The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

IAEA International Atomic Energy Agency

2240-Exercise

Advanced School on Scaling Laws in Geophysics: Mechanical and Thermal Processes in Geodynamics

23 May - 3 June, 2011

Deformation Length-scales for Boundary-driven flow in a Viscous Sheet

G. Houseman
Univ. of Leeds
UK

Advanced School on Scaling Laws in Geophysics: Mechanical and Thermal Processes in Geodynamics

Exercise: Deformation Length-scales for Boundary-driven flow in a Viscous Sheet

Greg Houseman

In this exercise we use the graphical display package called *sybil* to examine the deformation fields in a thin viscous sheet, calculated using a finite element program called *basil*.

1. The exercise commences with a demonstration of how *sybil* works.

    logging on
    starting up sybil
    opening a solution file
    plotting velocity (arrow plots and contoured component plots)
    plotting strain-rate or stress components
    profiles
    labels
    log files and obtaining hard copy
    advancing to solutions at later time
    finite strain markers
    crustal thickness

2. The set of experiments is designed to study the deformation that occurs when the boundary of a viscous half-space is disturbed by a prescribed boundary condition, in which the velocity varies harmonically along the boundary with wavelength L. We set up the problem so that the deforming boundary is $x = 0$, and assuming that the deformation is periodic in the y-direction, we solve only for $0 \le y \le L/2$. All such deformation problems are characterised by a field that decays with distance from the boundary, so that deformation is usually negligible at distances from the boundary much greater than L. We therefore assume that there is a boundary at $x = 3L/2$, beyond which there is no displacement (the length unit in these experiments = $L/2$). The constitutive relation for the experiments is a non-Newtonian viscous flow law in which strain-rate is proportional to the nth power of deviatoric stress
[$\tau_{ij} = B\dot{E}^{(1-n)/n}\dot{\varepsilon}_{ij}$, where $\dot{E} = \sqrt{\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij}}$ is the 2nd invariant of the strain-rate tensor].

The above constraints are common to the set of experiments but we consider:

(a) whether the deformation is
plane-strain (IN - experiments) or plane-stress (TV- experiments)

(b) whether the boundary velocity is
perpendicular to the boundary (INC -, TVS - experiments) or
parallel to the boundary (INX -, TVX - experiments)

(c) The value of n in the power-law constitutive relation: $\tau_{ij} = B\dot{E}^{(1-n)/n}\dot{\varepsilon}_{ij}$
(the number follows n in the filename), and

(d) The evolution with time of the deformation field in each experiment

3. Using sybil to interrogate the set of experiments which have previously been run, examine the deformation fields, as described below, and answer the questions (unless otherwise stated, assume that the questions refer to the time-zero solutions).

   (a) commencing with TVSn1A0: [plane-stress, n = 1, strain-rate proportional to stress]

      a-1. Construct an arrow plot of the velocity field.

      a-2. Profile the x-component of velocity, along $x = 0$. What is its maximum value ?

      a-3. Profile the x-component of velocity, along $y = 1$.

      a-4. At what distance from the boundary does u_x drop to 20% of its boundary value.

      a-5. Where is the y-component of velocity maximum ? and what is its approximate value ? (First contour the field, then profile through the maximum)

   (b) now compare the above plane-stress solution with a similar plane-strain problem INCn1A0. Repeat steps (a-1) through (a-5). In (a-3) overlay the two $y = 1$ profiles of u_x (use different colours for TVS and INC). What differences are observed in the two experiments ? Both these experiments represent a boundary that is at one location being indented (e.g., by continental collision), and elsewhere being extended (e.g., by retreat of subduction zone). INC (incompressible) represents the case in which there is assumed to be no vertical strain anywhere, whereas TVS (thin viscous sheet) represents the case in which the sheet can thicken or thin in response to horizontal force.

   (c) for TVSn1A0, construct contour plots of $\dot{\varepsilon}_{xx}, \dot{\varepsilon}_{yy}, \dot{\varepsilon}_{xy}$ and $\dot{\varepsilon}_{zz}$. Determine in each case where the maximum of the strain-rate component occurs, and what is its approximate value. What differences (qualitatively) are observed for the same type of plots with INCn1A0 ? Examine the shape of the finite strain markers in the final record of TVSn1A0, to reinforce your understanding of the strain-rate components contoured at time zero.

   (d) The rate at which velocity decays away from the boundary depends on the stress versus strain-rate exponent, n. For the TVS series (n1A0, n3A0, n10A0), construct profile plots (overlain in 3 colours) of the x-component of velocity, along $y = 1$, and estimate, for each, the distance at which the velocity decreases to 20 % of its maximum value.

4. If time remains, look now at the experiments where the boundary is driven by imposed shear (TVX and INX series).

   (a) Plot the velocity vectors and make contour plots of the velocity components for $n = 1$.

   (b) For the TVX series, repeat step 3(d) by profiling along $y = 0.5$, and compare the width of the deformation zones between TVS and TVX solutions

This exercise is based on the paper:

England, P., G.Houseman, and L. Sonder, Length scales for continental deformation in convergent, divergent and strike-slip environments: analytical and approximate solutions for a thin viscous sheet model, *J. Geophys. Res.*, 90, 3551-3557, 1985.

and uses the *basil/sybil* software package developed by: G. Houseman, T. Barr, and L.Evans, described in *Microdynamics Simulation*, Ed. P. Bons, D. Koehn, M.Jessell, Springer, pp138-154, 2008.