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2240-Exercise

Advanced School on Scaling Laws in Geophysics: Mechanical and Thermal Processes in Geodynamics

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Stokes Flow - sphere approaching a free boundary

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Exercise: Stokes Flow — sphere approaching a free boundary

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1. Directory *stokes* provides you with a set of time-zero calculations for the problem of a buoyant sphere rising through a viscous fluid. In these solutions Re « 1, gravity, $g = 1$; radius of sphere, $r = 1$; radius of container, $R = 10$; height of container, $H = 20$; viscosity of fluid, $\eta = 1$; relative density of sphere, $\Delta\rho = -1$; viscosity of sphere is 1000 (solid) for files named *stx10_??*, viscosity of sphere is 0.1 (bubble) for files named *sty10_??* These calculations assume axisymmetry, and we see only a single slice through the vertical cylinder with the axis of symmetry on $x = 0$. The base and the wall of the container are assumed rigid (zero velocity). The top of the cylinder is assumed sliding (zero normal velocity, zero shear stress). In each case a single instantaneous solution is provided in one file for a sphere whose centre is on the axis at y_c (y_c is the last number in the file name; $y = 0$ is the central height of the cylinder).

2. Open up *stx10_0* and check that the calculation is as described in the above paragraph, i.e., use Xyplot → Boundary / Internal boundary to see where the sphere is, check viscosity distribution, check density distribution, use the Profile → Mark function to confirm the coordinates of the sides and ends of the container, what is the radius of the sphere. Plot the velocity components and check that they are consistent with the declared boundary conditions. Plot the shear stress also; is it zero on the upper boundary ? 18 experimental solutions are provided, so be alert for any inconsistency in the expected solution, which could indicate an error in a *basil* input file.

3. Compare the velocity fields (contoured components) for the sphere at $y = 0$ and at $y = 8.5$. What differences are evident ? In each case measure the velocity of the sphere by making a vertical profile of Uy along the axis of symmetry, and noting the peak velocity in that part of the graph where the sphere is intersected.

4. Measure the velocity of the solid sphere for all of the experiments with names *stx10_??*, and plot the velocity against the distance between the sphere and the free boundary. What happens to the velocity as the solid sphere approaches the free boundary ? Why ? Can you explain the observed scaling ?

5. Compare the axial profiles of the *y*-component of velocity for the solid sphere at $y = 0$, *stx10_0,* and the low viscosity sphere at $y = 0$, *sty10_0*. In the *sty10_??* experiments everything is the same except we changed the viscosity of the sphere from 1000 used above for the solid sphere to 0.1, used here to represent a low-viscosity bubble. Explain what is happening inside the bubble (looking at Ux or strain-rate components might help here).

6. repeat the velocity measurements for all of the experiments with names *sty10_??*, and plot the velocity against the distance between the sphere and the free boundary.

7. Make axial profiles of the pressure through *stx10_0* and *sty10_0*. Can you explain the features that you see on this graph ?

The above exercise suggested by Michael Manga. Relevant references are:....