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# Impurity dynamics in one-dimensional fermion gas

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# Impurity dynamics in one-dimensional quantum gas

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**Swiss National Foundation is acknowledged**

Theory(ex):

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Thierry Giamarchi Geneva University

Theory(now):

Charles Mathy Harvard University

Eugene Demler Harvard University

Experiment:

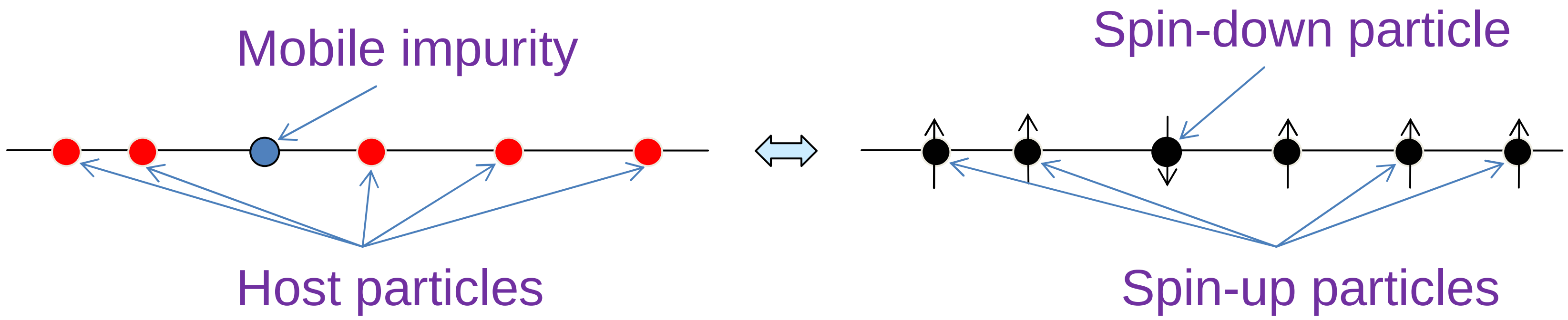
Elmar Haller,

Manfrend Mark,

Hanns-Christoph Naegerl

Innsbruck University

# Mobile impurity = itinerant ferromagnet

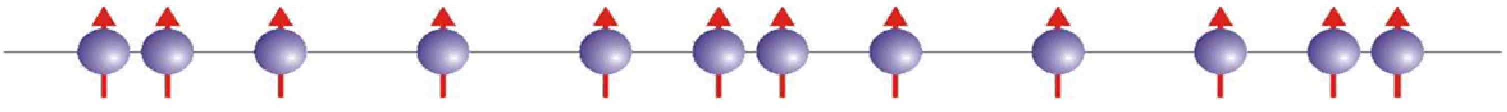


# Excitations in one-dimensional itinerant ferromagnets

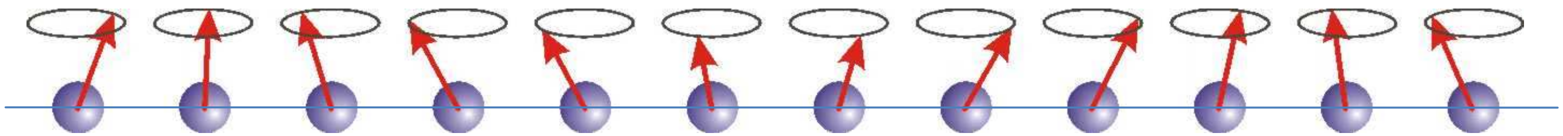
Take bosons or fermions in 1D carrying spin (1/2 for simplicity)

Consider ferromagnetic state (which is not necessarily ground state)

Excitations: longitudinal spin waves (linear dispersion = plasmons)



Excitations: transverse spin waves (quadratic dispersion = magnons)



$$\boxed{\varepsilon(q) = \frac{q^2}{2m_*} \qquad q \to 0}$$

Effective mass

# δ interacting bosons/fermions: Bethe-Ansatz solvable

(iso)spin *1/2* bosons/fermions, spin-independent interaction:

$$H = \sum_{j=1}^{N} \frac{p_j^2}{2m} + \sum_{i<j} U(x_i - x_j)$$

$$U(x) = \tilde{U}(x) + \alpha\delta(x)$$

Bethe-Ansatz solvable when $\tilde{U}(x) = 0$ Yang - Gaudin model

➡ Wave functions, spectrum and thermodynamics are known

C. N. Yang, PRL **19**, 1312 (1967);
M. Gaudin, Phys. Lett. A **24**, 55 (1967)

Dimensionless coupling: $\boxed{\gamma = \frac{m\alpha}{\hbar^2\rho}}$

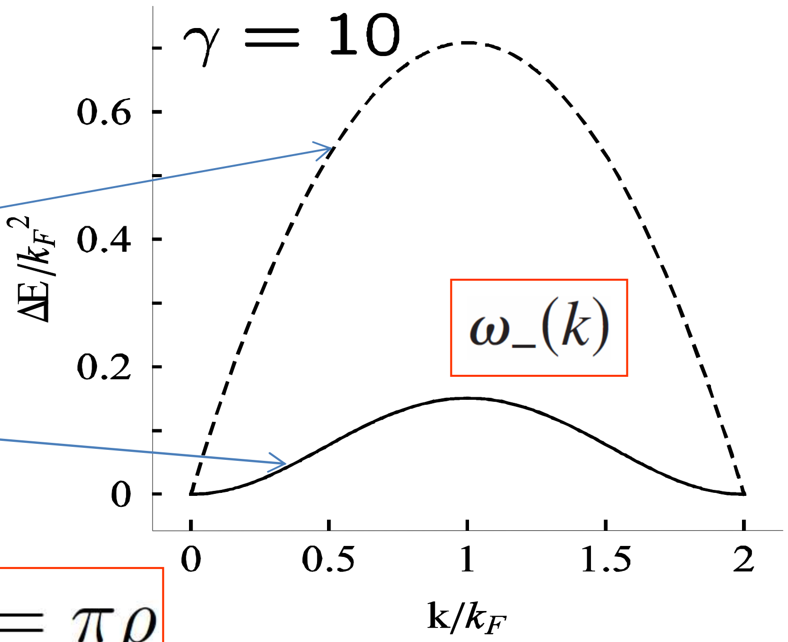
Dispersion of spinless boson gas:

$$\omega_-(k) = vk \qquad k \to 0$$

Dispersion of bosons with spin:

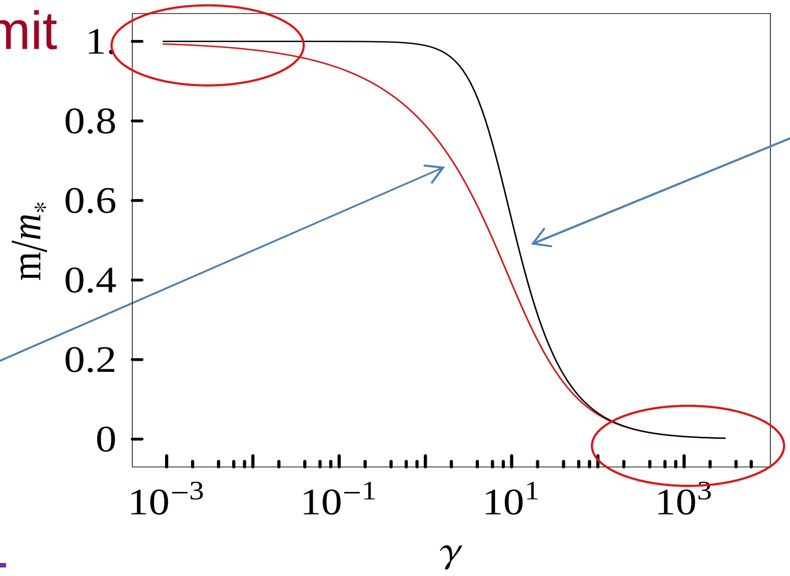
$$\omega_-(k) = \frac{k^2}{2m_*} \qquad k \to 0$$

$\boxed{k_F = \pi\rho}$



# Effective mass in the Yang-Gaudin model

Bogoliubov limit (bosons)/ no name (fermions)




This plot is for fermionic Yang-Gaudin model

J. B. McGuire, J. Math. Phys. **6**, 432 (1965)

Tonks-Girardeau limit

This plot is for bosonic Yang-Gaudin model

J. N. Fuchs et. al., PRL **95**, 150402 (2005)

Tonks-Girardeau (TG) limit: short-range potential $\gamma \to \infty$

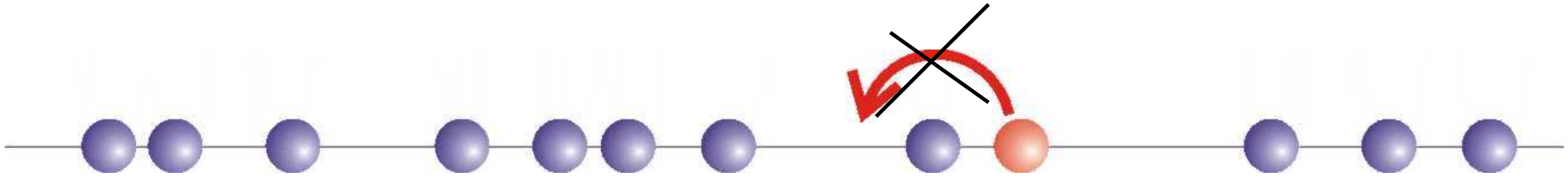
Effective mass diverges in the TG limit: $m_* = m\frac{3\gamma}{2\pi^2}, \qquad \gamma \to \infty$

(it costs no energy to flip a spin when $\gamma = \infty$)

# Limit of infinite repulsion (TG): logarithmic diffusion

$$G_\perp(x,t) = \langle \Uparrow | s_+(x,t) s_-(0,0) | \Uparrow \rangle$$

Spin flip = change particle color



Infinite repulsion: spin-down (red) particle cannot exchange the position with its neighbors

Results:

$$G_\perp(x,t) \simeq \frac{1}{\sqrt{\ln(t/t_{\rm F})}} \exp\left\{ -\frac{1}{K} \frac{(\pi\rho x)^2}{2\ln(t/t_{\rm F})} \right\}$$

as $x, t \to \infty$ or $x = 0,\ t \to \infty$

"logarithmic diffusion"

Here $t_{\rm F} = \hbar / E_{\rm F}$, $\quad E_{\rm F} = \dfrac{\hbar^2}{2m}(\pi\rho)^2$

Luttinger parameter $K \sim 1$

$\tilde{U}(x) = 0 \Leftrightarrow K = 1$

M. B. Zvonarev, V. V. Cheianov, T. Giamarchi, PRL **99**, 240404 (2007)

# How to get logarithmic diffusion: bosonization reminder

Consider equation of motion for the 1D fluid: $\partial_t^2 \phi - v^2 \partial_x^2 \phi = 0$

$\phi(x)$ gives deviation of the particle density $\rho(x)$ from average value, $\rho$

$$\boxed{\rho(x) = \rho - \frac{1}{\pi}\partial_x \phi(x)}$$

Quantization: $\theta(x)$ such that $[\partial_x \theta(x), \phi(y)] = -i\pi\delta(x-y)$ leads us to

$$\mathcal{H}(x) = \frac{v}{2\pi} : K(\partial_x \theta)^2 + \frac{1}{K}(\partial_x \phi)^2 :$$

Luttinger parameter $K$

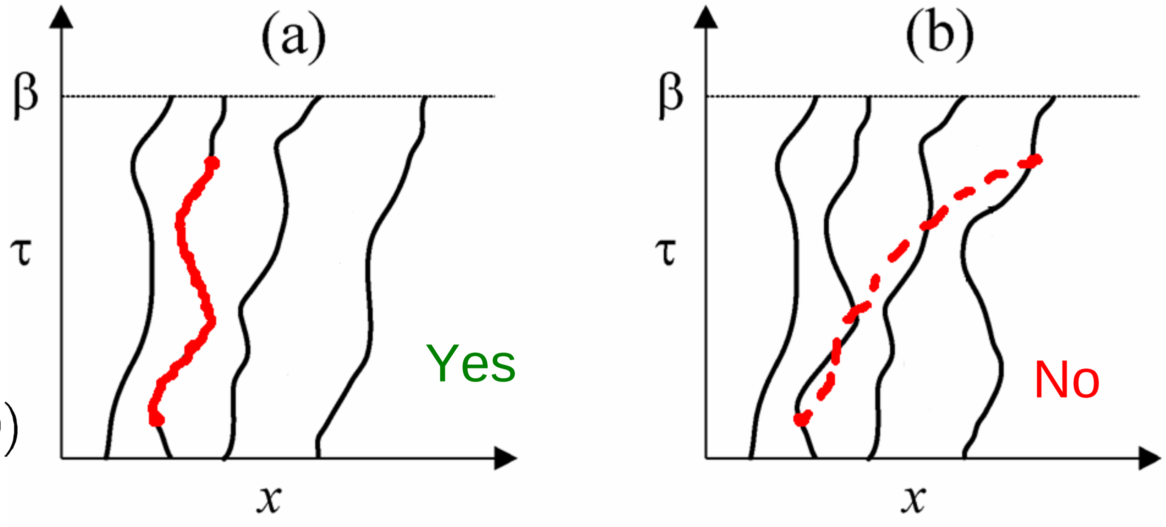
Compressibility $\kappa$

$$\Longrightarrow \quad K = \pi v \kappa, \quad \kappa = \left(\frac{\partial \rho}{\partial \mu}\right)_L$$

Boson creation operator: $\boxed{\psi^\dagger(x) \sim \sqrt{\rho}\, e^{-i\theta(x)}}$

# Solution: from spinful problem to spinless

Word lines of the particles (path integral formalism):
Solid curves: spin up come from $\langle \Uparrow | \cdots | \Uparrow \rangle$
Dashed curve: spin down comes from $s_+(x,t)s_-(0,0)$




Solid and dashed curves should not cross each other! – the only effect of spin

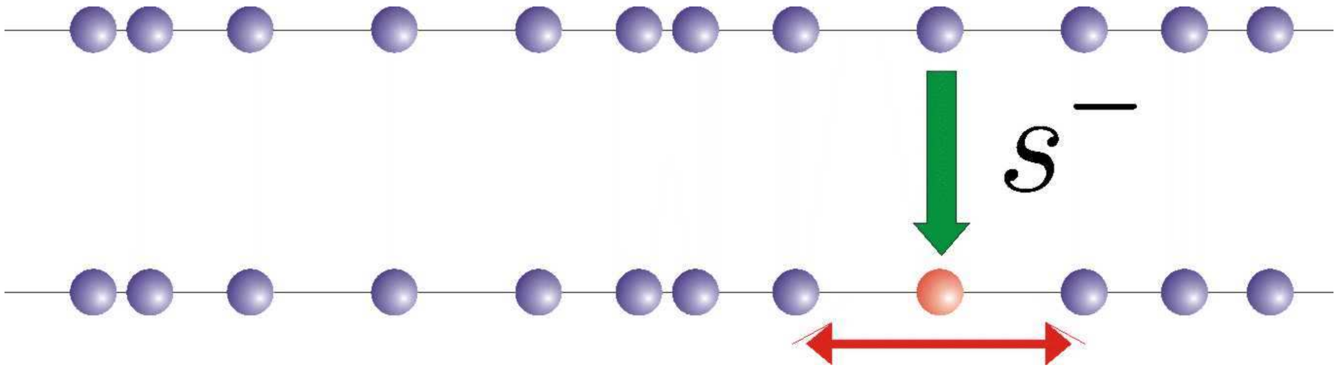
$$G_\perp(x,t) = \langle \delta[N(x,t)]\rho(x,t)\rho(0,0)\rangle \qquad \delta(N(x,\tau)) = \int d\lambda\, e^{i\lambda N(x,\tau)}$$

Operator $N(x,\tau)$ counts the number of crossings!

Solid lines do not disappear ⇨ continuity equation: $\partial_x j_x + \partial_\tau j_\tau = 0$

$$N(x,\tau) = \int_{(0,0)}^{(x,\tau)} j_\tau dx - j_x d\tau = \rho x - \frac{1}{\pi}[\phi(x,\tau) - \phi(0,0)]$$

# Estimate for the escape time



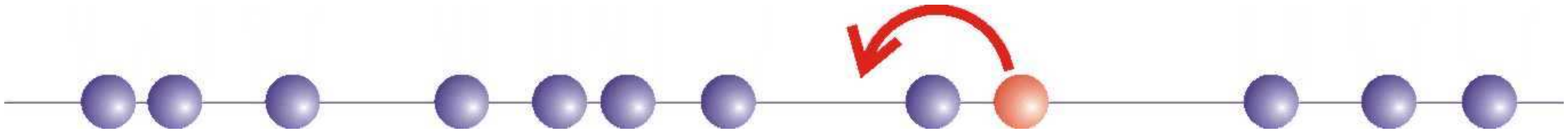

The "red" particle is trapped between impenetrable neighbors

Estimate for the escape time can be obtained from the Kronig-Penney model:

$$\boxed{t_* \sim \gamma t_{\mathrm{F}}, \qquad t_{\mathrm{F}} = \hbar / E_{\mathrm{F}}, \qquad \gamma \gg 1}$$

What happens when $t > t_*$ ?

"Red" and "blue" particles can exchange positions:



# Green's function for strong but finite repulsion

$$G_\perp(x,t) \simeq \left[\beta \ln\left(\frac{t}{t_F}\right) + \frac{it\hbar}{2m*}\right]^{-1/2} \exp\left\{\frac{im_* x^2}{2t\hbar - 4i\alpha m_* \ln(t/t_F)}\right\}$$

$$\alpha = \frac{K}{2(\pi\rho)^2} \qquad m_* = m\frac{3\gamma}{2\pi^2}, \qquad \gamma \to \infty$$

'Trapped' regime: $t_\mathrm{F} \ll t \ll t_*$

$$t_* \sim \gamma t_\mathrm{F}, \qquad t_\mathrm{F} = \hbar/E_\mathrm{F}, \qquad \gamma \gg 1$$

We recover limiting case

$$\gamma = \infty \Rightarrow m_* = \infty$$

$$G_\perp(x,t) \simeq \frac{1}{\sqrt{\ln(t/t_\mathrm{F})}} \exp\left\{-\frac{1}{K}\frac{(\pi\rho x)^2}{2\ln(t/t_\mathrm{F})}\right\}$$

'Open' regime: $t \gg t_*$

$$G_\perp(x,t) \simeq \exp\left\{-\frac{x^2}{2\ell^2}\right\} G_\perp^H(x,t)$$

$$\ell(t) = \frac{2K^{-1/2}}{\pi\rho}\frac{t/t_\mathrm{F}}{\sqrt{\ln t/t_\mathrm{F}}}\frac{m}{m_*}$$

$$G_\perp^H(x,t) \simeq \frac{1}{\sqrt{t}} \exp\left\{\frac{im_* x^2}{2t\hbar}\right\}$$

Green's function of the free particle

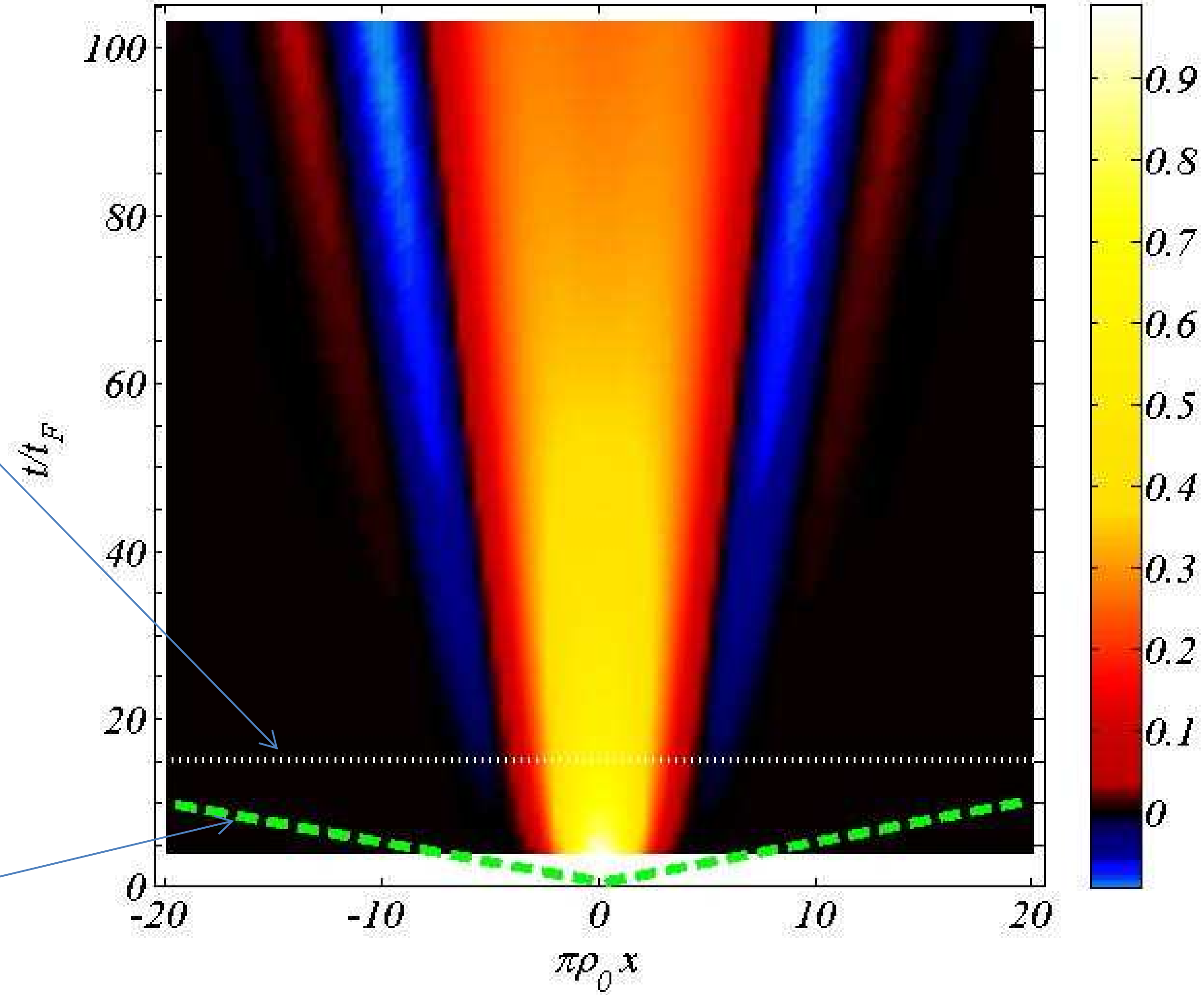
M. B. Zvonarev, V. V. Cheianov, T. Giamarchi, PRL **99**, 240404 (2007)

# Dynamics of spin excitations at finite interaction strength

Intensity plot of $\mathrm{Re}\, G_\perp(x,t)$ in the $x, t$ plane at $\gamma = 100$

White dotted line: crossover from 'trapped' to 'open' regime: $t_* \simeq 15 t_\mathrm{F}$

Dashed green lines: lightcone $x = vt$ corresponding to density excitations of spinless bosons



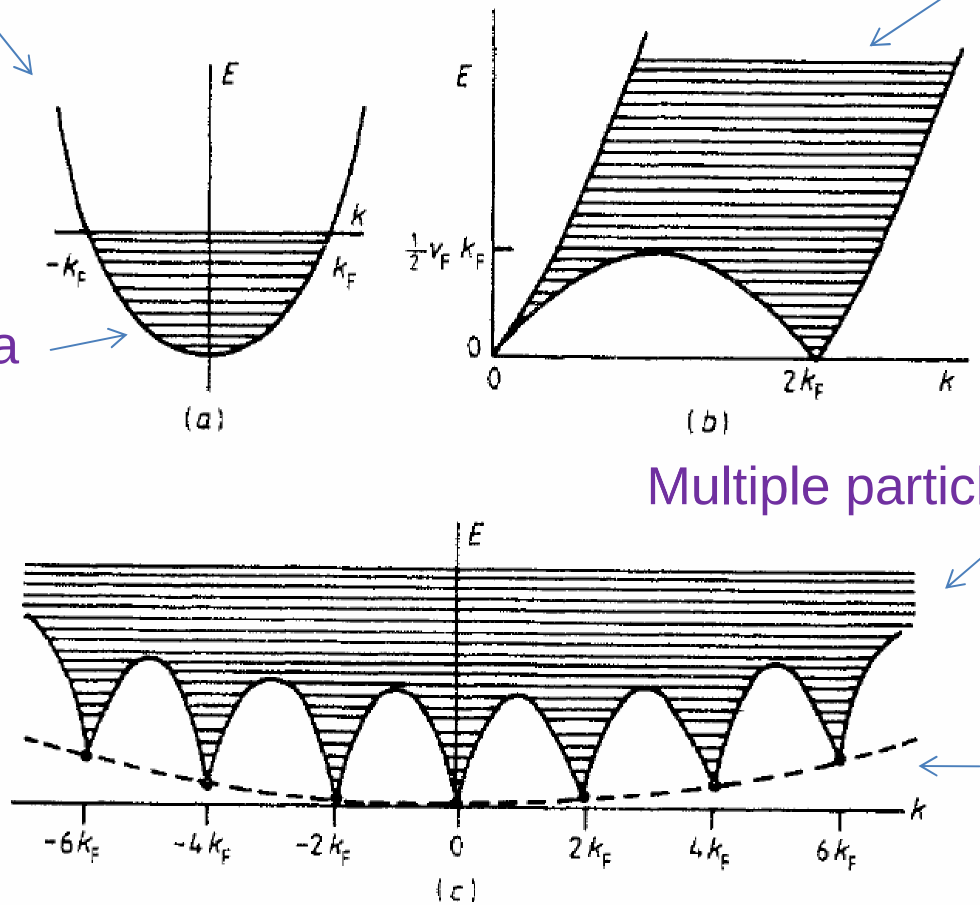
# Universal dynamics & edge singularity in the momentum space

# Excitation spectrum of the 1D non-interacting Fermi gas

Single particle dispersion

Single particle-hole pair spectrum

Fermi sea




Multiple particle-hole pair spectrum

Has a **gap** at
finite momentum

**Figure 1.** (*a*) Single-particle spectrum of the free Fermi gas in 1D; (*b*) Particle–hole pair spectrum; (*c*) full zero-charge (multiple particle–hole) excitation spectrum (energy differences $E(n) = 2\pi v_F n^2/L$ of extremal states at $k = 2nk_F$ greatly exaggerated).

# Excitation spectrum of the 1D **interacting** Fermi gas

Like in the non-interacting case, has a **gap** at all finite momenta:
(except for multiple integers of $2k_F$ )

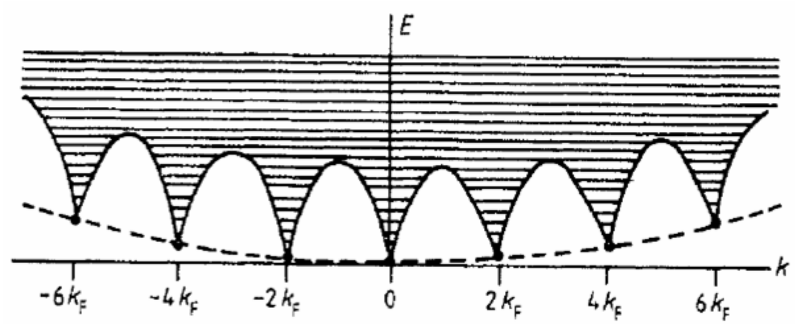
Proved by perturbation theory, numerics, exactly solvable models

⇨ Threshold energy $\omega_-(q) = \min_\nu E_\nu(q)$

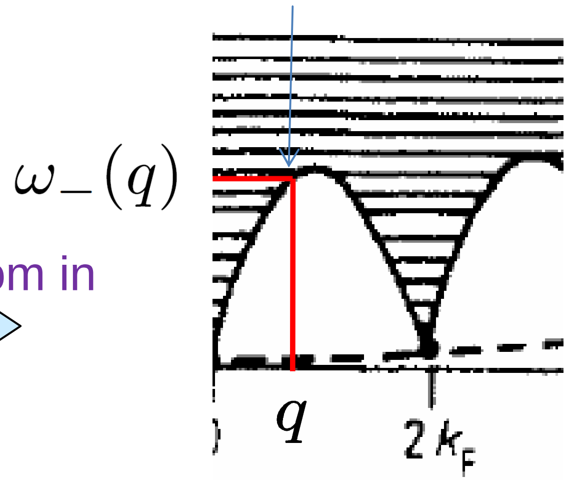
$\nu$ enumerates all the states with total momentum $q$

No excitations below $\omega_-$

Multiple particle-hole pair spectrum




Zoom in ⇨



# Spectral functions: definition & non-interacting case

Dynamic structure factor: gives absorption rate of a photon with

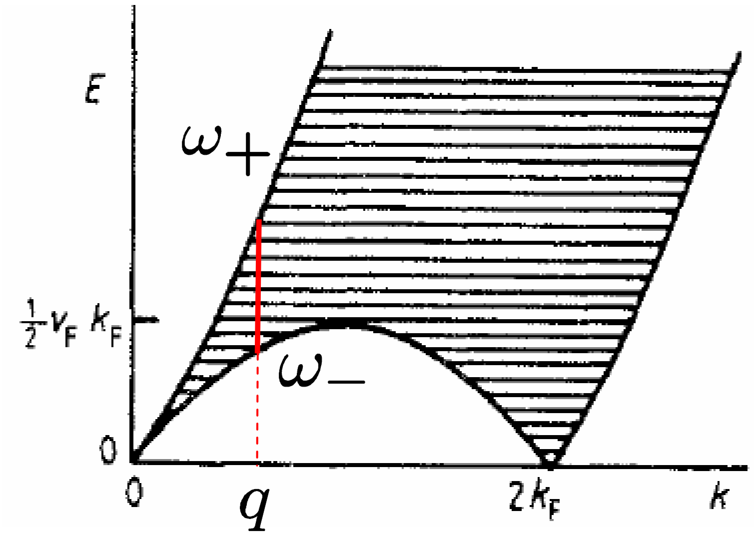
Energy

Momentum

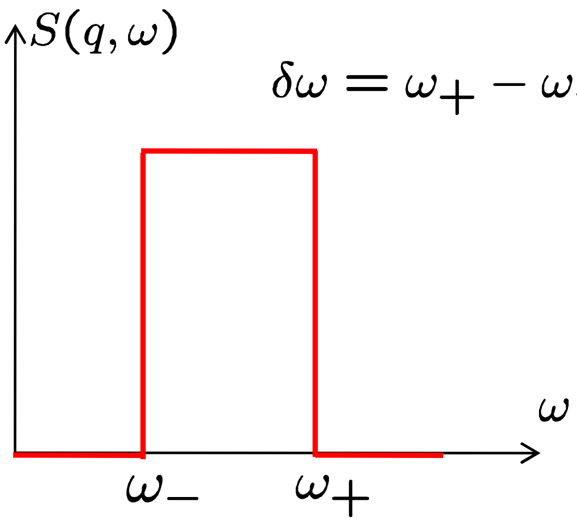
$$S(q,\omega) = \sum_{\nu} |\langle \nu, q | n_q^{\dagger} | \mathrm{gs} \rangle|^2 \delta(\omega - E_{\nu}(q))$$

$$n_q^{\dagger} = \sum_k \psi_k^{\dagger} \psi_{k-q}$$

No interaction ⇨ Photon creates single particle-hole pair:




⇨



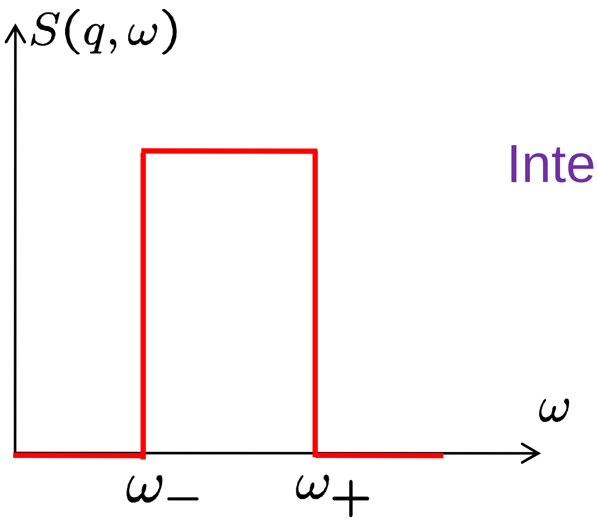

$$\delta\omega = \omega_+ - \omega_- \sim q^2$$

$q \to 0$

# Spectral functions: effects of the interaction

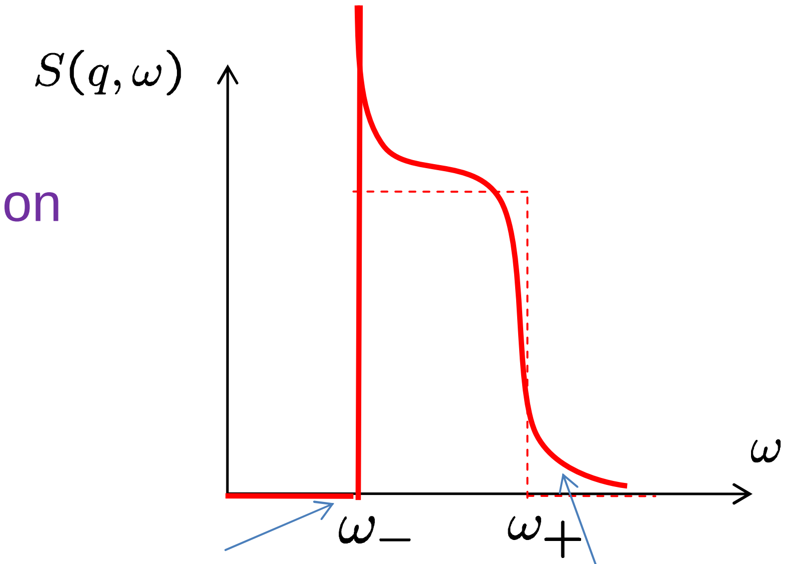
Interaction ⇨ Multiple particle-hole pairs can be excited

⇨ Some spread of structure functions is expected




Interaction

⇨




Threshold remains

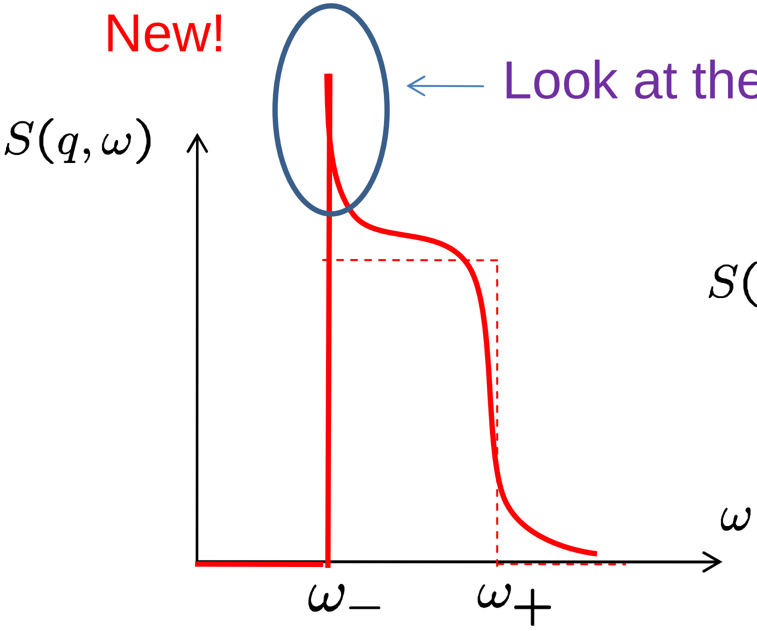
High momentum tail spreads

# Spectral functions: threshold singularities

Old!

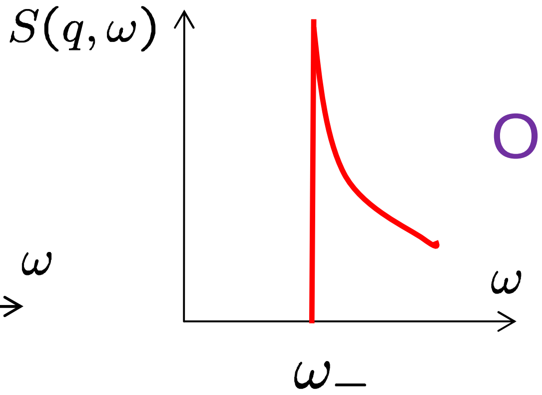
Looking for the universal properties ⇨ Looking at low energies

New!

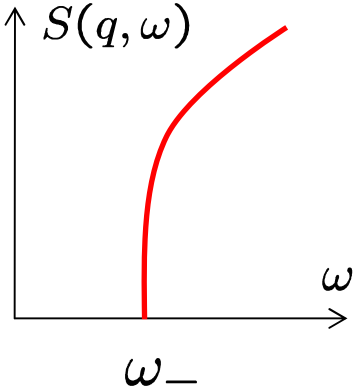



Look at the behavior near the edge of the spectrum

Zoom in




Or




⇨ Universal behavior:

Power law singularity:

$$\boxed{S(q,\omega) \simeq c(q)\theta(\omega-\omega_-)(\omega-\omega_-)^{\Delta(q)}}$$

Momentum is **arbitrary** !!! **<u>The result of the last 4 year's activity of many! people</u>** (see Imambekov's talk)

# Back to real space: get $G_\perp(x,t)$ from $S(q,\omega)$

Fourier transform: $G(x,t) = \int dq\, e^{iqx} \int d\omega\, e^{-i\omega t} S(q,\omega)$

We know $S(q,\omega) \simeq c(q)\theta(\omega-\omega_-)(\omega-\omega_-)^{\Delta(q)}$ near threshold

⇨ we know $S(q,\omega)$ as $q,\omega \to 0$

Using symmetry $S(q,\omega) = S(-q,\omega)$ we expand $\Delta(q)$ and $\omega_-(q)$ at $q \to 0$

$$\Delta(q) = -1 + \alpha q^2 + \cdots$$
$$\omega_-(q) = q^2/2m_* + \cdots$$

⇨ $x,t \to \infty$

Logarithmic diffusion for $m/m_* \ll 1$

… ??? … for $m/m_* \simeq 1$

How Luttinger physics can appear as $q \to 0$ ?

$$\Delta(q) = -1 + \alpha|q| + \cdots$$
$$\omega_-(q) = v_s|q| + \cdots$$

⇨ $x,t \to \infty$

$$G(x,t) \sim \frac{1}{(x^2 - v^2t^2)^\mu}$$

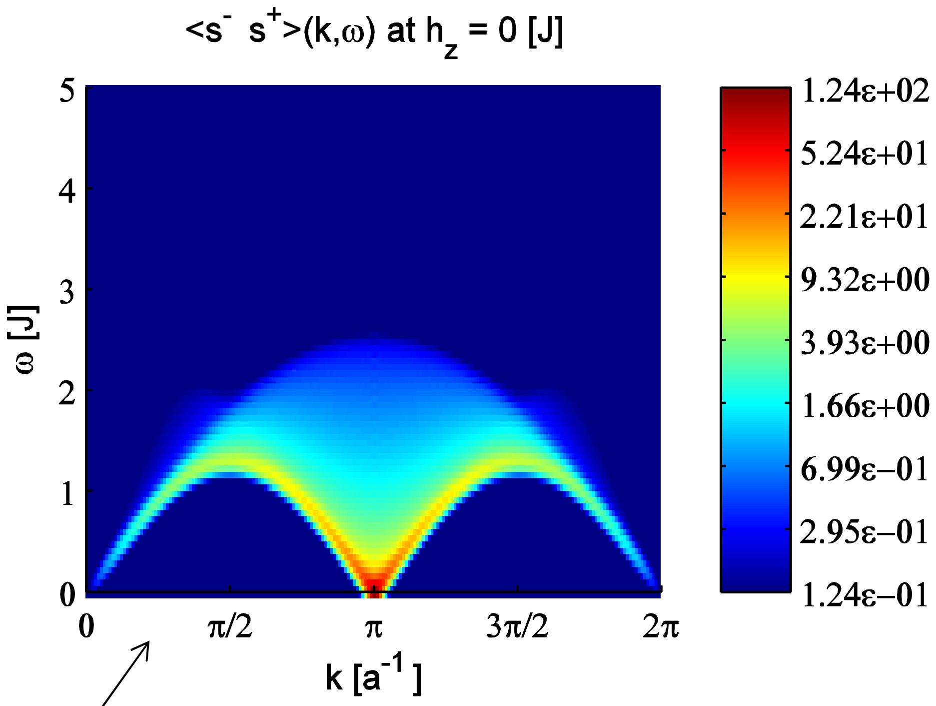
Luttinger form of the Green's function!

# Do we have experiments/numerics for

Momentum space ⇨ $S(q, \omega)$

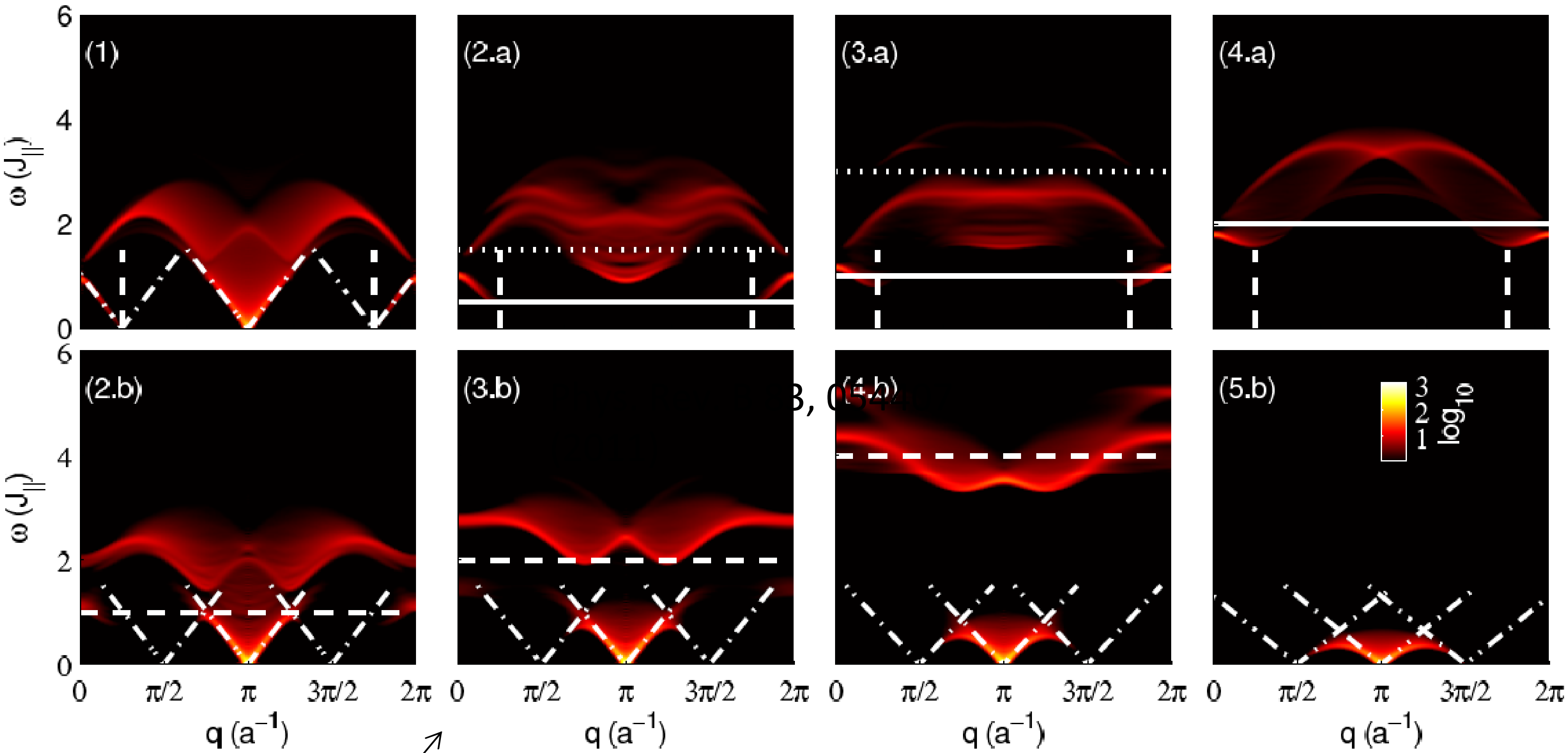
Real space ⇨ $G_\perp(x, t)$

# Experiment/numerics in momentum space




"Typical" numerical plot of the structure factor (this one is for the Heisenberg magnet).

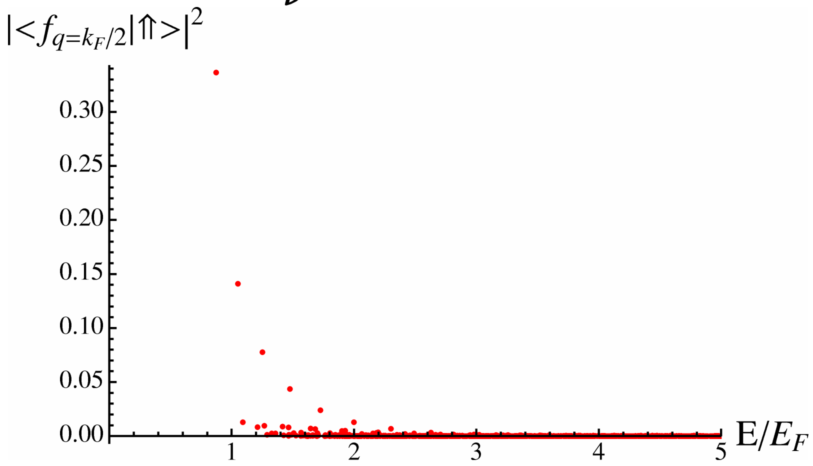
# More numerics in momentum space

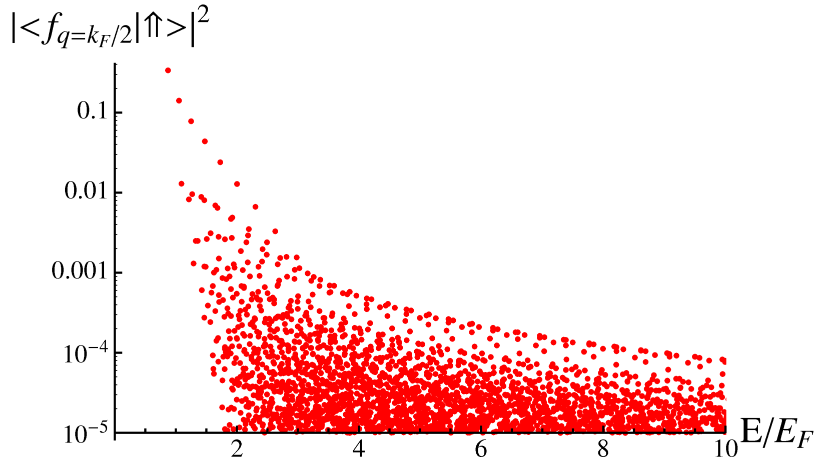


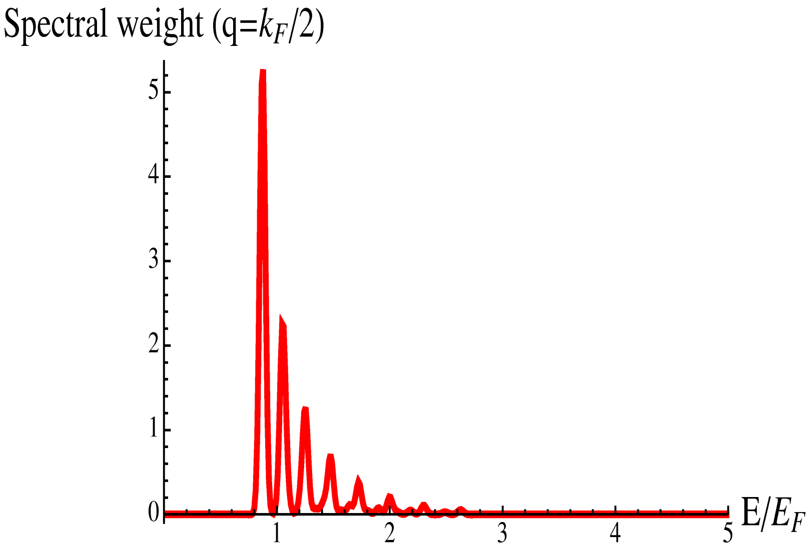

Some structure factors for the spin ladder

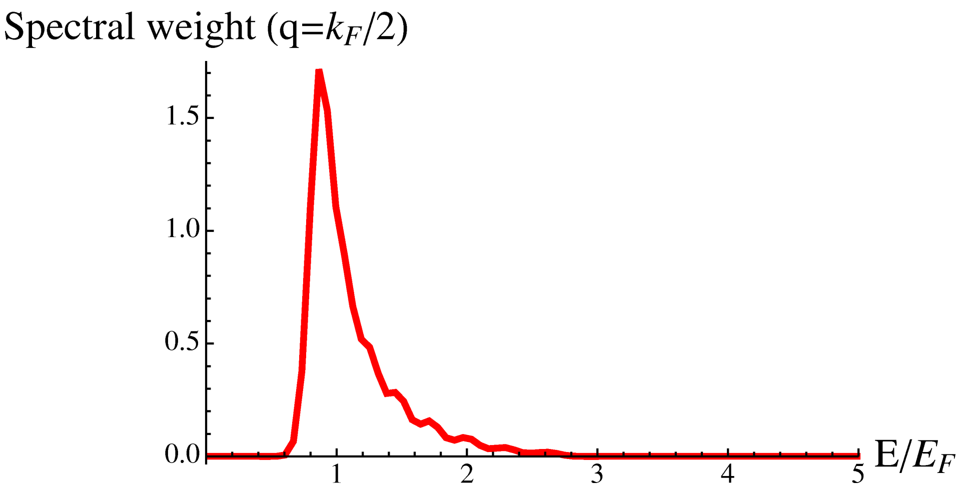
Pierre Bouillot *et. al.*, Phys. Rev. B **83**, 054407 (2011)

$$S(q,\omega)=\sum_{\nu}|\langle\nu,q|n_q^\dagger|\mathrm{gs}\rangle|^2\delta(\omega-E_\nu(q))$$ Example of numerics




How to extract exponents from numerics of such precision - see Imambekov's talk)

There is no "no go" theorem but!

To resolve the exponents of the edge singularity of the structure functions is a difficult and open problem for both numerics and "solid state" experiment

May be experiments in cold gases could do it better?

Are so far done with the impurity subjected to the external (gravity) force

=

driven motion problem in the real space

Why should we always stick to the momentum space?

# First (cold gases) experiment

Stefan Palzer, Christoph Zipkes, Carlo Sias, and Michael Köhl, PRL 103, 150601 (2009)

**Quantum Transport through a Tonks-Girardeau Gas**

## Quantum impurity is driven by the gravity force

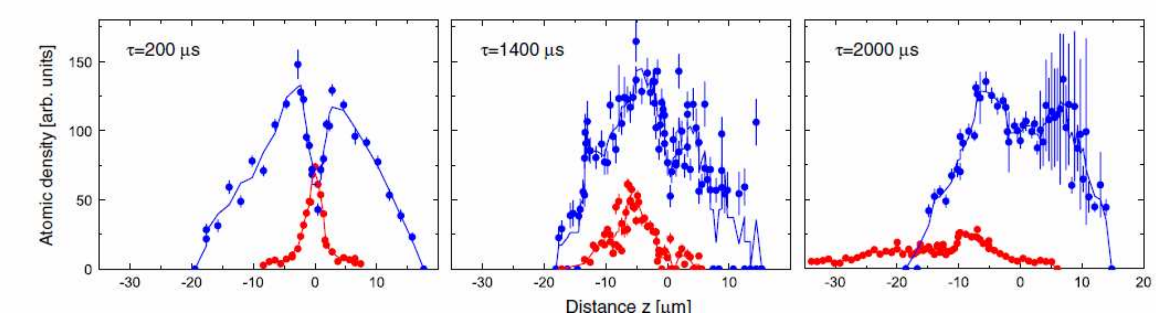



FIG. 2 (color online). *In situ* measurement of the time evolution of both the trapped component (upper curve) and the impurity (lower curve) for different times $\tau$. The data are taken for $\gamma = 7$. The solid line is a two-point average of the data to guide the eye.

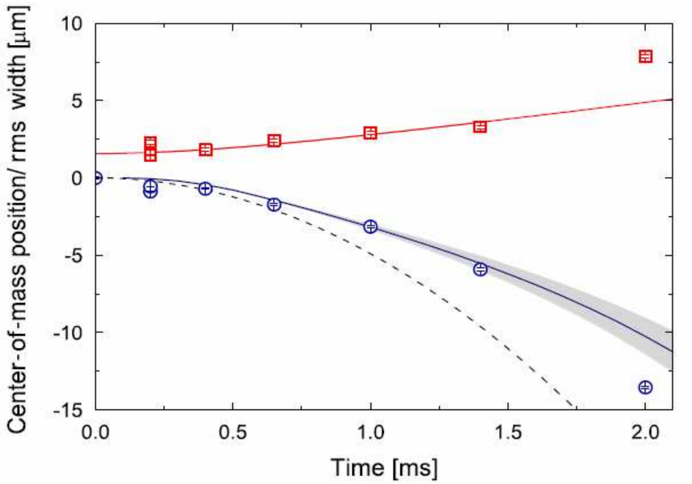



FIG. 3 (color online). The circles show the measured center-of-mass position taken for $\gamma = 7$. The error bars are the statistical error of the center of mass of the measured density distribution. The solid line is the prediction according to the model described in the text. The gray shaded area indicates the regime of uncertainty of 10% of $n_{1D}$ given by our experimental parameters. The dashed curve indicates purely ballistic motion. The squares show the increase of the width of the impurity wave packet. The data point at 2 ms contains atoms which have already left the trapped gas which is not taken into account by the theory.

# Details of the experiment

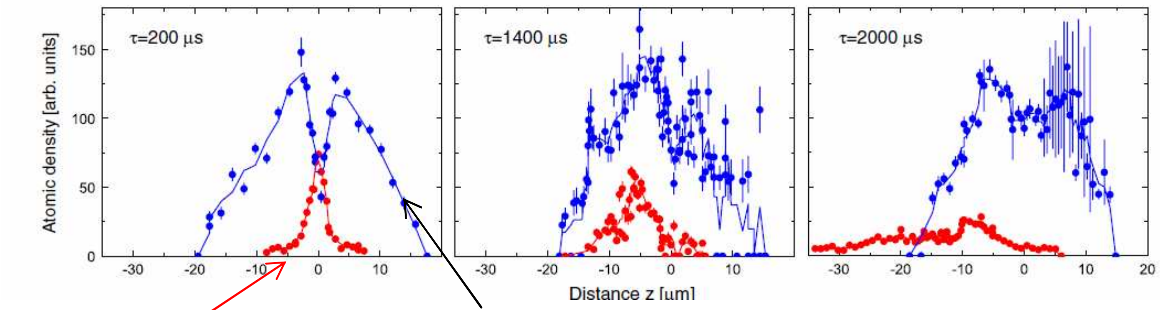
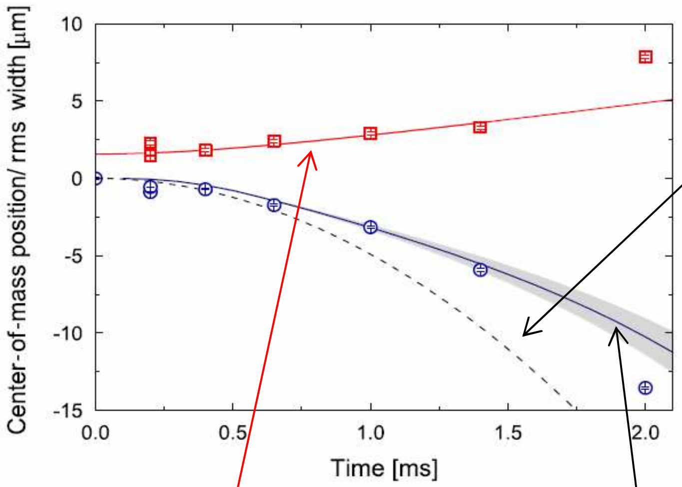



FIG. 2 (color online). *In situ* measurement of the time evolution of both the trapped component (upper curve) and the impurity (lower curve) for different times $\tau$. The data are taken for $\gamma = 7$. The solid line is a two-point average of the data to guide the eye.

Red: impurity; blue: host particles




dashed line: free falling particle

$$\gamma = 7 \Rightarrow m/m_* \simeq 0.45$$

Looks like particle falling with effective mass, $a = \frac{m}{m_*} g$

Finale velocity is more than 2 x sound velocity

Red: host centre mass;
blue: impurity centre mass

# Do we have a theory for such a problem?

VOLUME 87, NUMBER 10 PHYSICAL REVIEW LETTERS 3 SEPTEMBER 2001

**Superfluidity versus Bloch Oscillations in Confined Atomic Gases**

H. P. Büchler,[1] V. B. Geshkenbein,[1,2] and G. Blatter[1]
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[2]*Landau Institute for Theoretical Physics, 117940 Moscow, Russia*
(Received 22 December 2000; published 20 August 2001)

PHYSICAL REVIEW A **70**, 013608 (2004)

**Motion of a heavy impurity through a Bose-Einstein condensate**

G. E. Astrakharchik[1,2] and L. P. Pitaevskii[1,3]
[1]*Dipartimento di Fisica, Università di Trento and Istituto Nazionale per la Fisica della Materia, I-38050 Povo, Trento, Italy*
[2]*Institute of Spectroscopy, 142190 Troitsk, Moscow Region, Russia*
[3]*Kapitza Institute for Physical Problems, 119334 Moscow, Russia*
(Received 29 March 2004; published 20 July 2004; corrected 26 August 2004)

PRL **103,** 085302 (2009) PHYSICAL REVIEW LETTERS week ending 21 AUGUST 2009

**Drag Force on an Impurity below the Superfluid Critical Velocity in a Quasi-One-Dimensional Bose-Einstein Condensate**

Andrew G. Sykes,[1] Matthew J. Davis,[1] and David C. Roberts[2]
[1]*The University of Queensland, School of Mathematics and Physics, ARC Centre of Excellence for Quantum-Atom Optics, Brisbane 4072, Australia*
[2]*Theoretical Division and Center for Nonlinear Studies, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA*
(Received 3 April 2009; revised manuscript received 15 July 2009; published 18 August 2009)

The above cited papers are about the impurity of <u>infinite</u> mass moving with <u>constant</u> velocity

# What do we know about the finite mass impurity under the external force?

PRL **102,** 070402 (2009) PHYSICAL REVIEW LETTERS week ending 20 FEBRUARY 2009

**Bloch Oscillations in a One-Dimensional Spinor Gas**

D. M. Gangardt[1,*] and A. Kamenev[2]

[1]*School of Physics and Astronomy, University of Birmingham, Edgbaston, Birmingham, B15 2TT, United Kingdom*
[2]*School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455, USA*
(Received 21 November 2008; revised manuscript received 19 January 2009; published 18 February 2009)

A force applied to a spin-flipped particle in a one-dimensional spinor gas may lead to Bloch oscillations of the particle's position and velocity. The existence of Bloch oscillations crucially depends on the viscous friction force exerted by the rest of the gas on the spin excitation. We evaluate the friction in terms of the quantum fluid parameters. In particular, we show that the friction is absent for integrable cases, such as an $SU(2)$ symmetric gas of bosons or fermions. For small deviations from the exact integrability the friction is very weak, opening the possibility to observe Bloch oscillations.

The above paper further develops the ideas of some previous works

# This paper is about effective hydrodynamic theory

Suggested equation of motion for the impurity: $\partial_t q = F - \kappa V$

2 crucial statements:

1: viscosity coefficient $\kappa \sim T^4$

2: velocity $V = \dfrac{\partial \varepsilon(q)}{\partial q}$ $\varepsilon(q)$ is the edge of the excitation spectrum

$$\Rightarrow \quad V_{\text{drift}} = \kappa^{-1} \begin{cases} F; & F < F_c, \\ F - \sqrt{F^2 - F_c^2}; & F > F_c, \end{cases} \qquad F_c = \kappa \hbar n / M^*$$

In the strong forcing regime, $F > F_c$, the drift motion is superimposed with the Bloch oscillations with the period

$$\Delta T = 2\pi \hbar n / \sqrt{F^2 - F_c^2}$$

# How hydrodynamic theory is solved for strong repulsion

Bosons with spin-independent interaction:

$$H=\sum_{j=1}^{N}\frac{p_j^2}{2m}+\alpha\sum_{i<j}\delta(x_i-x_j)$$

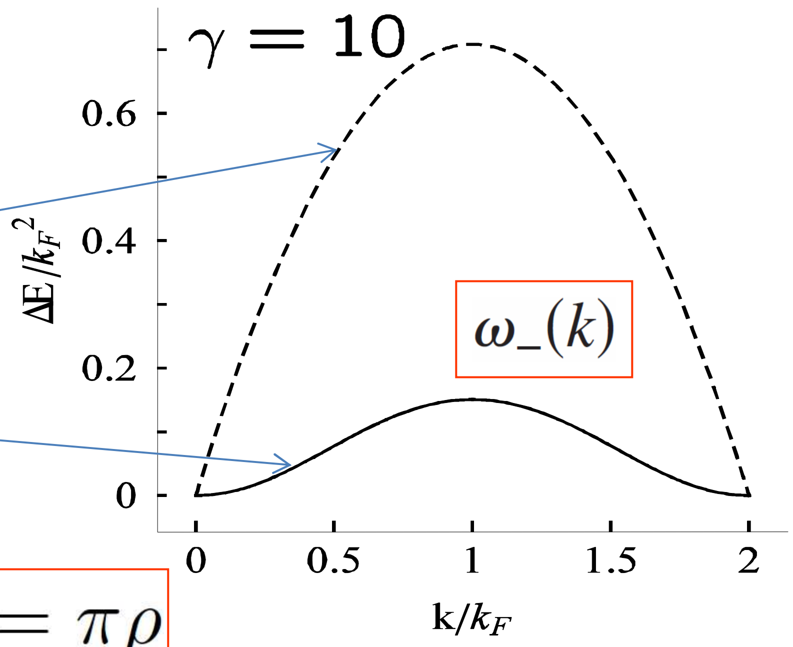
Dimensionless coupling: $\boxed{\gamma=\frac{m\alpha}{\hbar^2\rho}}$

Dispersion of spinless boson gas:

$$\omega_-(k)=vk \qquad k\to 0$$

Dispersion of bosons with spin:

$$\omega_-(k)=\frac{k^2}{2m_*} \qquad k\to 0$$




$\boxed{k_F=\pi\rho}$

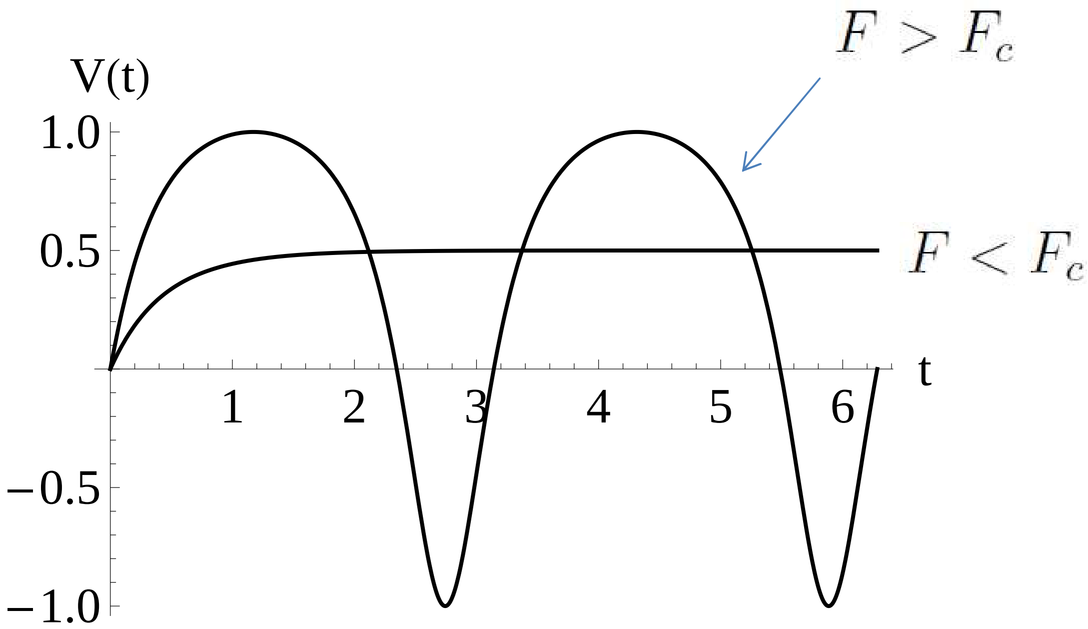
Strong repulsion: $\dfrac{\varepsilon(q)}{\varepsilon_F}=\dfrac{4m}{\pi^2 m_*}\sin^2\left(\dfrac{\pi}{2}\dfrac{q}{q_F}\right) \qquad m_*=m\dfrac{3\gamma}{2\pi^2}, \qquad \gamma\longrightarrow\infty$

# How hydrodynamic theory is solved for strong repulsion II

$$V = \kappa^{-1} F_c \sin(2\beta q)\,, \qquad F_c = \kappa \frac{2}{\pi}\frac{m}{m_*}\frac{\varepsilon_F}{q_F} = \kappa\frac{q_F}{\pi m_*}, \qquad \beta = \frac{\pi}{2q_F} \qquad \gamma \to \infty$$

It is said that the viscosity coefficient is independent of velocity, therefore

$$\frac{dq}{F - F_c \sin(2\beta q)} = dt$$



# Summary

Mobile impurity in the quantum gas has non-Luttinger dynamics in real space due to quadratic dispersion

This non-Luttinger behavior can be obtained from looking at the behavior of the structure functions near the edge of the excitations spectrum

Many details of the edge behavior of the spectral functions are yet difficult to get in the numerics&experiment

The hope is that the real-space dynamics of impurity have some pronounced features (e.g. logarithmic diffusion) which are "easy" to observe in the numerics&experiment.

Cold gases are good for working with real-time dynamics of impurity

There is only one published experiment on the 1D impurity motion (driven by constant external force), but several other experiments are on the way.

## Some details on the edge exponents not shown in the talk

# Excitations near threshold energy

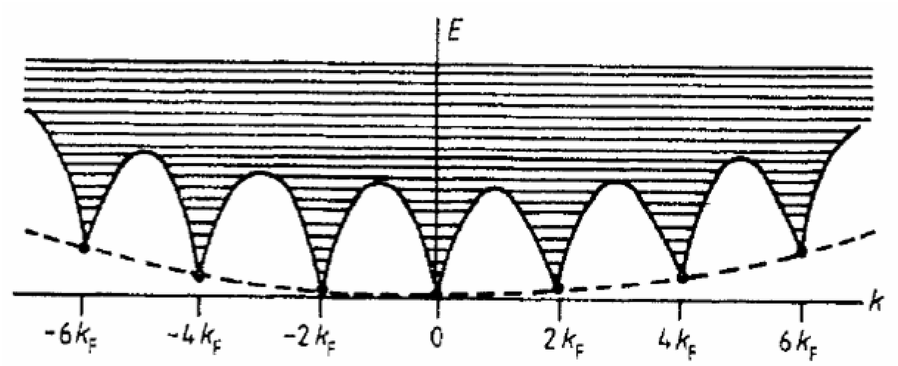
Consider spectral function $S(q,\omega) = \sum_{\nu} |\langle \nu, q | n_q^{\dagger} | \mathrm{gs} \rangle|^2 \delta(\omega - E_{\nu}(q))$

$$n_q^{\dagger} = \sum_k \psi_k^{\dagger} \psi_{k-q}$$

Problem: classify excitations with arbitrary $q$ and $E_{\nu}(q) \to \omega_{-}(q)$

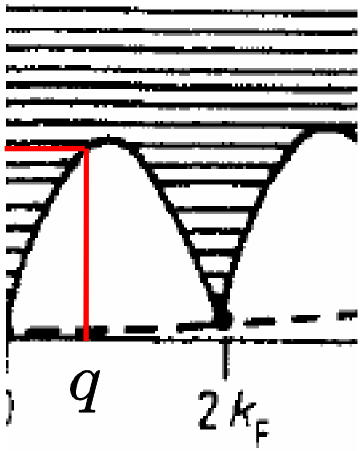
Hint: look at free fermions:

Multiple particle-hole pair spectrum

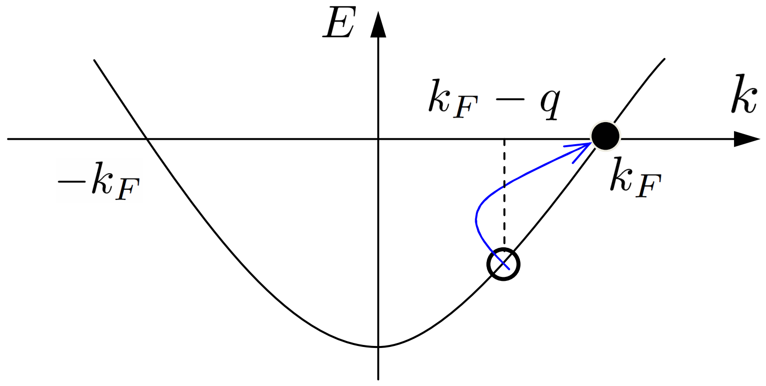



$\omega_{-}(q)$

Zoom in

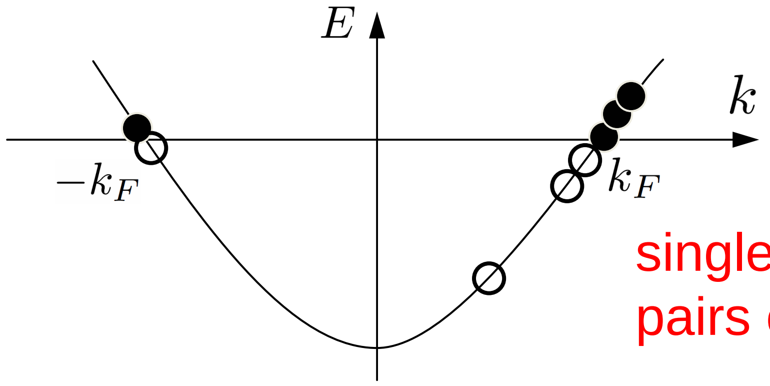


# Excitations near threshold energy (continued)




$\Leftrightarrow$ Excitation with momentum $q$ and energy $\omega_-(q)$

single deep hole




$\Leftrightarrow$ Excitations with momentum $q$ and energy close to $\omega_-(q)$

single deep hole and many particle-hole pairs close to Fermi points

Interacting (integrable) systems: like free fermion case!

**Conjecture**

# Effective theory near the edge of the spectrum

Should be a minimal theory of the 1D polaron problem:
free plasmons interacting with a deep hole

Free plasmons = particle-hole pairs near Fermi points = Luttinger Liquid:

$$H_{LL} = \int \frac{dx}{4\pi} \sum_{\alpha=R,L} v_\alpha (\partial_x \varphi_\alpha)^2 \qquad [\varphi_\alpha(x), \varphi_{\alpha'}(y)] = i\pi\alpha\delta_{\alpha\alpha'}\mathrm{sgn}(x-y)$$

Plasmons linearly coupled to the deep hole:

$$H_i = -\int \frac{dx}{2\pi} \sum_{\alpha=R,L} v_\alpha \beta_\alpha (\partial_x \varphi_\alpha) d^\dagger(x) d(x)$$

deep hole

$$\psi(x) = \sum_{\alpha=R,L} e^{i\alpha k_F x}\psi_\alpha(x) + e^{i(k_F-q)x} d^\dagger(x)$$

⇨

$$S(q,\omega) \simeq c(q)\theta(\omega-\omega_-)(\omega-\omega_-)^{\Delta(q)}$$

$$\Delta(q) = -1 + \frac{1}{4\pi^2}(\beta_+^2 + \beta_-^2)$$

$\beta_\pm$ depend on $q$ !!!

# Dynamical structure factor for $G_\perp(x,t) = \langle \Uparrow | s_+(x,t) s_-(0,0) | \Uparrow \rangle$

$$S(q,\omega) = \int dx\, e^{-iqx} \int dt\, e^{i\omega t} G_\perp(x,t)$$

$$S(q,\omega) = \sum_\nu |\langle \nu, q | s_q^- | \Uparrow \rangle|^2 \delta(\omega - E_\nu(q))$$

Exited states are made by one magnon and arbitrary number of plasmons (particle-hole pairs or density fluctuations)

⇨ Similar to a polaron problem

Effective model: magnon carries momentum $q$, its dispersion has a minimum around $q$ . Plasmons have a linear dispersion.

Effective Hamiltonian: $H_{\text{eff}} = H_{LL} + H_i$

Free plasmons = Luttinger Liquid

Plasmons linearly coupled to the magnon

# Effective theory & behavior in the momentum space

Free plasmons = Luttinger Liquid: $H_{LL} = \int \frac{dx}{4\pi} \sum_{\alpha=R,L} v_\alpha (\partial_x \varphi_\alpha)^2$

$$[\varphi_\alpha(x), \varphi_{\alpha'}(y)] = i\pi\alpha\delta_{\alpha\alpha'}\mathrm{sgn}(x-y)$$

magnon

Plasmons linearly coupled to the magnon

$$H_i = -\int \frac{dx}{2\pi} \sum_{\alpha=R,L} v_\alpha \beta_\alpha (\partial_x \varphi_\alpha) \tilde{s}^z(x)$$

$$s^\pm(x) = e^{\mp iqx}\tilde{s}^\pm(x) \qquad s^z(x) = \tilde{s}^z(x) + \rho_0/2$$

$$S(q,\omega) \simeq c(q)\theta(\omega-\omega_-)(\omega-\omega_-)^{\Delta(q)}$$

⇨ $$\Delta(q) = -1 + \frac{1}{4\pi^2}(\beta_+^2 + \beta_-^2)$$

$\beta_\pm$ depend on $q$ !!!

Momentum is **arbitrary** !!!

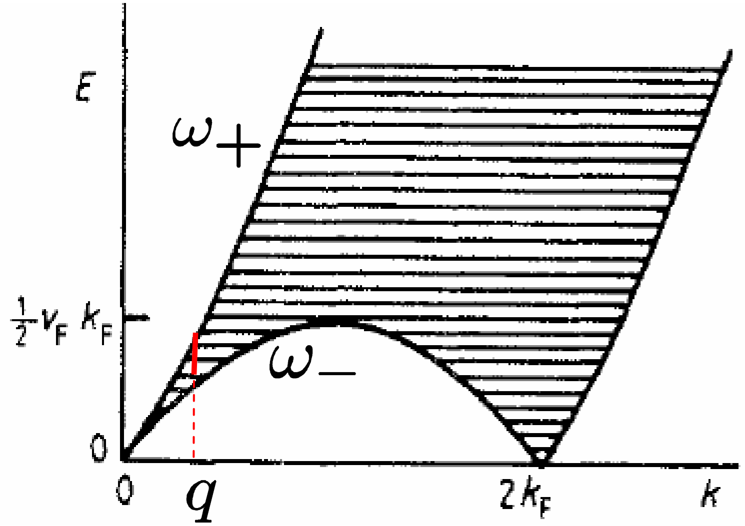
# Threshold singularities for interacting fermions

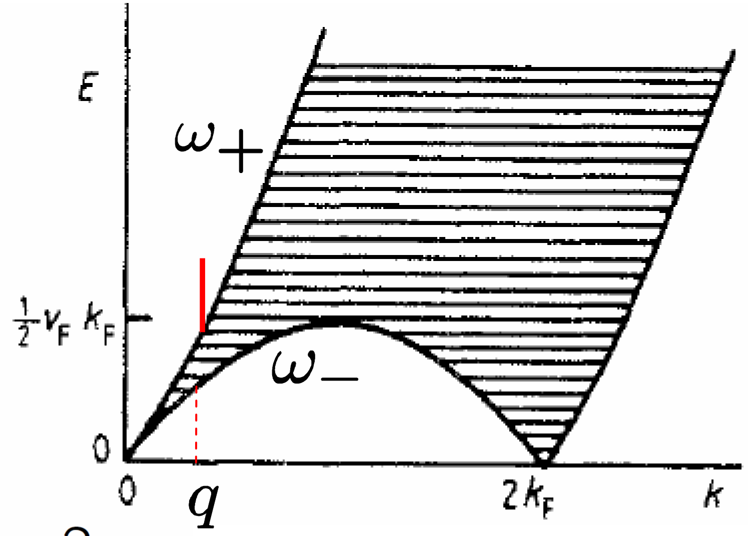
If $q \ll 2k_F$ ⇨ Luttinger Liquid theory works here:

Non-Luttinger

$$\omega_- < \omega < \omega_+ \qquad\qquad \omega_+ < \omega \ll E_F$$

$$H_{\text{eff}} = H_{LL} + H_i \qquad\qquad H_{\text{eff}} = H_{LL} + \cancel{H_i}$$

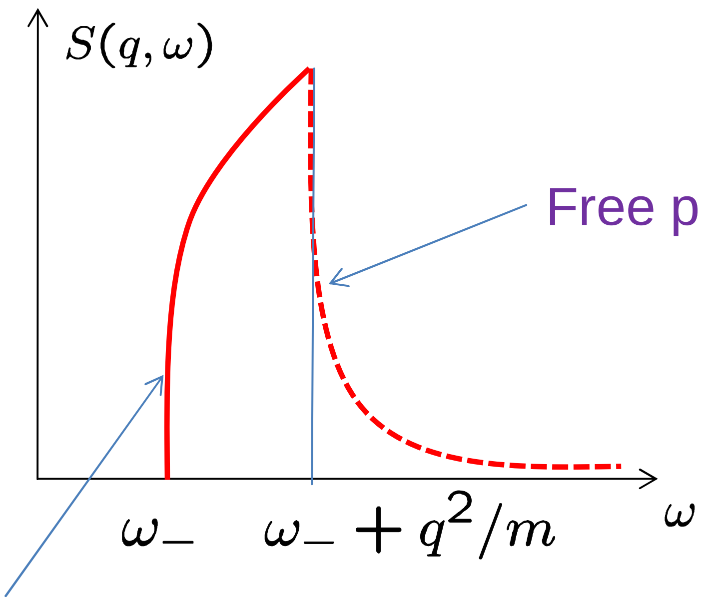



Polaron exists $\quad \omega_+ = \omega_- + q^2/m \quad$ Polaron washed out

# Threshold singularities for interacting fermions

$$S(q,\omega) \simeq c(q)\theta(\omega-\omega_-)(\omega-\omega_-)^{\Delta(q)}$$




Free plasmons, $H_{\text{eff}} = H_{LL}$

Free plasmons interact with mobile impurity: polaron problem

$$H_{\text{eff}} = H_{LL} + H_i$$