The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific and
Cultural Organization

IAEA
International Atomic Energy Agency

2239-4

Workshop on Integrability and its Breaking in Strongly Correlated and Disordered Systems

23 - 27 May 2011

Quantum Quenches, Thermalization and Many-Body Localization

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From adiabatic dynamics to thermalization in Many-Body systems

Rosario Fazio
Scuola Normale Superiore, Pisa

- Adiabatic dynamics in an open quantum critical system

- Quench and many-body localization

In collaboration with

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Elena Canovi	*Uni - Stuttgart*
Davide Rossini	*SNS - Pisa*
Luigi Amico	*DMFCI - Catania*
Alessandro Silva	*ICTP - Trieste*
Giuseppe Santoro	*SISSA - Trieste*

- Phys. Rev. Lett. **101**, 175701 (2008)
- Phys. Rev. B **80**, 024302 (2009)
- Phys. Rev. B **83**, 094431 (2011)

Non-equilibrium quantum many-body systems

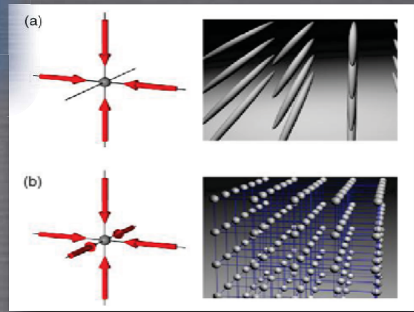
Artificial many-body systems (nanoscience, cold atoms)

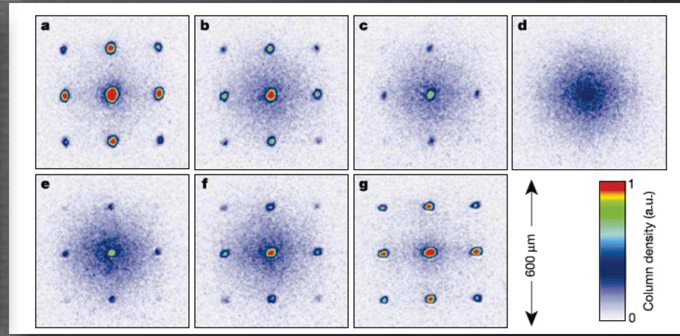
Driven systems

Time-dep Hamiltonians

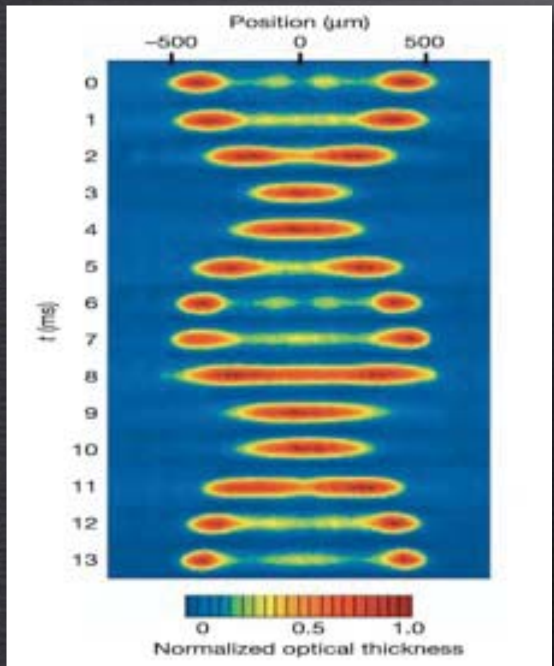
see the review by
A. Polkovnikov, K. Sengupta, A. Silva, M. Vengalattore, ArXiv:1007.5331

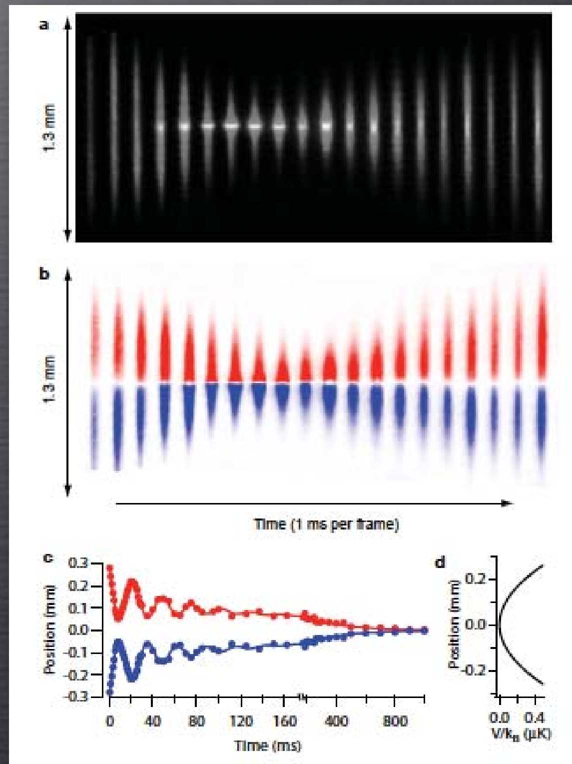
Cold atomic gases





M. Greiner, O. Mandel, T. Esslinger, T.W. Hänsch, and I. Bloch, Nature 415, 39-44 (2002)





T. Kinoshita, T. Wenger, and D. S. Weiss, Nature 440, 900 (2006)

A. Sommer, M. Ku, G. Roati and M.W. Zwirlein Nature 472, 201 (2011)

Time-Dep Hamiltonians

$$\mathcal{H} = \mathcal{H}_0 + \lambda(t)\mathcal{H}_1$$

Quench

Sengupta, Powell, Sachdev ('04)
Calabrese and Cardy ('07)
Rigol et al, ('06)
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Adiabatic

Adiabatic

Zurek, Dorner, Zoller ('05)
Polkovnikov ('05)
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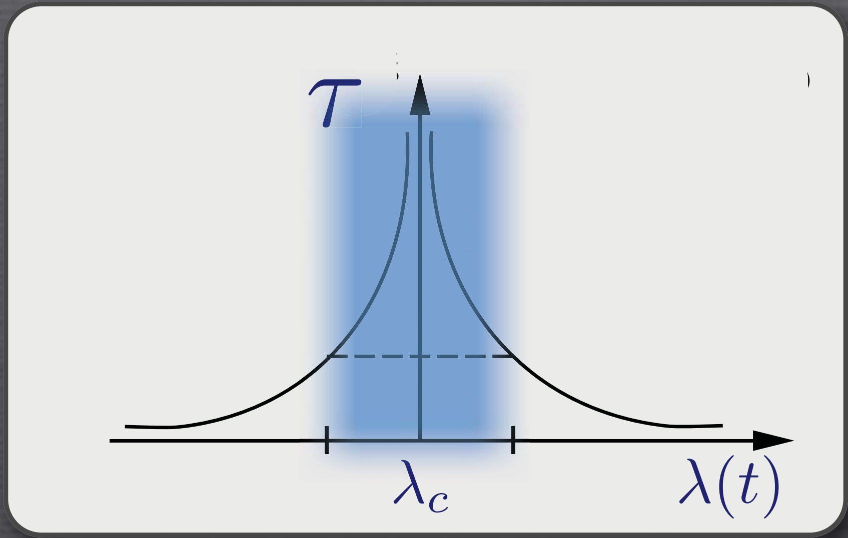
Adiabatic Dynamics Close to a Critical Point



- Adiabatic quantum computation
- Quantum annealing
- Defect formation passing through a critical point

Topological Defect Formation

Simulation of phase transitions in the early universe in condensed matter systems (superfluids and Josephson junctions)

Th: Zurek '85-'88

Exps:Bauerle et al '96,Ruutu et al'96}

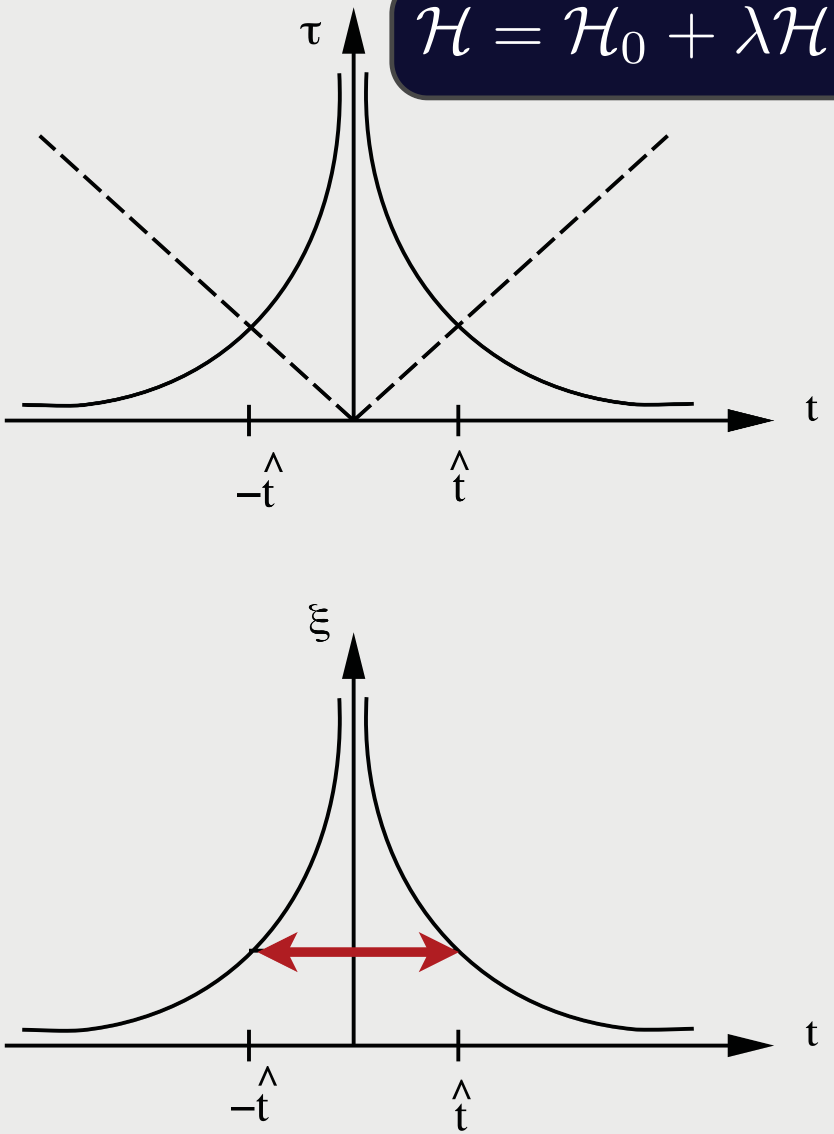
Extension to quantum phase transitions

Zurek, Dorner, Zoller '05

Polkovnikov '05

Defect density

W. Zurek '85
W. Zurek, U. Dorner and P. Zoller '05
A. Polkovnikov '05



$$\mathcal{H} = \mathcal{H}_0 + \lambda \mathcal{H}_1$$

$$\lambda - \lambda_c = vt$$

The adiabatic approximation breaks down when

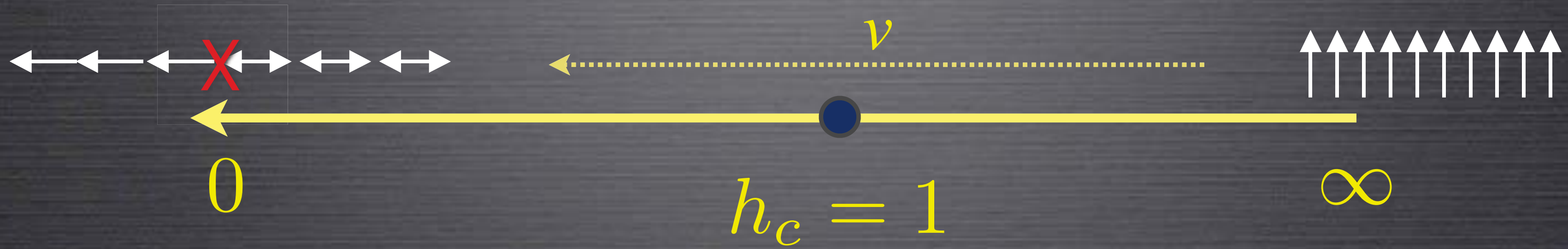
$$\frac{\dot{\lambda}}{\lambda} \sim \tau$$

$$\rho_{def} \sim \hat{\xi}^{-d} \sim v^{\frac{d\nu}{z\nu+1}}$$

$$\mathcal{E}_{res} \sim \mathrm{J}\rho_{def}$$

1D Ising Model

$$H = -\frac{J}{2}\sum_{j}^{N}\{\sigma_j^x\sigma_{j+1}^x + h(t)\sigma_j^z\}$$

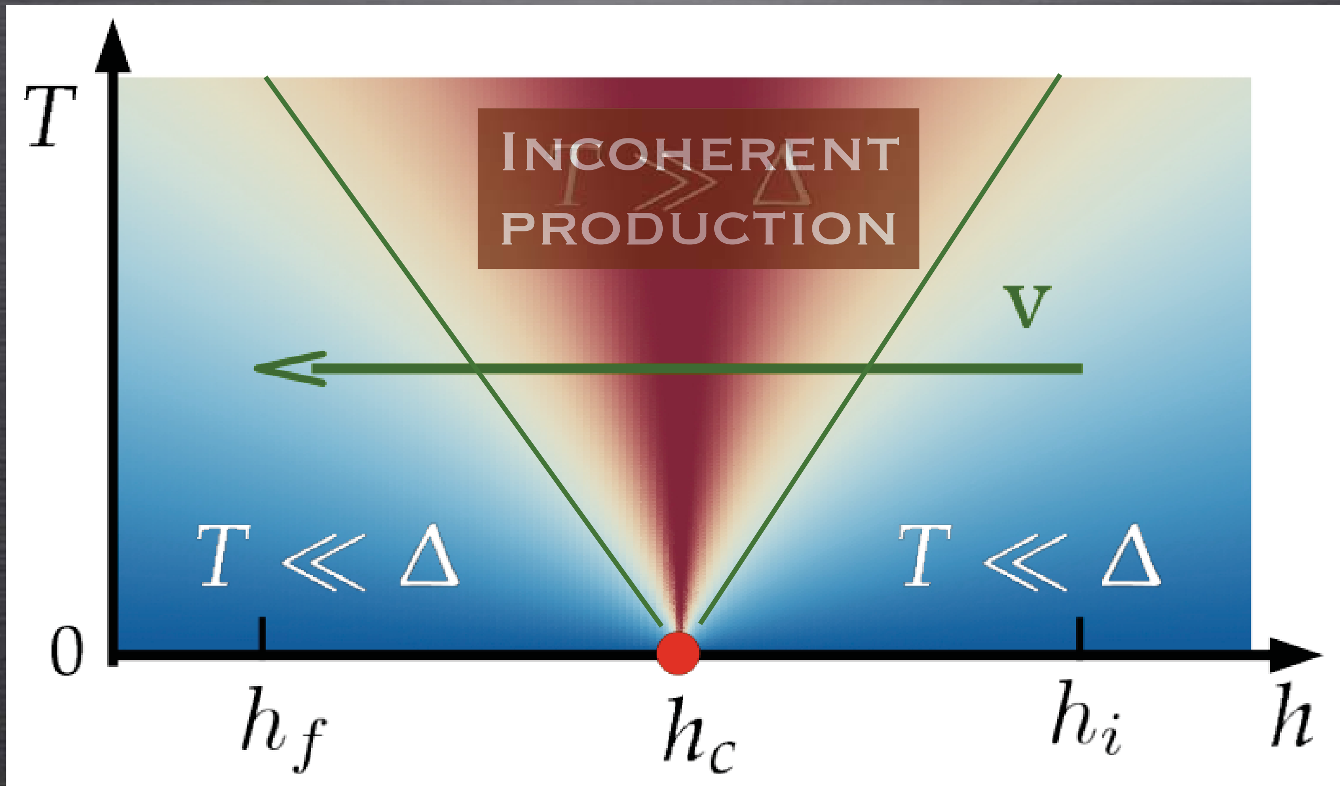


$$\mathcal{E}_{res} \sim \sqrt{v}$$

CONSIDER A QUANTUM SYSTEM COUPLED TO AN ENVIRONMENT AT A TEMPERATURE T

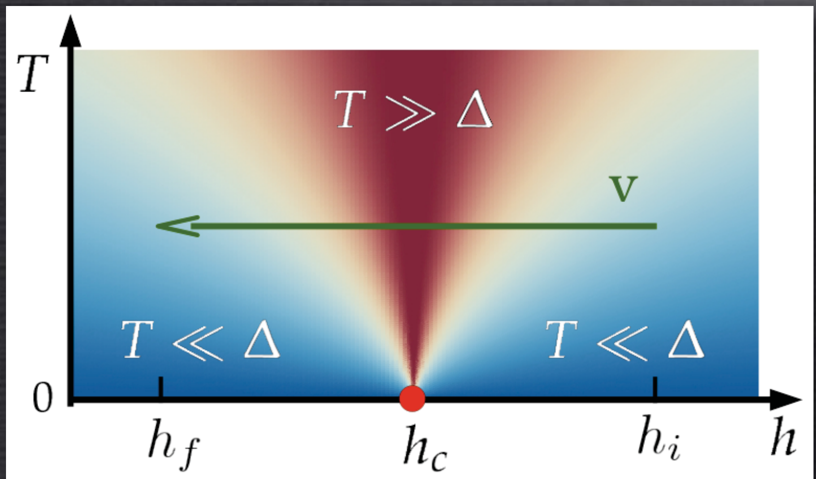
Is it possible in the presence of dissipation and dephasing to describe universally the production of defects in an adiabatic quench ?

Quantum Critical Region



"Incoherent" Defects

- ✓ Density of defects $\mathcal{E} \simeq \mathcal{E}_{KZ} + \mathcal{E}_{inc}$
- ✓ The bath does not influence the system for $T \ll \Delta$
- ✓ Relaxation in the critical region $\tau_r^{-1} \propto \alpha T^{\theta}$



$$t_{QC} = 2T^{1/\nu z} v^{-1}$$

$$\mathcal{E} = \int \frac{d^d k}{(2\pi)^d} \mathcal{P}_k$$

$$\frac{d}{dt}\mathcal{P}_k = -\frac{1}{\tau}\left[\mathcal{P}_k - \mathcal{P}_k^{th}(h_c)\right]$$

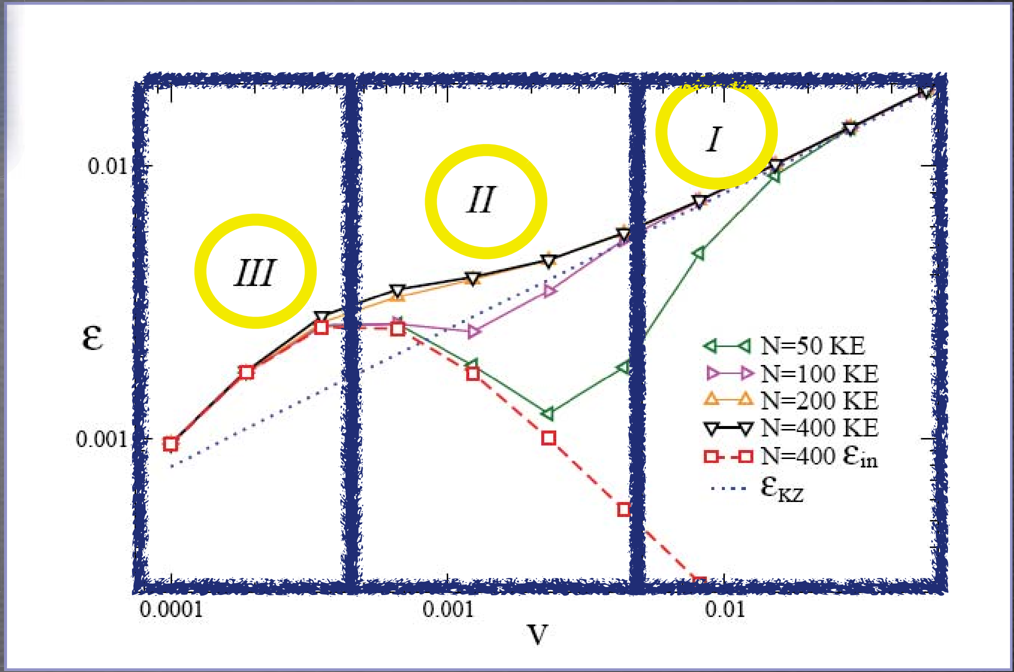
"Incoherent" Defects

$$\mathcal{E}_{inc} \propto \alpha v^{-1} T^{\theta + \frac{d\nu+1}{\nu z}}$$

$$v_{cross} \propto \alpha^{\frac{\nu z+1}{\nu(z+d)+1}} T^{\left(1+\frac{(\theta-1)\nu z}{\nu(z+d)+1}\right)\left(1+\frac{1}{\nu z}\right)}$$

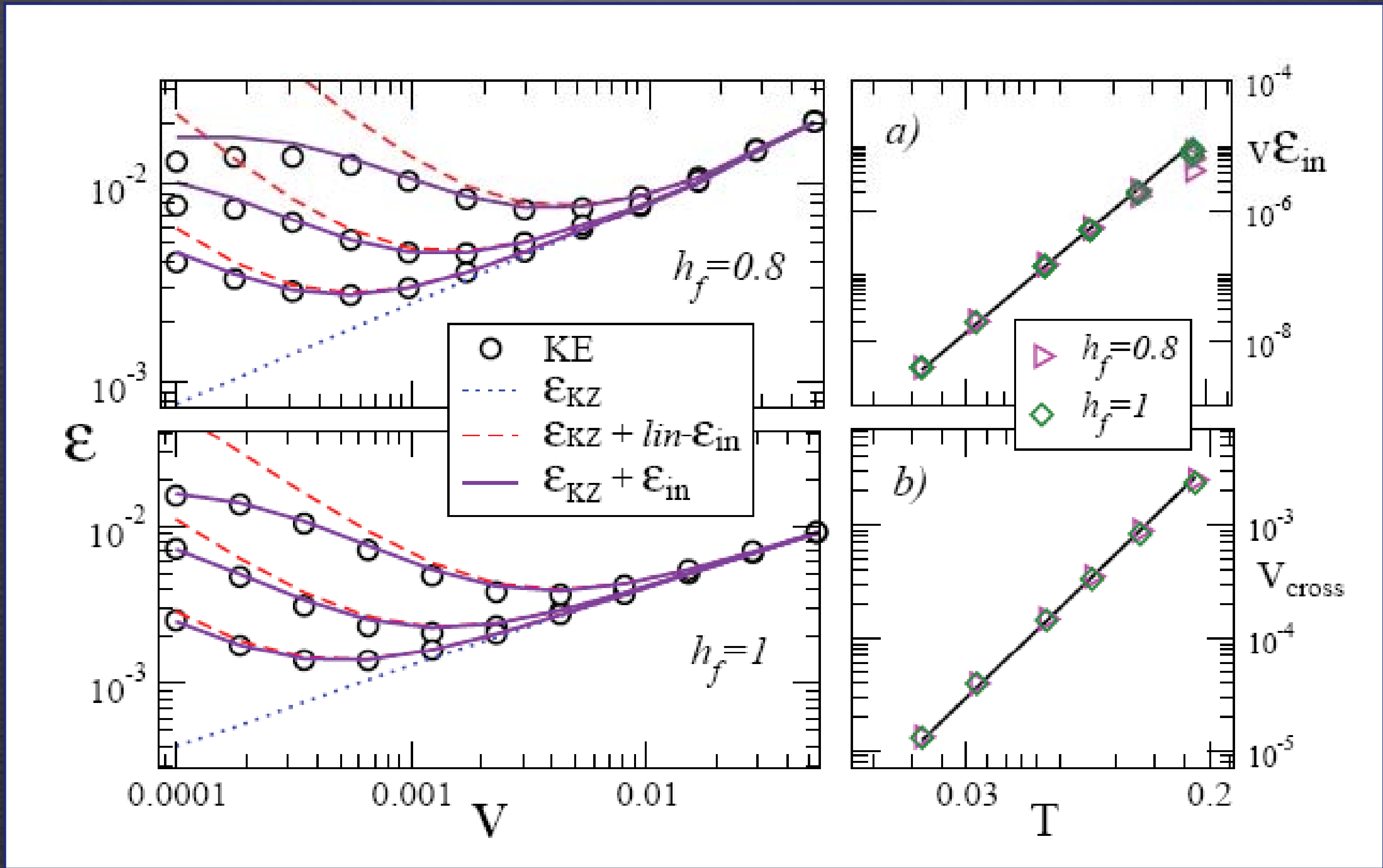
Density of Defects

The bath has two effects:
- It creates excitations near the critical point
- It relaxes the system to its ground state after leaving the quantum critical region



- For fast quenches the bath is unable to affect the system and the KZ scaling is preserved.
- For very slow quenches only thermal excitations contribute to defect formation is dominated by the coupling to the environment.
- In the crossover region both thermal and non-adiabatic excitations contribute.

Comparison of The Kinetic Equations with The Scaling analysis



Time-Dep Hamiltonians

$$\mathcal{H} = \mathcal{H}_0 + \lambda(t)\mathcal{H}_1$$

Quench | **Quench** | **Adiabatic**

Sengupta, Powell, Sachdev ('04)
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Rigol et al, ('06)
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Zurek, Dorner, Zoller ('05)
Polkovnikov ('05)
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Quantum Quenches and Thermalization

Deutsch, *PRA (1991)*
Srednicki, *PRE (1994)*
Rigol, Dunjko, Yurovsky, & Olshanii, *PRL (2007)*
Rigol, Muramatsu, & Olshanii, *PRA (2006)*
Cazalilla, *PRL (2006)*
Calabrese & Cardy, *PRL (2006)*, *JSTAT (2007)*
Gangardt & Pustilnik, *PRA (2008)*
Eckstein *& Kollar, PRL (2008), PRA (2008)*
Iucci & Cazalilla, *arXiv (2009)*
Barther, Schollwoeck, *PRL (2008)*
Rigol, Dunjko, & Olshanii, *Nature (2008)*
Kollath, Lauchli & Altman, *PRL (2007)*
Manmana, Wessel, Noack, & Muramatsu, *PRL (2007)*
Rigol *PRL (2009), PRA (2009)*
Biroli et al. *arXiv 0907.3731*
.....

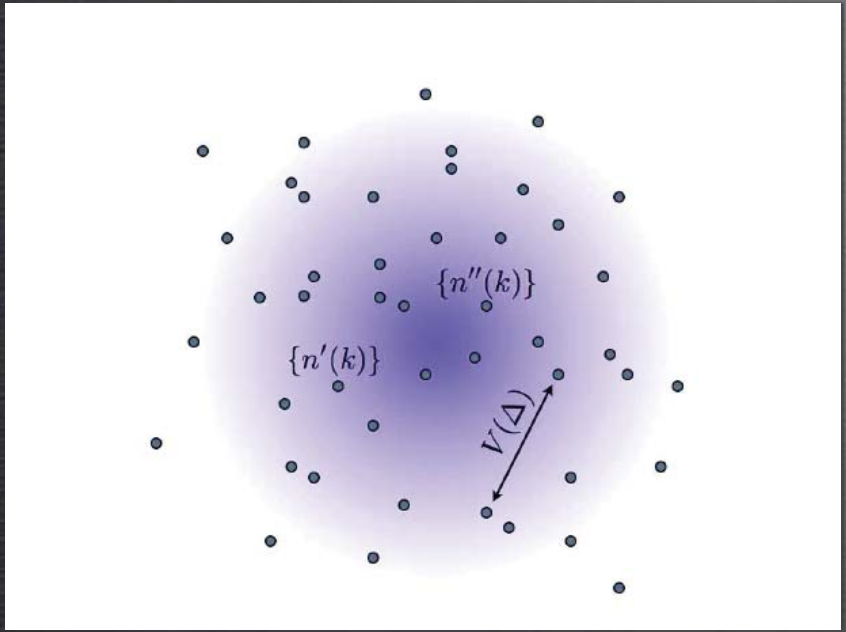
- The system acts as its own thermal bath ?
- In integrable systems the time-averaged steady state described by a generalized Gibbs ensemble
- Far from integrability → thermalization

↓

Many-BOdy localization

Many-Body Localization

Altshuler, Gefen, Kamenev, Levitov, *PRL (1997)*
Basko, Aleiner, Altshuler, *Ann. Phys. (2006)*
Pal and Huse, PRB (2010)
Znidaric, Prosen, and Prelosek, PRB (2008)
Oganesyan and Huse PRB (2007)
...



Quasi-particle space

For integrable models all states are localized

Break integrability = connect lattice points

Localization/delocalization transition

Connection between the localization transition and the thermalization following a quantum quench

The Models

$$\mathcal{H}(t) \equiv \mathcal{H}_0[g(t)] + \mathcal{H}_{ib}$$

$$g(t) = \begin{cases} g_0 & \text{for} \quad t < 0 \\ g & \text{for} \quad t \geq 0 \end{cases}$$

$$\mathcal{H}_0(J_z) = \sum_{i=1}^{L-1} \left[J\left(\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y\right) + J_z \sigma_i^z \sigma_{i+1}^z \right],$$

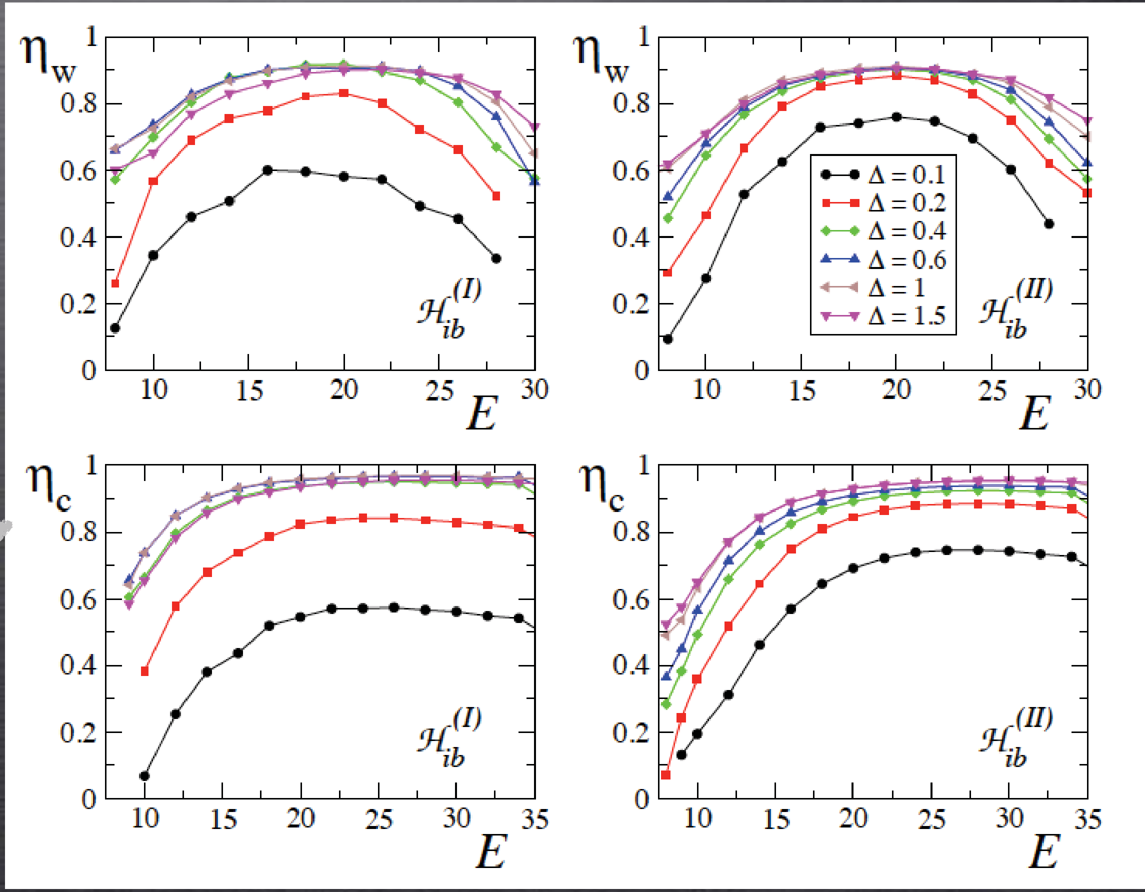
$$\mathcal{H}_{ib} = \begin{cases} \Delta \sum_{i=1}^{L} h_i \sigma_i^z & (I) \\ \Delta \sum_{i=1}^{L-1} h_i \sigma_i^z \sigma_{i+1}^z & (II) \\ \Delta \sum_{i=1}^{L-2} h_i \sigma_i^z \sigma_{i+2}^z & (III) \\ \Delta \sum_{i=1}^{L-2} \sigma_i^z \sigma_{i+2}^z & (IV) \end{cases}$$

Level spacing statistics

$$\eta \equiv \frac{\int_0^{s_0}[P(s) - P_P(s)]ds}{\int_0^{s_0}[P_{WD}(s) - P_P(s)]ds}$$

In a shell of energy E

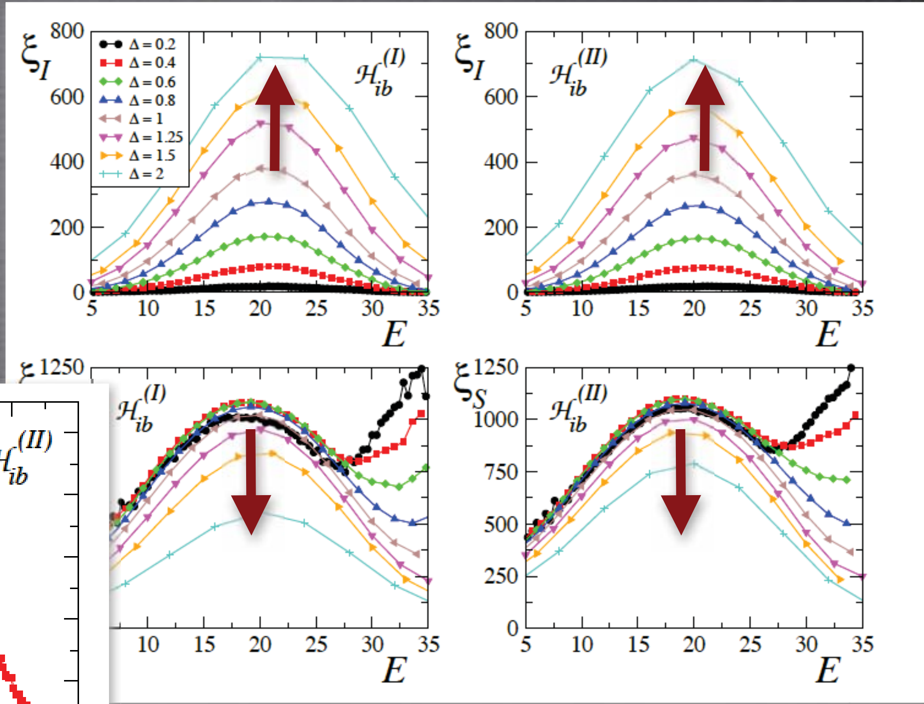
Including all the energies < E

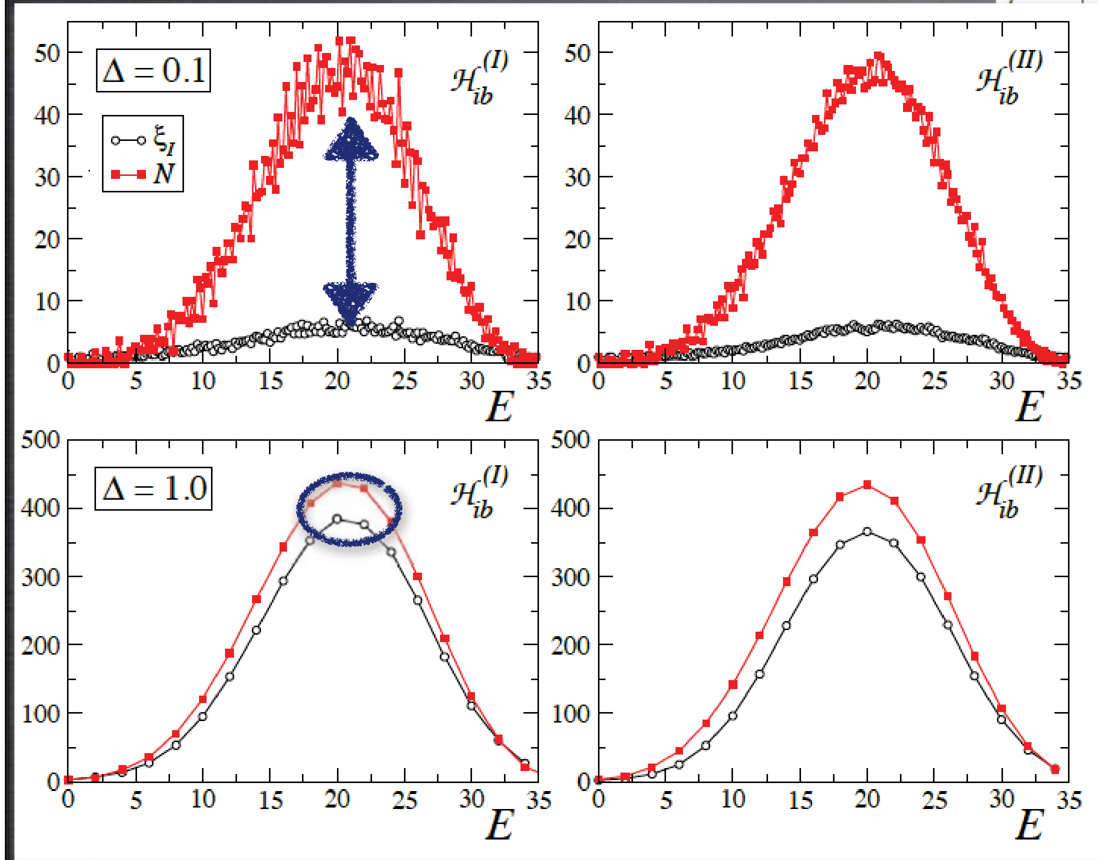


IPR $\xi^{-1} = \sum_{n=1}^{N} |<n|\psi>|^4$

I - in the integrable basis

S - in the computational basis



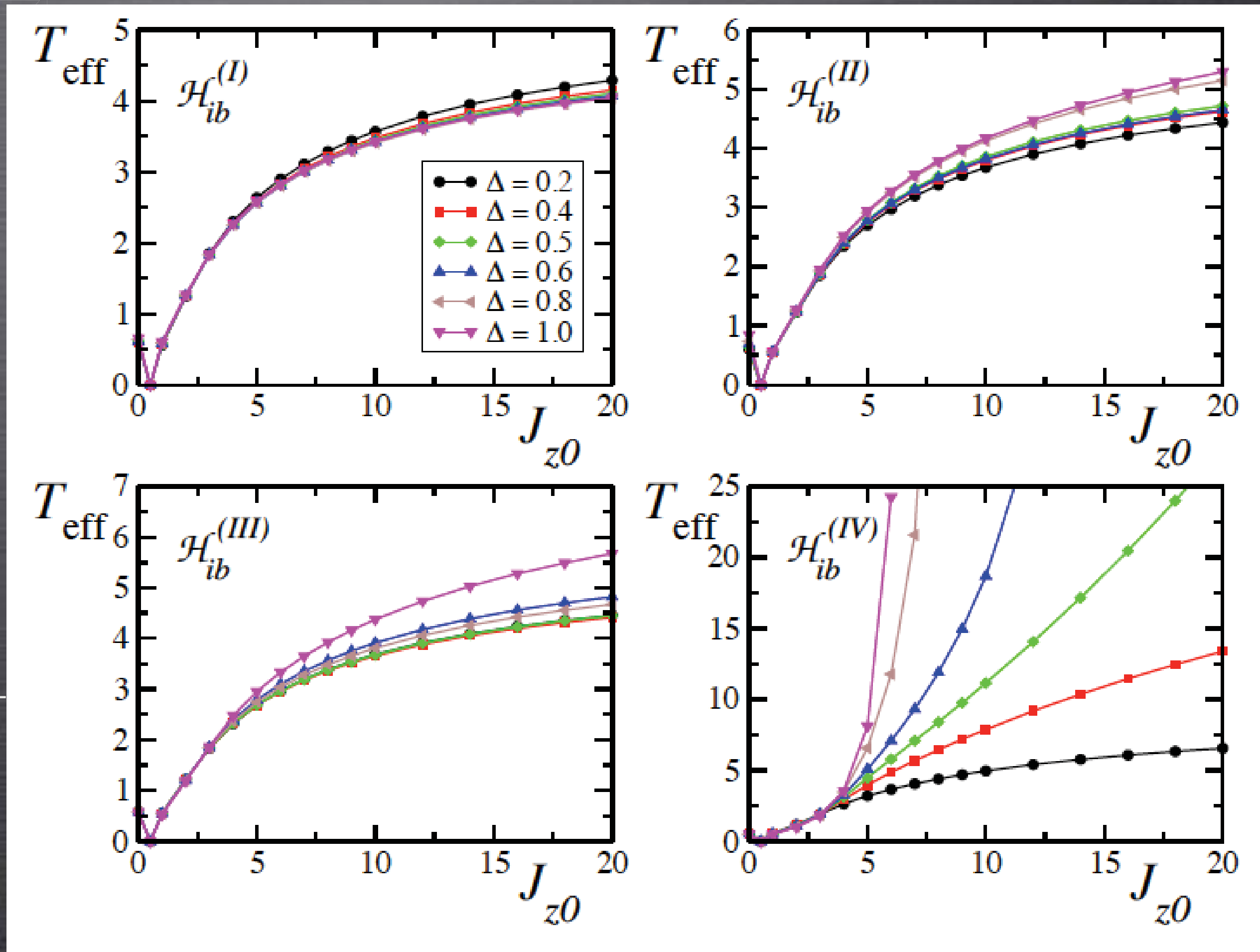


Increasing Disorder

Effective Temperature

see Rossini, Silva, Mussardo and Santoro PRL (2009)

Compare the long-time dynamics with an effective thermal average



$$\rho(T_{\text{eff}}) = \frac{e^{-\mathcal{H}(J_z)/T_{\text{eff}}}}{\text{Tr}[e^{-(J_z)/T_{\text{eff}}}]}$$

$$E_0 = \langle \psi_0 | (J_z) | \psi_0 \rangle$$

$$E_0 \equiv \langle (J_z) \rangle_{T_{\text{eff}}} = \text{Tr}\left[\rho(T_{\text{eff}})\,(J_z)\right]$$

Correlation functions

$$n_k^\alpha \equiv \frac{1}{L}\sum_{j,l=1}^{L} e^{2\pi i(j-l)k/L}\sigma_j^\alpha\sigma_l^\alpha\,, \qquad (\alpha = x, z)$$

n_k^z **LOCAL**

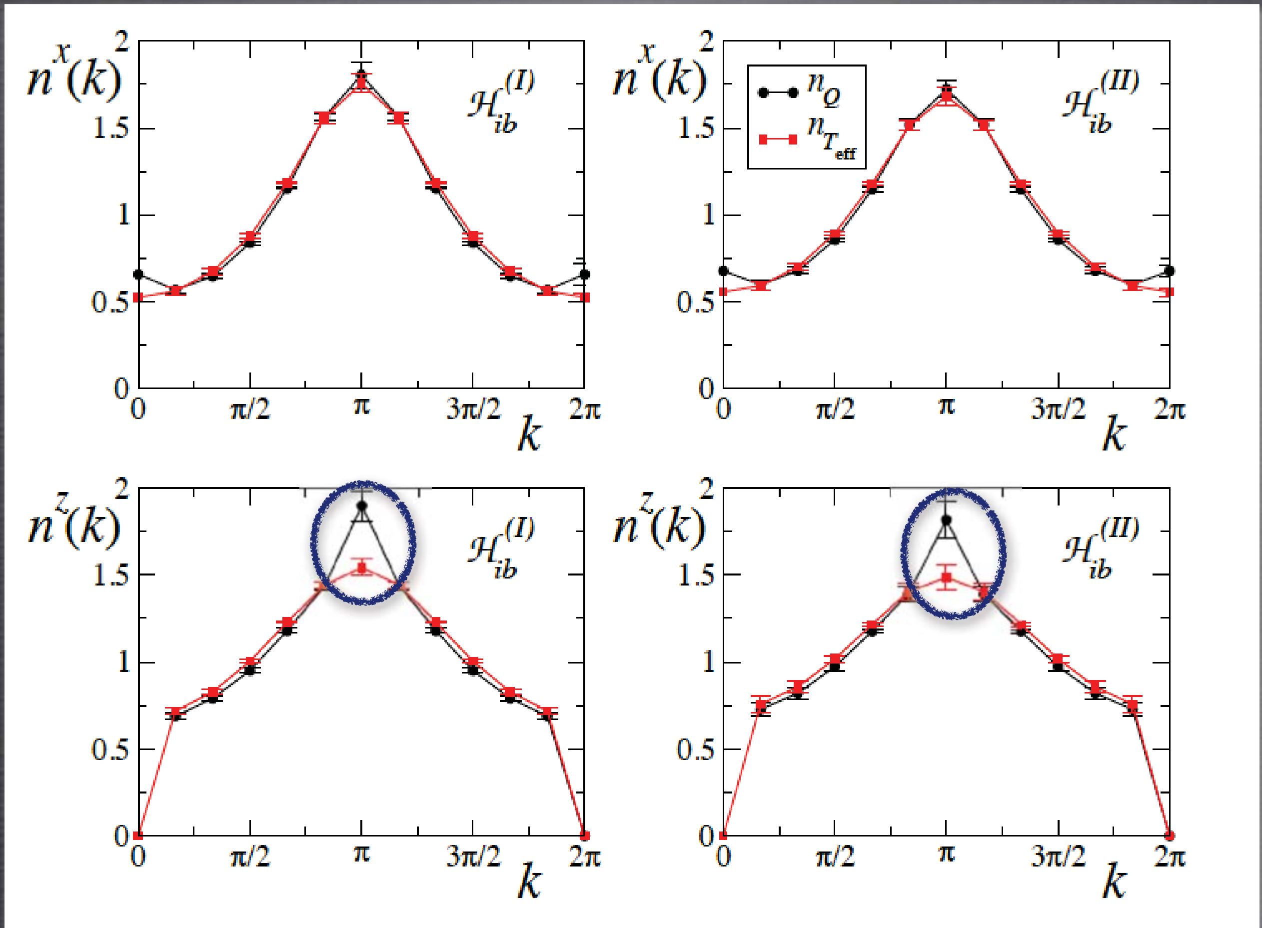
n_k^x **NON LOCAL**

Nonlocal: integrability IS NOT important

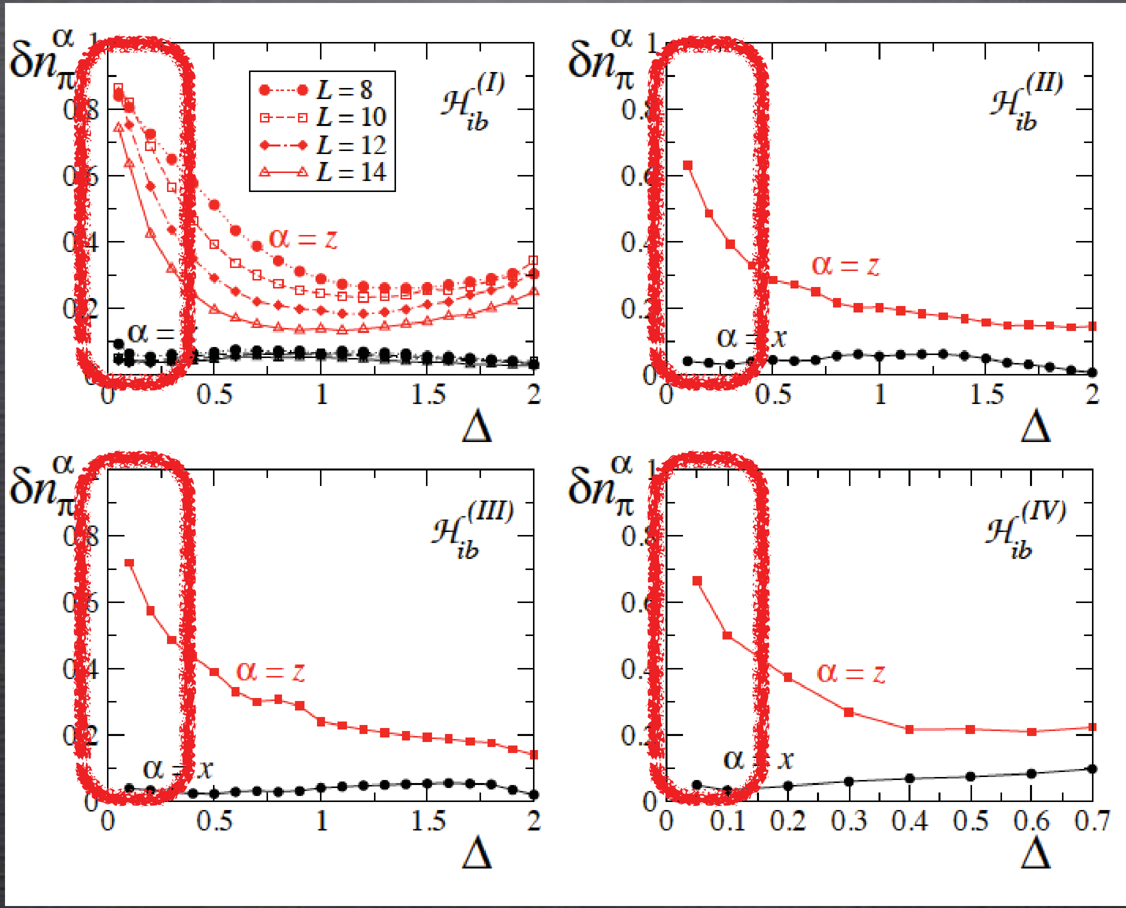
(see Rossini et al (2009))

Correlation functions

$$n_k^\alpha \equiv \frac{1}{L}\sum_{j,l=1}^{L} e^{2\pi i(j-l)k/L}\sigma_j^\alpha\sigma_l^\alpha\,, \qquad (\alpha = x, z)$$



Deviations from the thermal state



Conclusions

- Coherent vs incoherent defect production accross a critical point
- Thermalization is in correspondence with the localization/ delocalization transition in quasi-particle space
- In order to observe this transition one has to study a local observable in quasi-particle space