Defect-Driven Structures for Self-Assembly



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MJB and L. Giomi, Adv. Phys. 58, 449-563 (2009) (arXiv:0812.3064)

Introduction

Among soft and biological materials there are many compelling examples of objects with curvature and intrinsic order. These systems span a very broad range of length and energy scales and the physical mechanisms that lead to their assembly and mechanical stability can be very different.



Lengths (in meters)

$$1.42 \times 10^{-10}$$
 5×10^{-9} 1.2×10^{-8} 10^{-7} 10^{-6} 10^{-3}

Energy ranges between hundreds of k_BT (sp² bonds in graphene) to 1 k_BT (Van der Waals interactions).

Suppose we want to design nano to meso scale building blocks (super-atoms) for creating super-molecules and subsequent 3D structures without the restrictions of quantum mechanics

Requirements:

Distinguished regions for the attachment of ligandsHigh degree of specificity

For small scale objects the surface to volume ratio is large so surface topological defects can provide the distinguished regions

The effective valence will then be determined by the number of defective regions and the type of directional bonding will be determined by the relative arrangement of the defects.

Lubensky & Prost (1992)

The number of defective regions is an energetic question with underlying topological constraints

Strategy:

- Take a set of microscopic objects interacting on some fixed surface
 Map to an effective interacting defect problem by treating everything but the defects as a continuum
- Find the ground state of the defect Hamiltonian

Membranes





365,000X



J.F. Hainfeld and T.L. Steck, J. Supramol. Struct. **6**, 301 (1977) (courtesy Leo Van Hemmen) 10,000X

Membranes

Crystalline (elastic) membranes



$$\mathbf{E} = \mathbf{E}_{el} + E_{bend}$$

2d Elasticity of Sheet + Shape Change from Height Fluctuations

$$\begin{split} \mathrm{E}_{el} &= \frac{1}{2} \int d^2 x [2\mu u_{ij}^2 + \lambda u_{kk}^2] \\ \text{where } \mathrm{u}_{ij} &= \frac{1}{2} (\partial_i u_j + \partial_j u_i + \partial_i h \partial_j h) \text{ (strain tensor)} \\ \mathrm{E}_{bend} &= \frac{\kappa}{2} \int d^2 x (\nabla^2 h)^2 \end{split}$$

Energy minimized when
$$u_{ij} \approx 0$$

 $\implies \partial_j u_i + \partial_i u_j = -\partial_i h \partial_j h$

BUT

This is impossible for single-valued phonon fields $u_i(x_1, x_2)$

Defects, e.g; disclinations and dislocations, are an essential part of the ground state!

In fact,

$$\frac{1}{2}\epsilon_{im}\epsilon_{jn}\partial_m\partial_n(\partial_j u_i + \partial_i u_j) = s(x_1, x_2) = det(\partial_i\partial_j h) = K(x_1, x_2)$$

$$\uparrow$$
defect density Gaussian curvature

What are the defects? 1. Disclinations= Bond-Orientation Defects





5=+1



 $E \sim R^2$ (planar) $E \sim \kappa \log(R/a)$ (pure bend)



2. Dislocations (translational defects) \equiv 5-7 dimers



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Integrating out the phonons one finds

$$E = \frac{Y}{2} \int \int d^{2}x d^{2}y [K(x) \frac{1}{\Delta^{2}(x,y)} K(y)] + \kappa E_{bending}$$

K(x) = Gaussian curvature

$$Y = \frac{4\mu(\mu + \lambda)}{2\mu + \lambda}$$
 (2D Young's Modulus

$$\mathbf{E} = \frac{\mathbf{v}}{2} \iint d^{2}x d^{2}y [\mathbf{K}(\mathbf{x}) - \mathbf{s}(\mathbf{x})] \frac{1}{\Delta^{2}(\mathbf{x}, \mathbf{y})} [\mathbf{K}(\mathbf{y}) - \mathbf{s}(\mathbf{y})] + \mathbf{N}\mathbf{E}_{c} + \kappa E_{b}$$

$$\mathbf{S} = \frac{\pi}{3} \sum_{i=1}^{N} \mathbf{q}_{i} \delta(\mathbf{x}, \mathbf{x}_{i})$$

$$G(x, y) = \frac{1}{\Delta^{2}(x, y)} \sim |x - y|^{2} \ln |x - y|$$

MJB, D.R. Nelson and A. Travesset: PRB 62 (2000) 8738

MJB, A. Cacciuto, D.R. Nelson and A. Travesset, PRB 73 (2006) 024115

From planar packing to spherical packing: topological constraints



Shape of defective regions

Map particle interaction to (universal) defect Hamiltonian in a continuum elastic background with 2 parameters: bulk modulus Y and core energy E_c

SCARS

$$K(x) = 1/R^2$$

Disclination elastic energies grow like R^2

For small R/a (<5) the disclinations are localized – isolated point-like disclinations as in 2d melting from the hexatic to the fluid

For large R/a the disclinations are delocalized - leads to formation of freely terminating linear defect arrays

Large Scale Simulations



N=752 (V=1/r) D.J. Wales, H. Mackay and E.L. Altshuler, PRB (2009)

http://www-wales.ch.cam.ac.uk/~wales/CCD/Thomson2/table.html

Thomson Applet: <u>http://thomson.phy.syr.edu/thomsonapplet.htm</u> MJB, C. Cecka and A.A. Middleton

PMMA particles on curved surfaces, imaging and reconstruction (Charge stabilized: Chaikin & Irvine: NYU)









Thursday, May 12, 2011

Crystalline Order on CMC Delaunay surfaces



Irvine, Vitelli and Chaikin, Nature (2010) MJB, Z. Yao, EPL (2011)

Catenoids









Euler number 0



Stretching a capillary bridge









More specimens from sphereland



SV40 Viral Capsid



Adenovirus



C60



Bilayer vesicle of Ph5C60K molecules: Zhou et al



MP AuNP: DeVries et al (Stellacci)



Irvine & Chaikin

Changing the Order

For **p**-fold order the total number of distinct defective regions is 2**p**

р	Local order	Valence
6	Crystalline	12
2	Nematic	4
1	Vector	2
4	Tetradic	8



p=1



p=2



Spherical nematics (p=2) give rise to 4 elementary +1/2 defects.



Lubensky & Prost (1992) D.R. Nelson, Nano Lett.2 (2002) 1125

In the isotropic (one Frank constant) approximation the +1/2 defects would sit at the vertices of a tetrahedron



H. Shin, MJB and X. Xing: Phys. Rev. Lett. 101 (2008) 037802

Hard and soft rod MC fluid simulation produces jammed splay-dominated nematic state with four defects lying on a great circle!



$$F = \frac{K_1}{2} \int (D \cdot n)^2 + \frac{K_3}{2} \int (D \times n)^2$$

For hard rods splay is preferred over bend

Take the extreme limit of pure splay deformations

$$D \times n = 0 \implies n \cdot (Dn) = 0$$

Director is a completely determined integral curve (follows geodesics)

+1 disclinations are degenerate with 2 +1/2s

Why are all defects on a great circle?

There is a special bending-free ground state



Cut and rotate by an arbitrary angle Director field is continuous after surgery, except at defect cores All four ¹/₂ defects form a rectangle of arbitrary aspect ratio One parameter family of degenerate ground states

Making nematic shells: double emulsions

Fernandez-Nieves et al; PRL (2007); Vitelli and Nelson, PRE (2006)



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Toroidal Crystals

Toroidal crystals are two-dimensional crystalline assemblages of monodisperse objects on a torus. Example: toroidal micelles from self-assembly of amphiphilic dumbbell molecules (Kim *et al* 2006)



Amphiphiles form double-dumbbells of 16 nm in length and self-assemble in spherical and uncapped cylindrical micelles. These turn into toroidal micelles in the course of a week. Diameters: D_1 =70-300 nm, D_2 =16 nm.



Although defects not required topologically $(\chi = 0)$ they appear to **mininize** the energy



Luca Giomi and MJB Phys. Rev. E 78, 010601(R) (2008); Eur. Phys. J. E27, 275 (2008)

Toroidal Scars





E. Pairam and A. Fernandez-Nieves PRL 102, 234501 (2009)

Tilt & Smectic Order



SAM consisting of 2 types of thiolterminated ligands (1-nonanethiol and 4-methybenzenethiol) on Au

DeVries et al (Stellacci): Science (2007)





Functionalization by MUA



Polymer Physics with Nanoparticles

XY-model of tilt order: order-to-disorder transition as a function of nanoparticle radius. Gaussian curvature plays role of external magnetic field; MJB, L. Giomi and X. Ma



Smectic Polymer Vesicles

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PEG: flexible and hydrophilic PAChol: liquid crystalline and hydrophobic

Phase Sequence of PAChol Homopolymer



- 1. Glassy smectic at room temperature (irrelevant for vesicle formation)
- 2. Isotropic-Smectic first order transition, no intermediate nematic phase

Smectic Polymer Vesicles



Shape change driven by smectic order - defects inevitable

M.-H. Li, *et al*, to be published

Smectic Vesicles





Isotropic >Smectic



2 (+1) disclinations



Functionalize defects?

A Different Copolymer: PA6ester1



PA6ester1: IN Transition

- Isotropic vesicles first form at high concentration of dioxane, no in-plane order
- As dioxane concentration decreases, nematic order develops on the vesicles
- Continuous IN transition, sufficiently close to T_{IN}
- $\begin{array}{l} K_1, K_3 << \kappa \\ \hbox{Four disclinations form a regular tetrahedron} \end{array}$





- As one approaches the smectic transition K_3 diverges
- Two regimes:
- 1.
- Splay dominated spherical nematic: 4 +1/2s on a single great circle
- $K_1 << K_3, \kappa$



Faceting see nice poster of N. Ramakrishnan

- Second possibility:
- If $K_1, K_3 >> \kappa$ the system prefers a state with vanishing Frank free energy, i.e. the director every parallel (in the sense of Levi-Civita parallel transport)
- How is that possible on a sphere? Obstruction due to Riemann curvature
- Key: faceting and localization of Riemann curvature near four corners.





J=0.3



Self-interstitials (MJB, Irivine & Chaikin)



Curvature driven fractionalization

- MJB, H. Shin and A. Travesset, PRE 75 (2007) [cond-mat/0610819]
- MJB, D. R. Nelson and H. Shin, Phys. Chem. Chem. Phys. (2008) [arxiv:0707.1909]

http://thomson.phy.syr.edu/thomsonapplet.htm

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http://thomson.phy.syr.edu/thomsonapplet.htm























Where did the particle go?



Interstitial fractionalization on a catenoid

1

4

Interstitial fractionalization on a catenoid



Interstitial fractionalization on a catenoid



Conclusions

- Topology determines broad structure of ground state
- Energetics determines the detailed structure = shape of defective regions

Curvature-driven effects

- Disclination delocalization
- Novel structures (disclinations) in ground state
- Interstitial= "particle" fractionalization

Defects allow design of superatoms # Defective regions = valence Global Geometry of Defective regions determines type of directional bonding available (controllable via elastic moduli)

Structure controls shape in some cases: faceted liquid crystalline vesicles from block copolymers