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Derivation of GPE.2D stress balance for different crustal structure/boundary conditions

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Problems with gravitational potential energy (per unit area) (GPE) and force per unit length.

1. Calculate the difference in potential energy (per unit area) between a mid-ocean ridge and old oceanic lithosphere. Assume that a column of mass including 2.5 km of water (density of 1 x 10^3 kg/m^3), 7 km of rock of density 2.9 x 10^3 kg/m^3, and mantle of density 3.2 x 10^3 kg/m^3 beneath the mid ocean ridge is in isostatic equilibrium with 6 km of water, 7 km of oceanic crust, and more dense (colder) mantle beneath the old oceanic lithosphere. Assume that compensation occurs at a depth of 113 km (100 km of mantle lithosphere beneath the base of old oceanic crust). First, determine the average density of the old mantle lithosphere. Then, assume that it is constant, from a depth of 13 km (6 km of water and 7 km of crust) to a depth of 113 km, and calculate the GPE difference (or force per unit length).

This value is commonly called "ridge push." It gives the force per unit length that the buoyant column of material beneath a mid-ocean ridge applies to old oceanic lithosphere.

2. Consider the difference in GPE between normal lithosphere, for which the surface is at sea level, the crustal thickness is 35 km, its density is 2.8 x 10^3 kg/m^3, and that of the mantle is 3.3 x 10^3 kg/m^3, and a plateau overlying thickened crust, with an elevation of 4 km and in Airy isostatic equilibrium with the lower region. First, how deep is the crustal root beneath the plateau, and how thick is the crust beneath the plateau? Then what is the GPE difference. Note that it is notably larger than "ridge push."

3. Suppose that we have a plateau at a height of 4 km, with a crustal thickness of 65 km (61 km below sea level, and 4 km above it), a crustal density of 2.8 x 10^3 kg/m^3, and underlain by 110 km of mantle lithosphere of density of 3.3 x 10^3 kg/m^3, which also is more dense than asthenosphere by 30 kg/m^3. Now assume that that the mantle lithosphere, 110 km thick, is removed and replaced by asthenosphere. How much would the surface rise so as to maintain isostatic compensation? (Be careful: although the surface, at which the density of crust is 2.8 x 10^3 kg/m^3, rises by a certain amount, the Moho (crust-mantle interface) also rises by that amount too.) Then estimate the gain in GPE associated with removal of mantle lithosphere by asthenosphere. Note that it too is bigger than "ridge push"!