



**The Abdus Salam  
International Centre for Theoretical Physics**



**2240-Exercise**

**Advanced School on Scaling Laws in Geophysics: Mechanical and  
Thermal Processes in Geodynamics**

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**Rayleigh-Taylor Instability**

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# Advanced School on Scaling Laws in Geophysics: Mechanical and Thermal Processes in Geodynamics

## Exercise: Rayleigh-Taylor Instability

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1. Use `sybil` to examine a solution file which illustrates Rayleigh-Taylor instability. The example solution provided is called `RTn1`. It shows the instability that develops when a dense fluid layer representing the lithospheric mantle starts to drip into a less dense, less viscous layer representing the asthenosphere. In this example the developing instability is slowed by the fact that there is a low-density crustal layer above the unstable mantle layer and the density increases from 0 at the base of the layer to 2 at the model Moho. The instability grows slowly at first, going through an exponential growth phase, then towards the end develops super-exponential growth.
  - a. plot the domain outline at different times to see how the drip develops (use a different colour for each time and put the graphs on top of each other to see the development. Note that the calculation starts with an initial perturbation to layer thickness in the form of a cosine variation.
  - b. plot the internal boundary (model Moho) at the same set of times to see the influence of the instability on crustal thickness
  - c. plot the density distribution at time zero (use colour mapping which will show the relative density variation between 0 and 1 in the mantle, and let the large negative colour in the crust saturate). Compare this with the density distribution at large time to see how the drip has developed.
  - d. plot the strain markers at 2 or 3 different times to get another perspective on the internal deformation
  - e. plot the velocity vectors at small and large times to see how the velocity distribution varies
  - f. for the  $t=2$  solution, look at the variation of the strain tensor by plotting `edxx`, `edyy`, `edxy` and `msst` (maximum shear strain rate). Note that the orientation of principal stresses varies continuously as can be seen by plotting the stress arrows (`SIGD`). This is a 2D incompressible flow field so at any point in the continuum, stretching in one direction is balanced by shortening in the orthogonal direction.
  - g. `RTn2` is another calculation, which is the same except that viscosity increases from 0.02 on the base of the unstable layer to 0.2 at the model Moho (`RTn1` used a constant viscosity of 0.1). Plot the viscosity distribution at time zero, and compare it with the distribution of viscosity after the instability is well developed. Plot and compare `edyy` (vertical strain-rate distribution), `visc` (viscosity), `tayy` (vertical deviatoric stress), and `thdi` (thermal dissipation rate or rate of mechanical work) for `RTn1` and `RTn2` to see the effect of viscosity in localising stress and work.
2. Now we look at some time series arising from the above two calculations. The data produced by `basil` when the computation was done are placed in `FD.sols/RTn#.dat` and we use a program called `grafdat` to make some simple plots (this task could also be done using Octave or Matlab

if preferred). Run grafdat to make two plots produced in the postscript file grafdat.ps - the plots initially show the vertical displacement of the bottom of the drip (YMIN) versus time. In the upper graph we plot the logarithm of the displacement versus time. What is the growth rate after exponential growth is established? Why does the perturbation decay at first? Modify grafdat.inp to make a similar plot for RTn2. How does the growth rate change?

The above exercise is based on research described in the following publications:

- Houseman, G., and P. Molnar (1997), Gravitational (Rayleigh-Taylor) instability of a layer with non-linear viscosity and convective thinning of continental lithosphere, *Geophys. J. Int.*, 128, 125-150.
- Molnar, P., G. Houseman and C. Conrad (1998), Rayleigh-Taylor instability and convective thinning of mechanically thickened lithosphere: effects of non-linear viscosity decreasing exponentially with depth and of horizontal shortening of the layer, *Geophys. J. Int.*, 133, 568-584.
- Neil, E.A., and G.A. Houseman (1999), Rayleigh-Taylor instability of the upper mantle and its role in intraplate orogeny, *Geophys. J. Int.*, 138, 89-107.
- Molnar, P; Houseman, GA (2004) The effects of buoyant crust on the gravitational instability of thickened mantle lithosphere at zones of intracontinental convergence, *Geophys. J. Int.*, 158, 1134-1150. [doi:10.1111/j.1365-246X.2004.02312.x](https://doi.org/10.1111/j.1365-246X.2004.02312.x)