The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization
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International Atomic Energy Agency

2239-13

Workshop on Integrability and its Breaking in Strongly Correlated and Disordered Systems

23 - 27 May 2011

Nonlinear response of driven interacting systems

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Nonlinear current response of interacting fermions in metallic and insulating regimes

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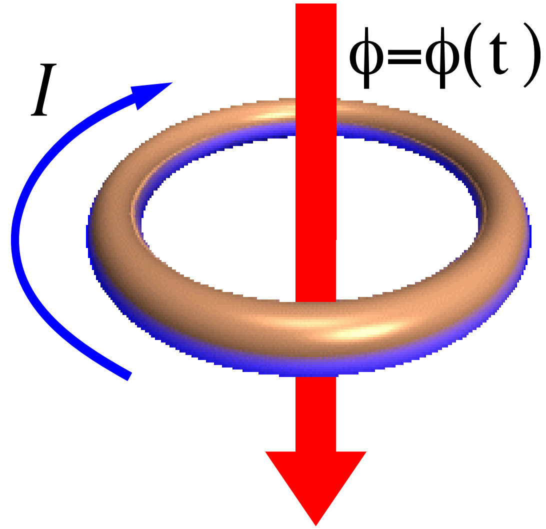
Motivation



Metallic and insulating regimes

- isolated 1D system: $\partial_t \rho(t) = -i[H(t), \rho(t)]$
- initially microcanonical ensemble with $\beta \ll 1$
- flux $\Phi(t) \longrightarrow$ electric field $F \propto \partial_t \Phi(t)$

Integrable vs. non–integrable systems for final F

- for $F = 0$: integrability $\longrightarrow$ conserved quantities $\longrightarrow$ relaxation
- in LR difference visible since response functions calculated at $F = 0$
- how the integrability–related properties of the LR change with F

Linear Response (LR) theory: $I_{LR}(t) = \int_0^t d\tau \sigma(t-\tau) F(\tau)$

- what are the boundaries of LR regime? the Joule heating $\propto F^2$?
- how to generalize LR for stronger fields and/or longer driving ?

1D t–V model

$$H = H_k + H_I \qquad H_k = -t_h \sum_j \left\{ \mathrm{e}^{i\phi(t)}\, c^\dagger_{j+1} c_j + \mathrm{h.}c. \right\}$$

$$F(t) = -\frac{d\phi(t)}{dt} \qquad J = -\frac{1}{L}\frac{\partial H}{\partial \phi} = \frac{i}{L}\sum_j \left\{ \mathrm{e}^{i\phi(t)}\, c^\dagger_{j+1} c_j - \mathrm{h.}c. \right\}.$$

$$E(t) = \mathrm{Tr}[\rho(t) H[\phi(t)]] \qquad \dot\rho(t) = -i[H(t), \rho(t)]$$

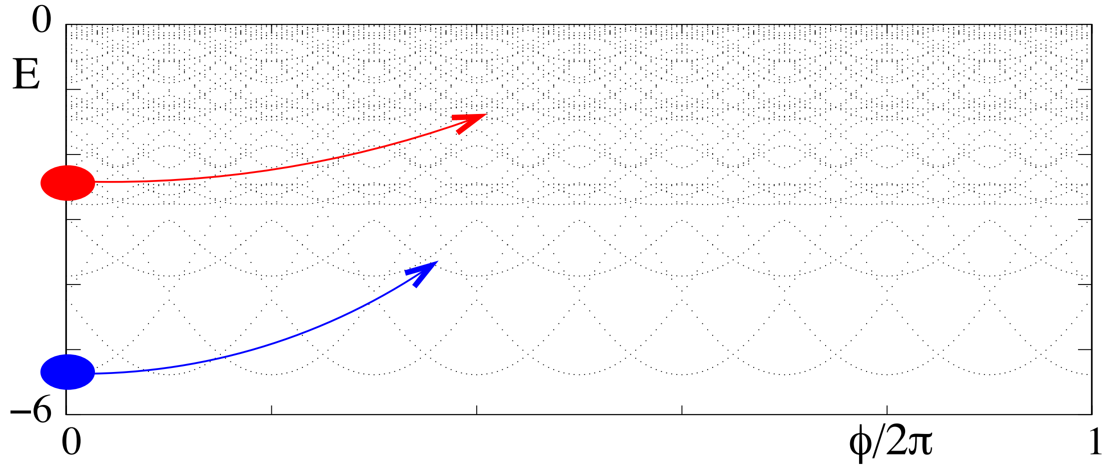
$$\begin{aligned} \frac{d}{dt}E(t) &= L\ F(t)\ I(t) \\ \frac{d}{dt}E_k(t) &= i\langle[H(t), H_k(t)]\rangle + L\ F(t)\ I(t) \\ \frac{d}{dt}I(t) &= i\langle[H(t), J(t)]\rangle - F(t)\frac{E_k(t)}{L} \end{aligned}$$

$H_I = 0 \longrightarrow$ the Bloch oscillations (F=const), $I(t) = I_0 \cos(Ft)$.

t–V model

$$H_I = V \sum_j \hat{n}_j \hat{n}_{j+1} + W \sum_j \hat{n}_j \hat{n}_{j+2}$$

$L = 10, V = 0.8, W = 0$, with $L/2$ particles

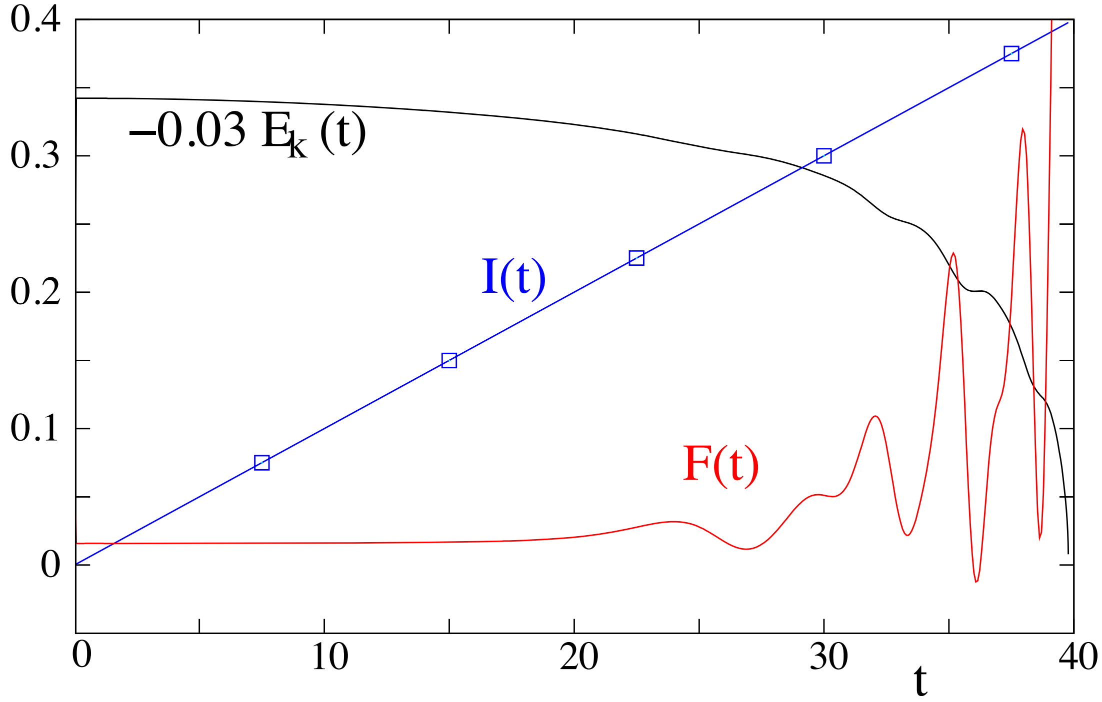


- $T = 5$ with $V = 1.4,\ W = 1$ or $V = 1,\ W = 0$
- the initial energy $\bar{E}_0$ from full diagonalization of small $L \sim 14$
- $\bar{E}_0$ rescaled for $L \sim 30$ and used in MCLM:
$$[H(0) - \bar{E}_0]^2 |\psi(0)\rangle = \varepsilon |\psi(0)\rangle$$
- $i\partial_t |\psi(t)\rangle = H[\phi(t)]|\psi(t)\rangle$, Park, Light, J. Chem. Phys. (1986)

Reverse problem

- standard: Assumed: $|\psi(0)\rangle$, $F(t)$; Calculated: $|\psi(t)\rangle$, $I(t)$
- reverse: Assumed: $|\psi(0)\rangle$, $I(t)$; Calculated: $|\psi(t)\rangle$, $F(t)$
- how: $$\frac{d}{dt}I(t) = i\langle[H(t), J(t)]\rangle - F(t)\frac{E_k(t)}{L}$$

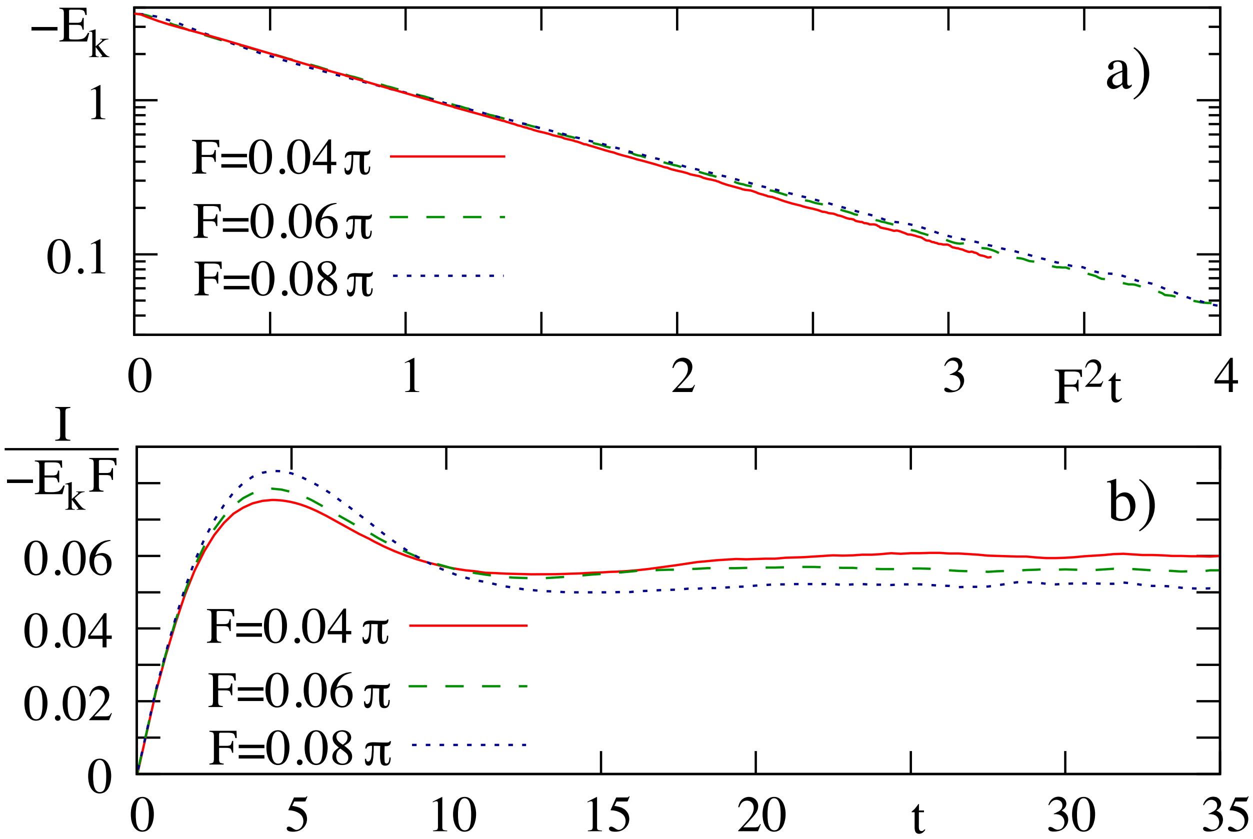
$L = 18, V = 1.4, W = 1$ with the ground state $|\psi(0)\rangle$



Beyond LR - nonintegrable metal

Expectations: the Joule heating $(\propto F^2) \longrightarrow$ increase of E_k

- LR: $\int d\omega\sigma(\omega) \propto -E_k$
- beyond LR: $\frac{d}{dt}I(t) = i\langle[H(t), J(t)]\rangle - F(t)\frac{E_k(t)}{L}$



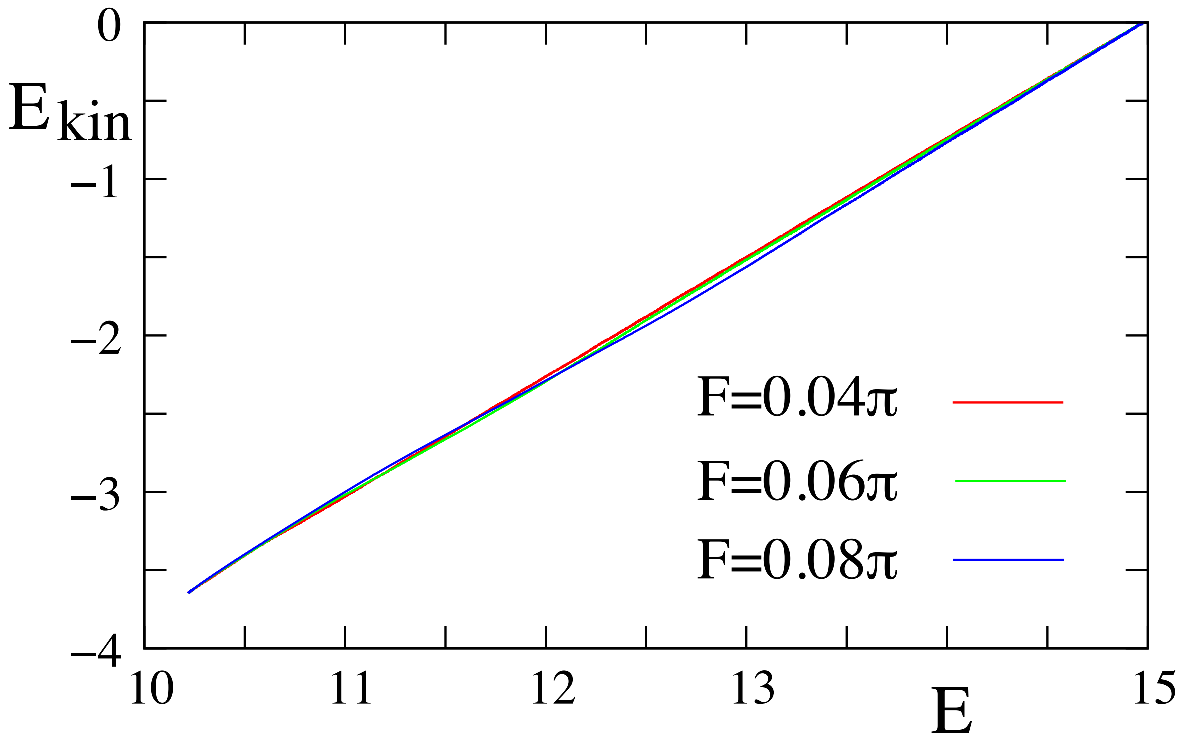
Beyond LR - nonintegrable metal

How to predict strongly nonlinear response without explicit solution of the von Neumann or the time–dependent Schrödinger equations?

Conjecture:

- $$I(t) \simeq I_{\mathrm{ER}}(t) = \frac{E_k(t)}{E_k(0)} I_{\mathrm{LR}}(t), \qquad I_{\mathrm{LR}}(t) = \int_0^t \mathrm{d}t' \sigma(t-t') F(t')$$

$$\frac{d}{dt} E_k(t) = i\langle [H(t), H_k(t)] \rangle + L\, F(t)\, I(t)$$



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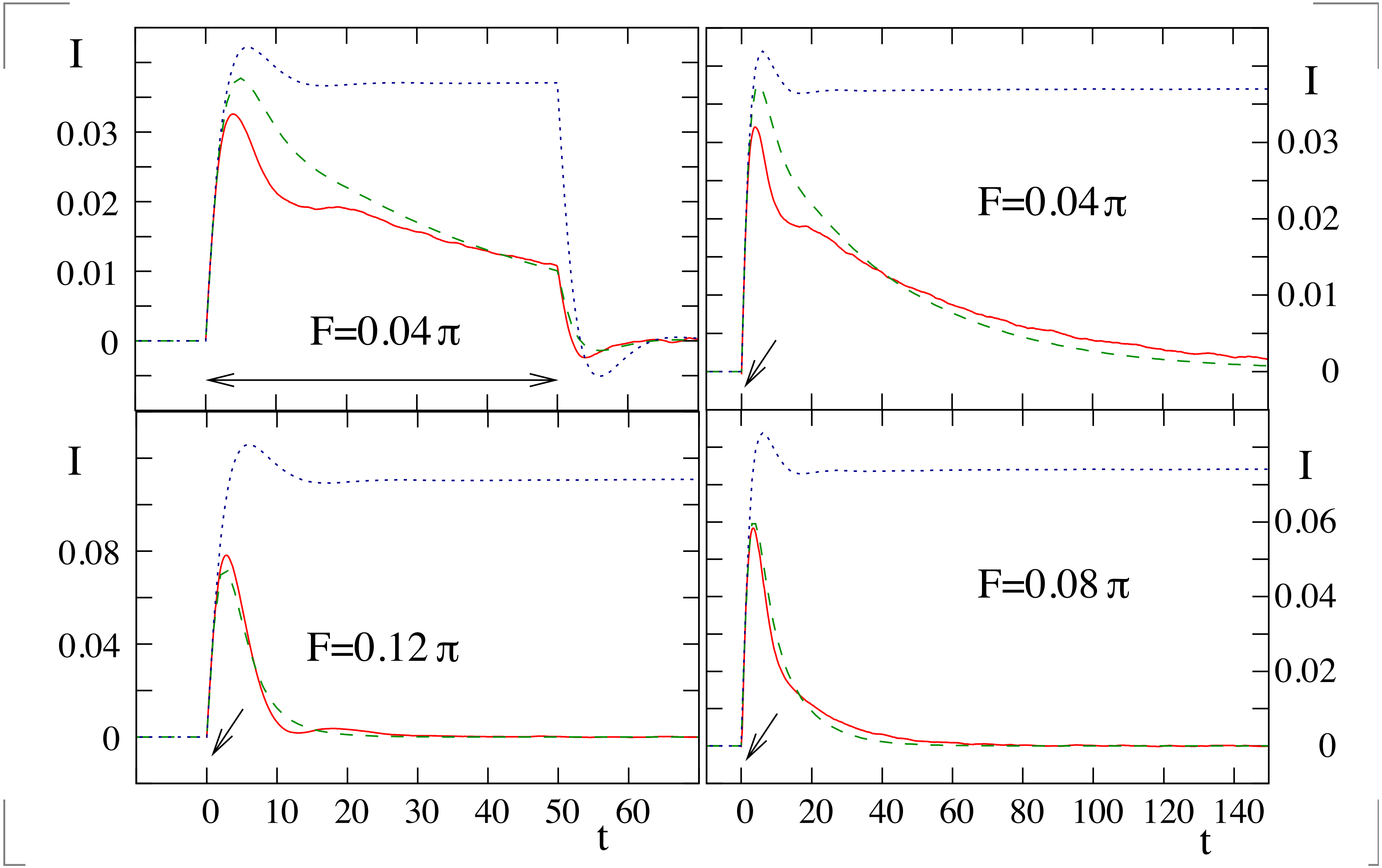
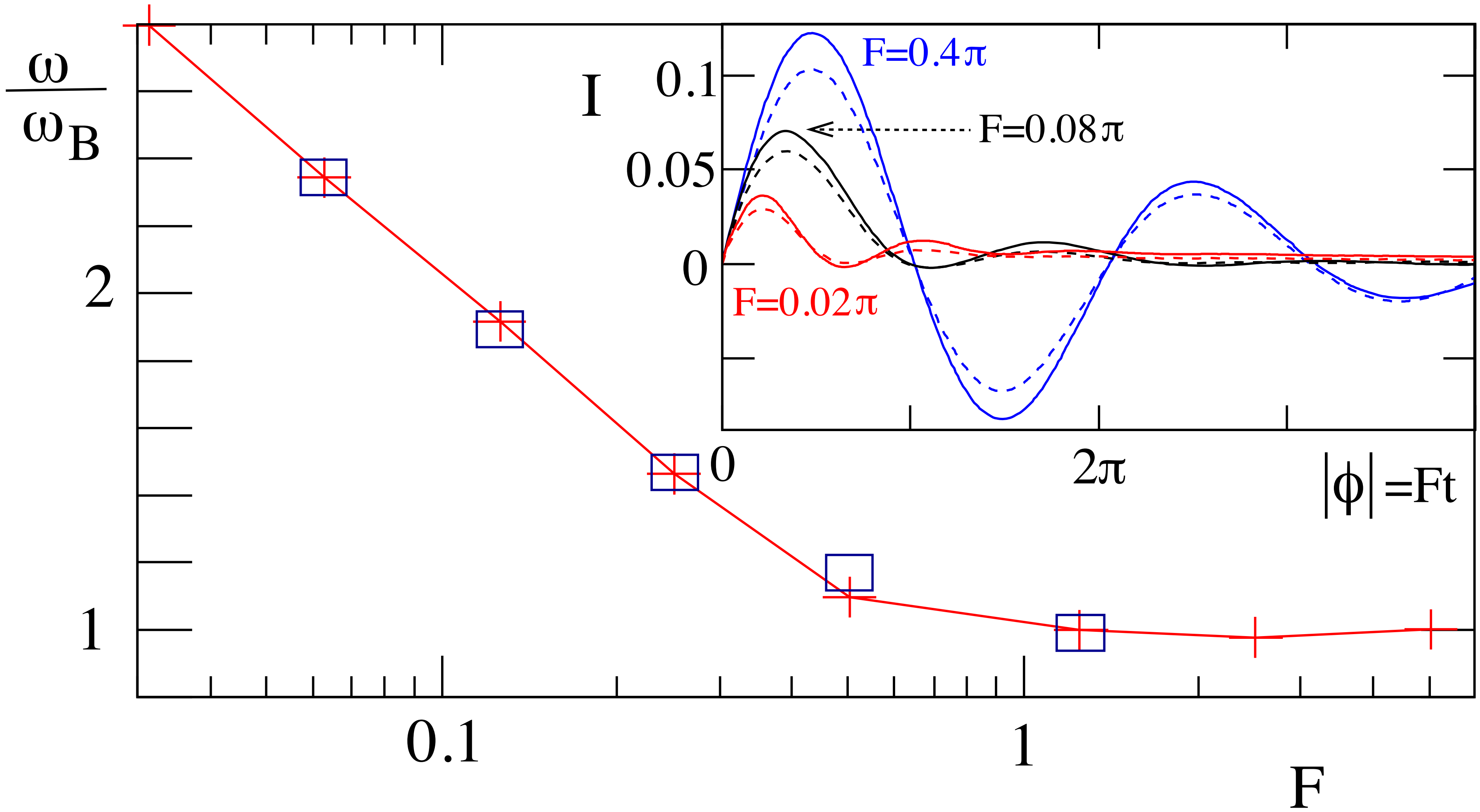
- $\dot{E}_k(t) = \gamma \dot{E}(t) = \gamma L F I_{\mathrm{ER}}(t)$
- In the long–time regime:

  $I_{\mathrm{ER}}(t) \propto -E_k(t) F$, then $-E_k(t) \propto \exp(-\alpha F^2 t)$ with $\alpha > 0$
-
  $$\gamma = \frac{E_k(t) - E_k(0)}{E(t) - E(0)} \simeq \frac{\partial E_k(0)}{\partial T} \left(\frac{\partial E(0)}{\partial T} \right)^{-1}.$$

Numerical check -nonintegrable metal

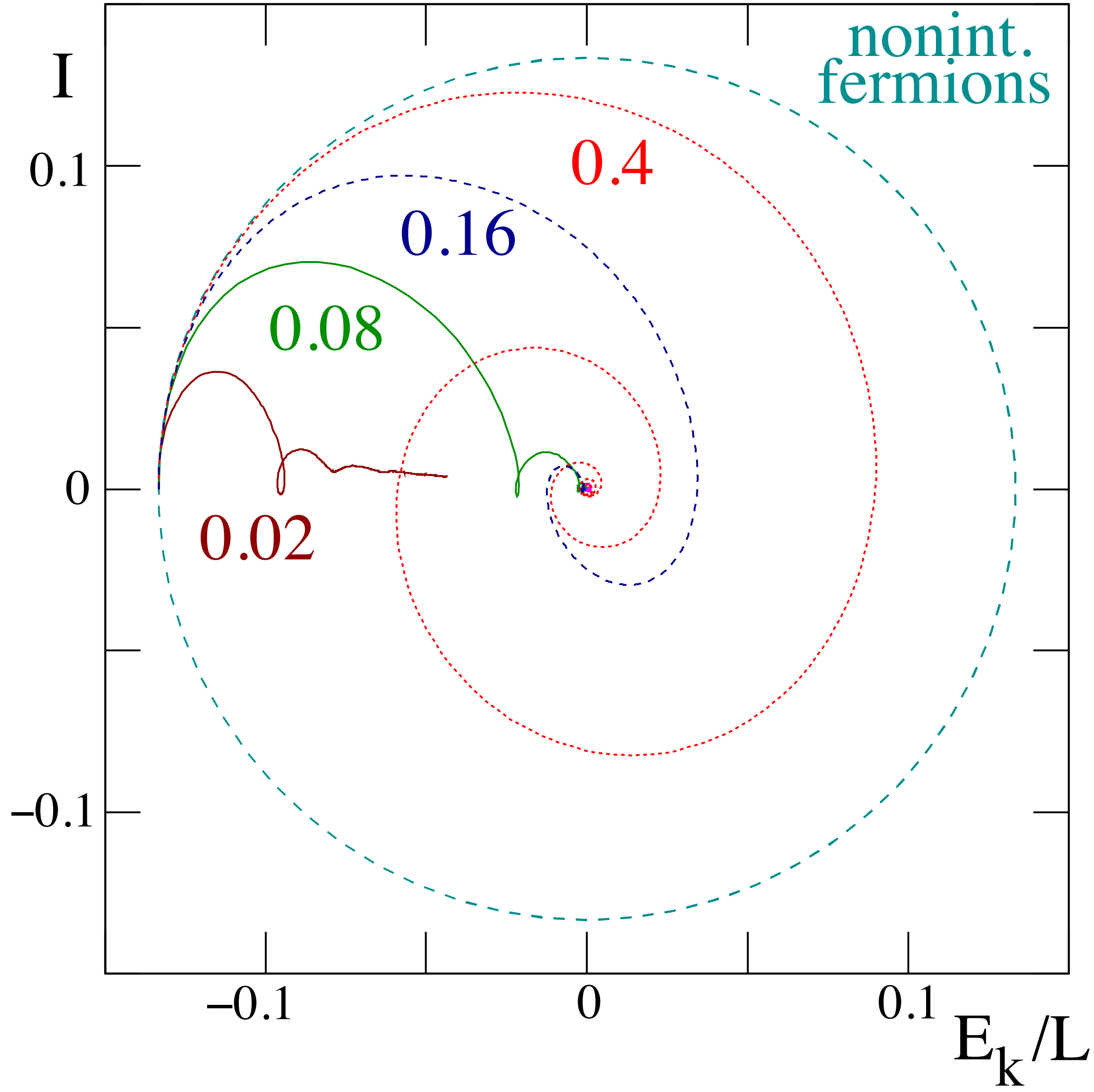


Integrable case

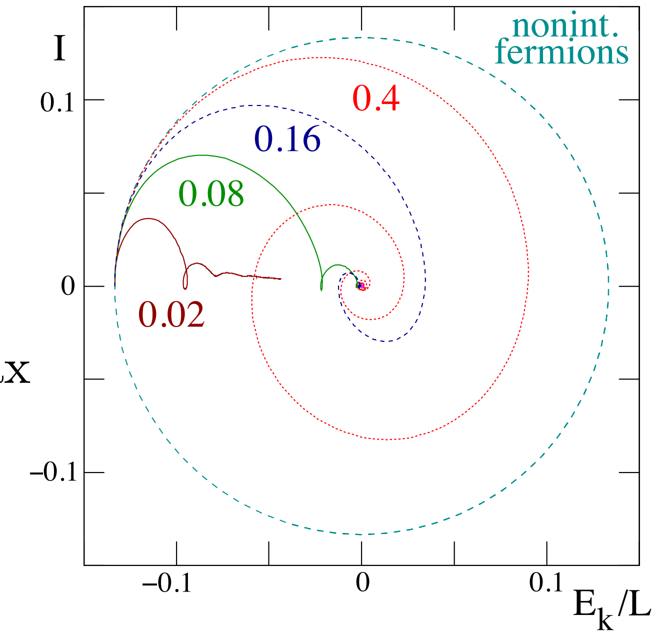


Interpretation: charge stiffness at any $T \rightarrow$ damped Bloch oscillations

Integrable case

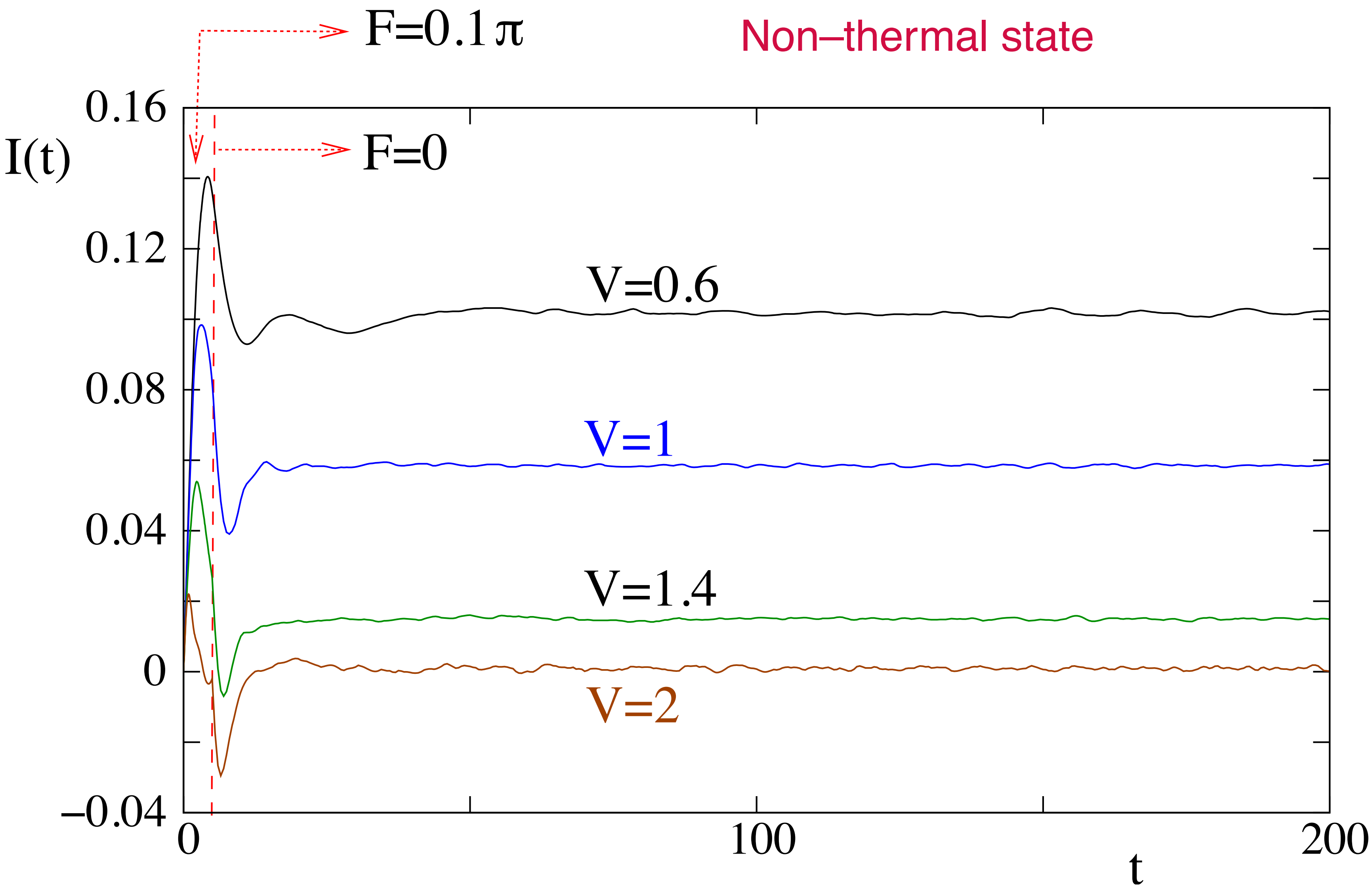


Short time and large field



- $\langle A(t)\rangle = \mathrm{Tr}\{\rho(t)A[\phi(t)]\}$
- $\phi(t)$: external parameter
- $\partial_t\rho(t) = -i[H(t),\rho(t)]$
- but eigenvalues of $H(t)$ bounded by $E_{\max}$
- for $t \ll E_{\max}^{-1}$, $\rho(t) \simeq \rho(0)$
- $I(t) \simeq \frac{1}{L}\sum_k 2\sin[k-\phi(t)]\langle c_k^\dagger c_k\rangle_{t=0}$
- $E_k(t) \simeq \sum_k 2\cos[k-\phi(t)]\langle c_k^\dagger c_k\rangle_{t=0}$
- $\left[\frac{E_k(t)}{L}\right]^2 + I^2(t) \simeq \mathrm{const}$
- for non-interacting particles valid for arbitrary t and $F(t)$
- $I(t)$ bounded by initial E_k

Absence of relaxation in integrable case



Summary on driving of metals:

Spinless fermions at half–filling, large (initial) temperature

- different responses of integrable and nonintegrable 1D metals
- nonintegrable: the Joule heating as a dominating nonlinear mechanism for large (initial) T
- nonintegrable: real–time current without formal solution of time–dependent problem
- integrable: the damped Bloch oscillations with (logarithmically) modified frequency

Reverse problem:

- Assume: $|\psi(0)\rangle$, $I(t)$
- Calculate: $|\psi(t)\rangle$, $F(t)$, as long as $E_k < 0$

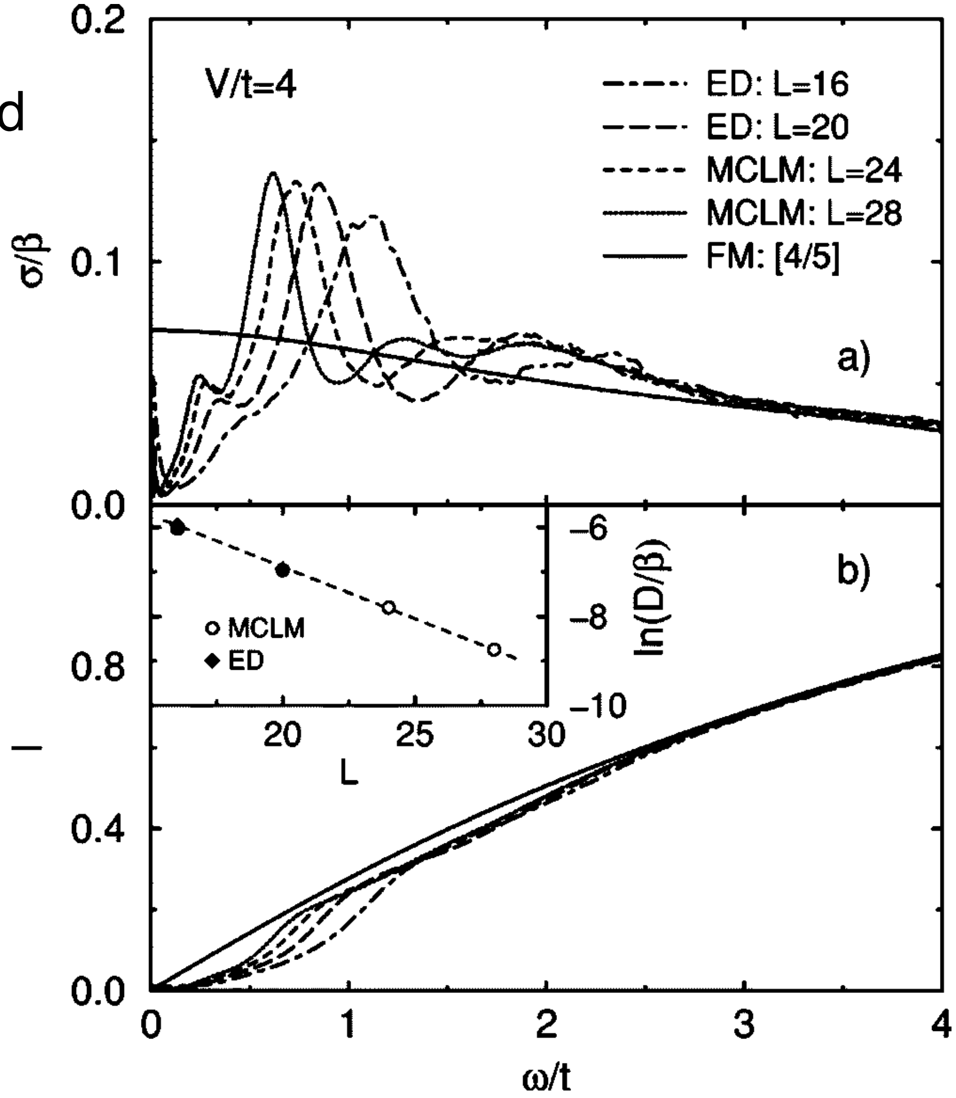
Details in: M.M. and P. Prelovšek, Phys. Rev. Lett. 105, 186405 (2010)

Driving insulators, V>2

Challenges

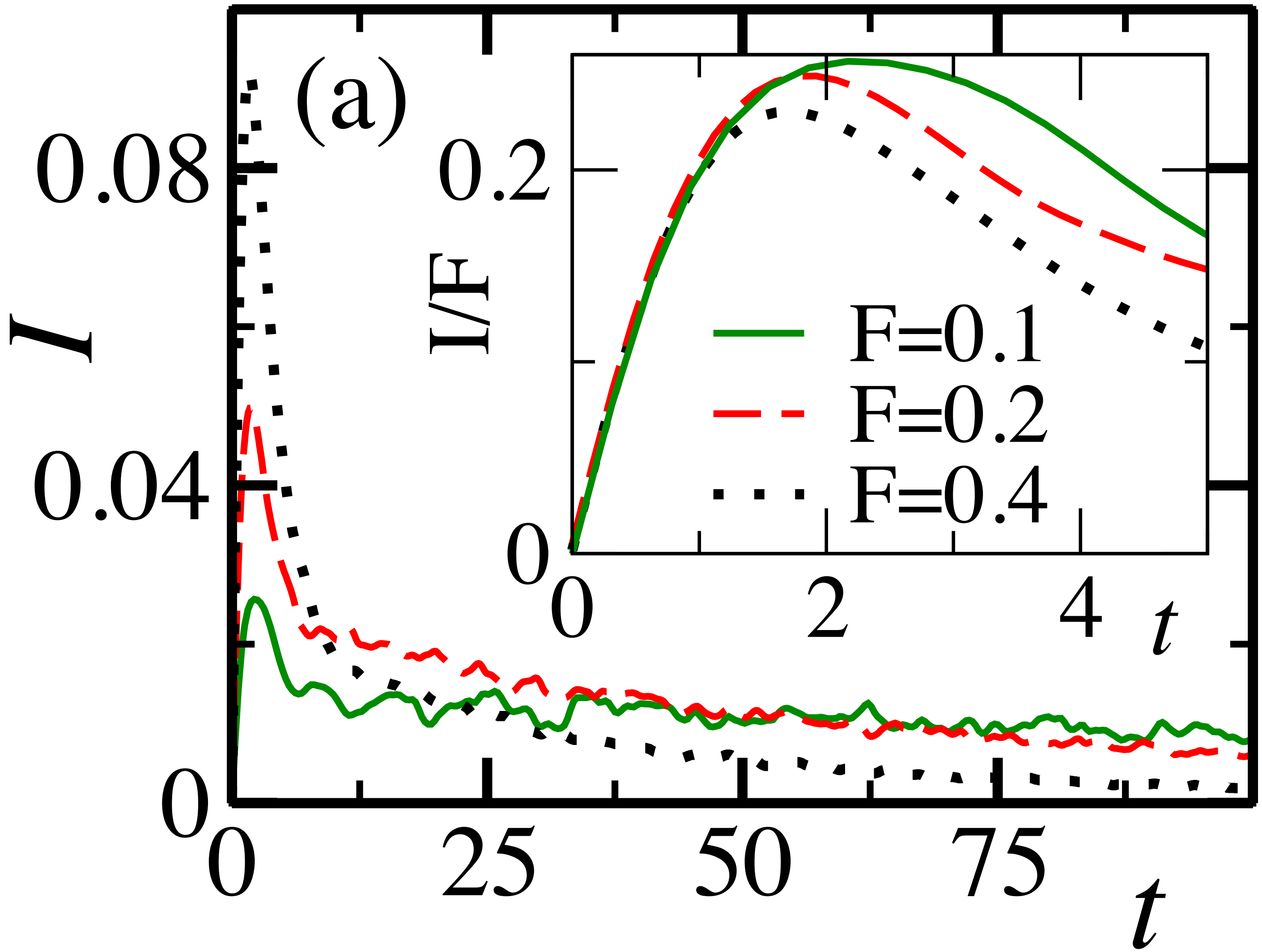
- Heating cannot be incorporated through E_k
- Involved finite–size scaling
- Literally dc response expected for open quantum systems
- Coupling to leads may break integrability
- Driving may cause inhomogeneous distribution of carriers and destroy Mott insulating state

P.Prelovšek *et al*, PRB 2004



Limits of LR in NI insulator

Real–time current non–integrable (NI) case : $V = 3, W = 1, L = 26$



Subtracting heating - NI insulator

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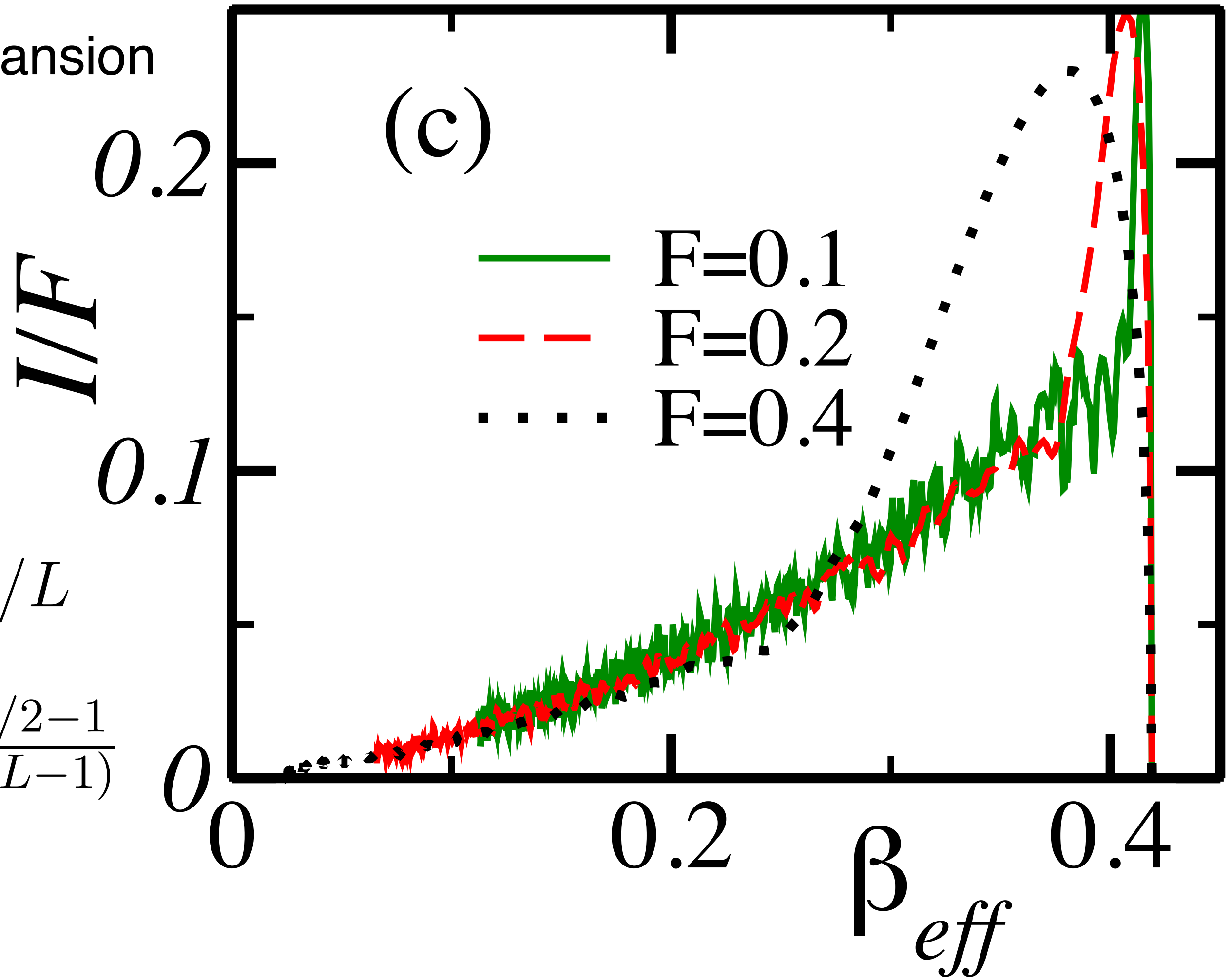
High–temperature expansion



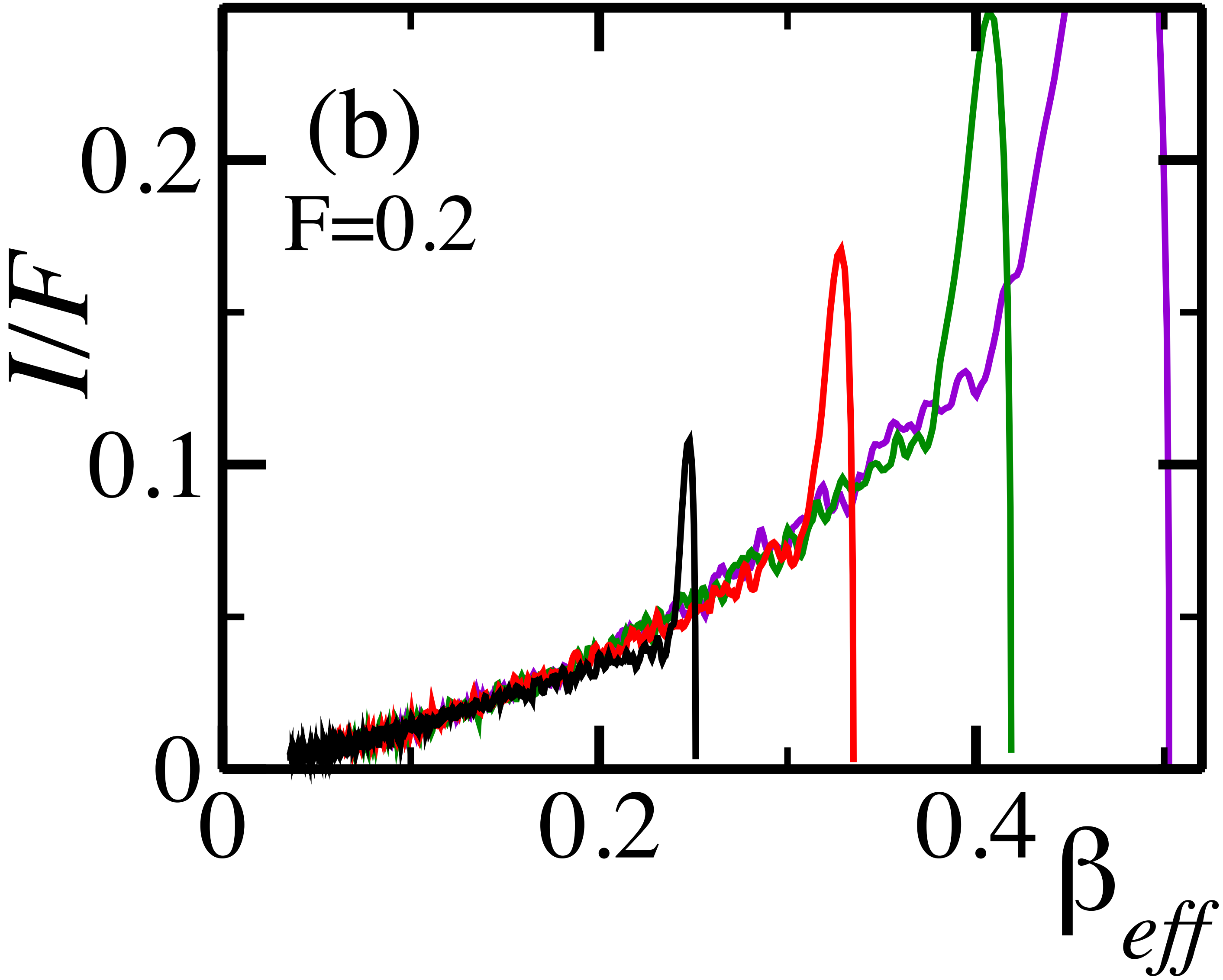
$\varepsilon(t) = [E_\infty - E(t)]/L$

$E_\infty = L(V+W)\frac{L/2-1}{2(L-1)}$

$\frac{\varepsilon}{\beta} = \frac{1}{2} + \frac{V^2+W^2}{16}$

$I(t) > 0 \rightarrow \beta_{eff}(t)$ is monotonic

Does $E(t)$ determine $I(t)$ in NI case?



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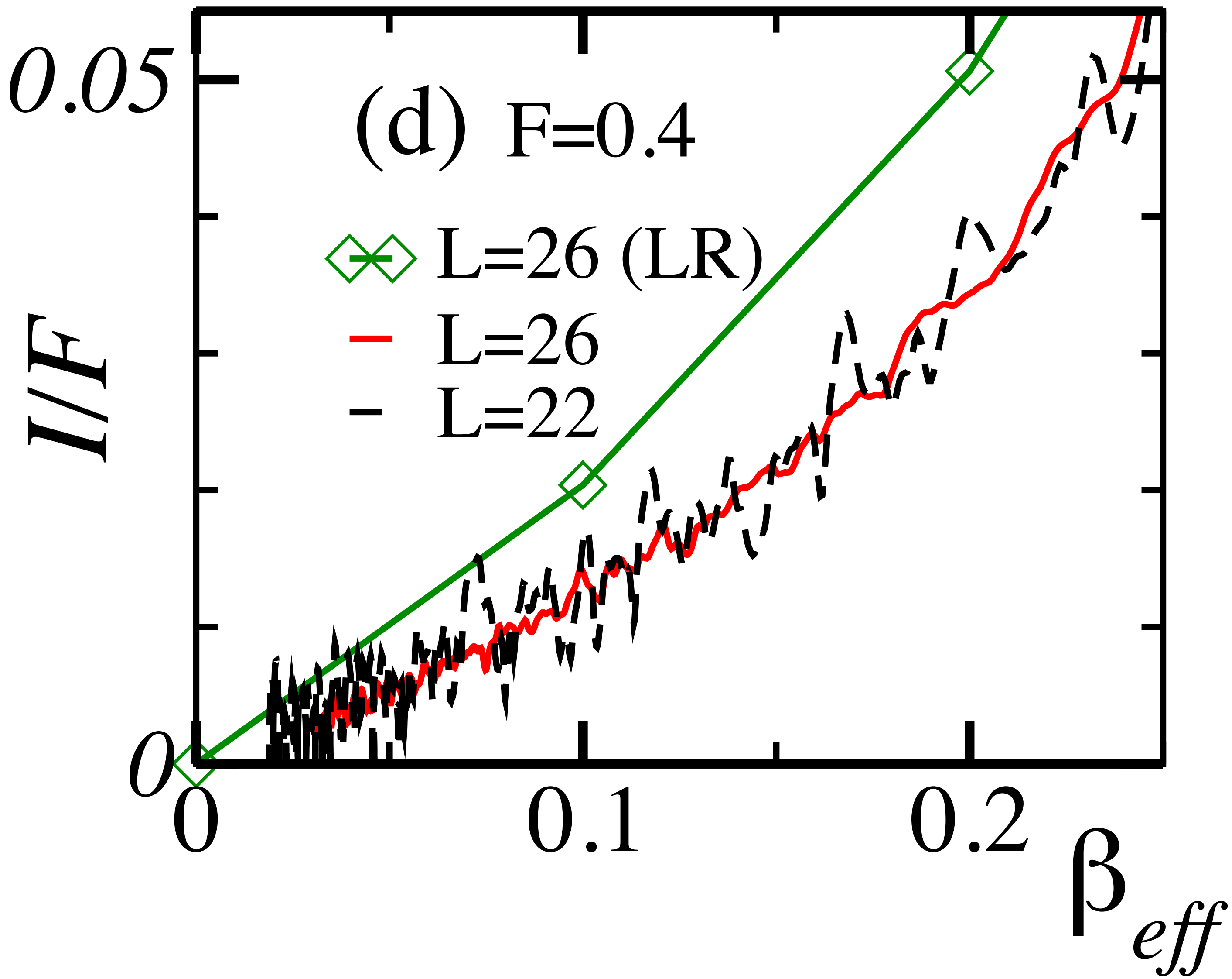
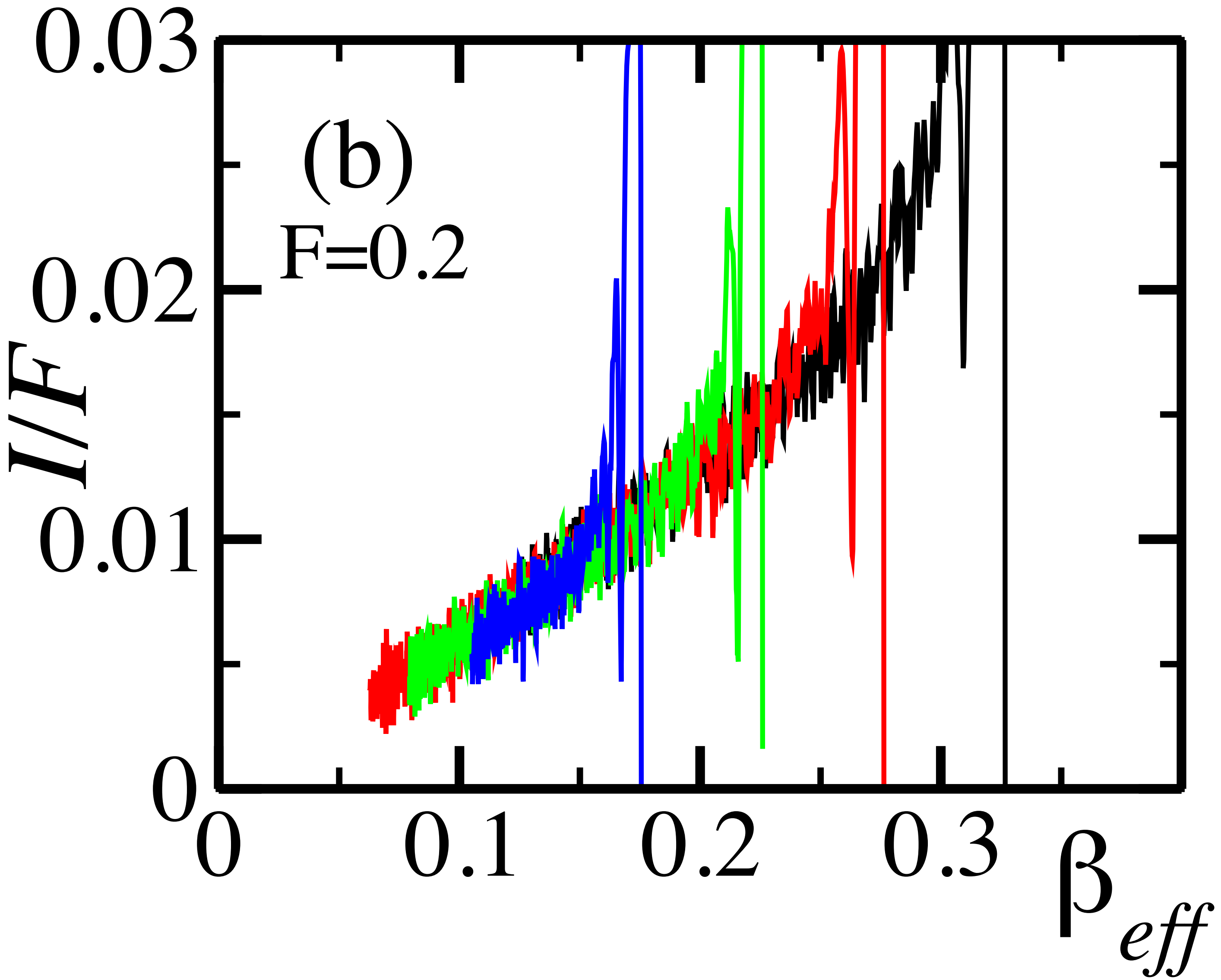


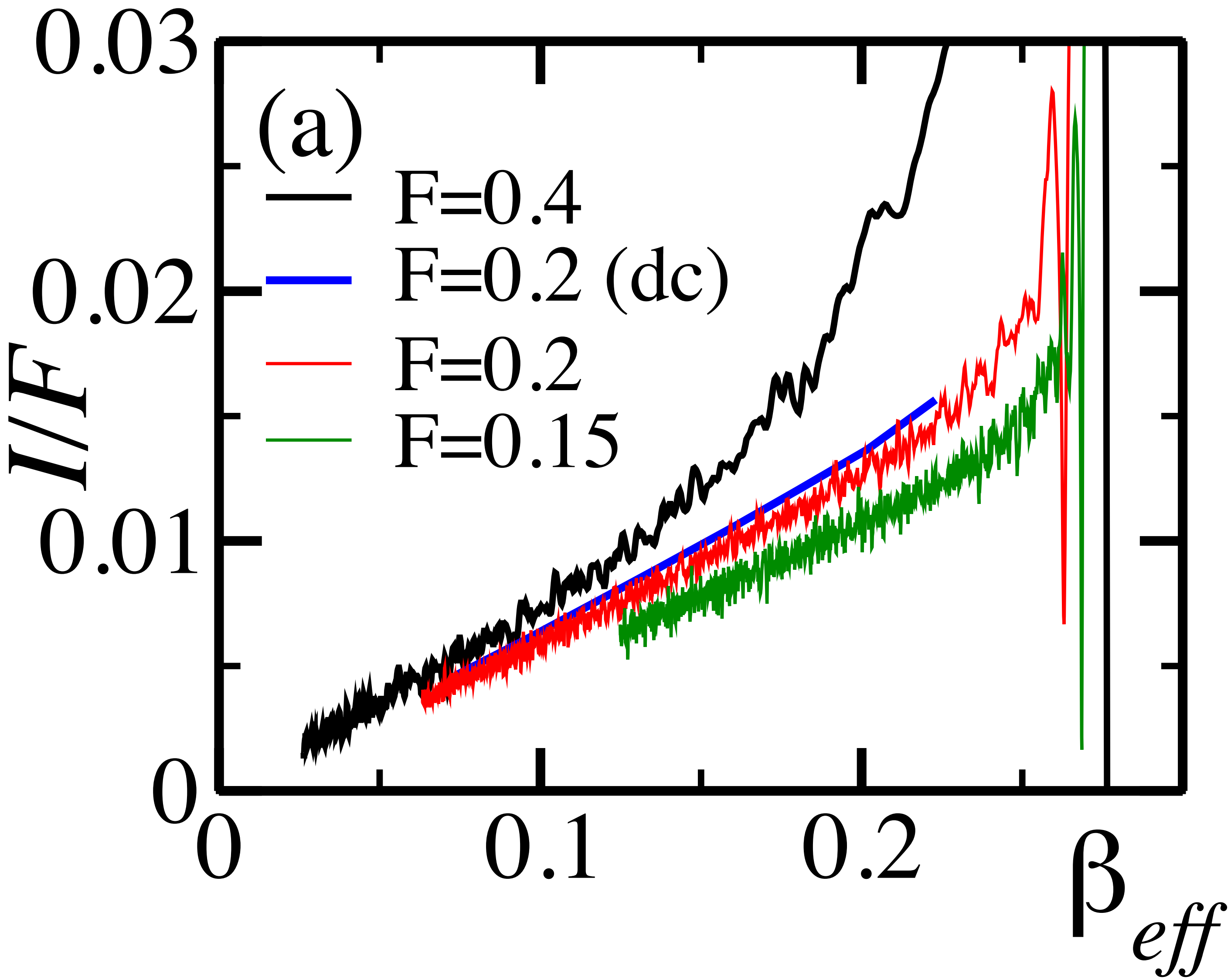
Integrable case and non–zero F

Integrable (I) case: $V = 3$, $W = 0$, $L = 28$



Integrable case and non–zero F

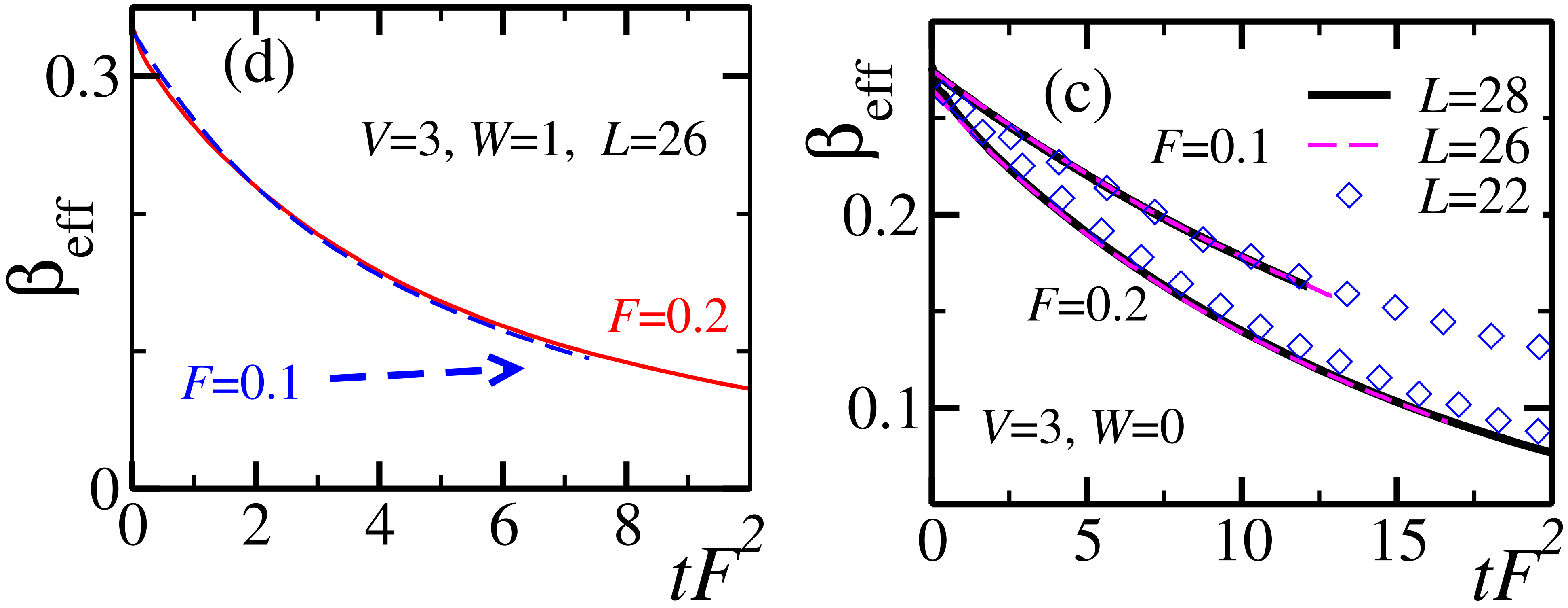
I case: $V = 3$, $W = 0$, $L = 28$



dc response from $E(t)$

$$\frac{d}{dt}E = L\ F\ I$$

$$\text{if}\ \ I(E,F)/F = \gamma(E)\ \ \text{then}\ \ \frac{d}{dt}E \sim F^2$$



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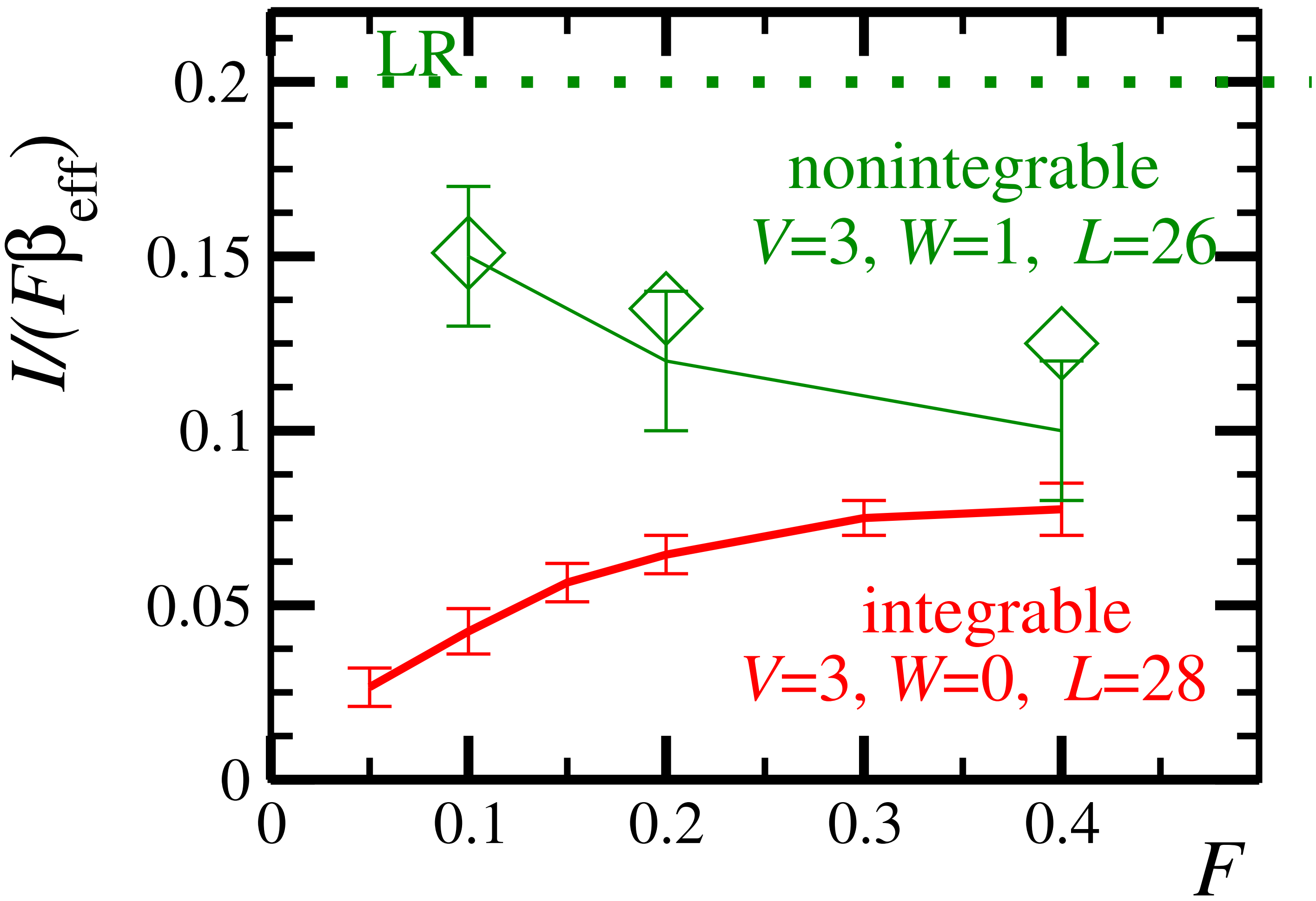
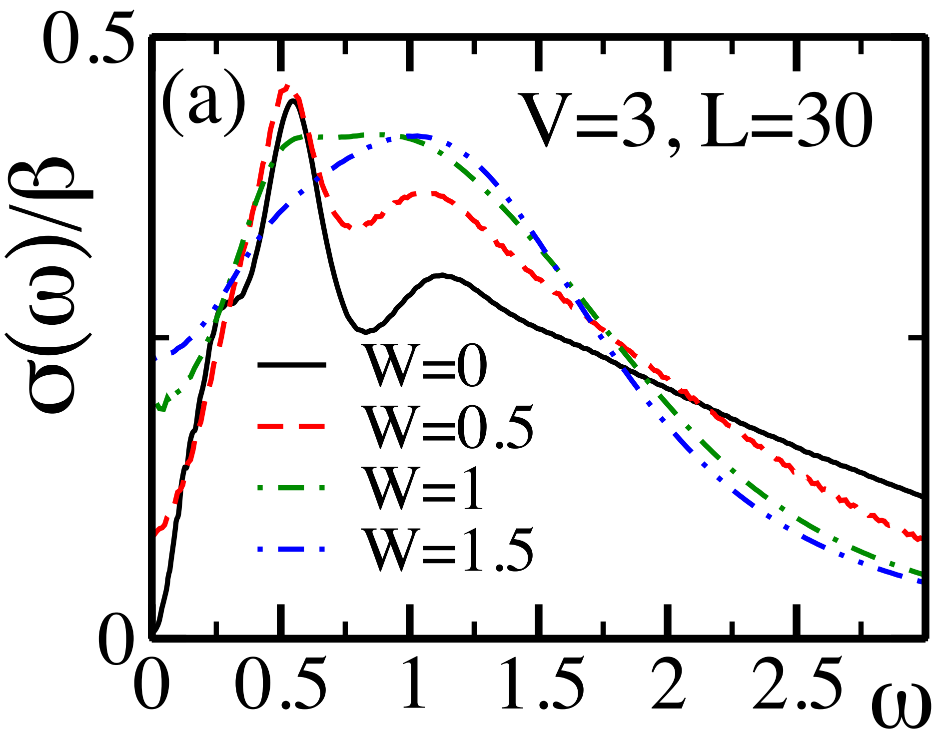
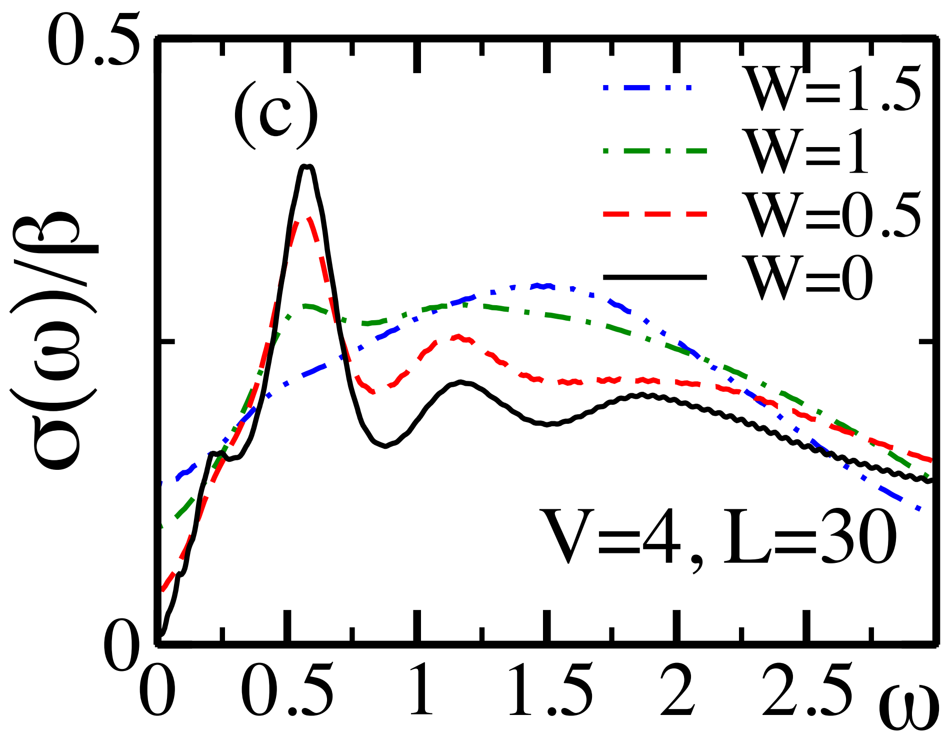


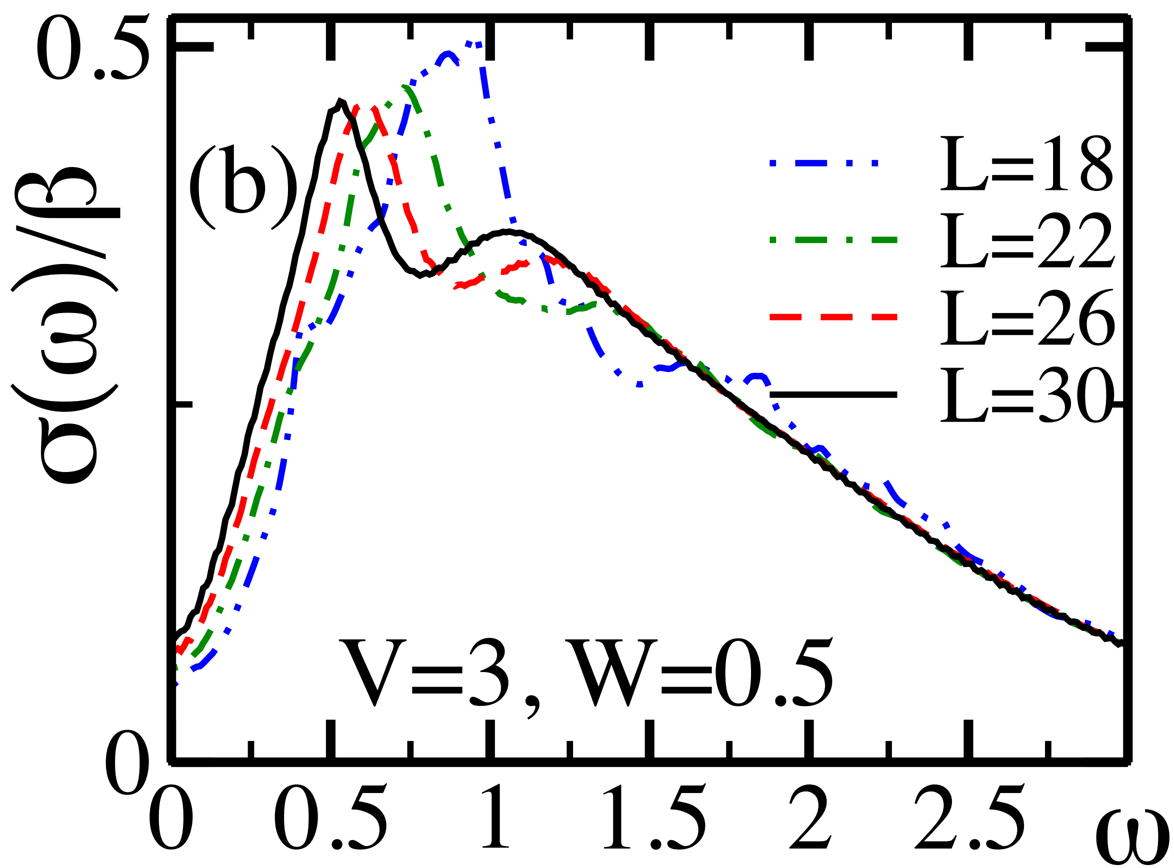
LR for various $V +$ small W

Does $\sigma(0)$ vanish for integrable case?





Finite–size effects



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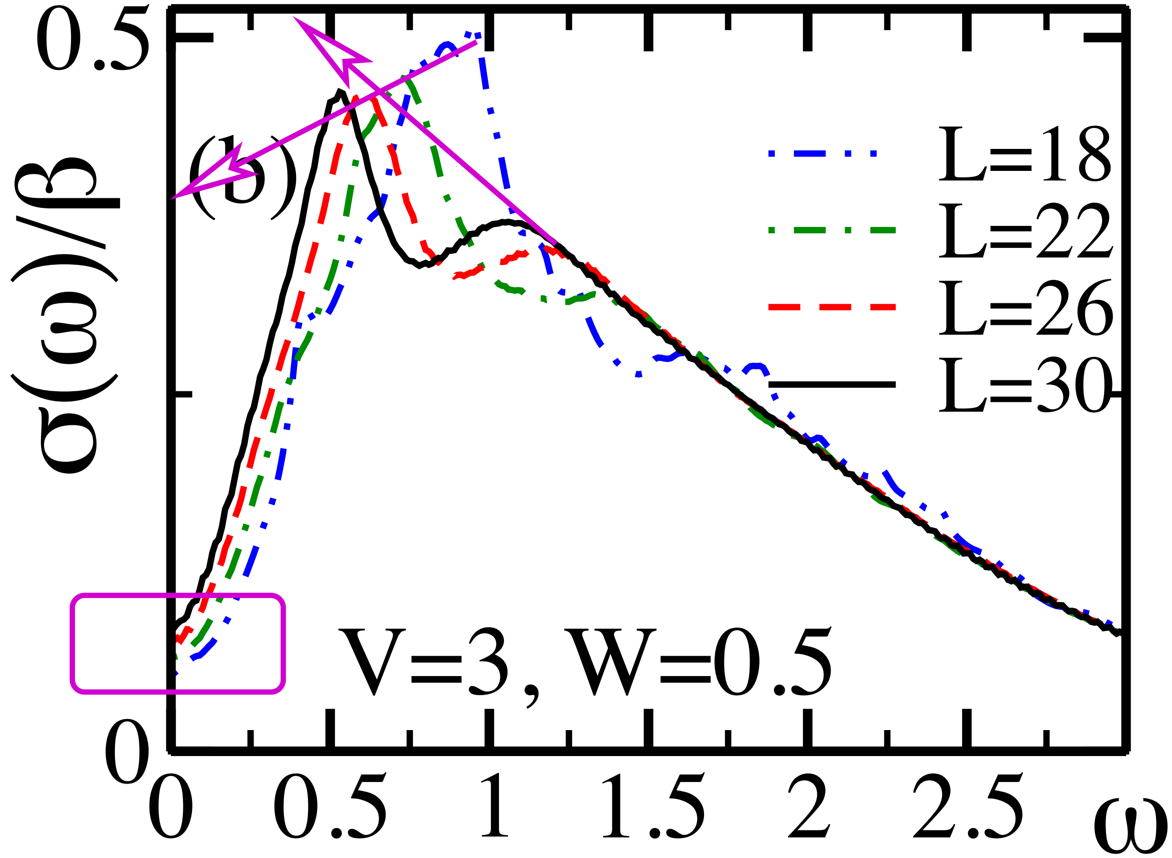
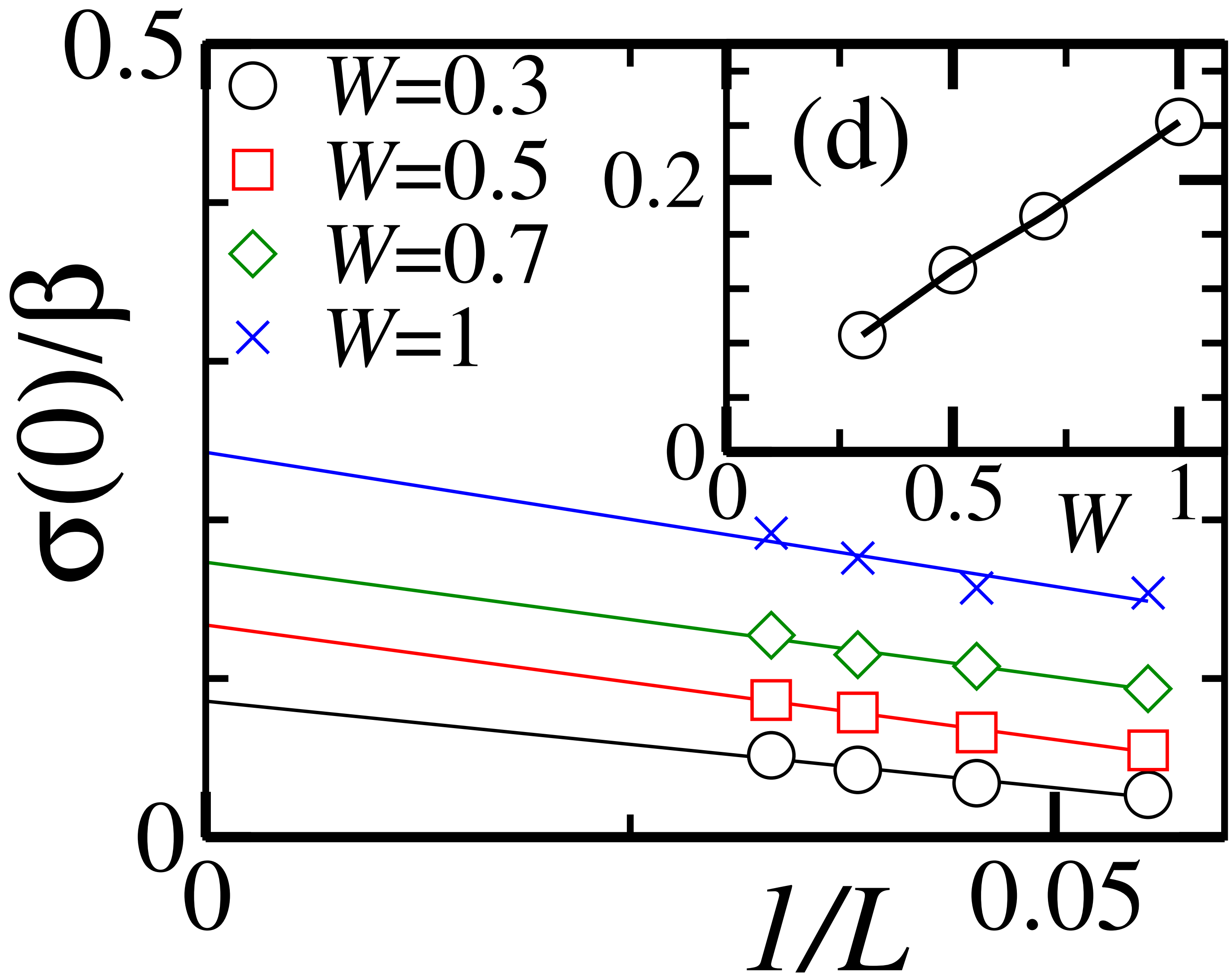


For $\omega > \omega_{FS} \simeq 1.25$ finite–size effects not visible.

Sum rule:: $\int_0^{\omega_{FS}} d\omega\sigma(\omega) + \int_{\omega_{FS}}^{\infty} d\omega\sigma(\omega) = \text{const}$

Finite–size effects



Summary on driving of insulators:

Setup under consideration:

- driving does not destroy half–filling even locally
- isolated system: microcanonical initial state + von Neuman equation

Results:

- clear difference with respect to band insulators at $T > 0$
- NI: the Joule heating central source of nonlinearity
- I : nonlinearity different and distinguishable from the Joule heating
- LR: $\sigma(0)|_{W \to 0} \to 0$

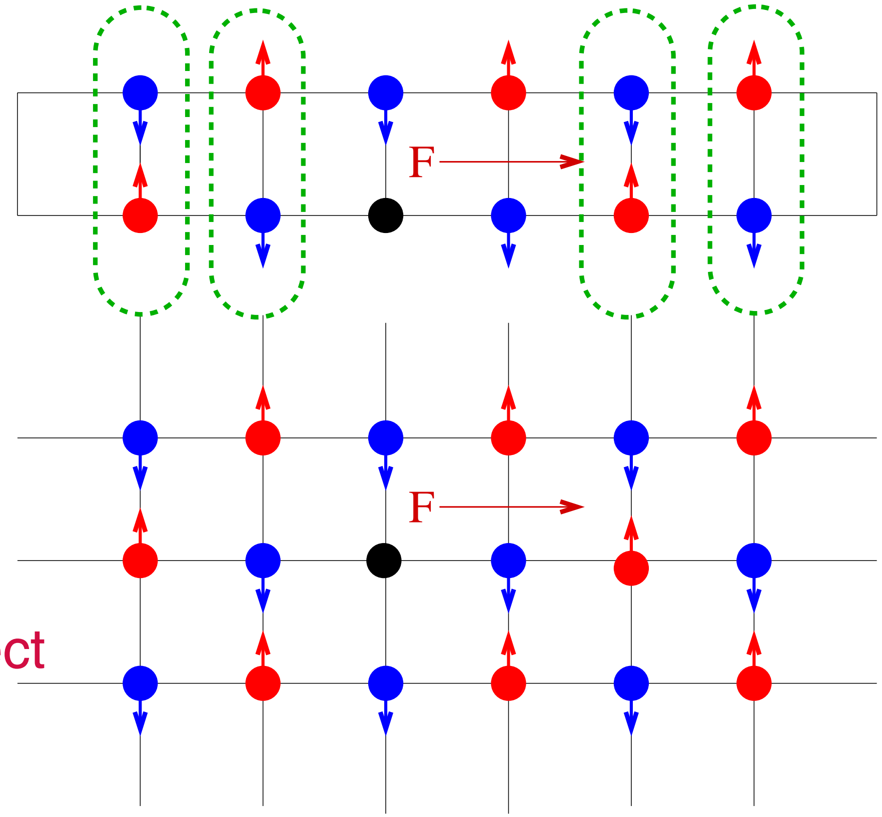
Interpretation:

- In like manner as W, finite F as well breaks integrability
- 'Ideal insulator' with LR regime determined by mechanisms which break integrability

Particle in a dissipative medium

$$H = -t_h \sum_{\langle lj \rangle, \sigma} \mathrm{e}^{i\phi_{lj}(t)}\ \tilde{c}^{\dagger}_{l,\sigma} \tilde{c}_{j,\sigma} + \mathrm{h.}c.$$

$$+J \sum_{\langle l,j \rangle} \mathbf{S_l S_j},$$

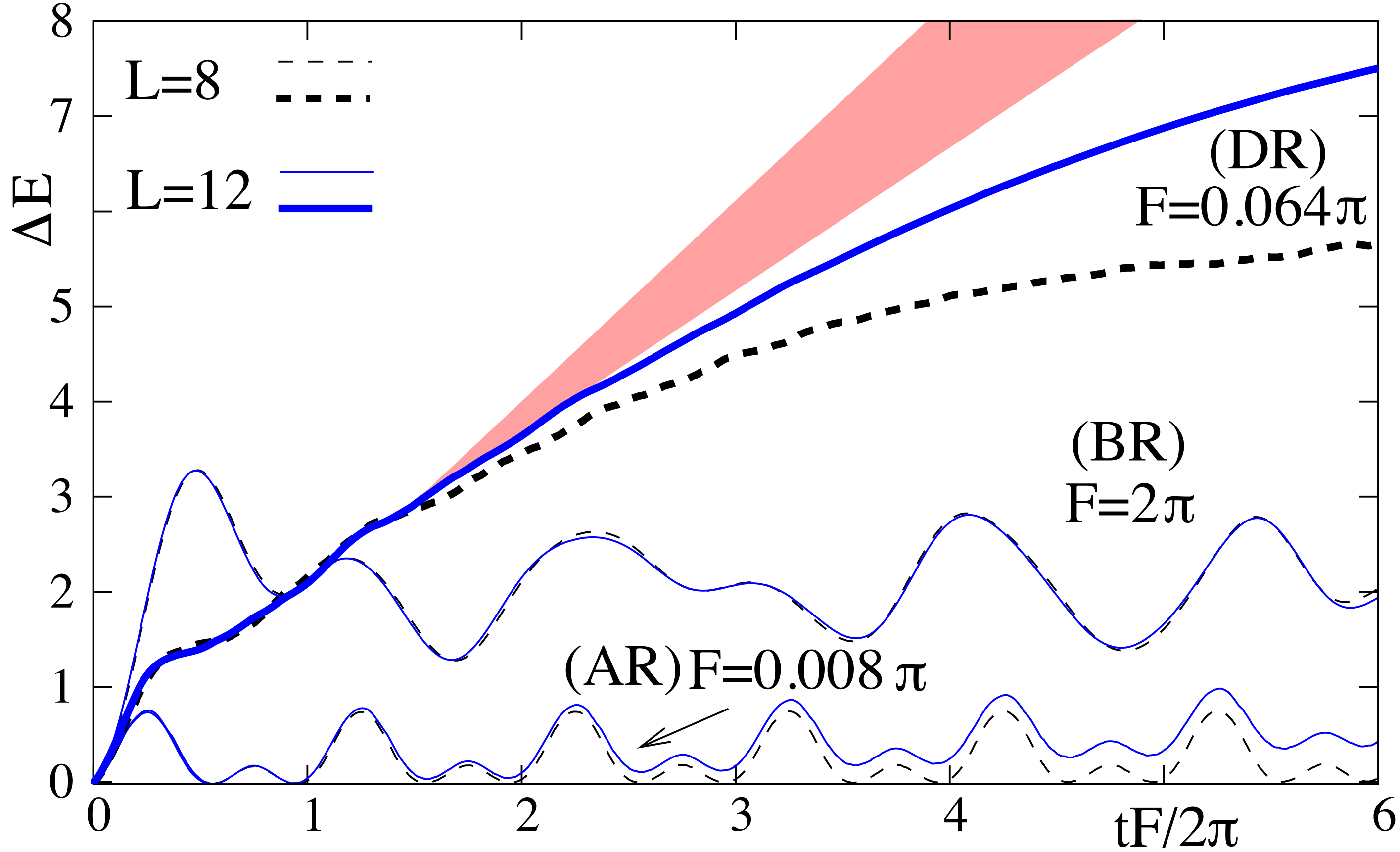


- hole in Mott insulator
- nonequilibrium physics of Mott insulators
- particle in a dissipative medium
- the Joule heating: finite–size effect
- scaled current $I(t) = L \langle \hat{I}(t) \rangle$.

t-J ladder

$$\dot{E}(t) = F(t)I(t), \qquad \bar{I}_t = \frac{1}{t}\int_0^t \mathrm{d}\tau I(\tau) = \frac{\Delta E(t)}{tF}, \qquad I_{dc} = \bar{I}_\infty$$

$J = 0.6$, $|\psi(0)\rangle$ -ground state



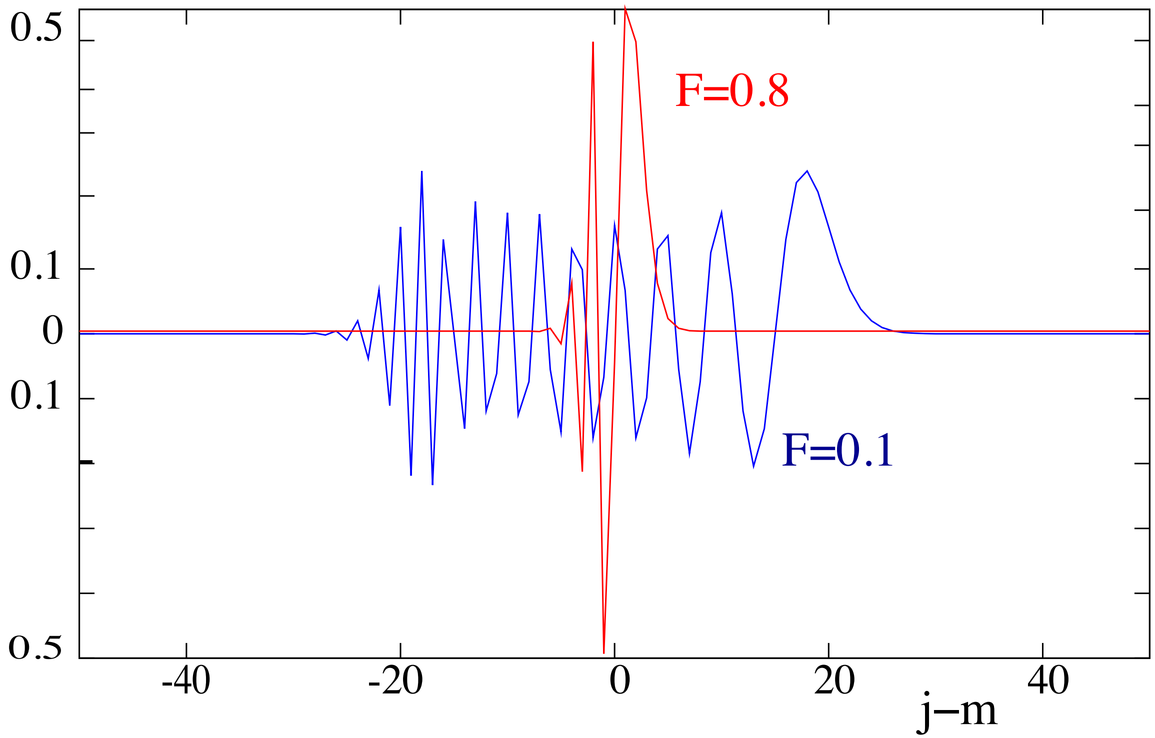
Interpretation of finite size–effects

Standard approach for open bc:

$$H = -\sum_j [c^\dagger_{j+1} c_j + h.c.] + F \sum_j j c^\dagger_j c_j$$

$$c_j = \sum_m J_{j-m}\left(\frac{2}{F}\right) c_m \qquad H = \sum_m F\, m\, c^\dagger_m c_m$$

$J_m(x)$ - Bessel function of the 1st kind; localization length: $l = \frac{4}{F}$



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Standard approach for open bc:

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$$c_j = \sum_m J_{j-m}(\frac{2}{F})\; c_m \qquad\qquad H = \sum_m F\; m\; c_m^\dagger c_m$$

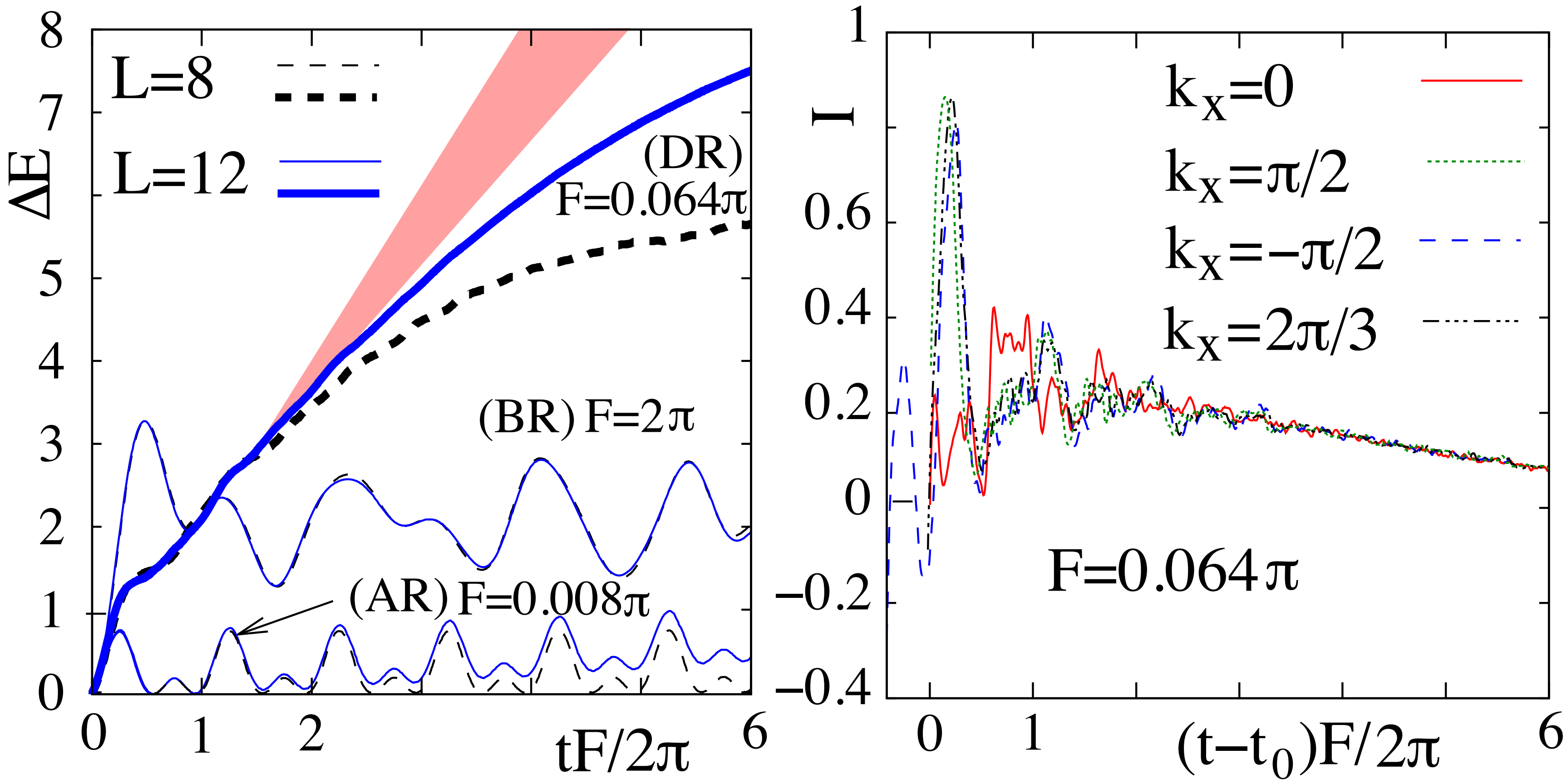
$J_m(x)$ - Bessel function of the 1st kind; localization length: $l = \frac{4}{F}$

Time–dependent flux with $F = \mathrm{const}$, single particle:

$$\dot{E}(t) = LFI(t) \to \Delta E(t) = E(t) - E(0) = FL\int_0^t \mathrm{d}\tau I(\tau)$$

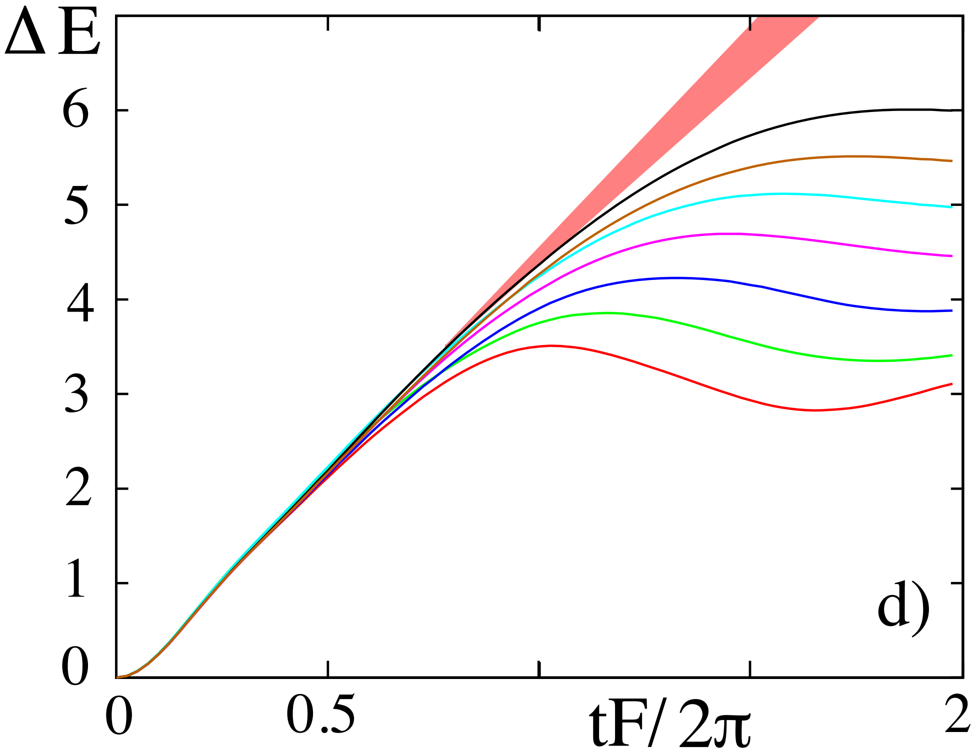
$$I(t) = \frac{1}{L} v(t) \to r(t) = L\int_0^t \mathrm{d}\tau I(\tau) = \Delta E(t)/F < \frac{4}{F}$$

t-J ladder: dissipative regime

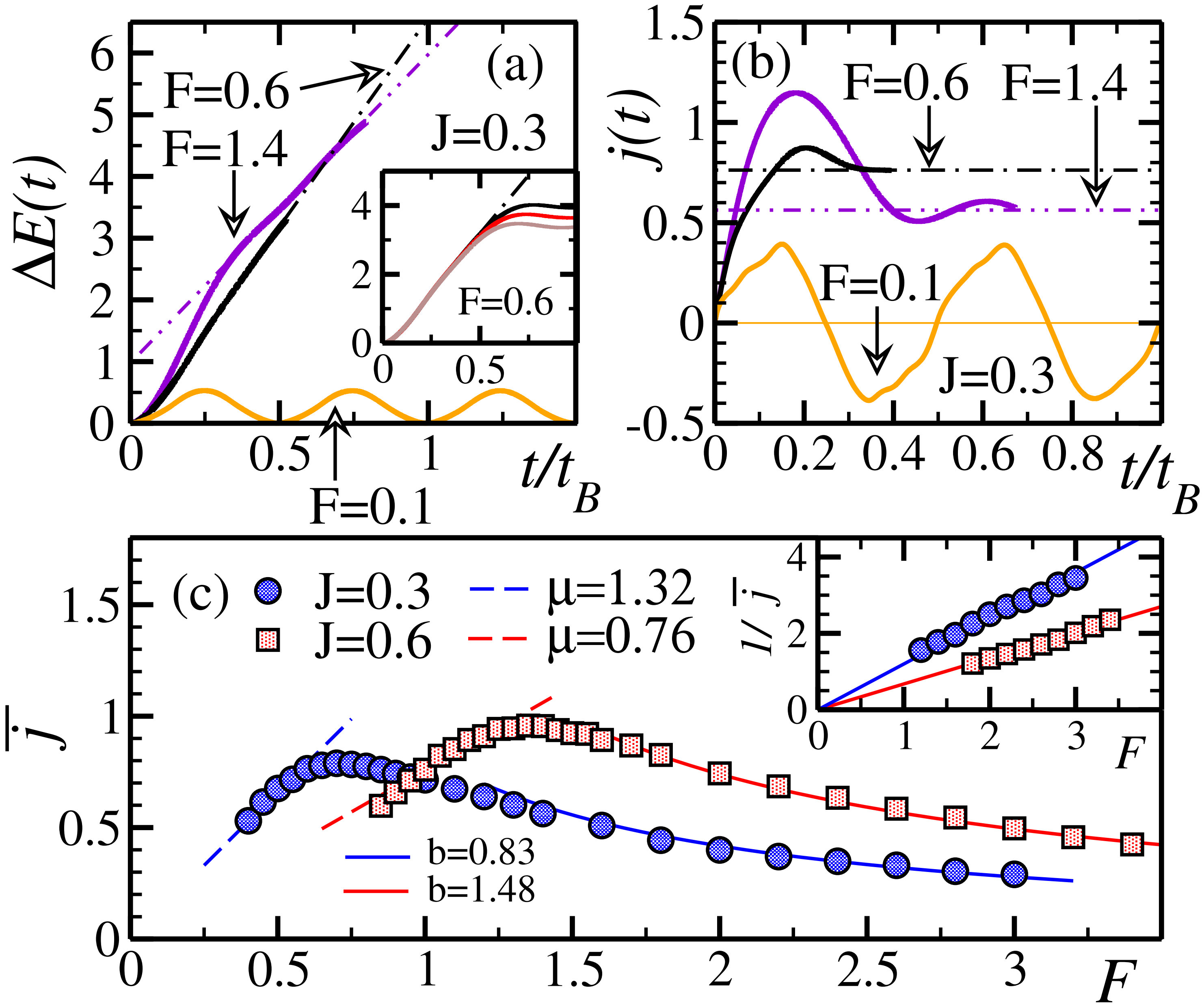


2D t-J model

- limited Hilbert space:
  J. Bonča, *et al* PRB (2007);
- $|\phi_0\rangle = c_{\mathbf{k}=(\pi/2,\pi/2)}|\mathrm{Neel}\rangle$
- $\{|\phi_l^{n_h}\rangle\} =$
  $[H_k + \tilde{H}_J]^{n_h}|\phi_0\rangle$
- $n_h = 1, ..., N_h$
- $N_h \leq 14$
- L (ladder) $\longrightarrow N_h$ (2D)
- AR,DR,BR - regimes of F

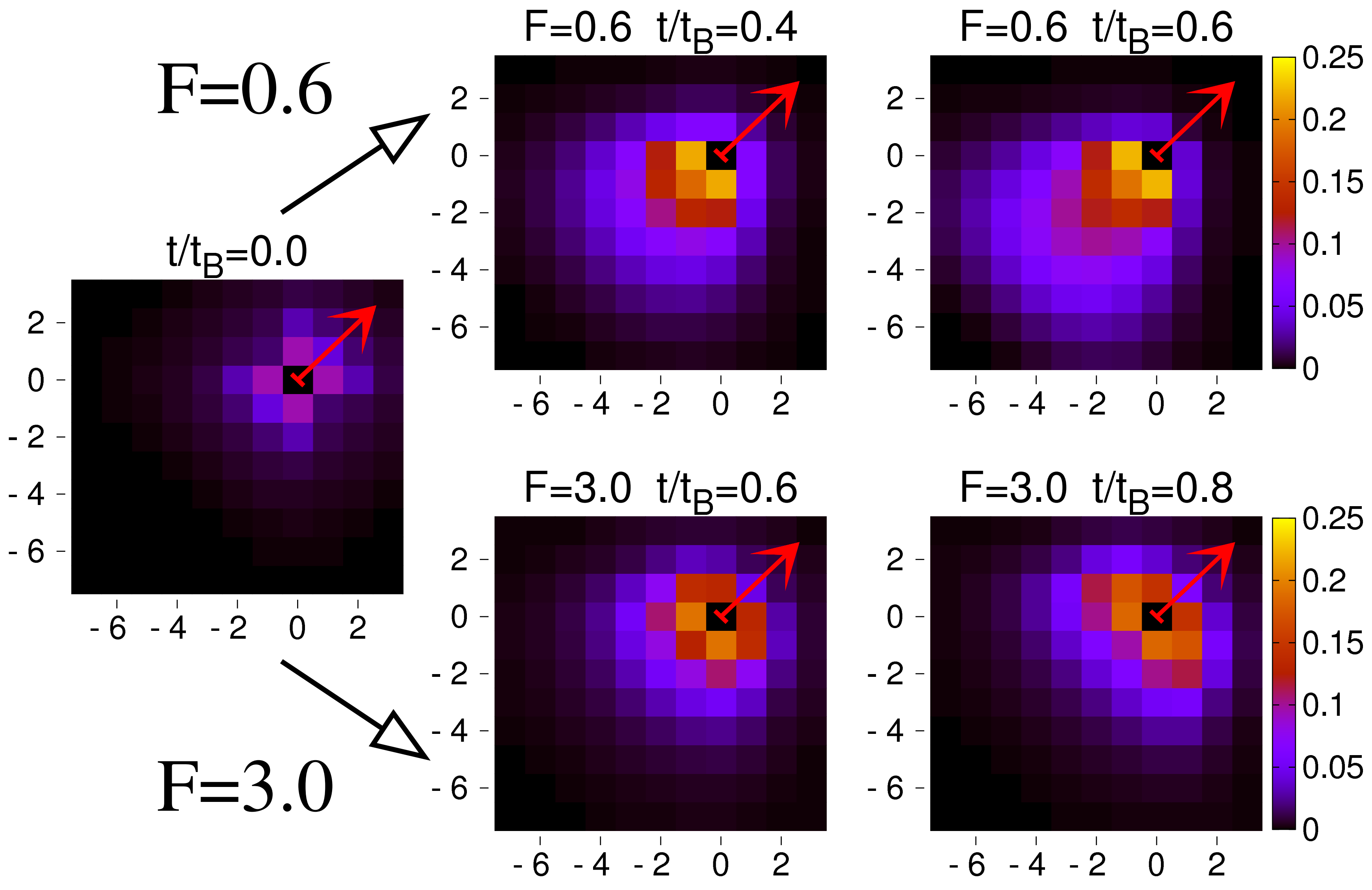


2D t-J model



(a)
F=0.6
F=1.4
J=0.3
F=0.6
F=0.1
$\Delta E(t)$
t/t_B
(b)
F=0.6
F=1.4
F=0.1
J=0.3
$j(t)$
t/t_B
(c)
J=0.3
J=0.6
μ=1.32
μ=0.76
b=0.83
b=1.48
$\overline{j}$
$1/\overline{j}$
F

2D t-J model



F=0.6
F=3.0
t/tB=0.0
F=0.6 t/tB=0.4
F=0.6 t/tB=0.6
F=3.0 t/tB=0.6
F=3.0 t/tB=0.8
0.25
0.2
0.15
0.1
0.05
0

Summary on driving a hole in t-J model

Similarities between hole dynamics on the ladder and in 2D lattice:

- Distinct field–regimes: adiabatic, dissipative and the Bloch–oscillation regime
- Similar carrier mobilities on the ladder and in 2D t–J lattice
- In DR: either positive or negative differential resistivity with crossover field that scales with J.

2D system only:

- Te crossover is accompanied by changing of the spatial structure of the spin polaron.

M. M., L. Vidmar, J. Bonca, P. Prelovsek, PRL **106**, 196401 (2011)

t-J ladder - supplementary