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**2239-6**

**Workshop on Integrability and its Breaking in Strongly Correlated and Disordered Systems**

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# Weak Interaction Quenches and Thermalization in Quantum Many-Body Systems

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# Non-Equilibrium Quantum Many-Body Systems: Universal Aspects of Weak Quenches

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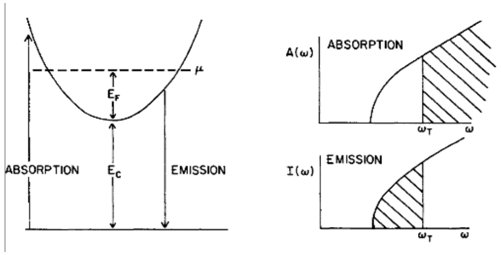
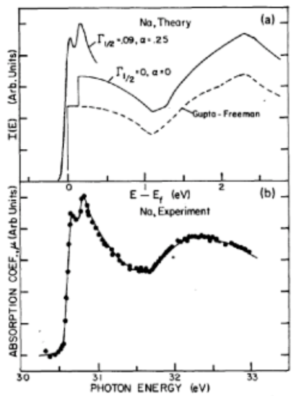
# X-Ray Edge Problem



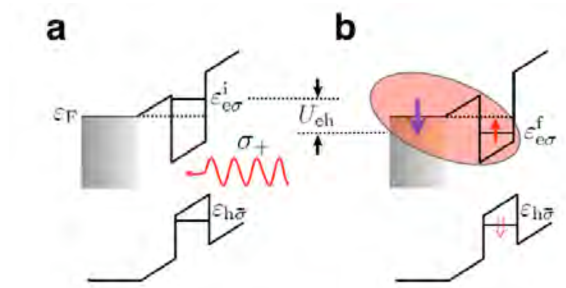

$$H_i = \sum_k \varepsilon_k c_k^\dagger c_k$$

$$H_f = \sum_k \varepsilon_k c_k^\dagger c_k + \sum_{k,k'} V_{kk'} c_{k'}^\dagger c_k$$




Mahan,
Many-Particle Physics

# Kondo Excitons

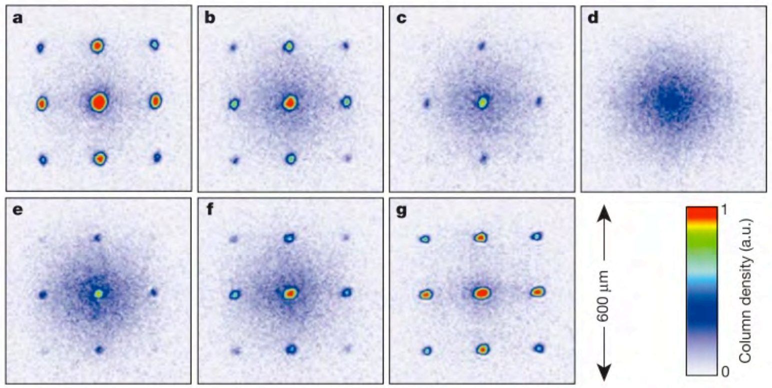



Türeci et al., PRL 106 (2011)
Latta et al., arXiv:1102.3982

**Initial Hamiltonian $H_i$: defines state $|\Psi_i\rangle$**

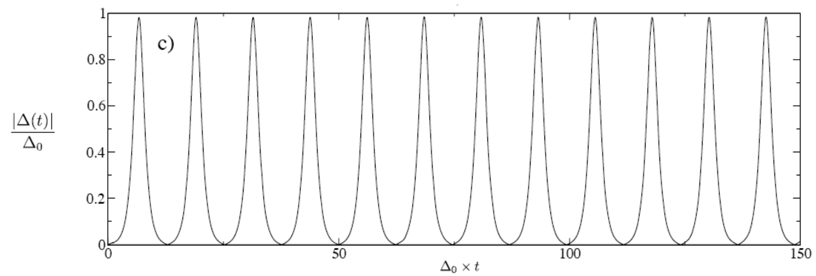
**Final Hamiltonian $H_f$: defines time evolution $|\Psi(t)\rangle = \exp(-iH_f t)\,|\Psi_i\rangle$**

# Collapse and Revival




Greiner et al., Nature 419, 51 (2002)

# Non-Equilibrium Cooper Pairing




Yuzbashyan et al., PRL 96 (2006)

$H_i$ , $|\Psi_i>$

Quench

$H_f$ , $|\Psi(t)> = \exp(-iH_f t)\,|\Psi_i>$

Work performed on system

Excitation energy in system

$$E_{ex} = \langle\Psi_i|H_f|\Psi_i\rangle - \langle\Psi_f|H_f|\Psi_f\rangle > 0$$

Long time limit $t\rightarrow\infty$

- Memory of initial state?
- Thermalization?

- Pure states remains pure

# Universal Aspects of Weak Quenches

Working definition:
"weak" = amenable to some (suitable) perturbation theory

Factor 2 enhancement

I) Ferromagnetic Kondo model ($d=0$)

II) Quenched Fermi gas ($d>1$)
(or: Beyond Landau adiabacity)

III) What about $d=1$?

# "Factor 2" for discrete Hamiltonians

- Perturbative Hamiltonian $H = H_0 + g\, H_{\text{int}}$
- Observable O(t) with (i) $O|\Omega_0\rangle = 0$ (ii) $[O, H_0] = 0$

- Ground states
  of $H_0$ $|\Omega_0\rangle$
  of H $|\Omega\rangle$

**M. Moeckel and S. K., Ann. Phys. (NY) 324, 2146 (2009)**

**In second order perturbation theory:**

$$\lim_{T\to\infty} \frac{1}{T}\int_0^T dt\, \langle \Omega_0(t)|O|\Omega_0(t)\rangle = 2\,\langle \Omega|O|\Omega\rangle$$

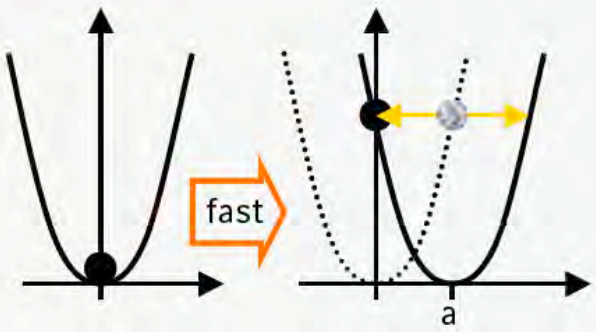
Expectation value of excited state (nonequilibrium)

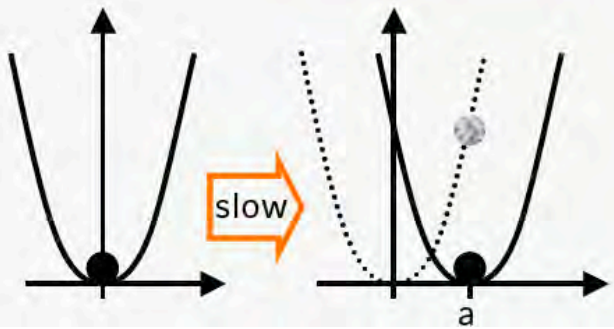
Ground state expectation value (equilibrium)

Sudden shift

$$V^{(0)} = \frac{1}{2}x^2 \;\rightarrow\; V^{(S)} = \frac{1}{2}(x-a)^2$$

Adiabatic shift



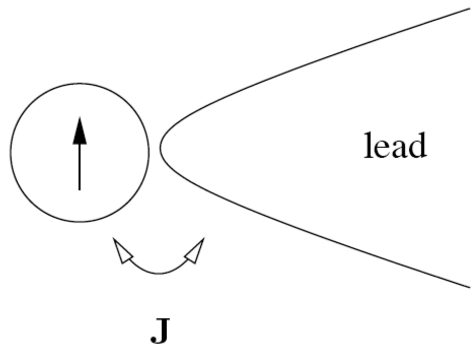



# Ferromagnetic Kondo Model

A. Hackl, D. Roosen, S. K., W. Hofstetter, Phys. Rev. Lett. 102, 196601 (2009)
A. Hackl, M. Vojta and S. K., Phys. Rev. B 80, 195117 (2009)

$$H_i = \sum_{k,\sigma} \varepsilon_k c^\dagger_{k\alpha} c_{k\alpha} - 0^+ S_z$$

infinitesimal magnetic field




⇒ **Product initial state:** $|\Psi_i\rangle = |\uparrow\rangle \otimes |FS\rangle$

$$H_f = \sum_{k,\sigma} \varepsilon_k c^\dagger_{k\alpha} c_{k\alpha} - 0^+ S_z + J\vec{S} \cdot \sum c_{k'\alpha} \vec{\sigma}_{\alpha\beta} c_{k\beta}$$

ferromagnetic coupling (J<0): Coupling constant flows to zero
⇒ Expansion becomes better (asymptotically exact) for long times

Nonequilibrium spin expectation value:

$$\langle S_z(t) \rangle = \frac{1}{2}\left(\frac{1}{\ln(t) - \frac{1}{\rho J}} + 1 + \rho J + O(J^2)\right).$$

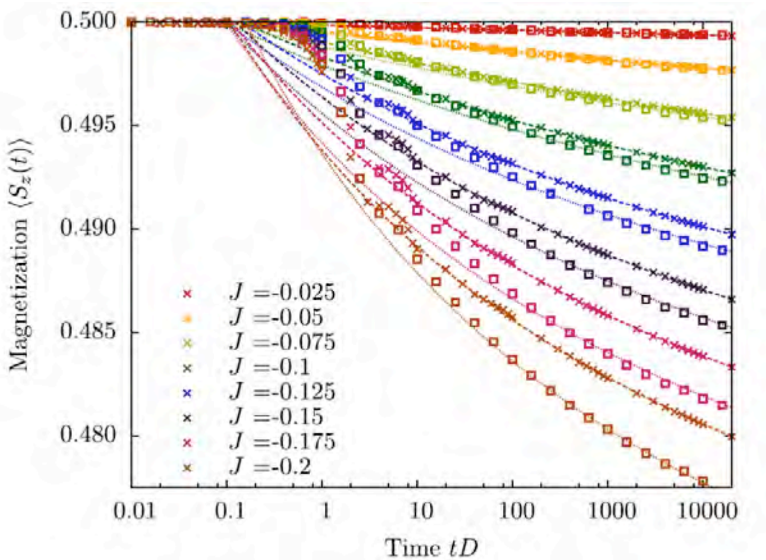
Equilibrium:

$$\langle S_z \rangle_{eq} = \frac{1}{2}\left(1 + \frac{\rho J}{2} + O(J^2)\right)$$

**Observable:**

$$O = S_z - \frac{1}{2}$$

Comparison with TD-NRG:
(Hackl et al., PRL 102)



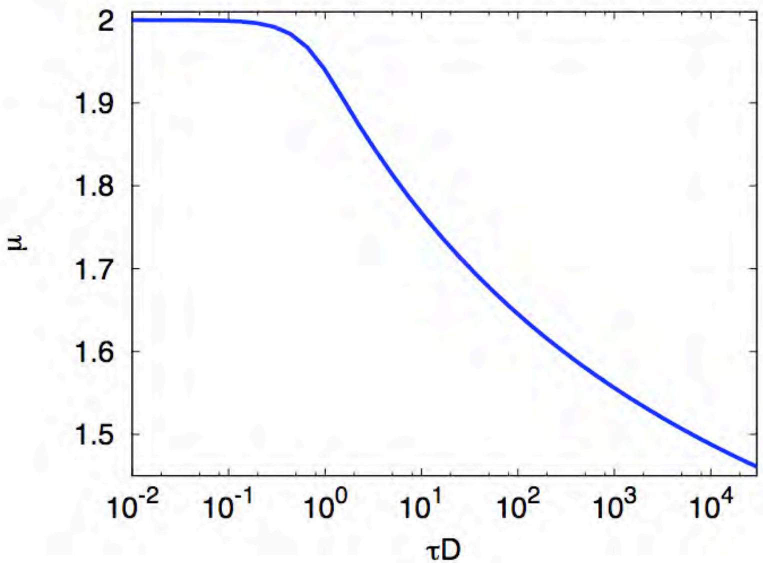

⇒ System remembers initial quantum state for all times

Crossover from adiabatic to instantaneous quenching:

Coupling J switched on on timescale τ

Measure of non-adiabacity:

$$\mu \stackrel{def}{=} \frac{\lim_{t\to\infty}\langle O(t)\rangle_{neq} - \langle O\rangle_0}{\langle O\rangle_{eq} - \langle O\rangle_0}$$



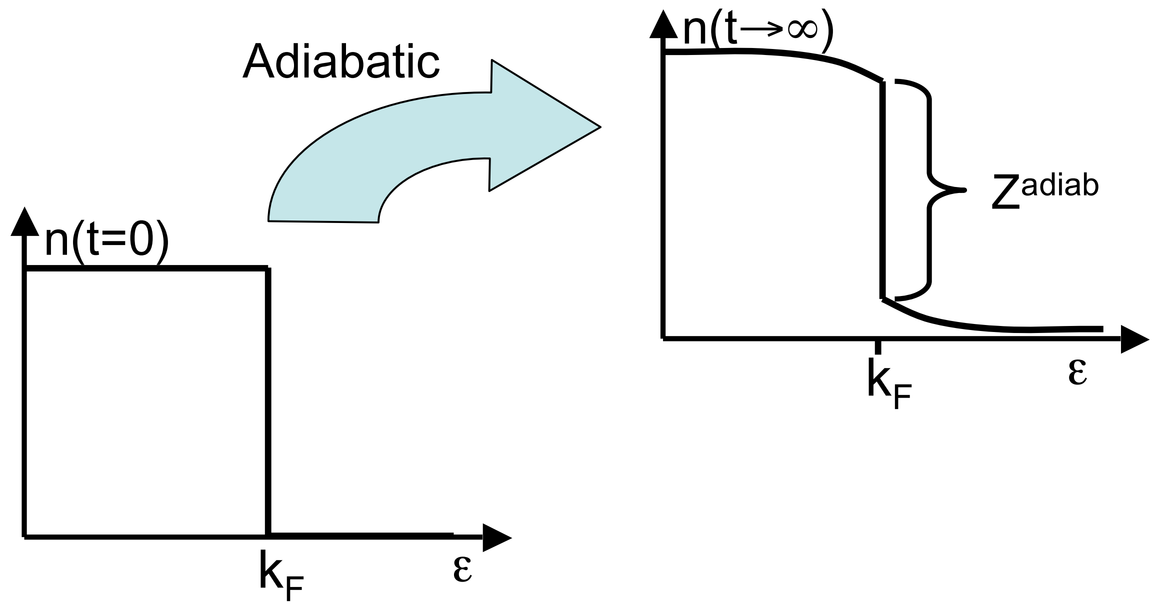

⇒ Crossover timescale nonperturbative (exponentially large) due to RG flow

[ C. Tomaras, S. K., Eur. Phys. Lett. 93, 47011 (2011) ]

# Sudden Interaction Fermi Liquid

**Landau Fermi liquid theory:**

**Adiabatic switching on of interaction**
**→ 1 to 1 correspondence between physical electrons and quasiparticles**

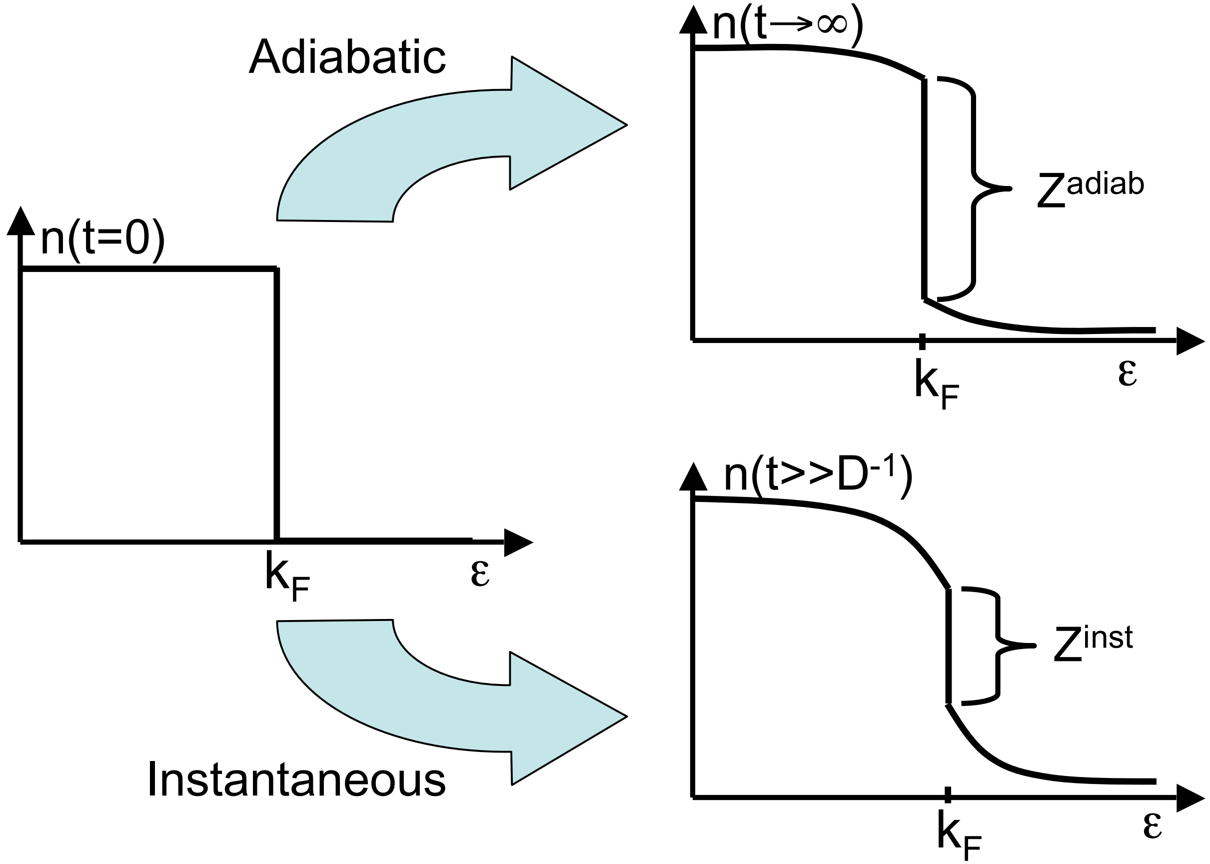



**What happens for sudden switching?**

**Initial Hamiltonian:** $$H_i = \sum_{k,\alpha} \varepsilon_k c^\dagger_{k\alpha} c_{k\alpha}$$

**Observable** $$O = \begin{cases} n_k & \text{for} \quad k > k_F \\ 1 - n_k & \text{for} \quad k < k_F \end{cases}$$

# Calculation to order $U^2$



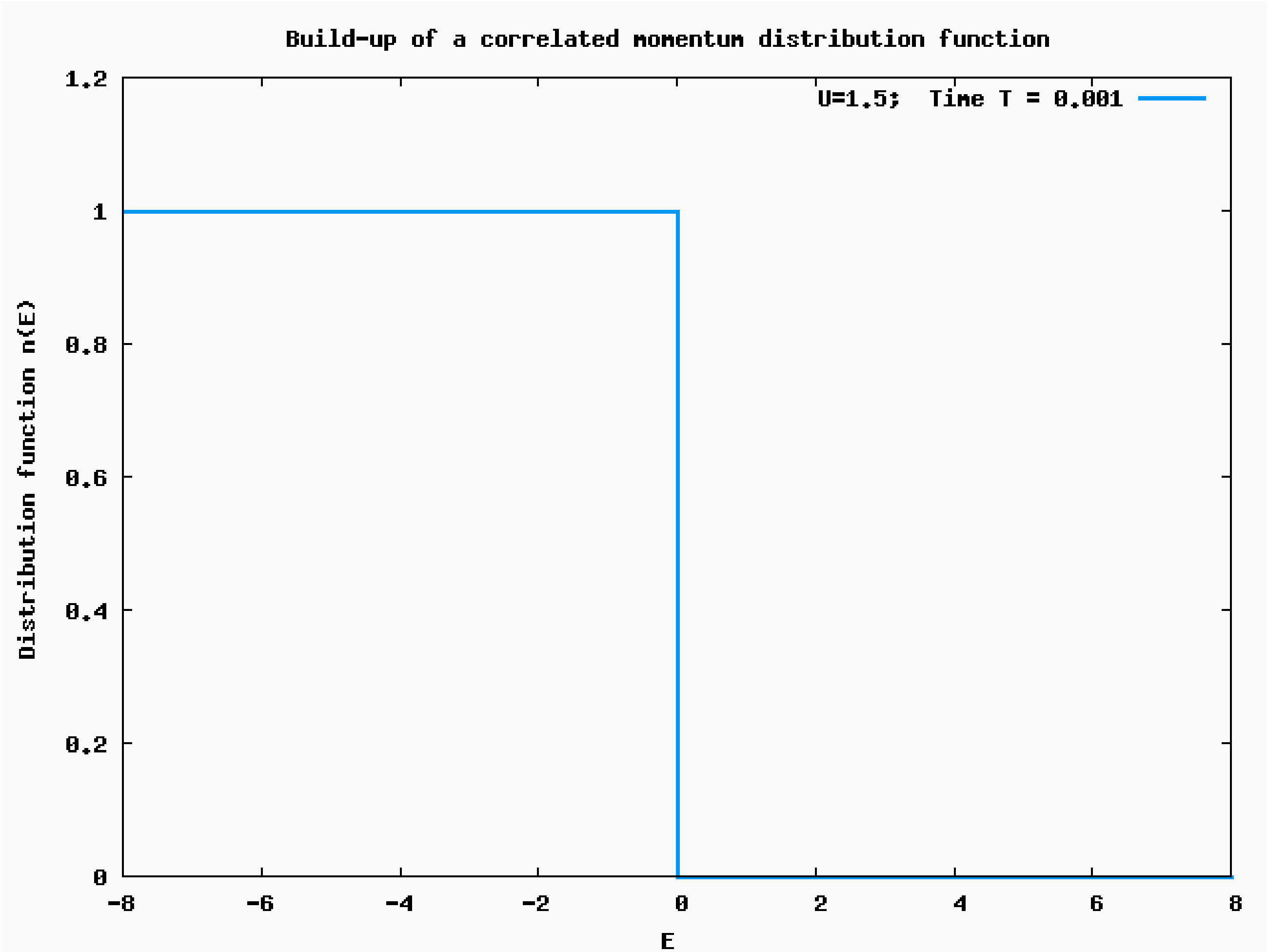

**Sudden switching looks like T=0 Fermi liquid with "wrong" quasiparticle residue:**

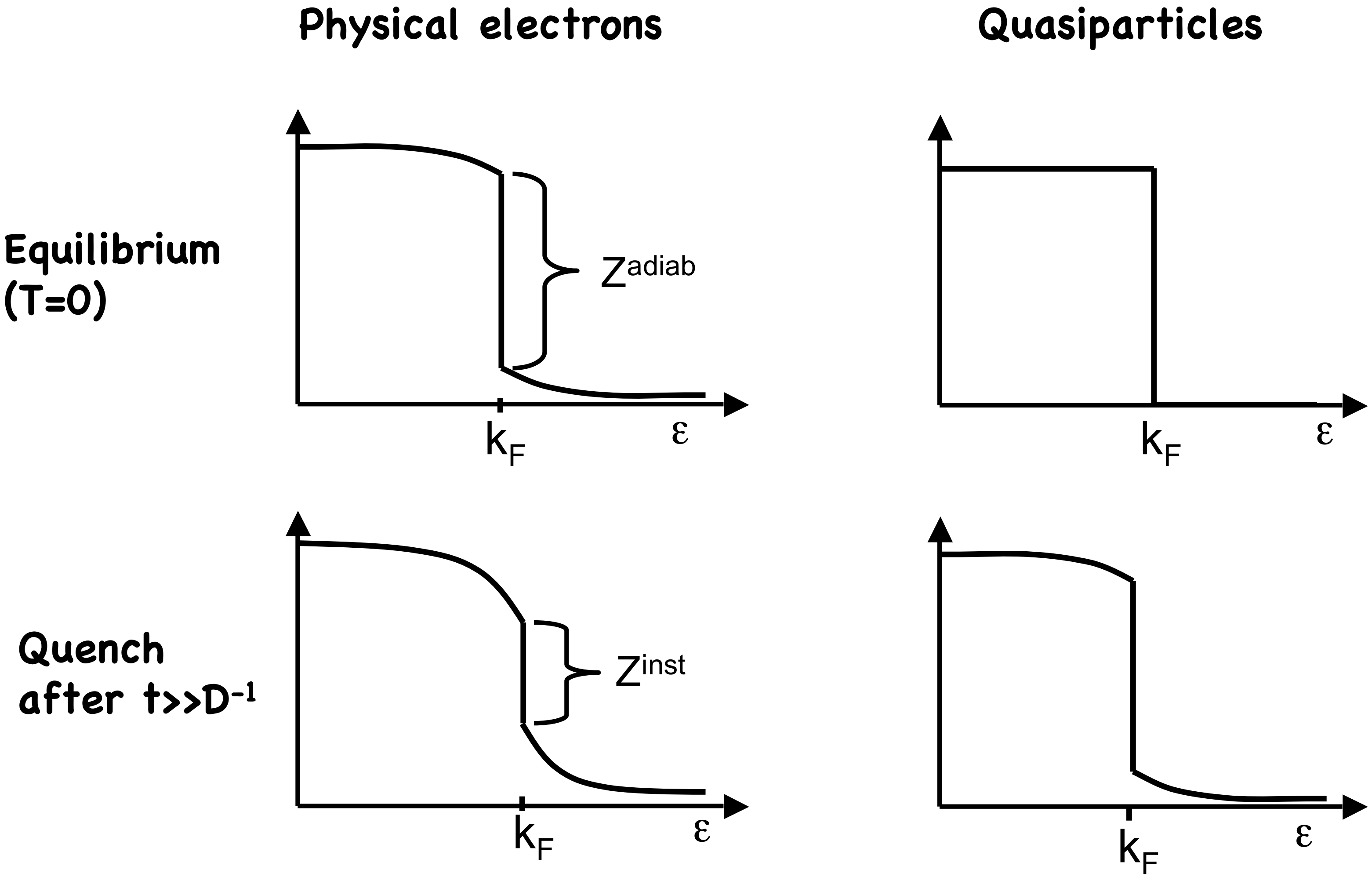
$$1-Z^{inst} = 2\,(1-Z^{adiab})$$

M. Möckel and S. K., Phys. Rev. Lett. 100, 175702 (2008);
Ann. Phys. 324, 2146 (2009)

Hubbard model in d≥2 dimensions:

$$H = \sum_{k,\alpha} \varepsilon_k c^\dagger_{k\alpha} c_{k\alpha} + U\,\Theta(t) \sum_i \left(n_{i\uparrow} - \frac{1}{2}\right)\left(n_{i\downarrow} - \frac{1}{2}\right)$$






**Nonthermal distribution function**

⇒ unstable under Quantum Boltzmann equation dynamics

⇒ thermalization on timescale $t \propto U^{-4}$

# Sudden quench (generic weak interaction g)

Time scale

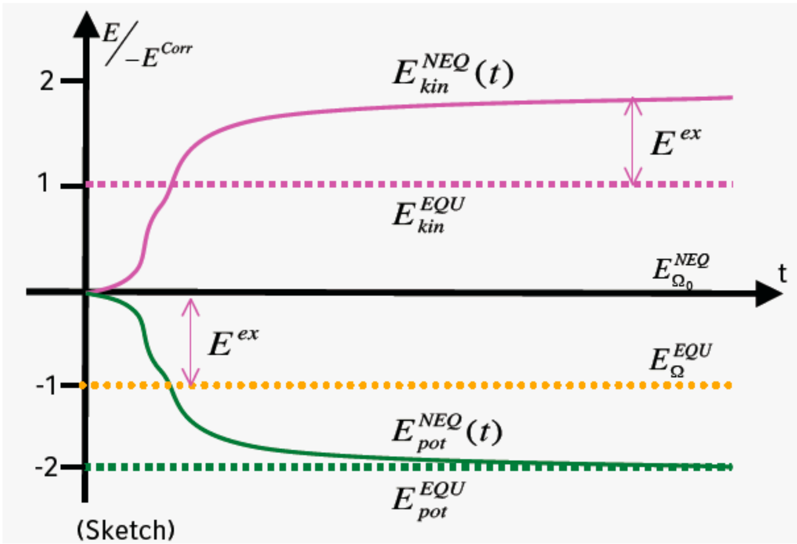
$t \propto D^{-1}$ • Formation of quasiparticles

$D^{-1} \ll t \ll g^{-2}$ • T=0 Fermi liquid with "wrong" quasiparticle residue:

$$1\text{-}Z^{inst} = 2\,(1\text{-}Z^{adiab})$$

$D^{-1} \ll t \ll g^{-4}$ • Quasi-steady state

• Prethermalization (Berges et al. 2004)




$t \propto g^{-4}$ • Quantum Boltzmann equation (quasiparticles explore available phase space):

Thermalization with $T_{eff} \propto g$

# Numerical Studies

M. Eckstein, M. Kollar and P. Werner, Phys. Rev. Lett. 103, 056403 (2009); Phys. Rev. B 81, 115131 (2010)

Non-equilibrium DMFT with real time QMC for interaction quench in half-filled Hubbard model

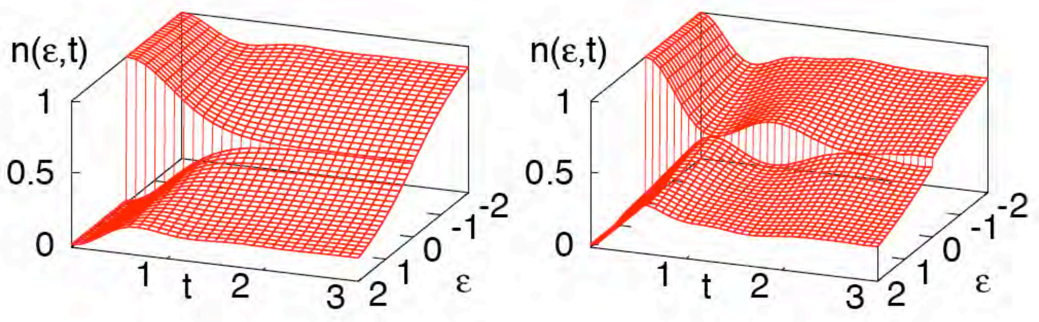
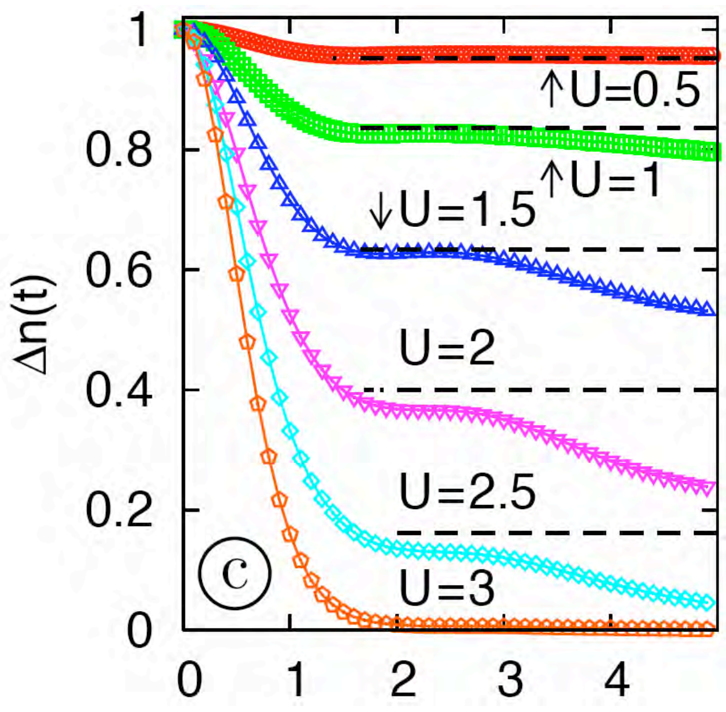



FIG. 1: Momentum distribution $n(\epsilon_{\boldsymbol{k}}, t)$ for quenches from $U = 0$ to $U = 3$ (left panel) and $U = 5$ (right panel).

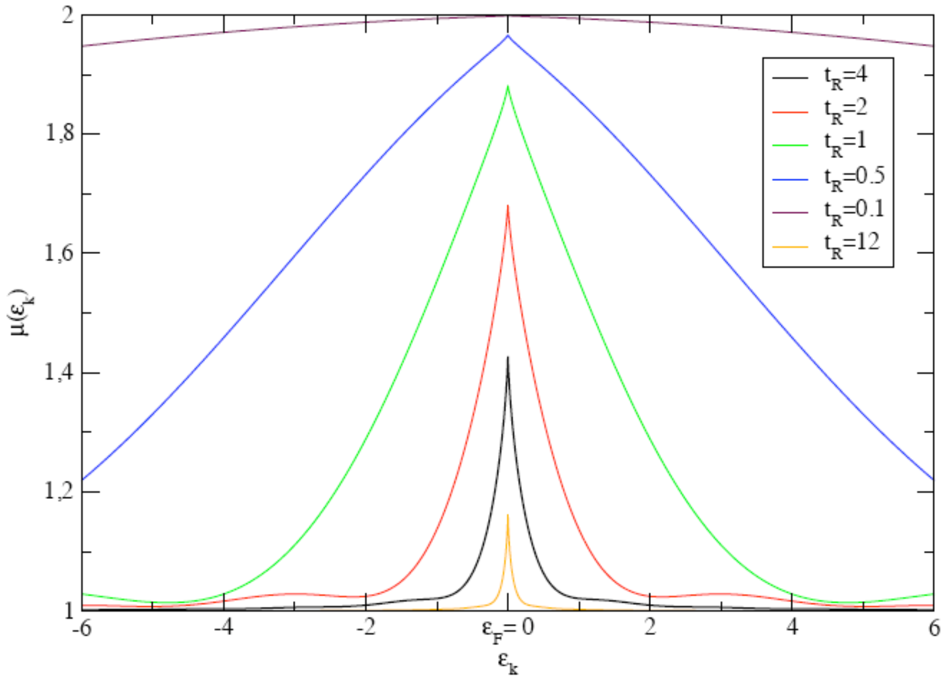


# Crossover from adiabatic to instantaneous quenching

**M. Moeckel and S. K., New J. Phys. 12, 055016 (2010)**

**Linear ramping on timescale $t_R$:**

$$U(t) = U \begin{cases} 0 & t \leq 0 \\ t/t_R & 0 < t < t_R \\ 1 & t > t_R \end{cases}$$



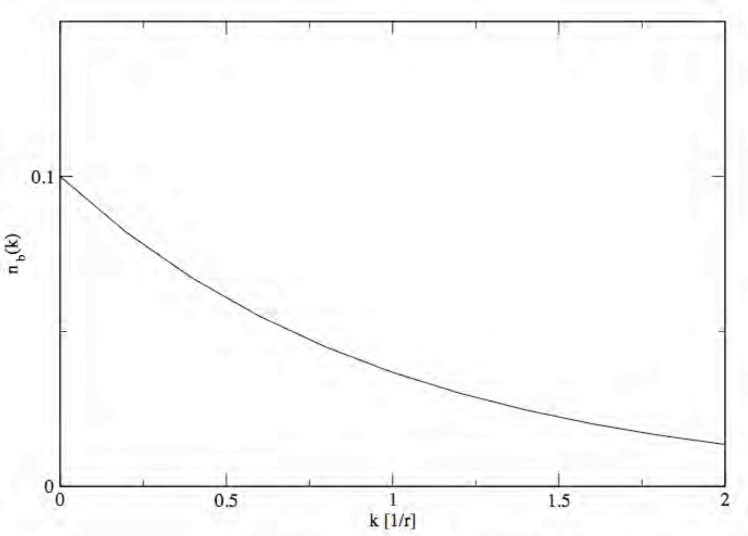

**Uniform approach to equilibrium also at Fermi surface!**

# Interaction Quench in a Luttinger Liquid

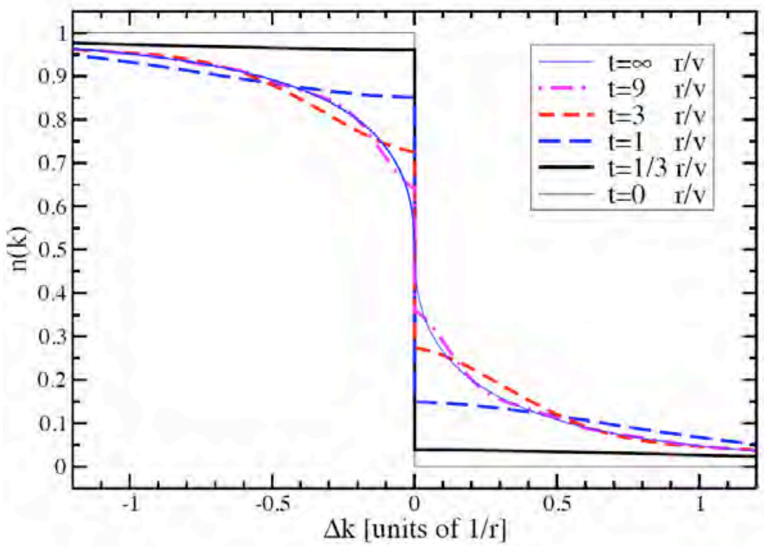
M. Cazalilla, Phys. Rev. Lett. 97 (2006):

Sudden forward scattering in 1d Fermi gas
$\Rightarrow$ Exact solution via Bogoliubov transformation to free bosons

No time evolution for free bosons (quasiparticles)




Time evolution for physical fermions (G. Uhrig, Phys. Rev. A 80)




Quasiparticle momentum distribution function time-invariant due to lack of quasiparticle interaction.

However, even Boltzmann dynamics from 2-quasiparticle interaction is generically ineffective in 1d!
(No time evolution beyond prethermalized regime.)

**Prethermalization:** Momentum-averaged quantities time-indep., but distribution over momentum modes non-thermal

Systems with well-defined quasiparticles

- Weak quenches for d>1:
  - Fast prethermalization
  - Thermalization with Boltzmann dynamics

- Weak quenches for d=1:
  - Fast prethermalization
  - Long time limit? (generically no 2-particle Boltzmann dynamics, constraints due to integrability, many-body localization)

No quasiparticle description possible

- Quench "to" Mott-Hubbard transition ($d=\infty$)
- Strong-coupling quantum critical points