aligning pullers are rheologically identical to flow-tumbling pushers

$$\eta_{app}(\alpha, \beta, \lambda) = \eta_{app}(-\alpha, \beta, 2-\lambda)$$





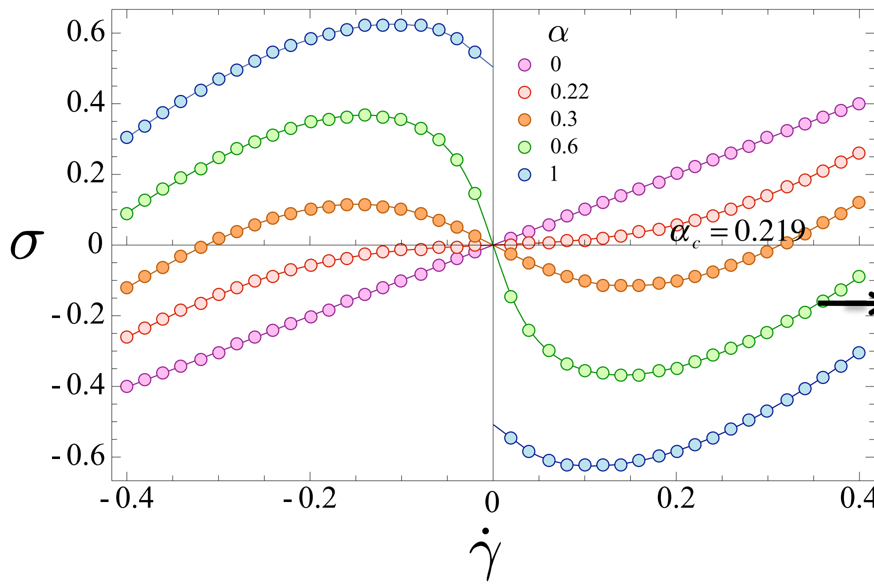
$$\beta = w = 0$$

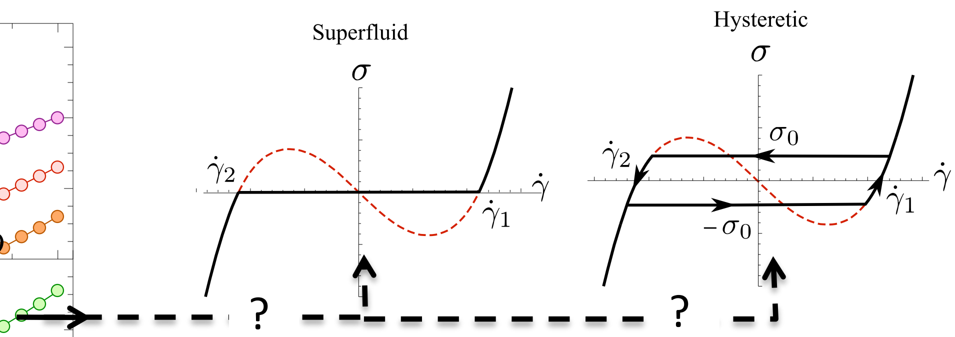
$$\eta_{app} = \frac{\eta}{\tan(kL/2)/kL/2}$$

$$k^2 = 2c_0\alpha(1-\lambda)/\eta$$

Nonlinear stress-strain curves for varying activity a

What would be the experimental stress-strain curve?

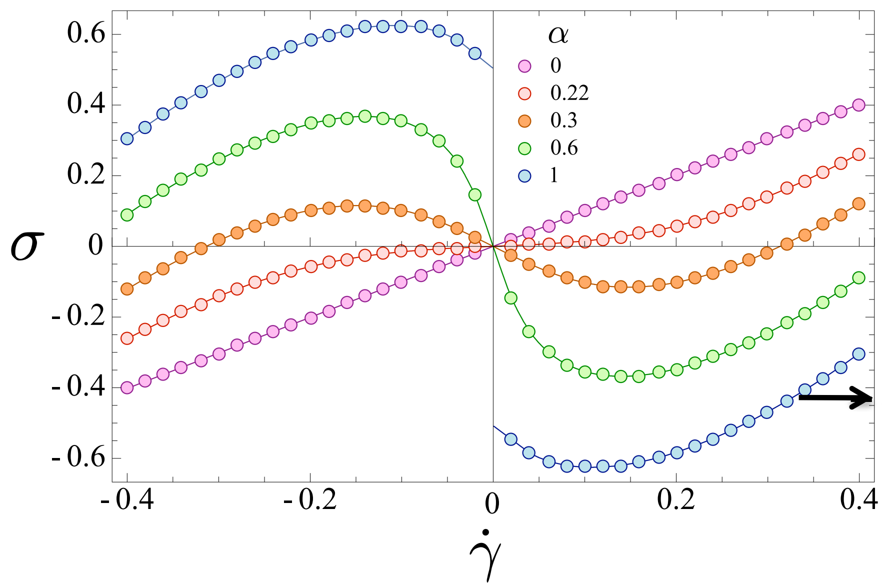


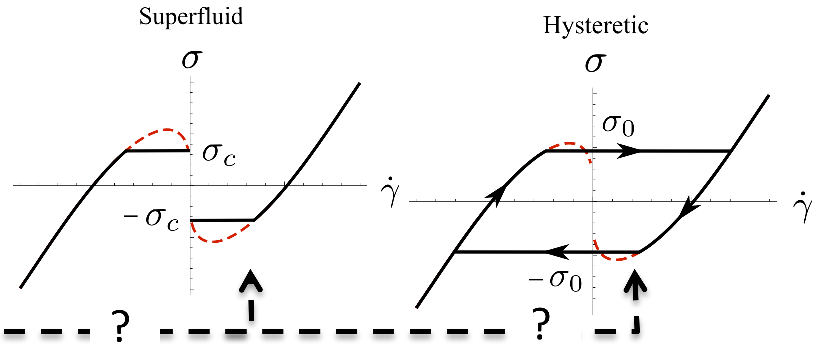


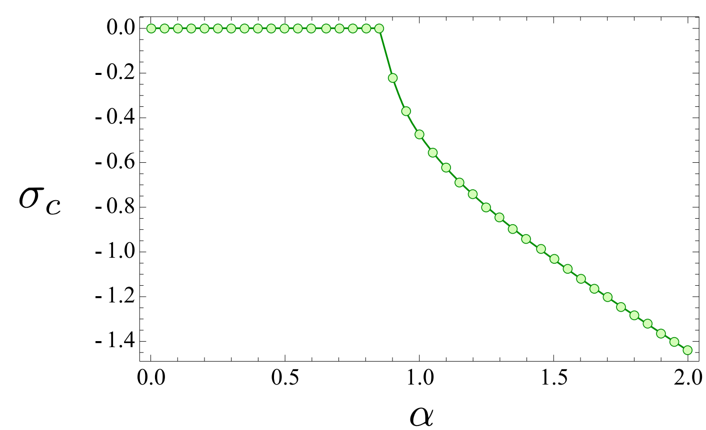
Bulk shear bands with opposite strain rates and zero stress.*

Hysteresis: coexistence of a range of strain rates with constant stress +/-s_0

*Marenduzzo et al, PRL 98, 118102 (2007); PRE 76, 031921 (2007) ; Cates et al, PRL 101, 068102 (2008); Giomi et al, PRE 2010.





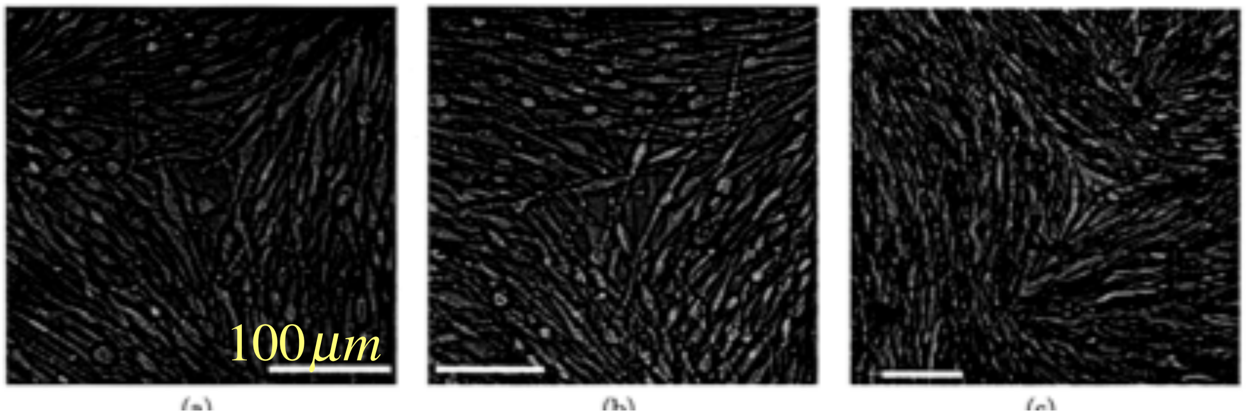


"Yield stress"

$$\sigma_c(\alpha) = \sigma(\dot{\gamma} = 0) \neq 0$$

Liverpool and MCM, PRL 97, 268101 (2006)

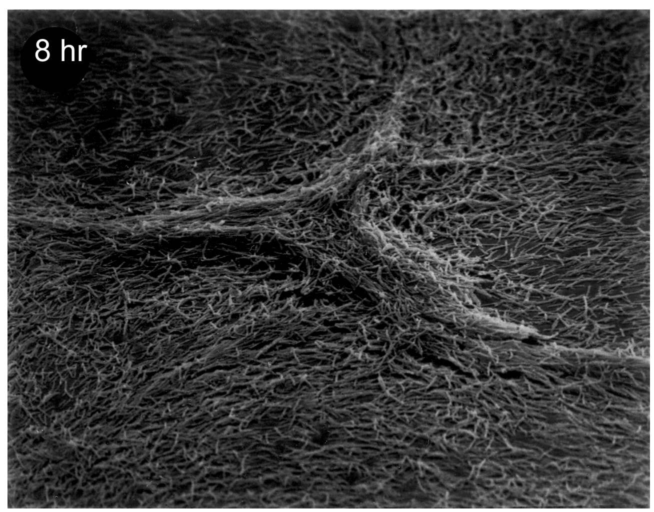
Patterns in living liquid crystals

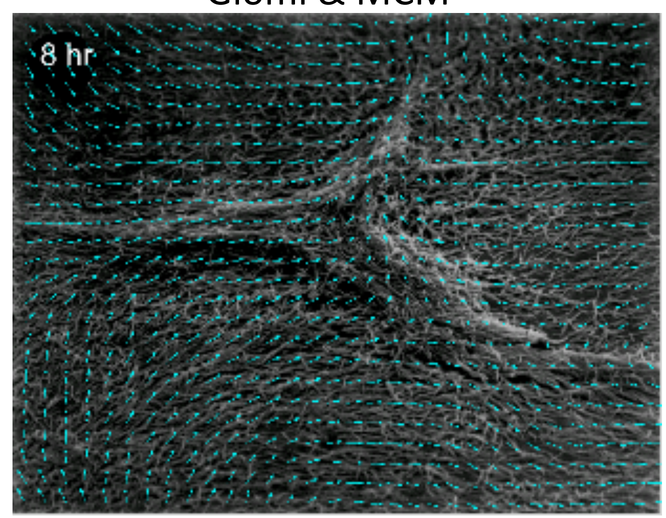


Melanocytes
Kemkemer et al EPJE 2000

Decoration by Bowick, Giomi & MCM

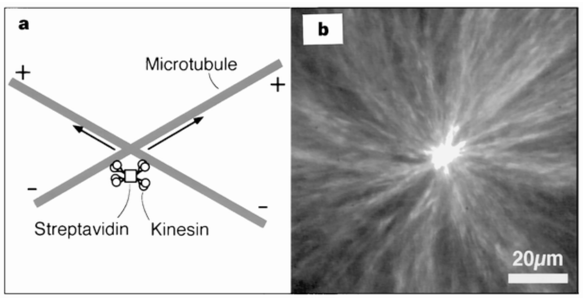
Myxobacteria aggregate formed 8 hours after a solution is deposited onto an agar substrate
(Kuner & Kaiser 1982)



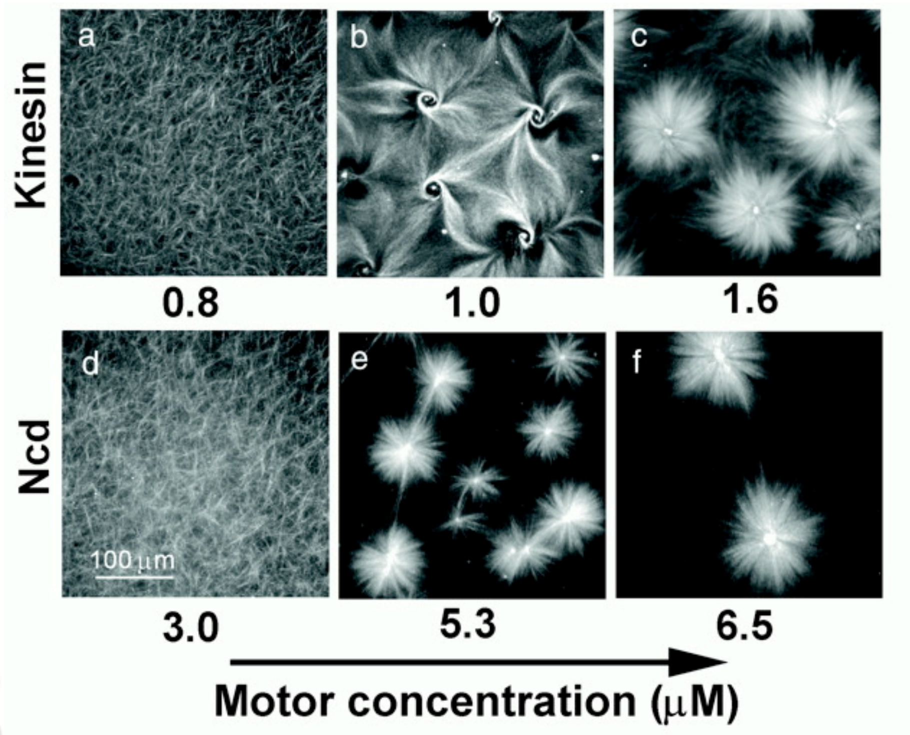


Activity-driven organization

Cell extracts of taxol-*stabilized* microtubules & motors show self-organization on mesoscopic scales.



microtubules and kinesin

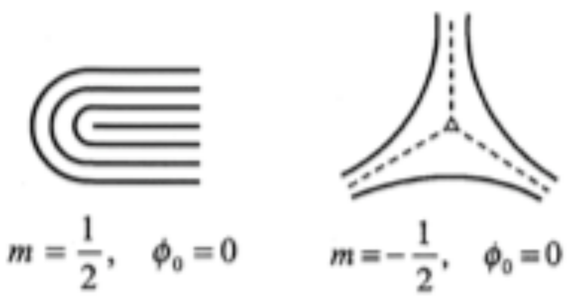


Nédélec et al, Nature **389**, 305 (1997)
Surrey et al, Science **292**, 1167 (2001)

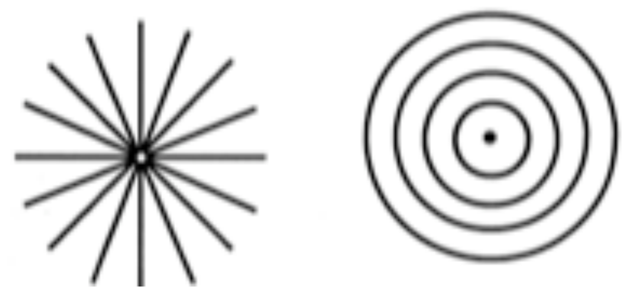
Topological defects

Activity-induced patterns in vitro bear striking resemblance to topological defect of ordered states

Because of n →-n symmetry, lowest energy defects in 2d nematic are m= ±1/2 disclinations



A polar state has the symmetry of a vector field with m=±1 defects



Defects in polarized state

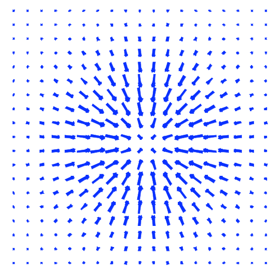
Kruse et al. 2005
Aranson & Tsimring 2005
MCM & TBL

neglect density modulations to study the topological defects of polarization field

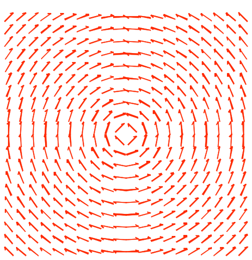
$$K_{1,3}^{\alpha} = K_{1,3} - \alpha(...)$$

$$\partial_t n_i = \delta_{ij}^T \left[K_3^{\alpha} \nabla^2 n_j + (K_1^{\alpha} - K_3^{\alpha}) \partial_j \vec{\nabla} \cdot \vec{n} \right] - \beta \vec{n} \cdot \vec{\nabla} n_i$$

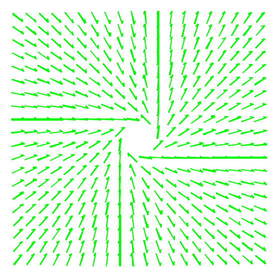
$$\vec{n} = (\cos\theta, \sin\theta)$$
$$\theta(r,\varphi) = \varphi + \Psi(r)$$



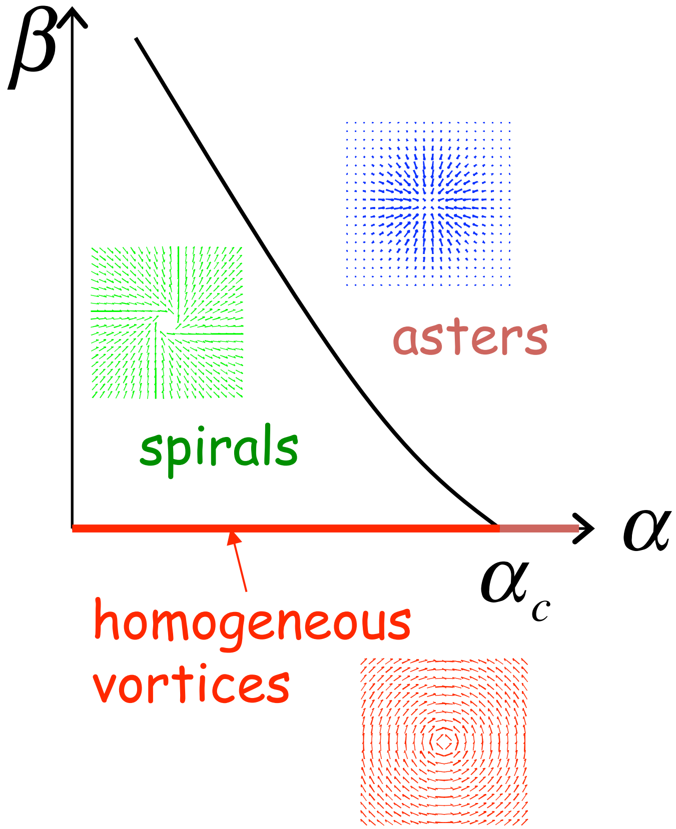
$\Psi = 0, \pi$
aster

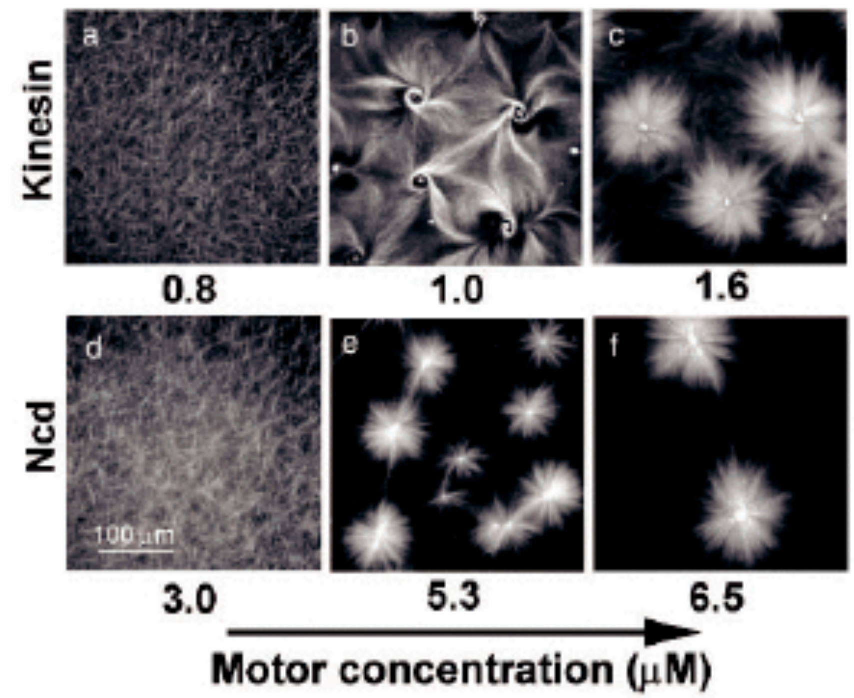


$\Psi = \pm\pi / 2$
vortex



spirals:
$\Psi(\mathrm{r}) = \sin^{-1}\left(r / r_0 \right)$





Consistent with simulation by F. Nedelec

β ~ motor speed ~ polarity

α ~ motor processivity (motor stalling at filaments' end)

See recent work by Elgeti, Cates and Marenduzzo, Soft Matter **7**, 3177 (2011)

Some References

(mainly review articles where more references can be found)

•J. Toner, Y. Tu & S. Ramaswamy, *Hydrodynamics and Phases of Flocks*, Ann. Phys. 318, 170 (2005).

•S Ramaswamy, *The Mechanics and Statistics of Active Matter*, Annu. Rev. Cond. Matt. Phys. 1, 323 (2010).

•K. Kruse, J.F. Joanny, F. Jülicher, *J. Prost, and K. Sekimoto, Generic theory of active polar gels: a paradigm for cytoskeletal dynamics* Eur. Phys. J. E **16**, 5–16 (2005).

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•T. Vicsek and A. Zafiris, *Collective motion* http://arxiv.org/abs/1010.5017