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2239-5

Workshop on Integrability and its Breaking in Strongly Correlated and Disordered Systems

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What is hiding beneath 1D Fermi surface?

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in collaboration with

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PRL 100, 206805 (2008), **Science** 323, 228 (2009), **PRL** 102, 126405 (2009), **PRL** 104, 116403 (2010), **Phys. Rev. B** 82, 245104 (2010), **arXiv**:1010.2268, **arXiv**:1103.4176

Outline

Nonlinear spectrum + interaction in 1D?

- Why bother? Qualitative considerations
- Beyond the low-energy limit
- ”Non-universal” prefactors and finite size size scalings
- Spin-Charge separation away from Fermi points
- Conclusions and Outlook

1D Hamiltonian and Correlators

1D fermionic Hamiltonian, zero temperature:

$$H = \sum \left(\frac{k^2}{2m} - \mu \right) \Psi_k^+ \Psi_k + \frac{1}{2} \sum V_q \Psi_{k+q}^+ \Psi_{k'-q}^+ \Psi_{k'} \Psi_k$$

Spectral function (particle part)

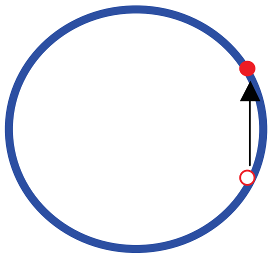
$$A_p(p,\omega) \propto \sum \left| \left\langle e \middle| \Psi^+(0,0) \middle| g \right\rangle \right|^2 \delta(p_e - p) \delta(E_{eg} - \omega)$$

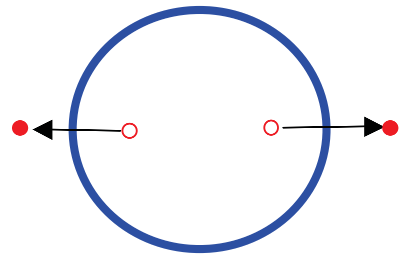
$$\left\langle \Psi(x,t) \Psi^+(0,0) \right\rangle = \int \frac{dp\,d\omega}{(2\pi)^2} e^{i(px - \omega t)} A_p(p,\omega)$$

Kinematics in higher dimensions and 1D

Higher dimensions – excitations with arbitrary positive energy and any momentum can be created

Fermi sea





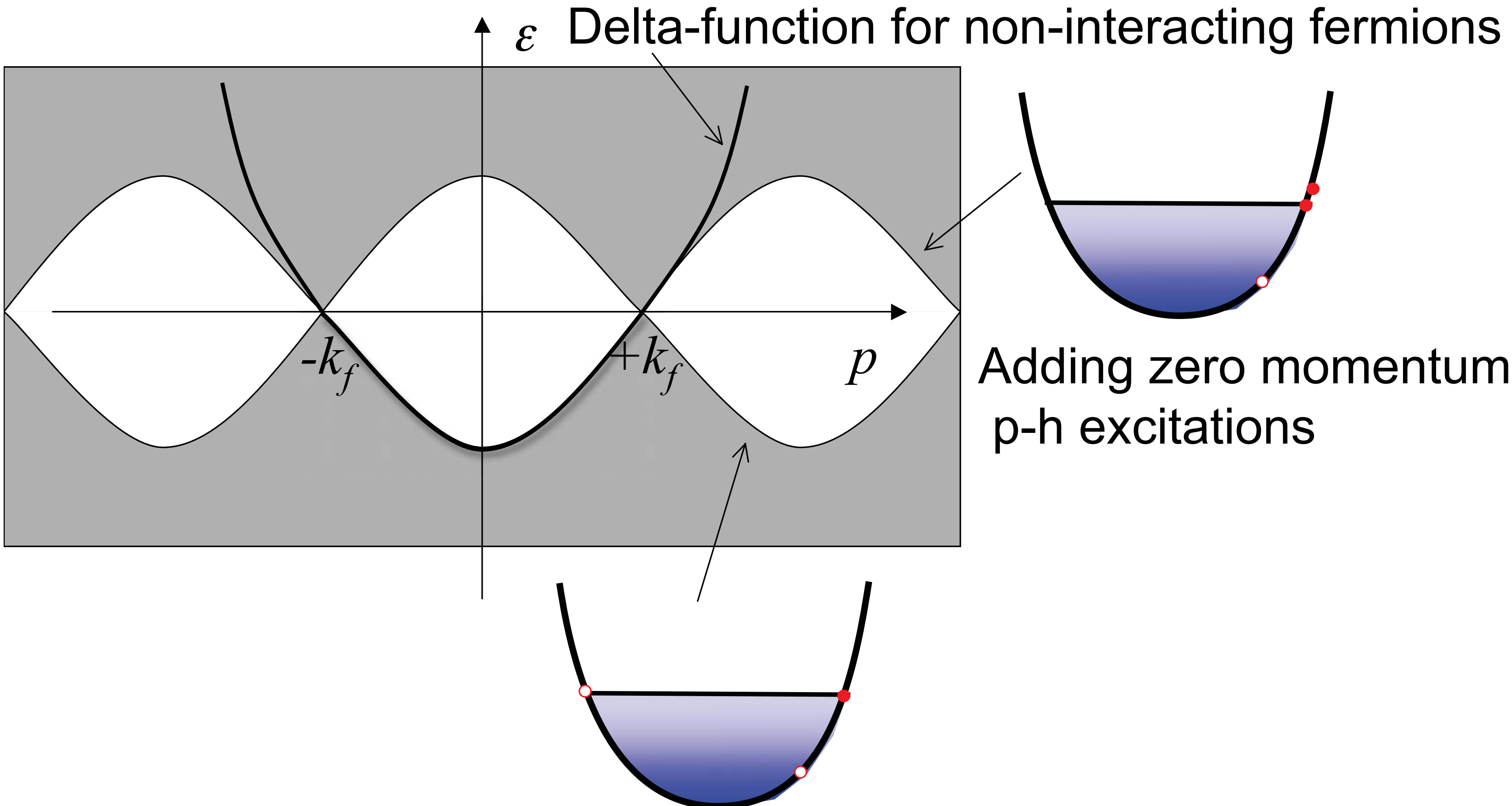
Zero energy, any momentum

any energy, zero momentum

1D: NO YES

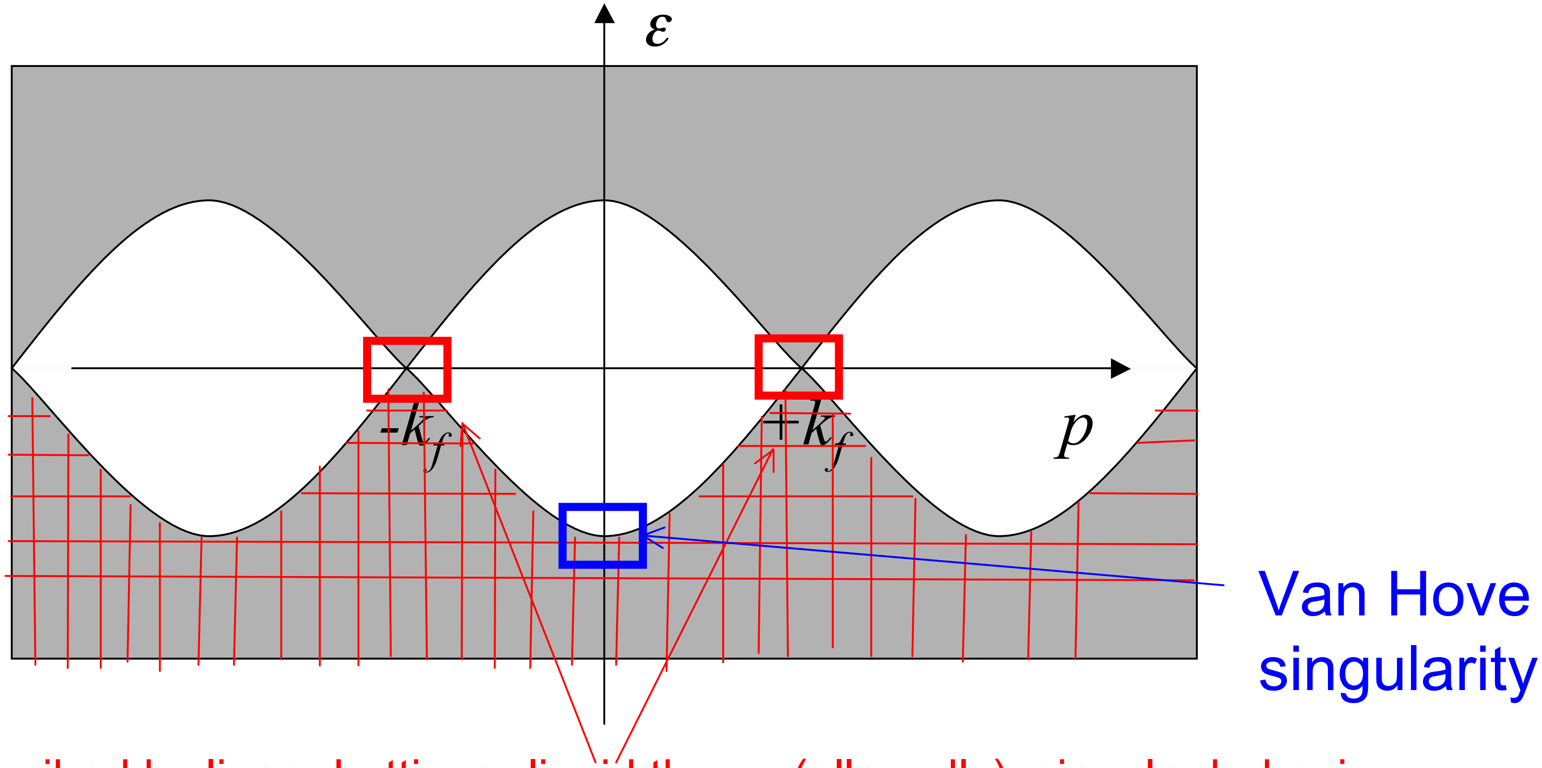
Zero energy, $2k_F$ momentum

Spectral function support in 1D



Spectral function support has sharp edges,
which are kinematically protected!

Large distance/time correlations



Described by linear Luttinger liquid theory (allegedly), singular behavior

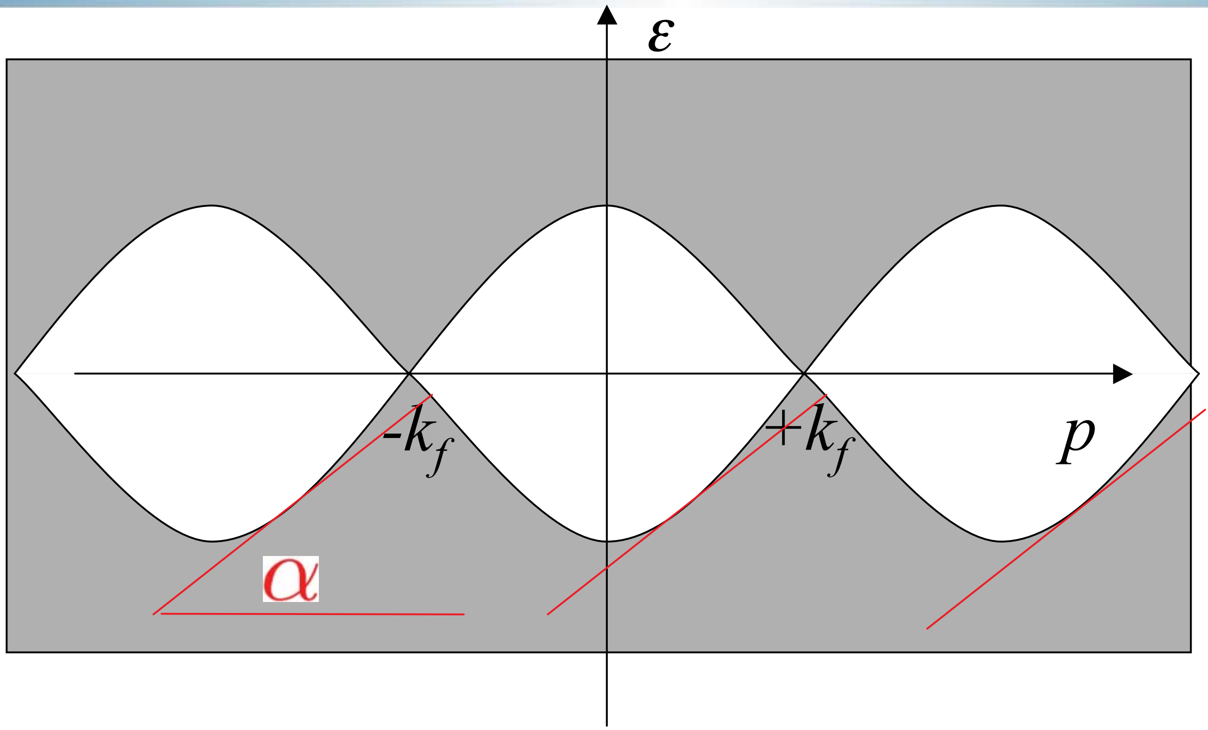
x=0: $\left\langle \Psi^{+}(0,t)\Psi(0,0)\right\rangle = \int \frac{d\omega}{2\pi} e^{-i\omega t} \int \frac{dp}{2\pi} A_h(p,\omega) =$

t=0: $\left\langle \Psi^{+}(x,0)\Psi(0,0)\right\rangle = \int \frac{dp}{2\pi} e^{ipx} \int \frac{d\omega}{2\pi} A_h(p,\omega) =$

$= \int \frac{d\omega}{2\pi} e^{-i\omega t} n(\omega)$ $\qquad$ $= \int \frac{dp}{2\pi} n(p) e^{ipx}$

$$\left\langle \Psi^{+}(x,t)\Psi(0,0)\right\rangle = \int \frac{dp\,d\omega}{(2\pi)^2} e^{i(px-\omega t)} A_h(p,\omega)$$

Large distance/time correlations

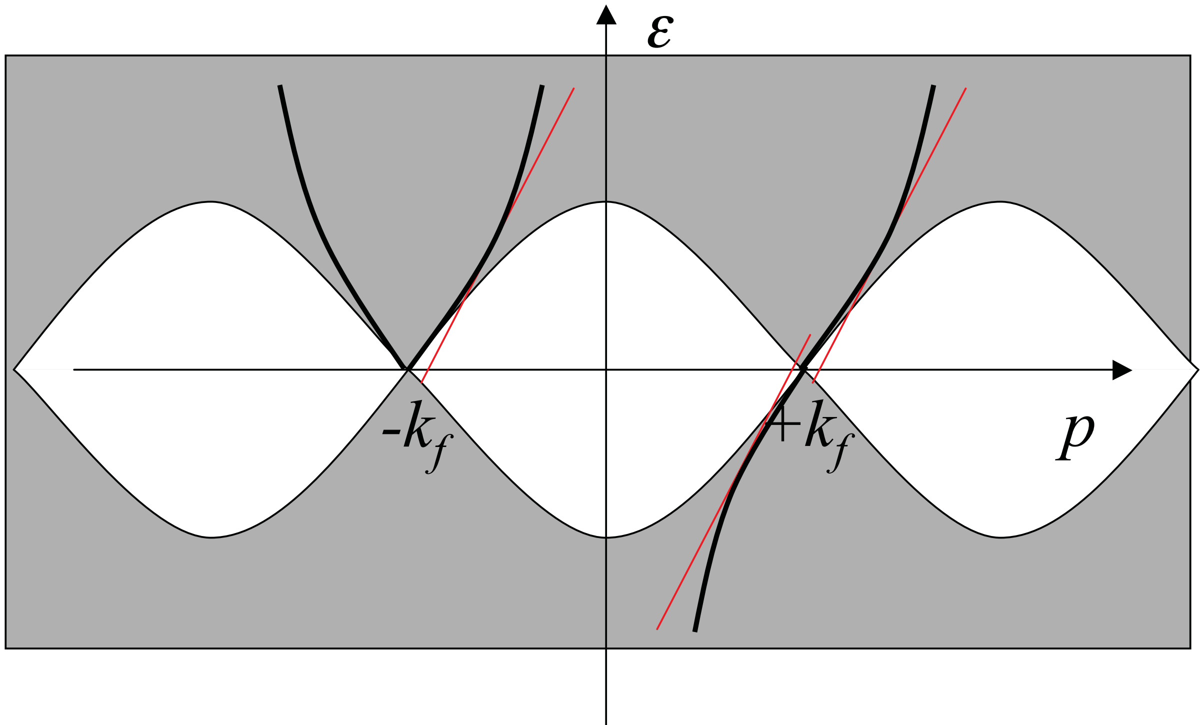


$\tan\alpha$ =x/t=fixed<sound velocity, time is sent to infinity: new power law terms

For nonint. fermions: $\propto 1/\sqrt{t}$ vs $1/t$ within linear theory

Large distance or time $\neq$ low energy

Integrable systems?



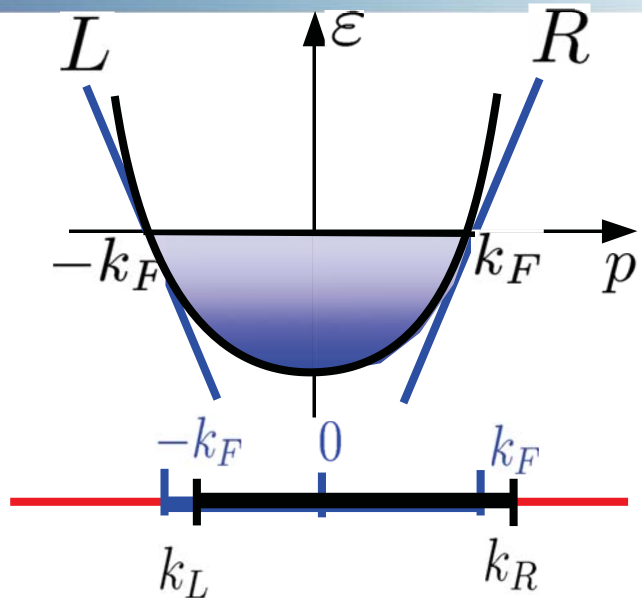
Singularities of response functions can exist within a continuum*

↓↓

New power laws for x/t>sound velocity

*proved for Lieb-Liniger model in **arXiv**:1010.2268

Linear Luttinger liquid theory



local current $\partial_x \theta \propto \delta k_R + \delta k_L$

local density $-\partial_x \varphi \propto \delta k_R - \delta k_L$

Linear hydrodynamic Hamiltonian:

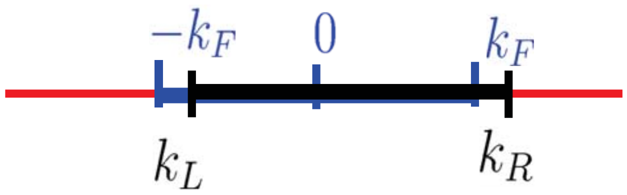
$$H_0 = \frac{\mathrm{v}}{2\pi} \int dx \left[K \left(\partial_x \theta \right)^2 + \frac{1}{K} \left(\partial_x \varphi \right)^2 \right]$$

v sound velocity

$K = \frac{\mathrm{v}_F}{\mathrm{v}}$

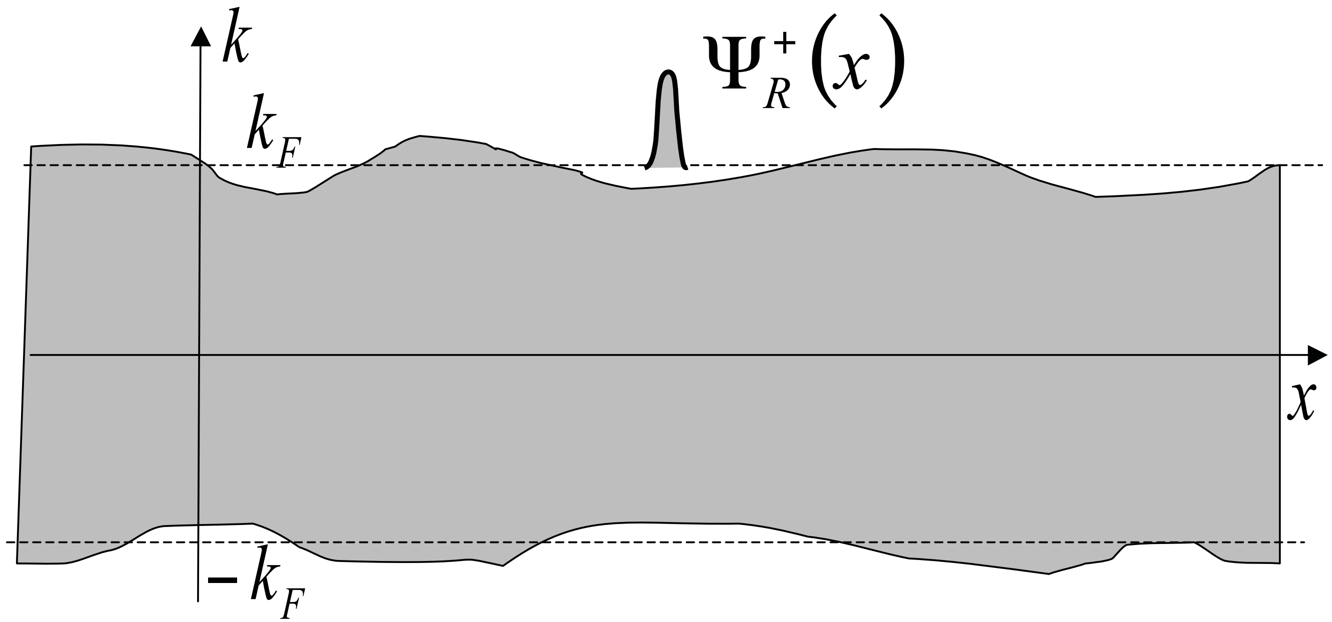
Luttinger Liquid parameter
For Galilean systems depends only on velocity renormalization, but controls correlation functions

Fermion in bosonization



local current $\partial_x \theta \propto \delta k_R + \delta k_L$

local density $-\partial_x \varphi \propto \delta k_R - \delta k_L$



Fermionic field: $$\Psi_R^+(x) \propto e^{i\varphi(x) - i\theta(x)}$$

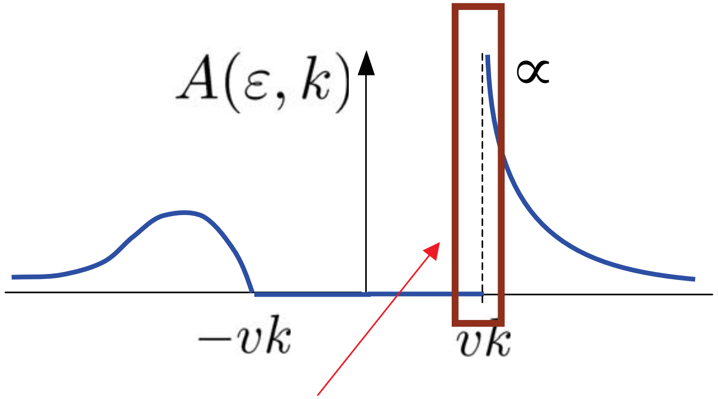
Analogy: single particle translation operator $$T = e^{ipa}$$

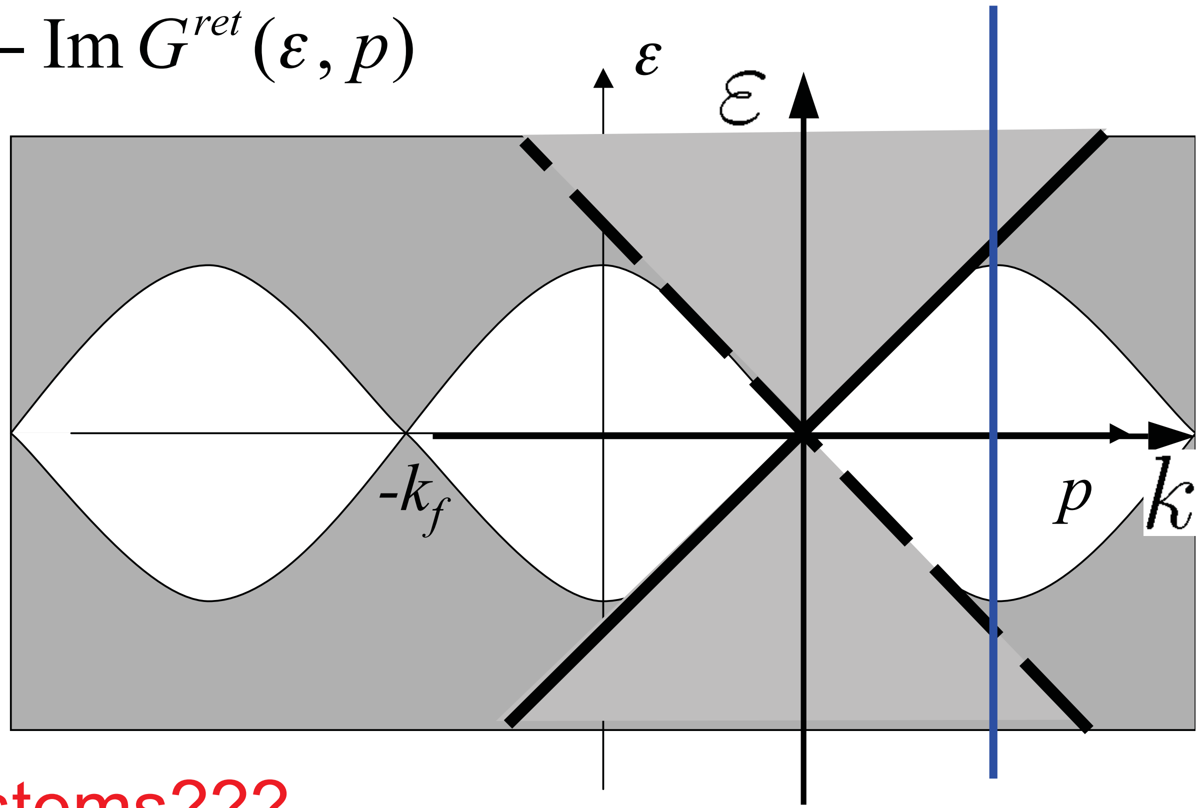
Spectral function in linear LLs

Fermionic field: $\Psi_R^+ \propto e^{i\varphi - i\theta}$

Spectral function: probability to tunnel in a particle (hole)

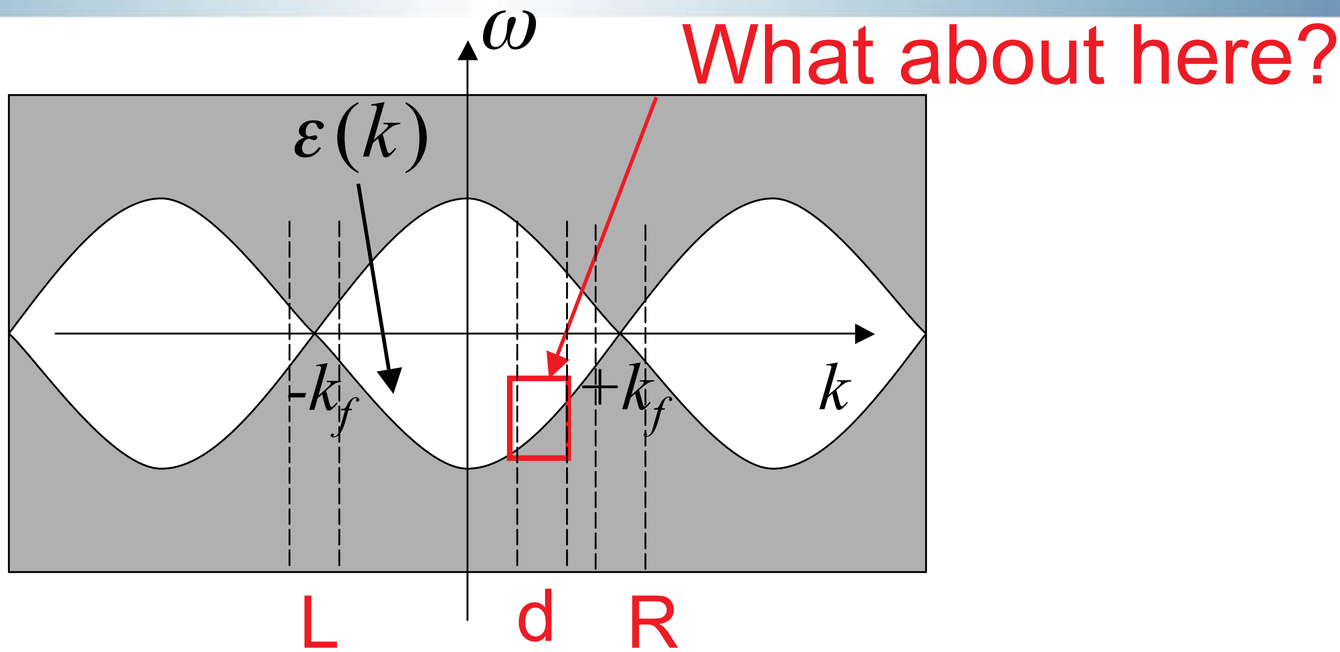
$$A(\varepsilon, p) \propto -\operatorname{Im} G^{ret}(\varepsilon, p)$$





Is it true for real systems???

Fermion beyond low energy limit



$$\Psi(x) \sim \underbrace{\Psi_R(x)e^{ik_F x} + \Psi_L(x)e^{-ik_F x}}_{\text{Bosonize!}} + \underbrace{e^{ikx}d(x)}_{\text{Effective mobile quantum impurity}}$$

Bosonize!

Effective mobile quantum impurity

M. Pustilnik et al, **PRL** 96, 196405 (2006)
R.G. Pereira et al, **PRL** 100, 027206(2008);V.V. Cheianov and M.Pustilnik, **PRL** 100, 126403 (2008).

Hamiltonian and Correlators

$$\Psi(x) \sim \Psi_R(x)e^{ik_F x} + \Psi_L(x)e^{-ik_F x} + e^{ikx}d(x)$$

Effective mobile quantum impurity Hamiltonian:

$$H_d = \int dx\, d^+(x)\left[\varepsilon(k) - i\frac{\partial \varepsilon}{\partial k}\nabla\right]d(x)$$

$$H_{\mathrm{int}} = \int dx \left[V_R \nabla \frac{\theta - \varphi}{2\pi} - V_L \nabla \frac{\theta + \varphi}{2\pi}\right] d(x) d^+(x)$$

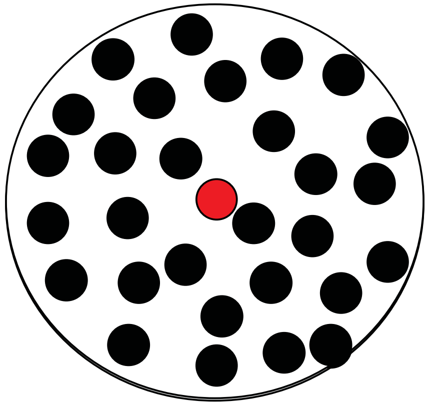
Correlation functions(e.g. spectral function):

$$A(k,\omega) \sim \iint dxdt e^{i\omega t}\left\langle d^+(x,t)d(0,0)\right\rangle_{H_0 + H_d + H_{\mathrm{int}}}$$

Analogy to Anderson’s orthogonality catastrophe (albeit with a mobile impurity), essentially a free fermionic problem.
Use Nozieres - De Dominicis solution, bosonization, boundary condition changing conformal operators, etc.

Anderson's orthogonality catastrophe

Single localized impurity in a Fermi gas (1967)



$$kR = \pi n$$

$$k'R + \delta = \pi n$$

$$\left|\langle \Psi(t) | \Psi(0) \rangle\right|^2 \propto \left|\frac{1}{E_F t}\right|^{\left(\frac{\delta}{2\pi}\right)^2} \rightarrow 0$$

Phase shifts define the exponents. In our case

$$A(k,\omega) \propto \theta\left(\varepsilon(k) - \omega\right)\left|\frac{1}{\varepsilon(k) - \omega}\right|^{1-\left[\frac{\delta_-(k)}{2\pi}\right]^2 - \left[\frac{\delta_+(k)}{2\pi}\right]^2}$$

Phenomenology beyond low energies

Momentum dependent phase shifts

For Galilean systems related to

$$\delta_-(k), \delta_+(k) \quad \longleftrightarrow \quad \frac{\partial \varepsilon(k)}{\partial k}, \frac{\partial \varepsilon(k)}{\partial \rho}$$

$$\frac{\delta_\pm(k)}{2\pi} = \frac{\frac{1}{\sqrt{K}}\left(\frac{k}{m} - \frac{\partial \varepsilon}{\partial k}\right) \pm \sqrt{K}\left(\frac{1}{\pi}\frac{\partial \varepsilon}{\partial \rho} + \frac{\mathrm{v}}{K}\right)}{2\left(\pm \frac{\partial \varepsilon}{\partial k} - \mathrm{v}\right)}$$

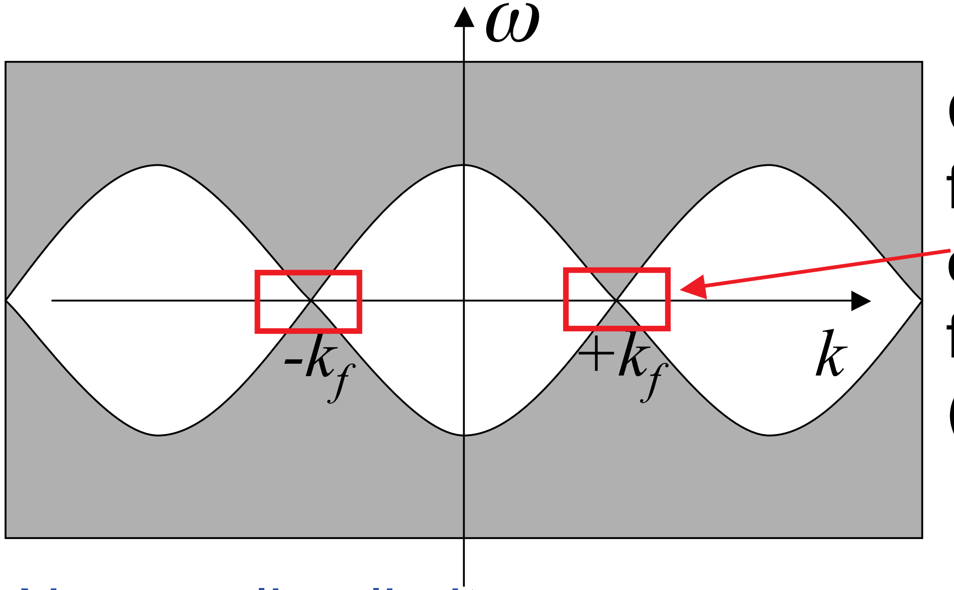
Position of the edge defines the singularities!

For integrable models, can fix phase shifts by other means as well: Pereira et al, Cheianov et al, Imambekov et al, Zvonarev et al, Essler

Checks for Lieb-Liniger, Calogero-Sutherland, Yang-Gaudin, Cheon-Shigehara, perturbation theory, finite size corrections, etc.

PRL 102, 126405 (2009)

Universal nonlinear Luttinger liquid



Can do better here,crossover from linear to nonlinear theory can be fully described by free fermionic quasiparticle theory (Mattis&Lieb'65; Rozhkov'05)

Science 323, 228 (2009)

New scaling limit

$$\frac{\varepsilon - \mathrm{v}\,k}{k^2/2m_*} \to \text{const} \qquad \frac{k}{k_F} \to 0$$

Universal crossover function

$$A(\varepsilon, k) \propto A\left(\frac{\varepsilon - vk}{k^2/2m_*}\right)$$

Universal phase shifts

$$\frac{\delta_+}{2\pi} = 1 - \frac{1}{2\sqrt{K}} - \frac{\sqrt{K}}{2} \qquad \frac{\delta_-}{2\pi} = \frac{1}{2\sqrt{K}} - \frac{\sqrt{K}}{2}$$

What about "non-universal" prefactors?

Let us warm up on linear Luttinger liquids:

$$\frac{\langle\hat\rho(x)\hat\rho(0)\rangle}{\rho_0^2} \approx 1 - \frac{K}{2(\pi\rho_0 x)^2} + \sum_{m\geq 1} \frac{A_m \cos(2mk_F x)}{(\rho_0 x)^{2m^2 K}},$$

$$\frac{\langle\hat\psi_F^\dagger(x)\hat\psi_F(0)\rangle}{\rho_0} \approx \sum_{m\geq 0} \frac{C_m \sin\left[(2m+1)k_F x\right]}{(\rho_0 x)^{(2m+1)^2 K/2 + 1/(2K)}}.$$

Finite size quantization of Lehmann representation using conformal invariance:

$$\langle\hat\psi_F^\dagger(x,t)\hat\psi_F(0)\rangle = \sum_{k,\omega} e^{i(kx-\omega t)} \left|\langle k,\omega|\hat\psi_F|N\rangle\right|^2$$

$$\left|\langle m, N-1|\hat\psi_F|N\rangle\right|^2 \approx \frac{C_m \rho_0}{2(-1)^m}\left(\frac{2\pi}{\rho_0 L}\right)^{\frac{(2m+1)^2K^2+1}{2K}}$$

Finite size field theory + a single matrix element (formfactor)

$\downarrow\downarrow$

"Non-universal" prefactors

arXiv:1010.2268, **arXiv**:1103.4176

Prefactors in response functions

Finite size quantization of effective impurity Hamiltonians:

$$A(k,\omega) = A_{0,-}(k) \int dx dt e^{i\delta\omega t} D(x,t) L(x,t) R(x,t),$$

$$L(R)(x,t) = (i(vt \pm x) + 0)^{-\left[\frac{\delta_{\mp}}{2\pi}\right]^2},$$

$$D(x,t) = \delta(x - v_d t) \to \sum_{n_D} e^{2i\pi n_D (x - v_d t)/L}$$

↓↓

At fixed k, sum of two incommensurate frequency ladders

$$A(k,\omega) = \sum_{n_\pm \geq 0} \delta\left(\delta\omega - \Delta E - \frac{2\pi n_+}{L}(v - v_d) - \frac{2\pi n_-}{L}(v + v_d)\right) \times$$

$$A_{0,+}(k) \frac{(2\pi)^{2-\mu_{0,-}}}{L^{1-\mu_{0,-}}} C\left(n_+, \left[\frac{\delta_+}{2\pi}\right]^2\right) C\left(n_-, \left[\frac{\delta_-}{2\pi}\right]^2\right),$$

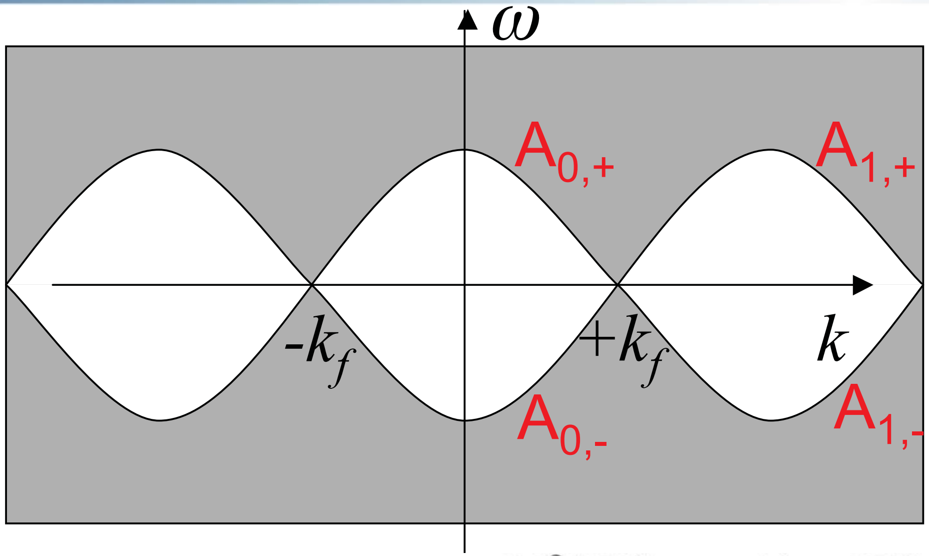
$$C(n,\mu) = \frac{\Gamma(\mu+n)}{\Gamma(\mu)\Gamma(n+1)}.$$

Finite size field theory + a single matrix element (formfactor)

↓↓

“Non-universal” prefactors

Perturbative calculation of prefactors



Finite size gaps allow to use "Landau-Lifshitz"-type perturbation series, e.g.

$$|N\rangle = |\mathrm{FS}\rangle + \sum_{|\alpha\rangle} \frac{\langle\alpha|\hat{V}|\mathrm{FS}\rangle}{E_{\mathrm{FS}} - E_\alpha}|\alpha\rangle + \ldots$$

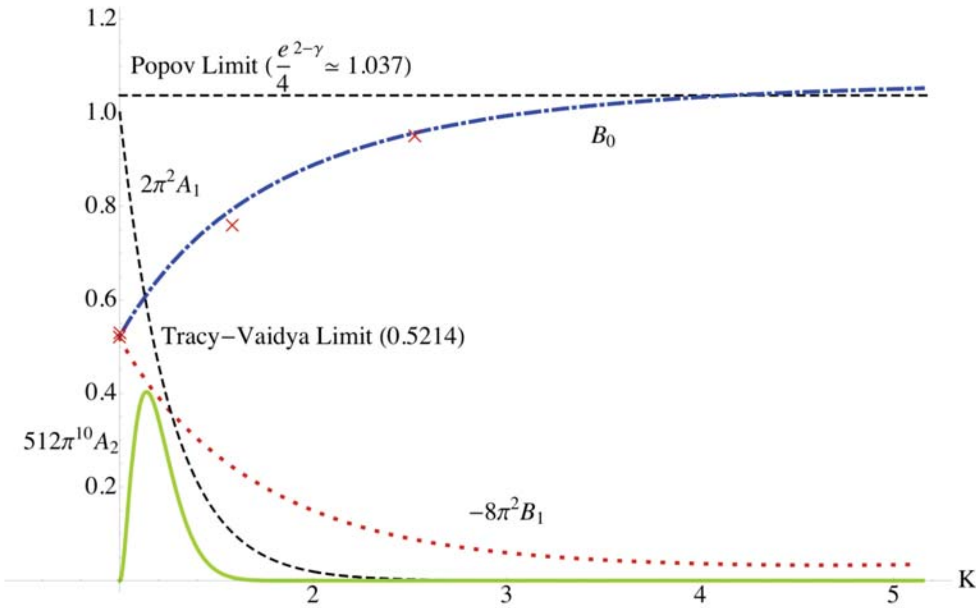
$$\begin{aligned}
A_{0,+}(k) &= \frac{M^2(V(k_F+k) - V(k_F-k))^2}{4\pi^2(k_F^2 - k^2)^2}, \\
A_{1,-}(k) &= \frac{M^2(V(2k_F) - V(k_F-k))^2}{16\pi^2 k_F^2 (k-k_F)^2}, \\
A_{1,+}(k) &= \frac{M^2 V'(k-k_F)^2}{4\pi^2(k-k_F)^4}, \\
C_1 &= \frac{M^2\rho_0 V'(2k_F)^2}{64\pi^7}, \\
A_2 &= \frac{M^2\rho_0(V'(2k_F) - k_F V''(2k_F))^2}{32\pi^{10}}.
\end{aligned}$$

arXiv:1103.4176

Exact prefactors in Lieb-Liniger model

$$\frac{\langle\hat\rho(x)\hat\rho(0)\rangle}{\rho_0^2} \approx 1-\frac{K}{2(\pi\rho_0 x)^2}+\sum_{m\geq 1}\frac{A_m\cos(2mk_F x)}{(\rho_0 x)^{2m^2K}}, \quad \frac{\langle\hat\psi_B^\dagger(x)\hat\psi_B(0)\rangle}{\rho_0}\approx\sum_{m\geq 0}\frac{B_m\cos(2mk_F x)}{(\rho_0 x)^{2m^2K+1/(2K)}}$$

From exact formfactors due to Slavnov (89,90):

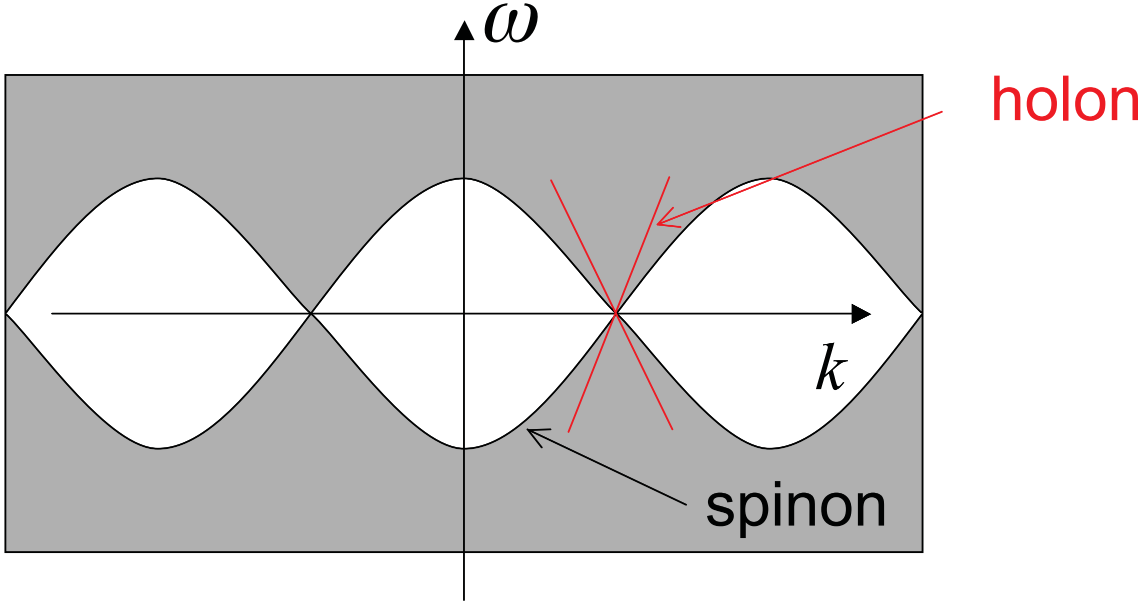


All Prefactors can be obtained in a closed analytical form

LX12341/Shashi Referee B: Can the 98% of the community, for whom the presented tremendous number of equations are meaningless, learn something from these results?

arXiv:1010.2268

Spin-charge separation(spin ½ fermions)



Repulsive interactions: mobile impurity has quantum numbers of a spinon, which leads to checkable consequences&phenomenology

PRL 104, 116403 (2010), **PRB** 82, 245104 (2010)

Checked for 1D Hubbard, F.H.L. Essler, **PRB** 82, 205120 (2010)

Many questions are not addressed : “mobile impurity Kondo effect” ? Universal nonlinear theory based on XXX model? Holon relaxation rates? Spin imbalanced systems?

Conclusions and Outlook

What is hiding: 1D physics beyond linear Luttinger liquid universality class

Future 1D directions: finite T, relaxation and kinetics, quantum quenches, …

Beyond 1D? Collective description of higher dimensional Fermi surface properties (patchwork bosonization)? Local Fermi surface curvature <-> nonlinear spectrum?

Hopefully, a review is coming soon