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Workshop on Integrability and its Breaking in Strongly Correlated and Disordered Systems

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Understanding Quantum Quenches through a Numerical Renormalization Group

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BROOKHAVEN NATIONAL LABORATORY

a passion for discovery

Office of Science U.S. DEPARTMENT OF ENERGY

Renormalization Group Improved Truncated Spectrum Approach (RGTSA)

A combined numerical/analytical technique to study strongly correlated systems in 1 and 2 dimensions.

This has been shown able to compute equilibrium quantities

- **spectrum**
- **correlation functions/matrix elements**

in a number of cases

- **perturbed minimal conformal models/sine-Gordon**

  RMK and Y. Adamov, PRL 98, 147205 (2007)
  G. Brandino, RMK, and G. Mussardo, J. Stat. Mech. T&E P07013 (2010)

- **semi-conducting carbon nanotubes**

  RMK, PRL 106, 136805 (2011)

We now show that it can be used to study quantum quenches.

Outline

1) Overview of the numerical renormalization group (NRG) as applied to continuum field theories

2) Applying the NRG to study quenches in Z_2 systems
   - address the connection thermalization and integrability /non-integrability (M. Rigol et al. Nature (2007))

3) Applying the NRG to study quenches in trapped 1D-Bose Gases

Overview of Truncated Spectrum Approach (TSA) for One Dimensional Systems

Basic idea is to study a known (i.e. integrable or conformal) continuum system together with some perturbation:

$$H = \underbrace{H_{known}}_{\text{i.e. critical quantum Ising}} + \underbrace{\Phi_{perturbation}}_{\text{magnetic field}}$$

Consider the model on a finite sized ring of circumference, R

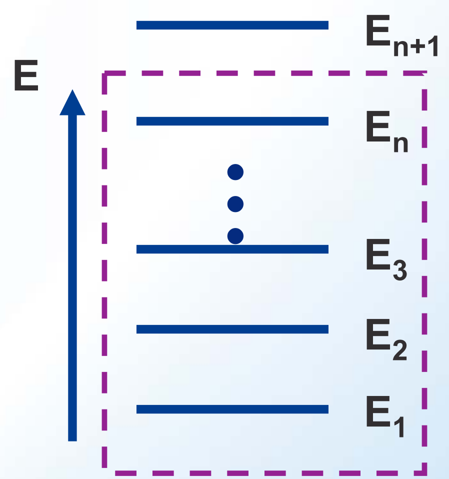
Spectrum of H_{known} then becomes discrete:

Input of strongly correlated information in the form of matrix elements:

$$\Phi_{ij} = \langle i | \Phi_{perturbation} | j \rangle \Big|_{\mathbf{H}_{\mathbf{Known}}}$$

Truncate Hilbert space, making it finite dimensional. This allows one to write full Hamiltonian as a finite dimensional matrix

$$\mathbf{H} = \begin{bmatrix} E_1 & \Phi_{12} & \cdots & \cdots & \Phi_{1n} \\ \Phi_{21} & E_2 & & & \vdots \\ \vdots & & \ddots & & \vdots \\ & & & E_{n-1} & \Phi_{n-1n} \\ \Phi_{n1} & \cdots & \cdots & \Phi_{nn-1} & E_n \end{bmatrix}$$



Diagonalize H numerically and extract spectrum

Example of the TSA: Quantum Critical Ising Chain in a Magnetic Field

Hamiltonian:

$$H = \underbrace{-J\sum_i \sigma_i^z \sigma_{i+1}^z + \sigma_i^x}_{\mathbf{H}_{\mathbf{known}}} \underbrace{- h\sum_i \sigma_i^z}_{\mathbf{\Phi}_{\mathbf{pert}}}$$

$\Downarrow$ **continuum limit**

Model is exactly solvable (A. Zamolodchikov) and has a spectrum with 8 excitations

$$H = \int dx(\underbrace{\psi\partial_x\psi + \bar{\psi}\partial_x\bar{\psi}}_{\mathbf{H}_{\mathbf{known}}} \underbrace{- h\sigma}_{\mathbf{\Phi}_{\mathbf{pert}}})$$

TSA Results keeping 39 states

Yurov and Zamolodchikov, 1991

Ratios of spectral gaps

	TSA	Exact (infinite volume)
Δ_2/Δ_1	1.61 ± .01	2 cos(π/5) = 1.618...
Δ_3/Δ_1	1.98 ± .02	2 cos(π/30) = 1.989...
Δ_4/Δ_1	2.43 ± .04	4 cos(π/5)cos(7π/30) = 2.405...
Δ_5/Δ_1	3.03 ± .07	4 cos(π/5)cos(2π/15) = 2.956...

Equivalent Exact Diagonalization Computation $\longrightarrow$ Chain with only five sites

Why does this work so well?

Two reasons: 1) Finite size errors are exponentially suppressed
2) Perturbation is highly relevant and Hilbert space is relatively simple

But there are problems:

1) With less relevant perturbations or more complicated Hilbert spaces (i.e. 1D atomic Bose gases) convergence of spectrum is slower

2) Matrix elements generically see slower convergence

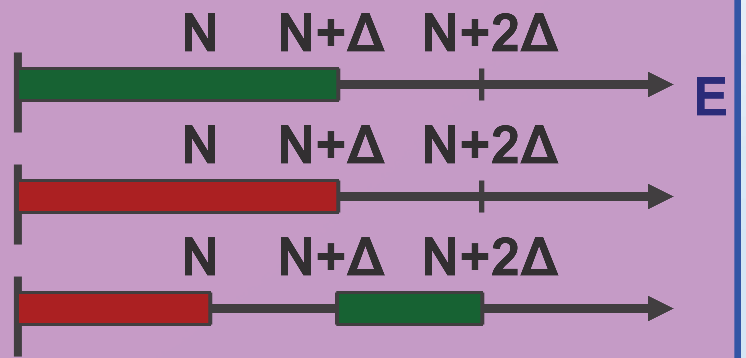
Using a Numerical Renormalization Group to Improve Results

RMK and Y. Adamov, PRL 98, 147205 (2007)
G. Brandino, RMK, and G. Mussardo, J. Stat. Mech. T&E P07013 (2010)

The TSA as is only can treat simple theories → Convergence issues surrounding truncation

However we have handled truncation in the crudest possible fashion: there is at least one way to improve on this → Numerical Renormalization Group (in the same spirit K. Wilson used it to study the Kondo problem)

NRG Recipe:



1) Take first N+Δ states of the theory
2) Compute the Hamiltonian and numerically diagonalize
3) Form a new basis of states using first N eigenstates (in red) plus next Δ states in original basis
4) Recompute Hamiltonian and numerically diagonalize
5) Repeat

How NRG works in quenches

Take prequench state, typically an eigenstate of a prequench Hamiltonian and express in the Hilbert space of H_{known}:

$$|pre - quench\rangle = \sum_{s \in H_{known}} c_s |s\rangle$$

Because post-quench states are also expressed in terms of states of H_{known},

$$|post - quench\rangle = \sum_{s \in H_{known}} d_s |s\rangle$$

we can expand one in terms of the other. As part of this we need, however, to accurately determine the post-quench spectrum over a wide range of energies. We can do so using a sweeping procedure (akin to the finite vol. DMRG algorithm).

G. Brandino, RMK, and G. Mussardo, J. Stat. Mech. T&E P07013 (2010

Quenches in Systems with Z_2 Symmetries

There has been considerable work on quenches in Ising systems:

D. Fioretto, G. Mussardo, New J. Phys. 12, 055015 (2010)
D. Rossini, S. Suzuki, G. Mussardo, G. Santoro, A. Silva, PRB 82, 144302 (2010)
D. Rossini, A. Silva, G. Mussardo, G. Santoro, PRL 102, 127204 (2009)
P. Calabrese and J. Cardy, PRL 96, 136801 (2006)
P. Calabrese, F.H.L. Essler, M. Fagotti, arXiv: 1104.0154

We will show that there for certain types of quenches, thermalization happens or does not happen independent of the underlying integrability/non-integrability of the model.

Number of examples of this general type of phenomena:

C. Gogolin, M. Mueller, J. Eisert, Phys. Rev. Lett. 106, 040401 (2011)
M. Banuls, J. Cirac, M. Hastings, Phys. Rev. Lett. 106, 050405 (2011)
C. Kollath, A. M. Lauchli, E. Altman, Phys. Rev. Lett. 98, 180601 (2007)

In our case this arises because of how the Z_2 symmetry determines the Hilbert space of the model.

Hilbert Space in Ordered and Disordered Phase

The Hilbert space of a Z_2 model always has two sectors:

Sector even under Z_2
Sector odd under Z_2

Ordered Phase
(with spontaneously broken symmetry): Two sectors are degenerate

Disordered Phase: Even and odd sectors are not degenerate

There are also 'spin' operators in the theory that connect the two sectors.

Example: Quantum Ising Hilbert Space

Hamiltonian: $$H = \int dx \psi \partial_x \psi + \bar{\psi}\partial_x \bar{\psi} + im\bar{\psi}\psi$$

Here the two sectors are known as the Ramond and Neveu-Schwarz:

free fermionic modes

even $$|k_1, k_2, \cdots, k_N\rangle_{NS} = a^\dagger_{k_1} \cdots a^\dagger_{k_N} |0\rangle_{NS}; \quad k_1, \cdots, k_N = \frac{2\pi n}{R}, \quad n \in Z + 1/2;$$

odd $$|l_1, l_2, \cdots, l_N\rangle_R = a^\dagger_{l_1} \cdots a^\dagger_{l_N} |0\rangle_R; \quad l_1, \cdots, l_N = \frac{2\pi n}{R}, \quad n \in Z;$$

$$m > 0 : \quad \mathrm{NS} \; - \; \text{states with even N} \quad \text{and} \quad \mathrm{R} \; - \; \text{states with N even}$$

in ordered phase sectors have states with the same number of particles

$$m < 0; \quad \mathrm{NS} \; - \; \text{states with even N} \quad \text{and} \quad \mathrm{R} \; - \; \text{states with N odd.}$$

in disordered phase sectors have states with the differing number of particles

$$\langle NS|\sigma(0)|R\rangle$$

only non-zero matrix elements connecting the sectors

Example: Tri-critical Ising Hilbert Space

Hamiltonian: $\mathcal{H} = -\sum_i [S_i^z S_{i+1}^z - D(S_i^z)^2 + H_{\mathrm{T}} S_i^x]$ *S_i are spin-1 operators*

In the scaling limit this model has a richer set of operators than Ising:

identity plus three non-trivial even (energy-like) operators: $I, \epsilon, \epsilon', \epsilon''$

two odd (spin-like) operators: σ, σ'

Correspondingly there are a richer set of integrable and non-integrable perturbations of the critical theory.

The even and odd sectors of the Hilbert space track the operators (there is a sector per operator).

The two spin operators connect the even and odd parts.

Ordered Phase to Disordered Phase Quench with Z_2 Preserved

Typical pre-quench state (with spontaneous symmetry breaking):

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|e\rangle \pm |o\rangle),$$

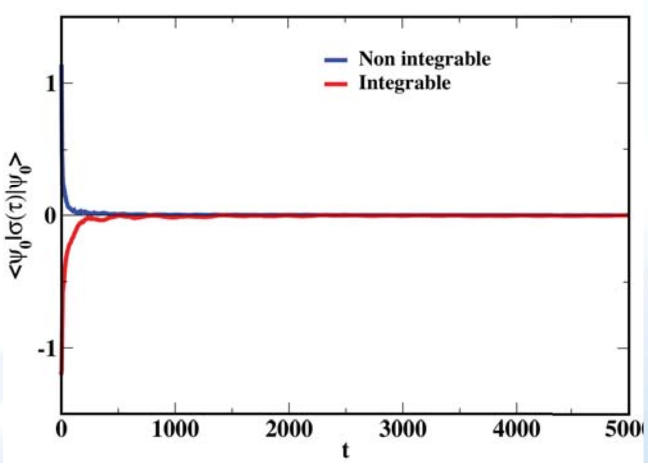
Post-quench this state becomes:

$$|+\rangle_{ordered} = \sqrt{\frac{1}{2}}\left(\sum_i \alpha_i |e\rangle_{i,disordered} + \sum_i \beta_i |o\rangle_{i,disordered}\right)$$

Long time evolution post-quench:

$$_{ordered}\langle +|\sigma(t)|+\rangle_{ordered} = \frac{1}{2}\sum_{ij}\left({}_{disordered}\langle e_i|\sigma(0)|o_j\rangle_{disordered} e^{i(E_i^{even}-E_j^{odd})t} + \text{h.c.}\right) \longrightarrow 0$$

This thermalization happens independent of integrability:



two different energy perturbations of tri-critical Ising

Disordered Phase to Ordered Phase Quench with Z_2 Preserved

Typical pre-quench state: $|e\rangle_{disordered}$

Post-quench expansion: $|e\rangle_{disordered} = \sum_i \alpha_i |e_i\rangle_{ordered}$

Expectation value of spin operator is zero (so state does not thermalize) regardless of integrability/non -integrability of theory:

$$_{disordered}\langle e|\sigma(t)|e\rangle_{disordered} = 0$$

Quench with Z_2 Broken

We consider an action where the spin operator is a perturbation:

$$H = H_{critical\ Ising} + h \int dx \sigma$$

Pre-quench state: $\alpha_-|e\rangle - \beta_-|o\rangle$ low energy eigenstate.

Post quench expansion of pre-quench state under quench h → -h:

dominant contributions:

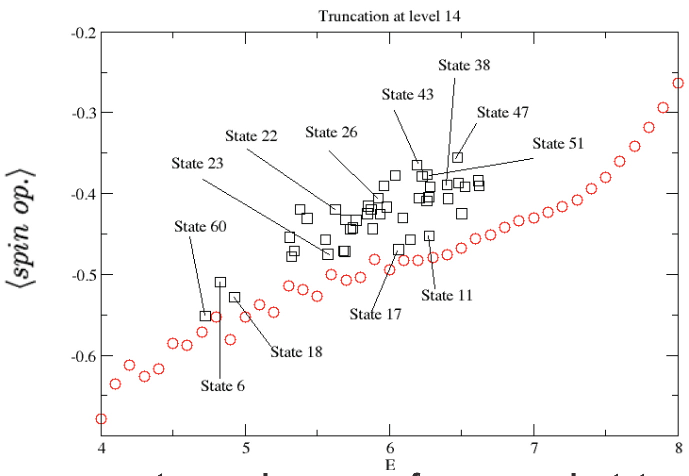
$$
\begin{aligned}
(\alpha_-|e\rangle - \beta_-|o\rangle)_{\mathrm{pre-quench}} = & \sum_{\mathrm{i \in post-quench\ high\ energy\ states}} c_i(\alpha_{i+}|e_i\rangle - \beta_{i+}|o_i\rangle) \\
& + \sum_{\mathrm{i \in post-quench\ low\ energy\ states}} c_i(\alpha_{i+}|e_i\rangle + \beta_{i+}|o_i\rangle) \\
& + \sum_{i \in \mathrm{states\ with\ middling\ energy}} c_i|i\rangle.
\end{aligned}
$$

This will lead to broad or bi-modal distributions of pre-quench states in terms of the post-quench ones, and so a general lack of thermalization (independent of the integrability/non-integrability of the theory).

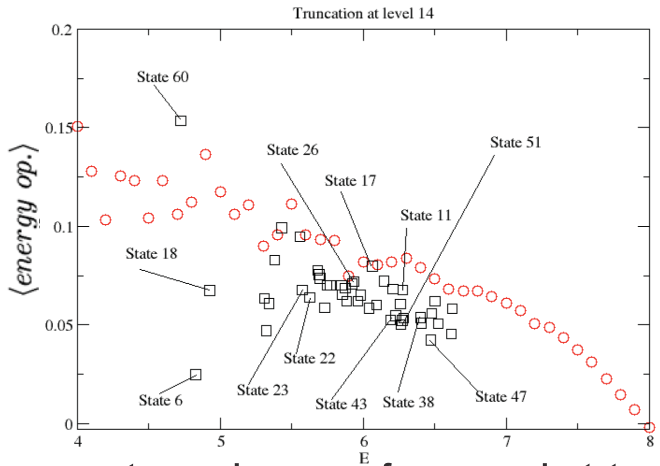
Example: Tri-critical Ising (quench h ⟶ -h)

$$H = H_{tri-critical\ Ising} + h \int dx\ \sigma$$

model is non-integrable



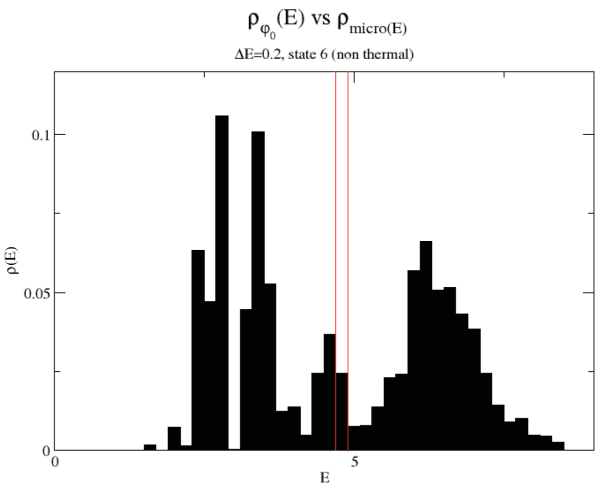
post-quench energy of pre-quench state

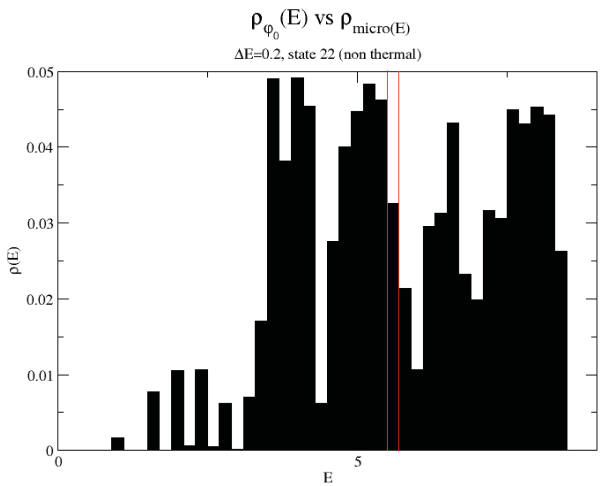


post-quench energy of pre-quench state

O: microcanonical ensemble

□: diagonal ensemble





distribution of non-thermalizing states over post-quench basis: $|pre-quench\rangle = \sum_E c_E |E, post-quench\rangle$

Quenches of 1D Bose Gases in Traps

The Lieb-Liniger model with a one-body potential:

$$H = -\sum_{j=1}^{N} \frac{\partial^2}{\partial z_j^2} \underbrace{+ 2c \sum_{1 \geq j < k} \delta(z_j - z_k)}_{H_{known}} + \underbrace{\sum_j V(z_j)}_{\Phi_{pert}}$$

we will work at unit density and so γ=c; m=1/2

Motivation: T. Kinoshita, T. Wenger, and D. Weiss, Nature 440, 900 (2006)
M. Rigol, V. Dunjko, V. Yurovsky, and M. Olshanii, PRL 98, 050405 (2007)

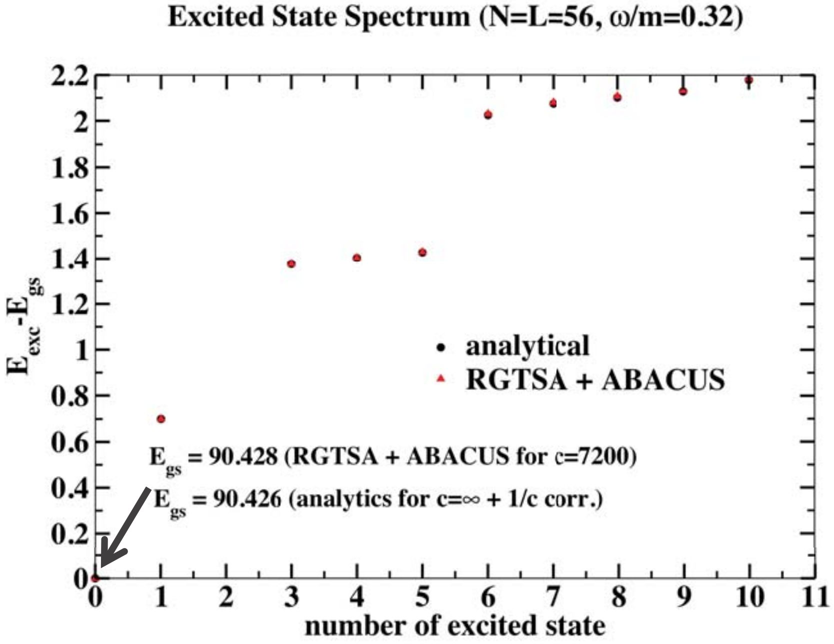
NRG on a non-relativistic system with an N-particle ground state is much more difficult numerically. The operational size of the Hilbert space is much larger. We thus equipped the NRG with a "variational metric" to allow it to better find it's way in this Hilbert space.

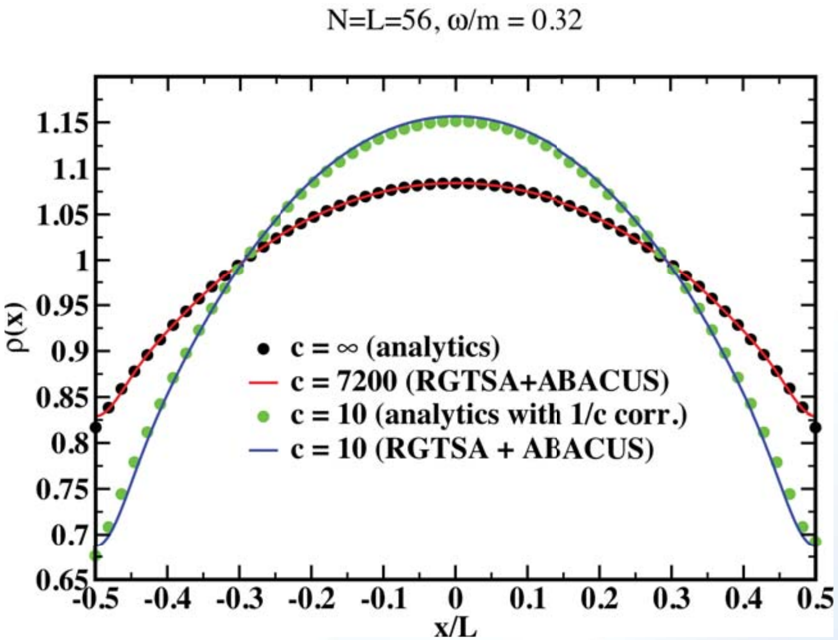
Matrix elements are handled with the algebraic Bethe ansatz (ABACUS).

Type of quench we will consider here:

$$V(z) = \frac{1}{4}\omega^2 z^2 \longrightarrow 0$$

Benchmarking Equilibrium Properties of Bose Gas in Trap



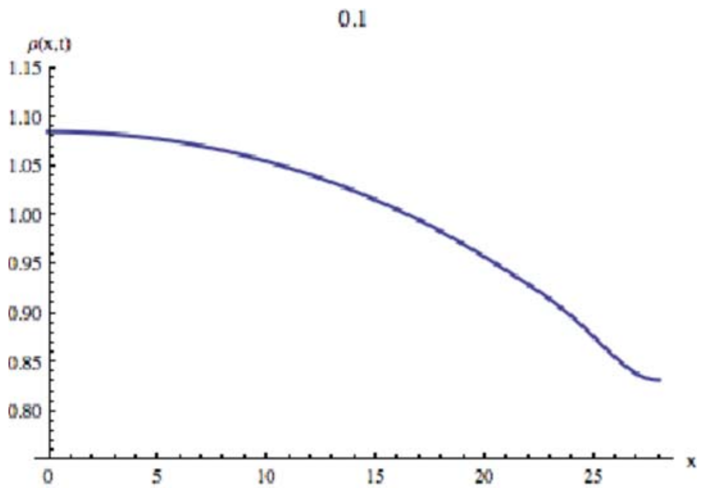


ground state and excited state energies can be accurately predicted

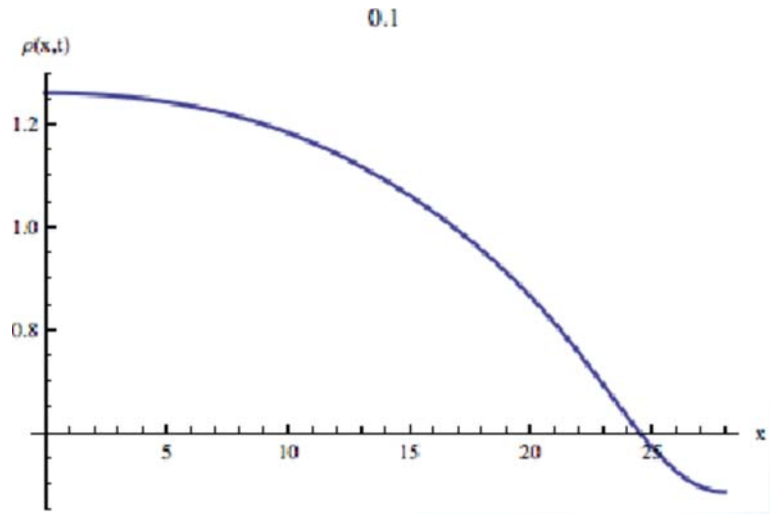
as can density profile in trap

Time Evolution of Gas after Release of Trap

N=L=56, ω/m=0.32



c = 7200



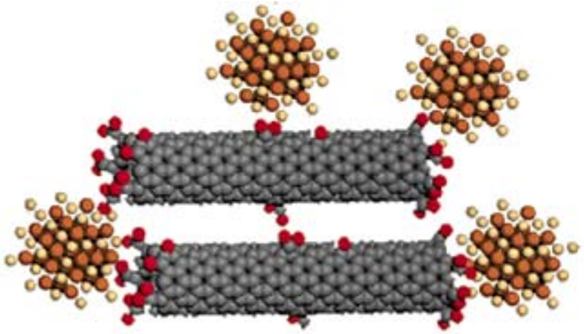
c = 1

Next task: Determine momentum distribution function and compare against thermal counterpart

After that: $V(z) = \frac{1}{4}\omega^2 z^2 \longrightarrow a\cos(\Omega z)$

Conclusions

The NRG can be used to study quenches in a variety of systems.



Interested in particular in studying exciton dynamics in nanotubes functionalized with quantum dots.

Quenches in Z_2 symmetric systems:

For models with a Z_2 symmetry, you can use the symmetry to classify a set of quenches that thermalize/do not thermalize, independent of the model's underlying integrability or lack thereof.

Quenches in Trapped 1D Bose Gases:

We can handle equilibrium properties – quench dynamics to follow shortly.