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Workshop on Integrability and its Breaking in Strongly Correlated and Disordered Systems

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Dynamical Theory of Superfluidity in One Dimension

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A Dynamical Theory of Superfluidity in 1D

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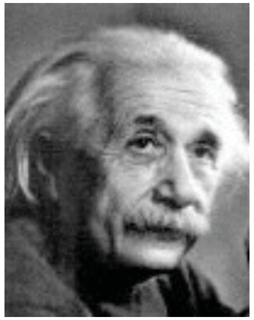
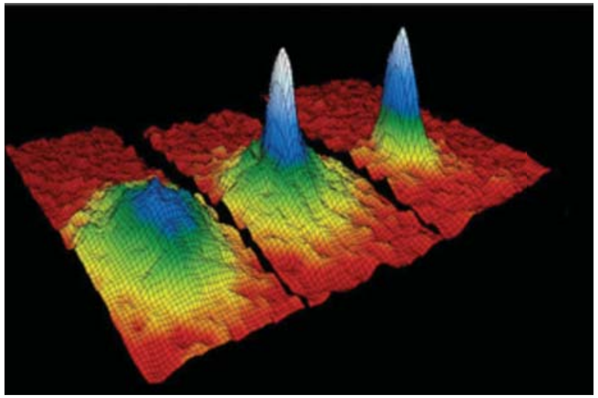
Reference: T Eggel, MAC & M Oshikawa arxiv:1104.0175 (2011)

Criteria for Superfluidity to occur are hard to define:

* It needs interactions
* Few low energy states
* BEC is not required

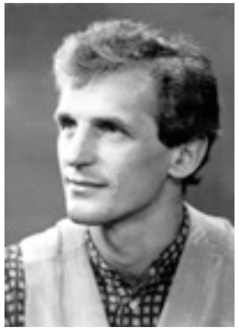
Superfluid ≠ Bose-Einstein Condensate (BEC)





$$\lim_{|\mathbf{r}-\mathbf{r}'|\to+\infty} \langle \Psi^\dagger(\mathbf{r})\Psi(\mathbf{r}')\rangle = |\Psi_0|^2 \neq 0$$

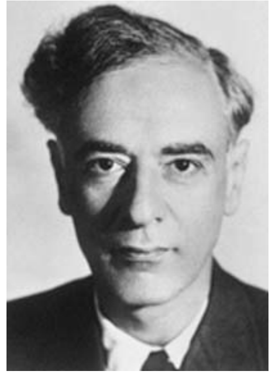
Worth a Nobel Prize (2001)



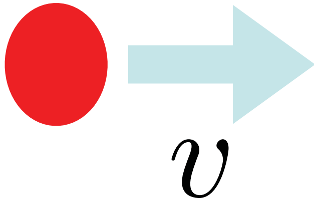
But BEC is not the same as Superfluidity!!

(Although in 3D BEC and SF are intimately related...)

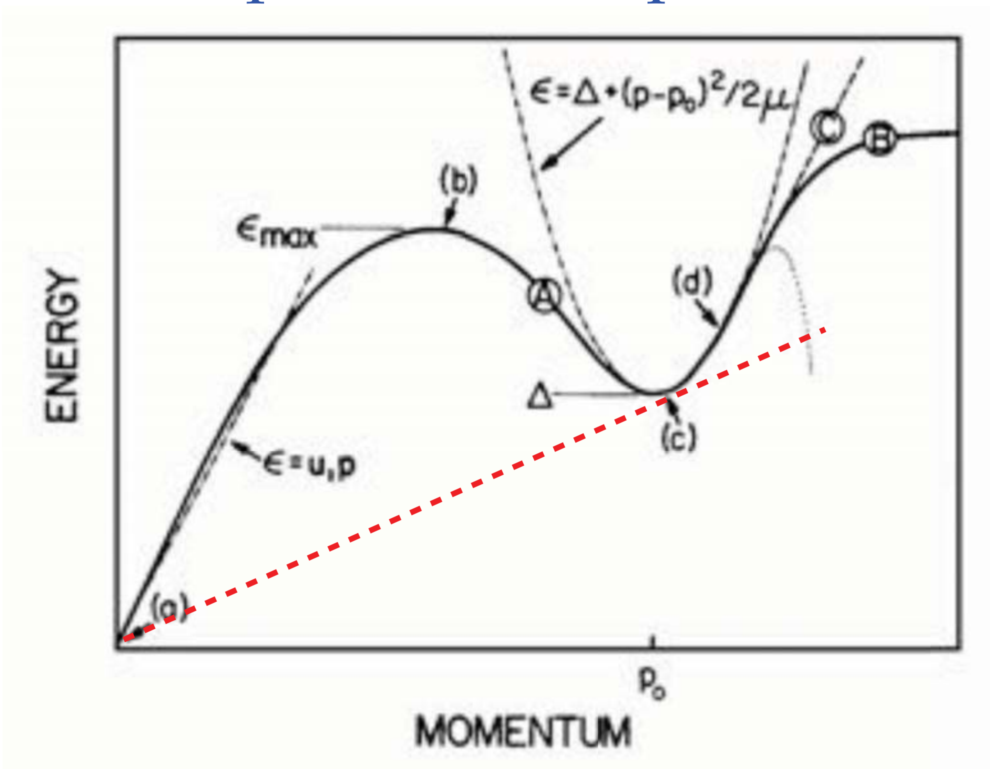
Landau's criterion



Consider a moving object:



Spectrum of liquid ^{4}He



Finite critical velocity

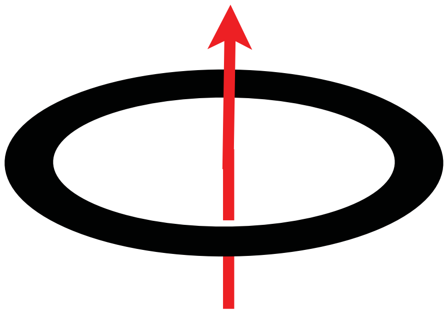
$$\min\left\{\frac{\epsilon(p)}{p}\right\} = v_{\mathrm{Landau}} > 0$$

Problem: e.g. How to define the SF properties at T > 0?

Fisher's criterion

Thermodynamics:

Superfluidity = <u>non-vanishing</u> Helicity Modulus



Twisted BC's

$$\hat{\Psi}(x+L,y,z) = e^{i\varphi}\hat{\Psi}(x+L,y,z)$$

$$\Upsilon(T) = \lim_{L\to+\infty} \frac{L}{S}\left(\frac{\partial^2 F(\varphi)}{\partial\varphi^2}\right)\Bigg|_{\varphi=0} \neq 0$$

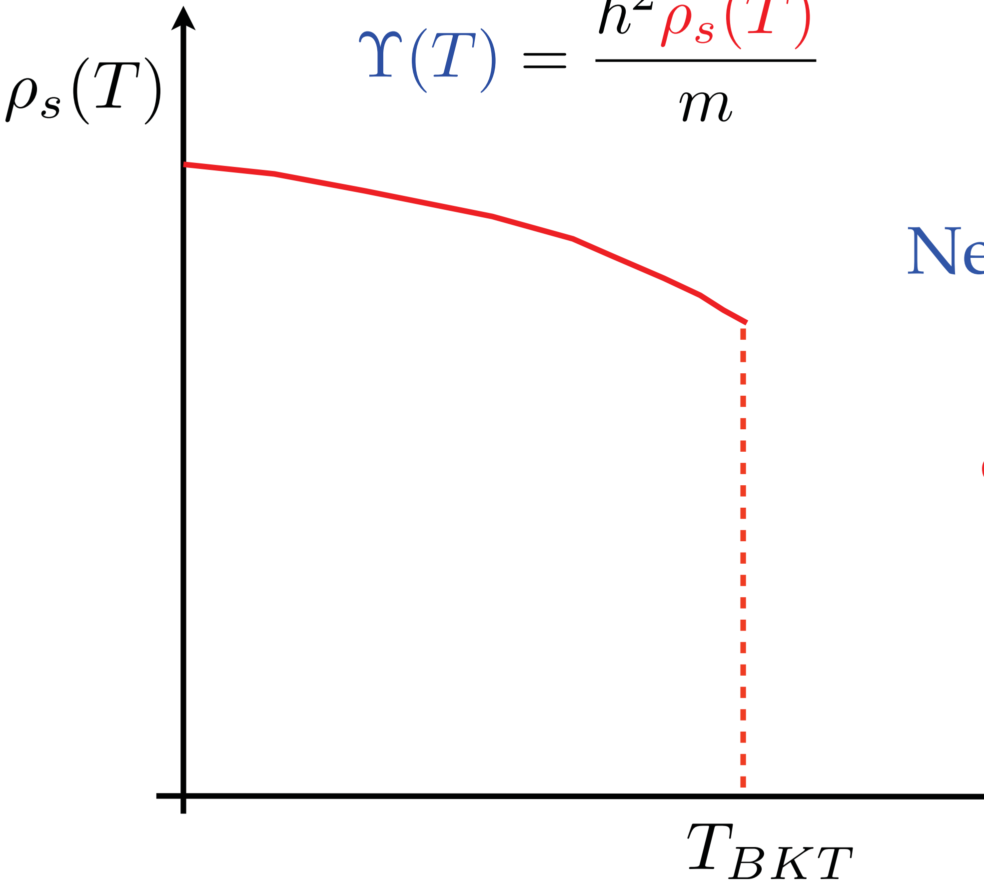
ME Fisher
et al PRA (1973)

Superfluid density: $\Upsilon(T) = \dfrac{\hbar^2 \rho_s(T)}{m}$

Interacting Bose fluids (BEC) in 2D

Absence of BEC (T > 0) $\langle \Psi^\dagger(\mathbf{r})\Psi(\mathbf{r}')\rangle \sim |\mathbf{r}-\mathbf{r}'|^{-\frac{1}{2K(T)}} \to 0$

$$\Upsilon(T) = \frac{\hbar^2 \rho_s(T)}{m}$$

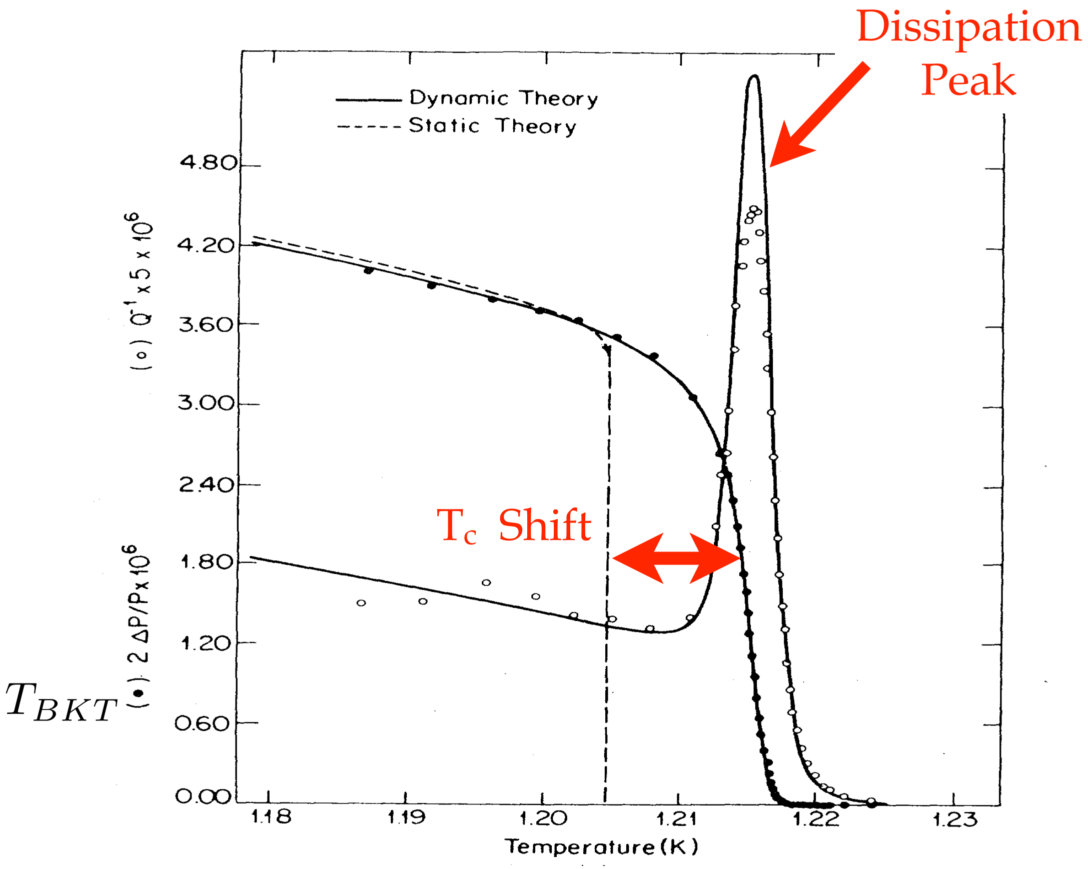


Nelson & Kosterlitz
PRL (1977)

Universal Jump
of the SF density

Superfluidity in 2D (Experiments)

2D ^{4}He films: Torsional oscillator measurements

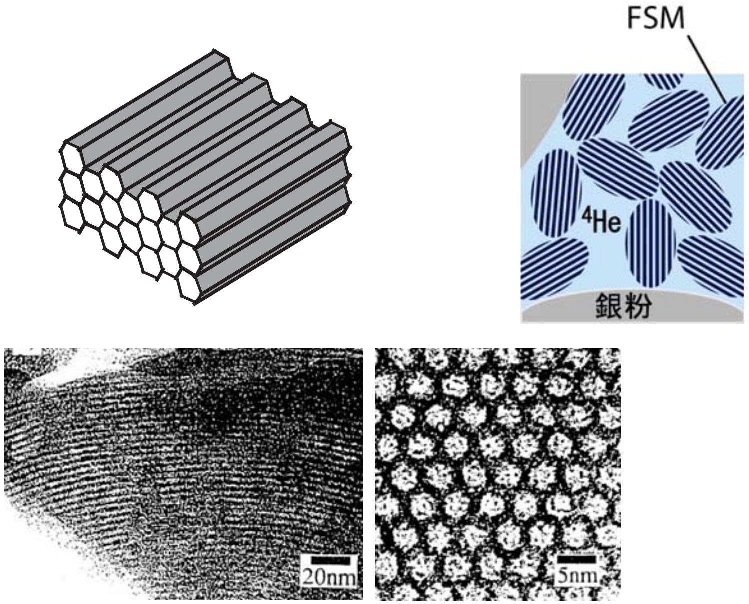


Experiment: DJ Bishop & JD Reppy PRL (1978)
Theory: Ambegaokar, Halperin, Nelson & Siggia PRL (1978)

Superfluidity in 1D & Fisher's criterion

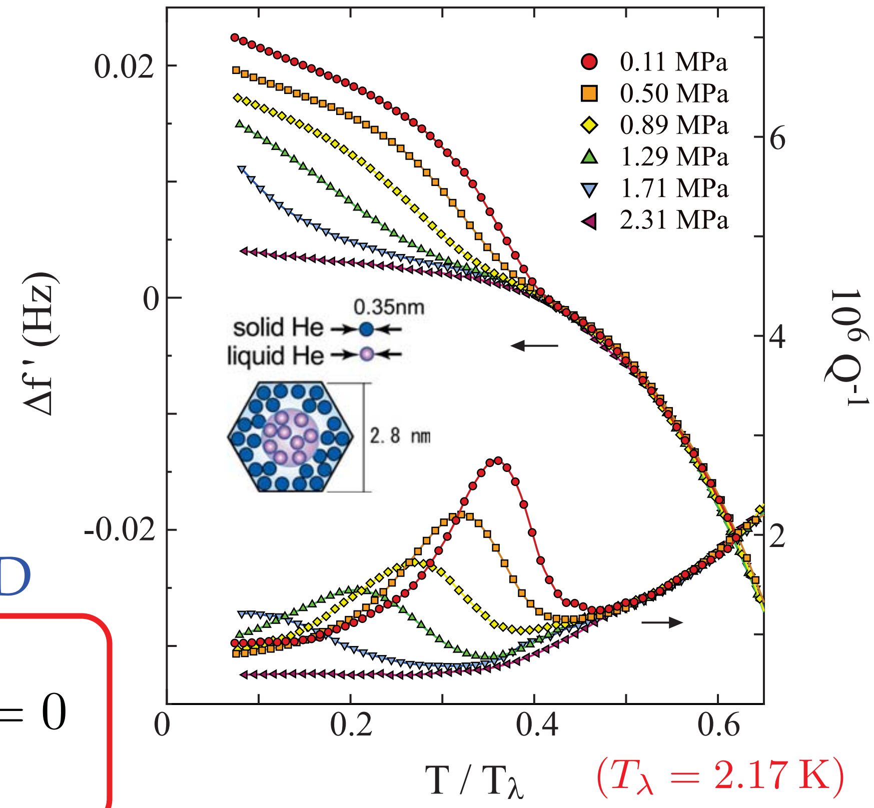
Brought to you by

FSM-16 Pore size: 2.8 nm



TEM of FSM16 (2.8 nm)

J Taniguchi *et al* PRB 2010



$(T_\lambda = 2.17\ \mathrm{K})$

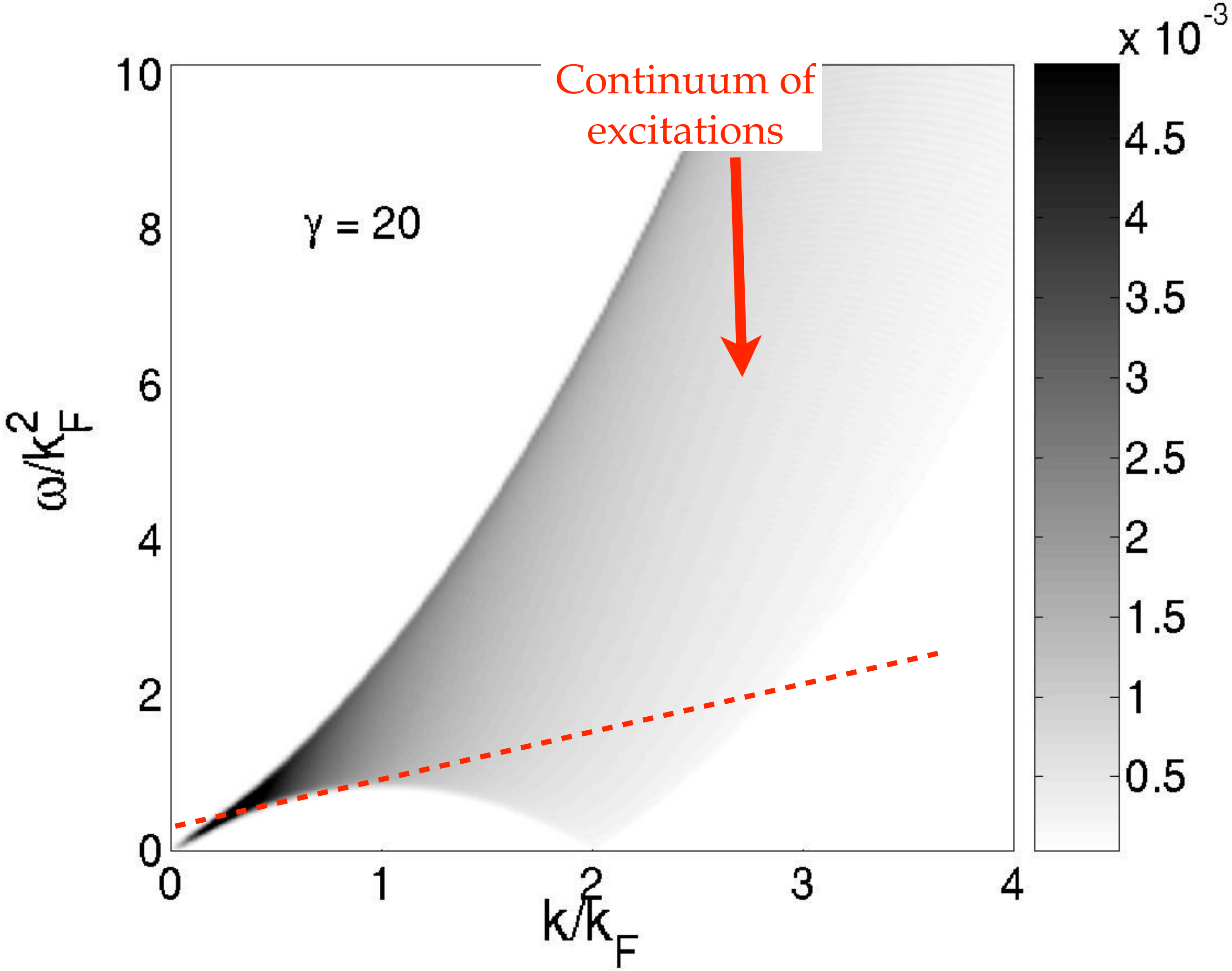
The helicity modulus vanishes in 1D

$$\Upsilon_{1D}(T) = \lim_{L\to+\infty} L\left(\frac{\partial^2 F(\varphi)}{\partial\varphi^2}\right)\Bigg|_{\varphi=0} = 0$$

So what is the origin of this SF signal?

Landau's criterion is violated in 1D

Support of the dynamic structure factor $S(k,\omega)$



[For the Lieb-Liniger model, JS Caux and P Calabrese, PRA(R) (2006)]

Laughlin’s criterion

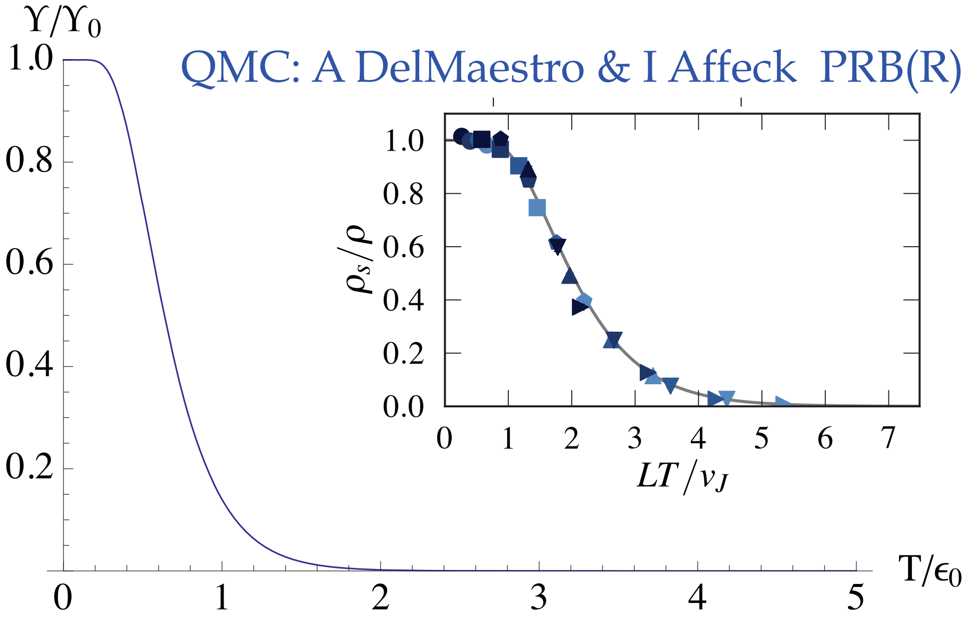
“Superfluidity... it’s like pornography
I can’t define it but I know it when I see it.”

R. B. Laughlin in “Mesoscopic Protectorates”,
talk at KITP (2000)

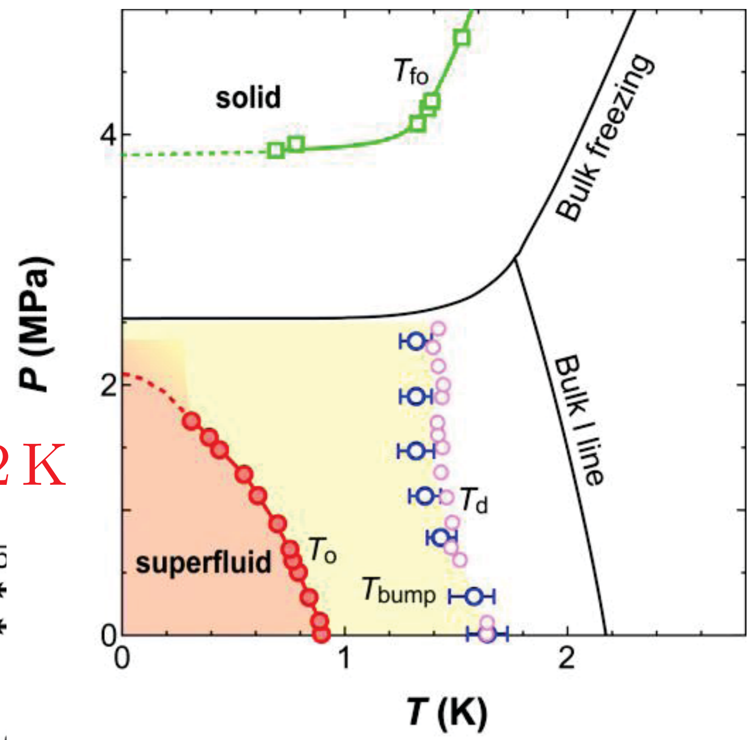
A detective story

Is it a finite size effect?

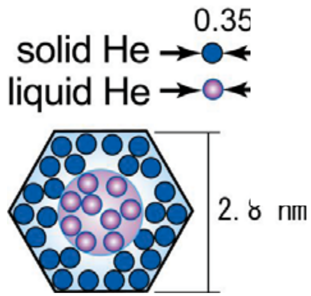
QMC: A DelMaestro & I Affeck PRB(R) (2010)



J Taniguchi *et al* PRB 2010



$$\frac{\epsilon_0}{k_B} = \frac{\hbar v K}{L} < \frac{\hbar^2 \pi \rho_0}{m(^4He)L} = T_{\mathrm{onset}}^{\mathrm{max}} \simeq 0.2\,\mathrm{K}$$

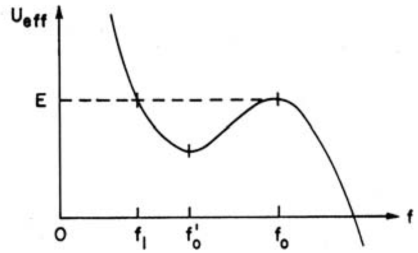


Could it be a dynamical effect? Phase Slips

Thermal Phase Slips (from GL theory)

$$\Gamma_{\mathrm{TPS}} = \Omega(T) e^{-\frac{\Delta F(T)}{k_B T}}$$

$$|T - T_c|/T_c \ll 1$$



Langer-Ambegaokar PR (1967)
McCumber-Halperin PR (1970)

Quantum Phase Slips:

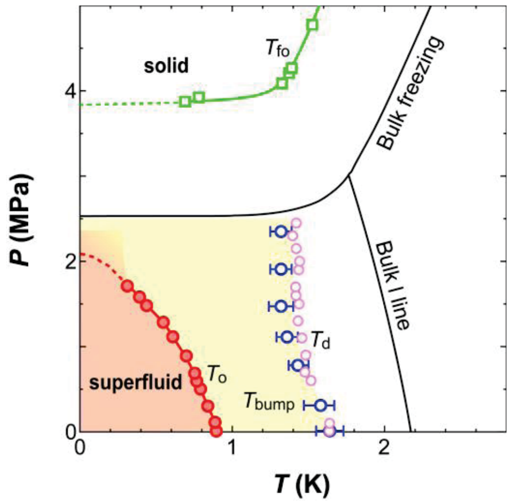
$$\rho = \rho_0 + \delta\rho$$

$$S = \int dx d\tau \left[i\rho\, \partial_\tau \theta + \cdots \right] = \int dx \left[i\rho_0 \partial_\tau \theta + \cdots \right]$$

Non-trivial Berry phase!

$$\Gamma_{\mathrm{QPS}} \sim e^{-\frac{\hbar \pi v \rho_0}{k_B T}}$$

Khlebnikov PRA (2005)

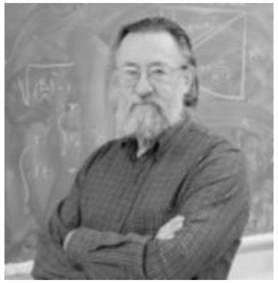


Γ_{PS} should be very small at low temperatures but it is contradicted by the experiment!
J Taniguchi *et al* PRB 2010

Torsional Oscillator (TO)

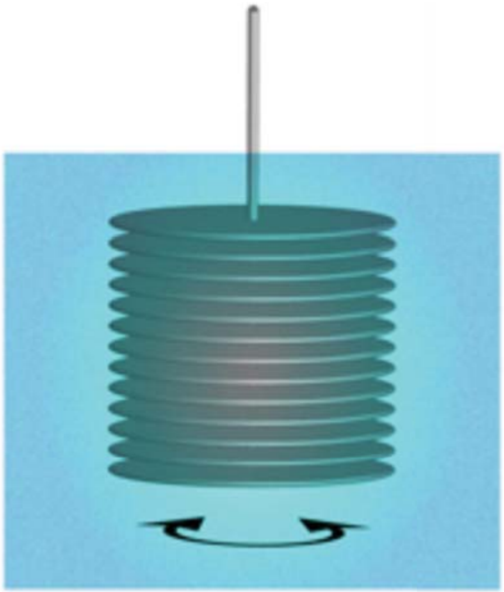
Modern torsional oscillator

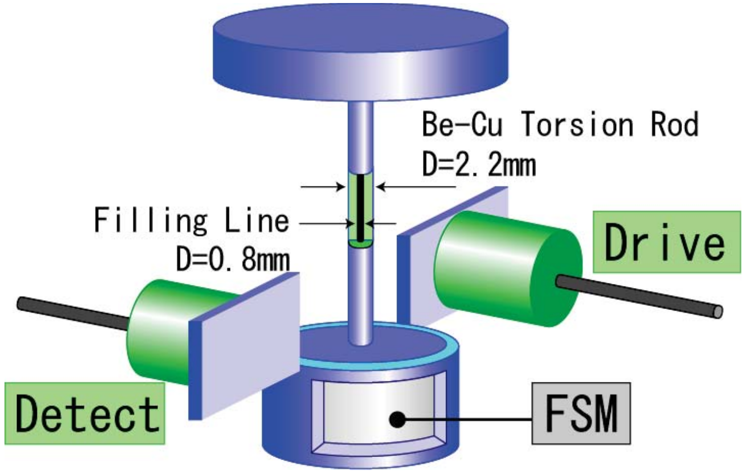
(As devised by JD Reppy)



Andronikashvili's Experiment

(As suggested by Landau)



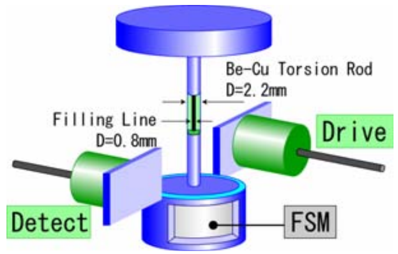


What is probed by the TO?

Rate of change of angular momentum — Torques

$$\frac{d}{dt}\left[I_0\dot{\varphi}(t) + \mathcal{L}_z(t)\right] = -k\varphi(t) - \eta\dot{\varphi}(t) + \tau_{\text{ext}}(t)$$

Empty TO — Normal Fluid — Restoring Torque — Friction — Driving Torque



Linear response theory

$$\mathcal{L}_z(t) = \int d\mathbf{r}(\hat{\mathbf{z}}\times\mathbf{r})\cdot\langle\mathbf{\Pi}(\mathbf{r},t)\rangle$$

$$\mathbf{\Pi}(\mathbf{r}) = \frac{\hbar}{2i}\left[\Psi^\dagger(\mathbf{r})\nabla\Psi(\mathbf{r}) - \nabla\Psi^\dagger(\mathbf{r})\Psi(\mathbf{r})\right]$$

TO response function

$$\chi_n(\omega;T) = \sum_{\mu,\nu}\int d\mathbf{r}d\mathbf{r}'\,(\hat{\mathbf{z}}\times\mathbf{r})_\mu(\hat{\mathbf{z}}\times\mathbf{r}')_\nu\,\chi_{\mu\nu}(\mathbf{r},\mathbf{r}',\omega;T)$$

$$\chi_{TO}^{-1}(\omega) = \omega^2\left[I_0 - \boxed{\chi_n(\omega;T)}\right] - k + i\eta\omega.$$

$$\delta\omega(T) \Leftrightarrow \text{Re}\,\chi_n(\omega_0;T)$$

$$\delta Q^{-1}(T) \Leftrightarrow \text{Im}\left[-\chi_n(T;\omega_0)\right]$$

$$\omega_0 \approx 2000\text{ Hz}$$

T Eggel, MAC & M Oshikawa
arxiv:1104.0175 (2011)

Momentum Response in d > 1

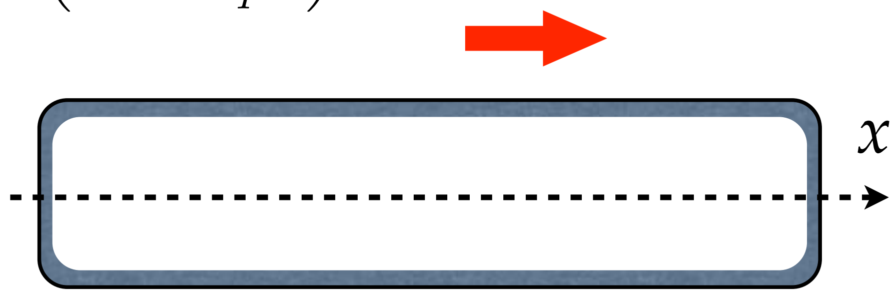
Response to the moving walls of the container

$$\chi_{\mu\nu}(\mathbf{q},\omega) = -\frac{i}{\hbar}\int_0^{+\infty} dt \int d\mathbf{r}\, e^{i(\omega t - i\mathbf{q}\cdot\mathbf{r})}\,\langle[\Pi_\mu(\mathbf{r},t),\Pi_\nu(\mathbf{0},0)]\rangle$$

$$\chi_{\mu\nu}(q,\omega) = \left(\frac{q_\mu q_\nu}{q^2}\right)\overbrace{\chi_L(q,\omega)}^{\text{Longitudinal}} + \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\overbrace{\chi_T(q,\omega)}^{\text{Transverse}}$$

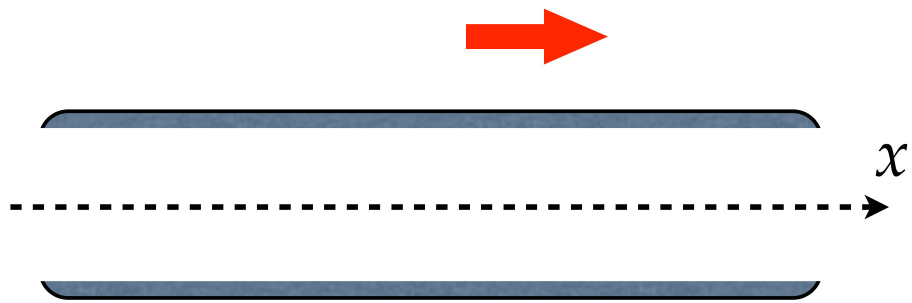
Longitudinal response

$$\lim_{q_x\to 0}\ \lim_{q_y,q_z\to 0} \chi_{xx}(q,\omega=0) = -m\rho_0$$



Transverse response

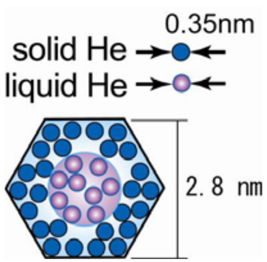
$$\lim_{q_y,q_z\to 0}\ \lim_{q_x\to 0} \chi_{xx}(q,\omega=0) = -m\rho_n(T)$$

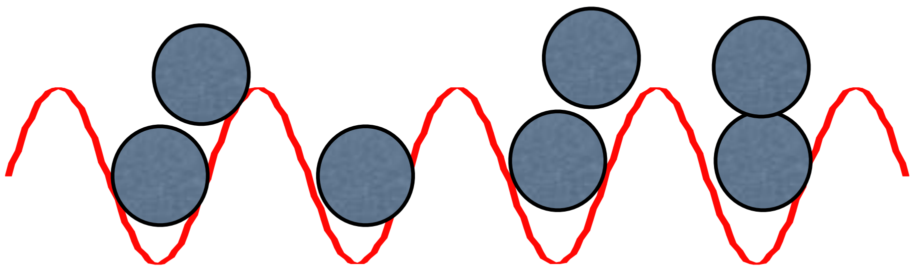


Momentum Response in d = 1

$$\chi(x,t) = -\frac{i}{\hbar}\theta(t)\left\langle[\Pi(x,t),\Pi(0,0)]\right\rangle$$

- No obvious transverse - longitudinal separation
- Depends on the wall-fluid potential





$$H(t) = H_0 + \sum_{i=1}^{N} V_{\rm ext}(x_i - X(t))$$

$$H'(t) = H_0 + \sum_{i=1}^{N} V_{\rm ext}(x_i) - \dot{X}(t)\Pi$$

Calculation of momentum response akin to conductivity

T Eggel, MAC & M Oshikawa arxiv:1104.0175 (2011)

Harmonic Fluid Description

FDM Haldane PRL (1981)

RG fixed point Hamiltonian

$$H_* = \sum_{q\neq 0} \hbar v|q|\, b^\dagger(q)b(q) + \cdots \frac{\hbar v}{2\pi}\int dx\left[K^{-1}\left(\partial_x\phi\right)^2 + K\left(\partial_x\theta\right)^2\right] = \int dx\, \epsilon(x)$$

$$P = \sum_{q\neq 0} \hbar q\, b^\dagger(q)b(q) + \cdots = \frac{\hbar}{\pi}\int dx\, \partial_x\phi\partial_x\theta = \frac{1}{v^2}\int dx\, j_\epsilon(x)$$ $\propto$ Energy current

$$J = \frac{mvK}{\pi}\int dx\, \partial_x\theta(x,t) = \int dx\, j(x,t)$$ Particle mass current

Momentum current
(including the leading irrelevant operator)

$$\Pi = J + \frac{vK}{v_F}P$$

J and P separately conserved by the fixed-point Hamiltonian

$$[H_*, J] = [H_*, P] = 0$$

Phase slips and Memory matrix

Phase Slips (for a periodic wall potential) Leading irr. operators

$$H_{PS} = \sum_{n>0,m} \frac{\hbar v g_{nm}}{\pi a_0^2} \int dx \cos\left(2n\phi(x) + \Delta k_{nm} x\right) \qquad \Delta k_{mn} = (2n\pi\rho_0 - 2mG)\hbar$$

$$[H_{PS}, J] \neq 0 \qquad [H_{PS}, P] \neq 0$$

J and P become coupled and acquire a finite decay rate

$$\chi(\omega;T) = \mathrm{Tr}\left\{V\left[\omega\mathbf{1} + iM(\omega;T)\right]^{-1} iM(\omega;T)\hat{\boldsymbol{\chi}}(T)\right\}$$

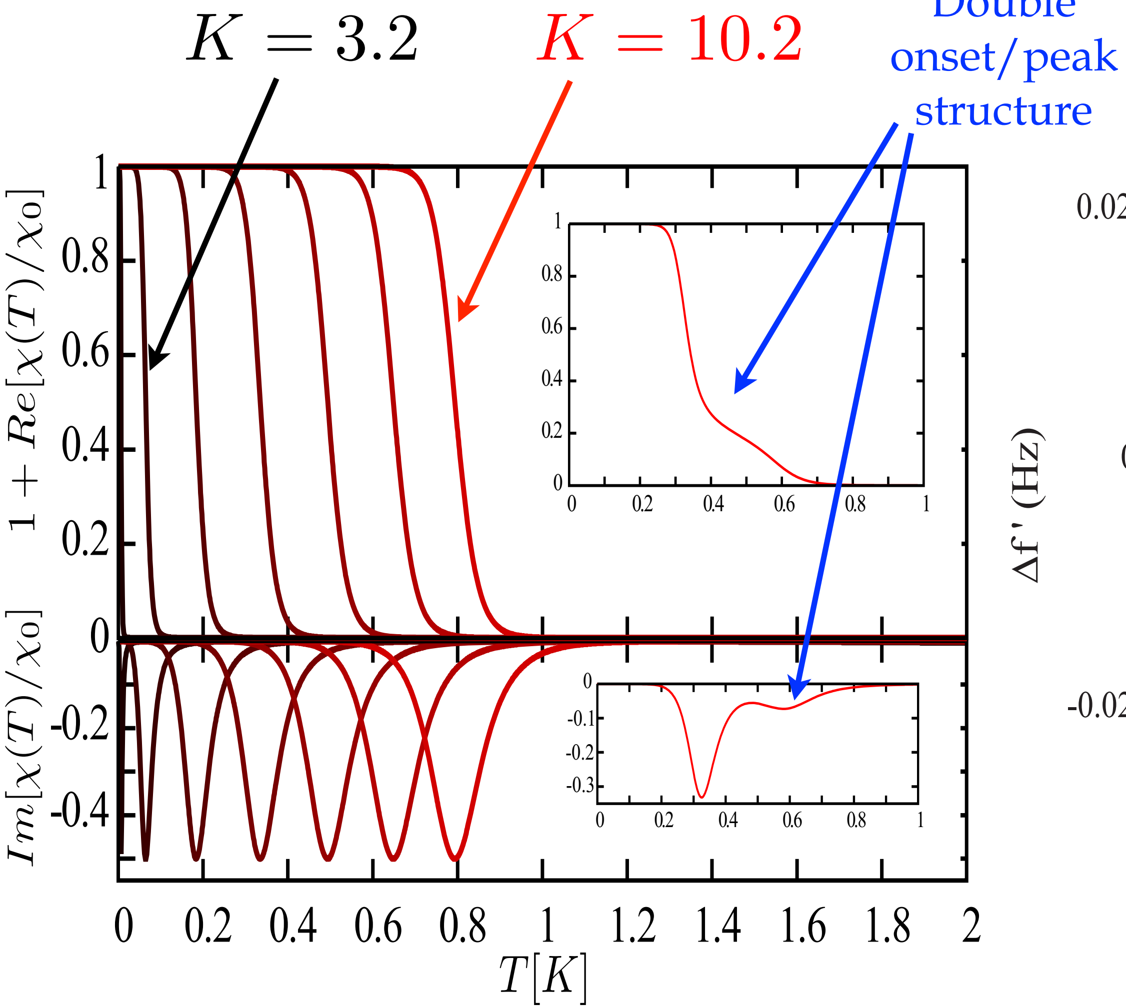
$$\hat{\boldsymbol{\chi}}(T) \simeq \mathrm{diag}\{\chi_{JJ}, \chi_{PP}(T)\} = -\mathrm{diag}\{\frac{M^2 vK}{\hbar\pi}, \frac{\pi(k_B T)^2}{6\hbar v^3}\} + \cdots$$

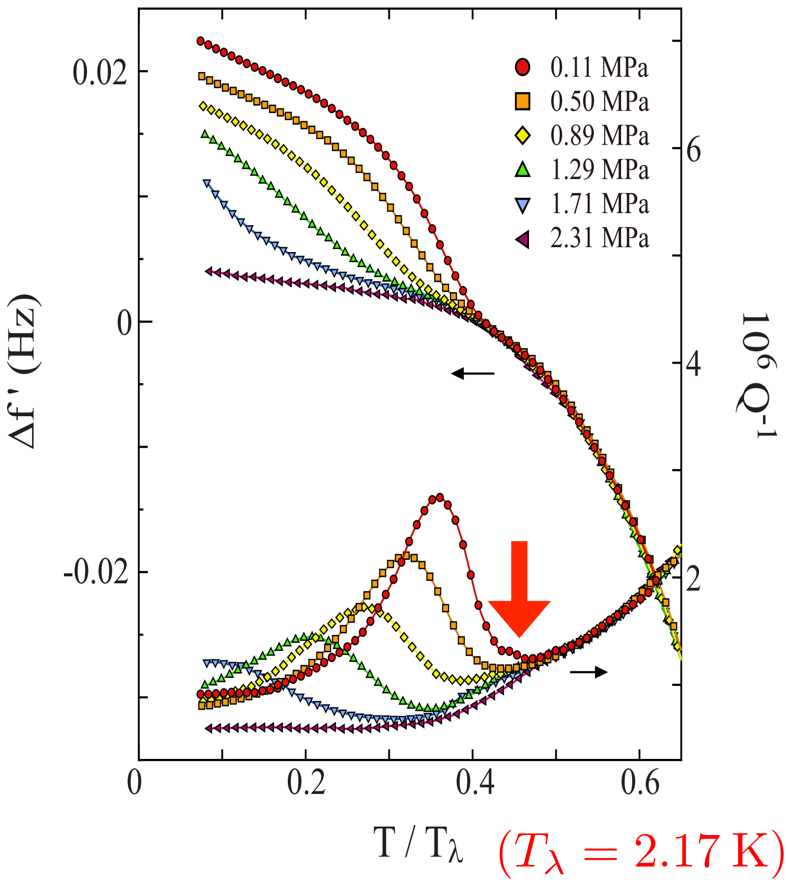
$M(\omega, T)$ is a 2 x 2 matrix whose eigenvalues are the decay rates (it can be evaluated perturbatively in H_{PS})

D Forster "Hydrodynamic fluctuations, broken symmetry, and correlation functions", W. A. Bejamin (1975)
A Rosch and N Andrei PRL (2005)

Results

Luttinger parameter dependence (Compressibility $\propto K$)



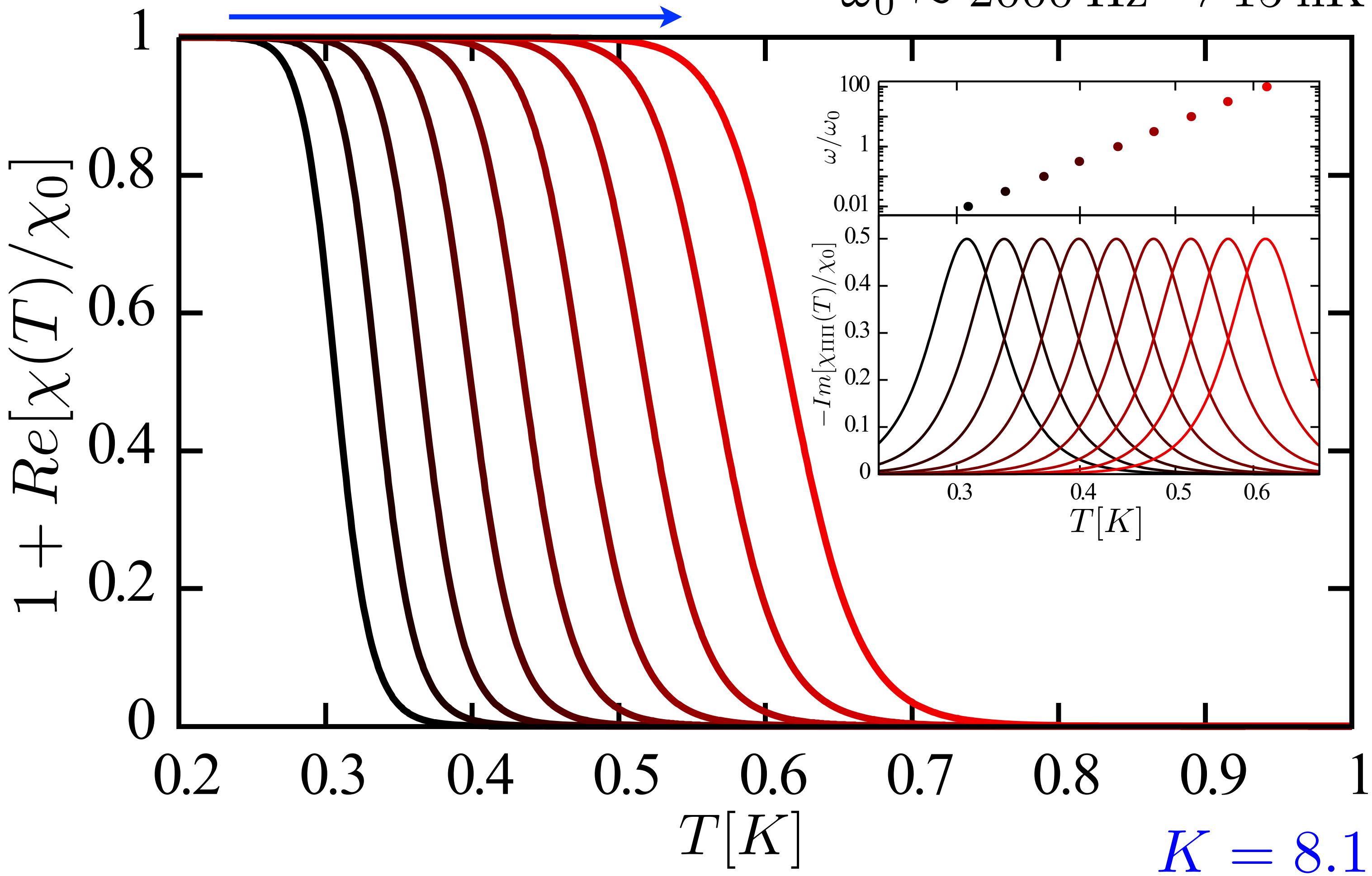


Prediction: Frequency dependence

Simplified model $\chi(\omega_0; T) \approx -\frac{|\hat{\chi}(T)|}{1 - i\omega_0 \tau(T)}$ $\quad \tau(T_{\text{onset}}) \sim \frac{1}{\omega_0}$

$\omega/\omega_0 \uparrow$

$\omega_0 \approx 2000\ \text{Hz} \rightarrow 15\ \text{nK}$

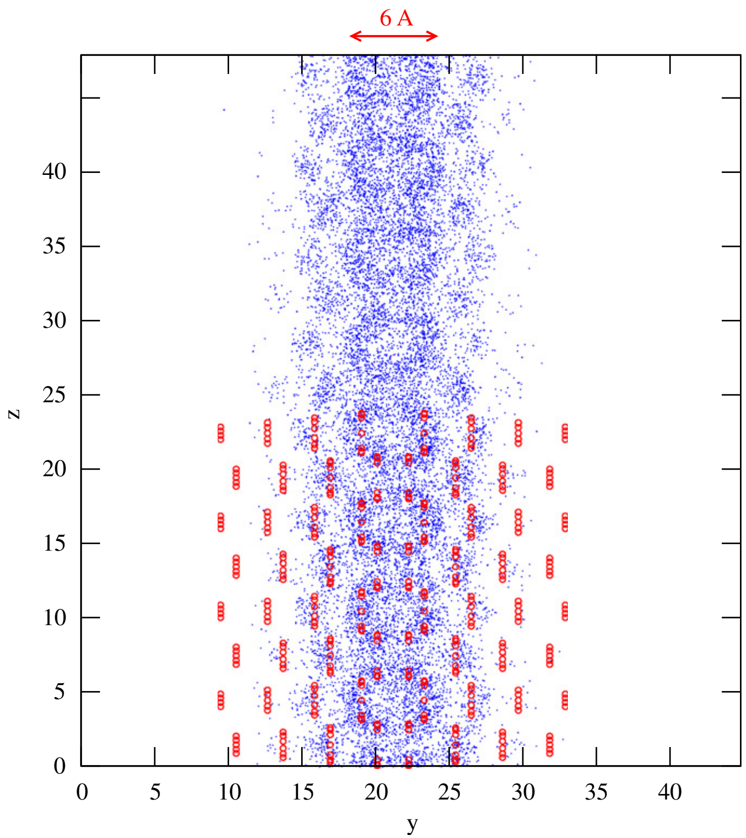


$K = 8.1$

Further applications: Super-Solid ^{4}He?

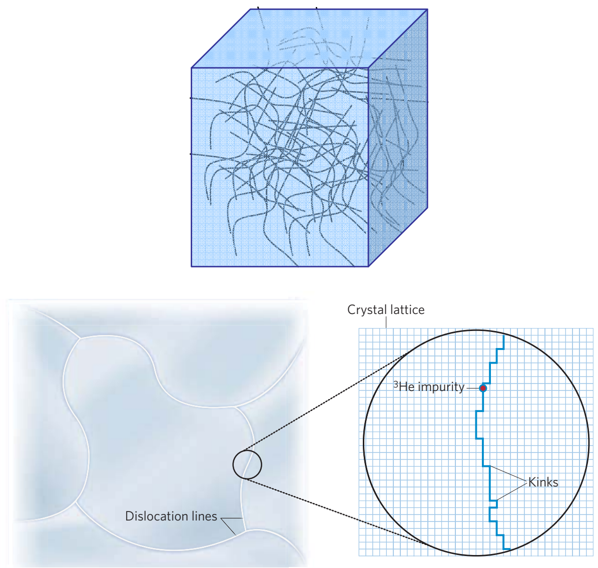
Skew dislocations behave as Tomonaga-Luttinger liquids

Luttinger parameter (QMC) $K \simeq 5$



M Boninsegni *et al* PRL (2008)

Dislocation network
(Shevchenko state)



S Balibar *Nature* (2010)

Conclusions

- The helicity modulus in 1D vanishes
- Superfluidity is a dynamical effect in 1D
- Importance of Phase slips
- Importance of coupling between particle and energy currents
- Possible relevance to the Shevchenko state in solid ^{4}He
- Possible relevance to ultracold atoms