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Advanced School on Scaling Laws in Geophysics: Mechanical and Thermal Processes in Geodynamics

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Molnar and England's S in thrust faulting

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Shear stresses on thrust faults, the temperature structure near thrust faults, and frictional (or dissipative) heating

1. Few quantities in the earth sciences are more poorly known than the magnitudes of deviatoric stress that cause deformation of the crust and mantle.
2. Friction can be as source of heating that is important in metamorphism and perhaps maturation of oil.

Basic equation for thermal structure

The rate that heat content at a point in a body changes, which consists of the rate of change in a parcel of material plus heat advected with it, equals the sum of heat produced within it plus the heat conducted into it.

$$\rho C_P \frac{dT}{dt} = \rho C_P \left(\frac{\partial T}{\partial t} + \vec{V} \cdot \nabla T \right) = A + \nabla \cdot \vec{q}$$

A = rate of heat production within a body

$\vec{q}$ = heat flux (a vector) at any point;

$\vec{q} = -k\nabla T$ (Fourier's Law) where k is the coefficient of thermal conductivity.

Simplifying assumptions

$$\rho C_P\left(\frac{\partial T}{\partial t}+\vec{V}\cdot\nabla T\right)=A-k\nabla^2 T$$

Steady state: $\partial T/\partial t=0$

No motion: $\vec{V}$ = 0

No internal heat generation: A = 0

So, the basic equation becomes:

$$k\nabla^2 T=0$$

We will use this later.

Steady state, one dimension

$k\nabla^2 T = 0$ becomes $k\dfrac{d^2T}{dz^2} = 0$

For a layer of thickness h and temperature T = 0 at the top and T = T_0 at the bottom, the solution is: $T(z) = T_0 z/h$

For a heat flux at the base: $q_0 = k\, dT/dz$, the solution is

$$T(z) = \frac{q_0}{k}\frac{z}{h}$$

Time dependent heat transfer

Now consider a half-space on the surface of which the temperature is changed abruptly from $T = 0$ to $T = T_0$.

With no motion and no heat sources, the basic, one-dimensional equation is:

$$\frac{\partial T}{\partial t} = -\frac{k}{\rho C_P}\frac{\partial^2 T}{\partial z^2} = -\kappa \frac{\partial^2 T}{\partial z^2}$$

Here κ is the thermal diffusivity.

Error function time dependence

The solution to $$\frac{\partial T}{\partial t} + \kappa \frac{\partial^2 T}{\partial z^2} = 0$$

is $$T(z,t) = T_0 erfc\left(\frac{z}{2\sqrt{\kappa t}}\right)$$

where

$$erfc\left(\frac{z}{2\sqrt{\kappa t}}\right) = \frac{2}{\sqrt{\pi}} \int_{\frac{z}{2\sqrt{\kappa t}}}^{\infty} \exp\left(-x^2\right) dx$$

Heat flux for error function solution

With this solution: $$T(z,t) = T_0 erfc\left(\frac{z}{2\sqrt{\kappa t}}\right)$$

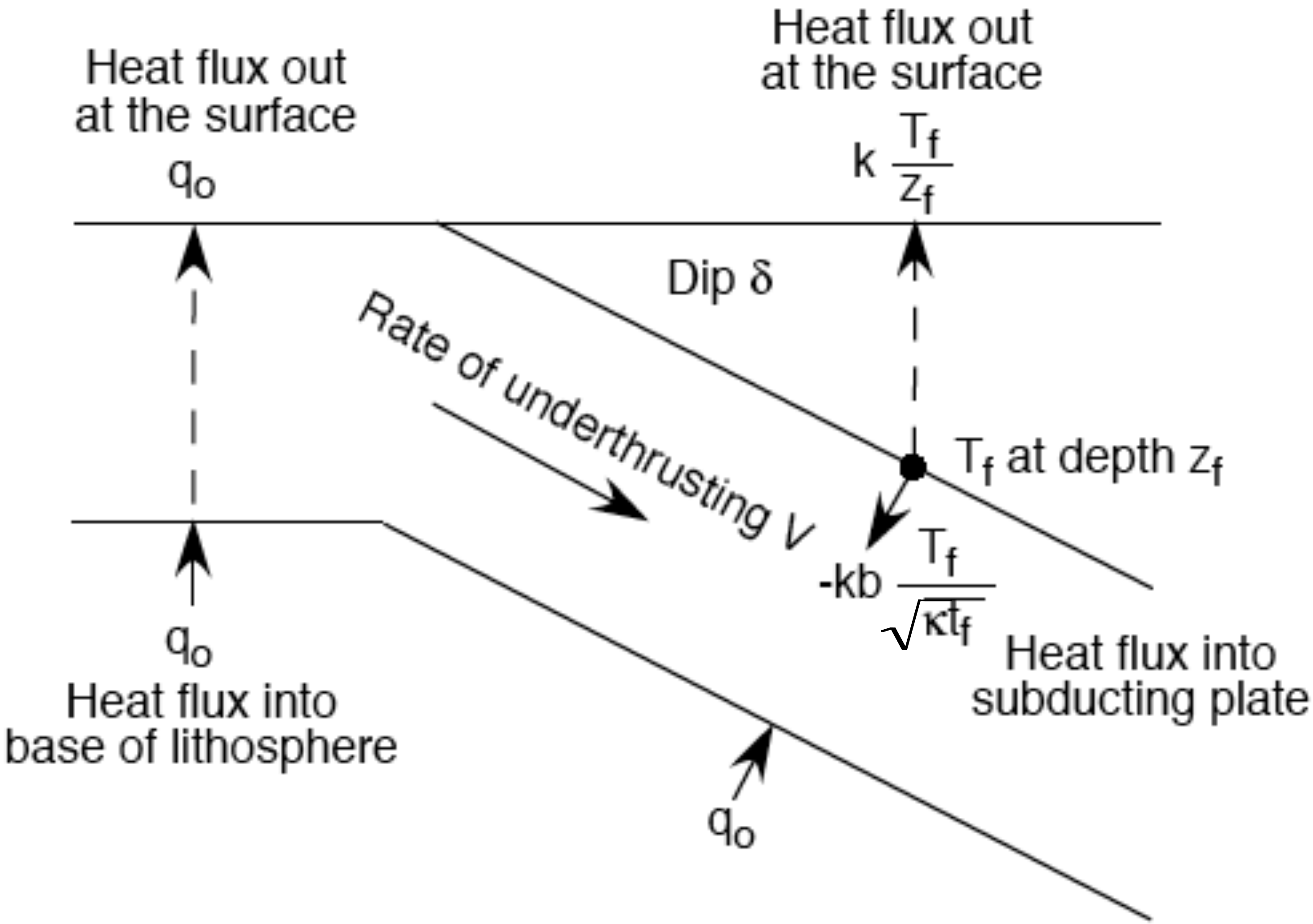
The flux of heat at z = 0 is simply:

$$q = k\frac{\partial T}{\partial z} = k\frac{T_0}{\sqrt{\kappa t}}$$

This expression contains the essential result that the scaling dimension for such heat transfer is simply: $\sqrt{\kappa t}$

We can now examine a thrust fault.

Simplified geometry and assumptions



Steady-State temperature in a thrust fault setting: no frictional heating (yet)

Consider the point z_f on the fault, where the steady-state temperature is T_f.

The heat flux upward through the upper thrust plate is simply: $k\, T_f / z_f$.

The flux from the underthrust plate consists of two parts. One is the flux from below: q_0. The other is a downward flux due to the warming of the top of the plate, which is approximately: $-k\,T_f \big/ \sqrt{\kappa t}$

Balance of heat fluxes

Balancing the fluxes and allowing for the approximation with a dimensionless coefficient b, we have:

$$k\frac{T_f}{z_f} = q_0 - bk\frac{T_f}{\sqrt{\kappa t}}$$

Rearranging gives

$$T_f = \frac{q_0}{k}\frac{z_f\sqrt{\kappa t}}{bz_f + \sqrt{\kappa t}}$$

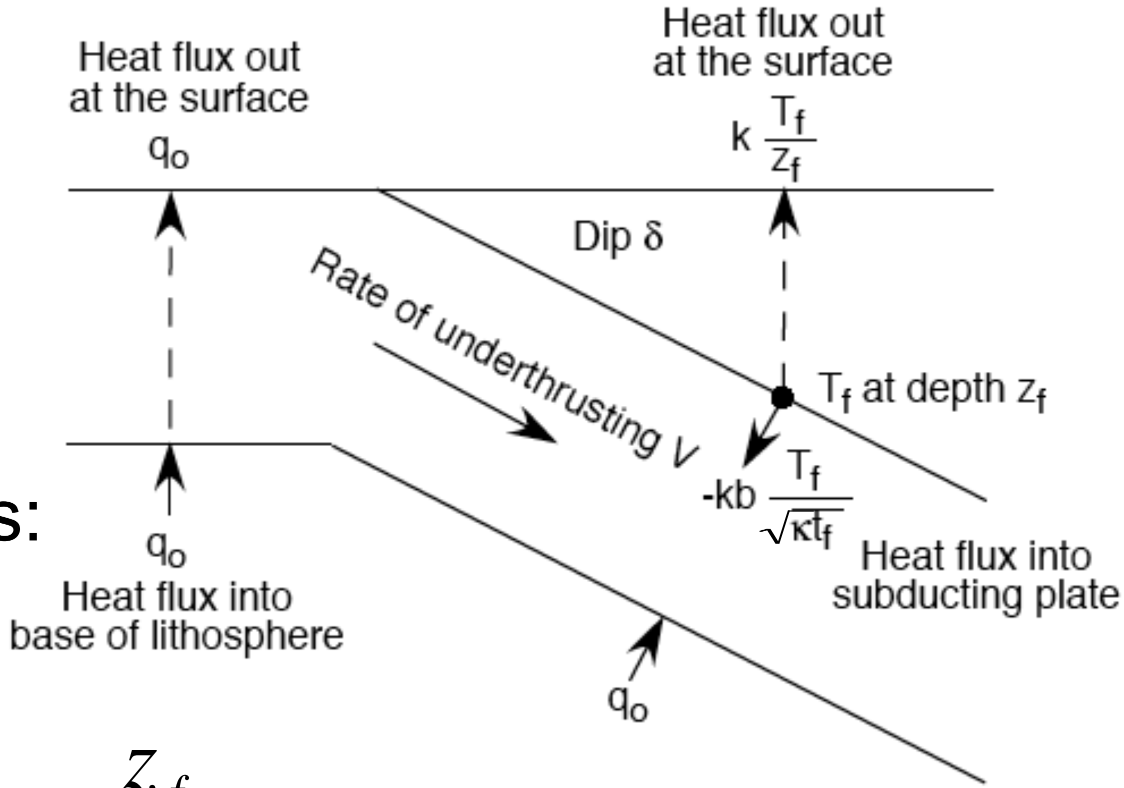
We need one more step, using the geometry.

Temperature on the fault

Using

$$t_f = z_f / V \sin\delta$$

The temperature
on the fault is:



$$T_f = \frac{q_0}{k} \frac{z_f}{1 + b\sqrt{\frac{z_f V \sin\delta}{\kappa}}}$$

Simple interpretation

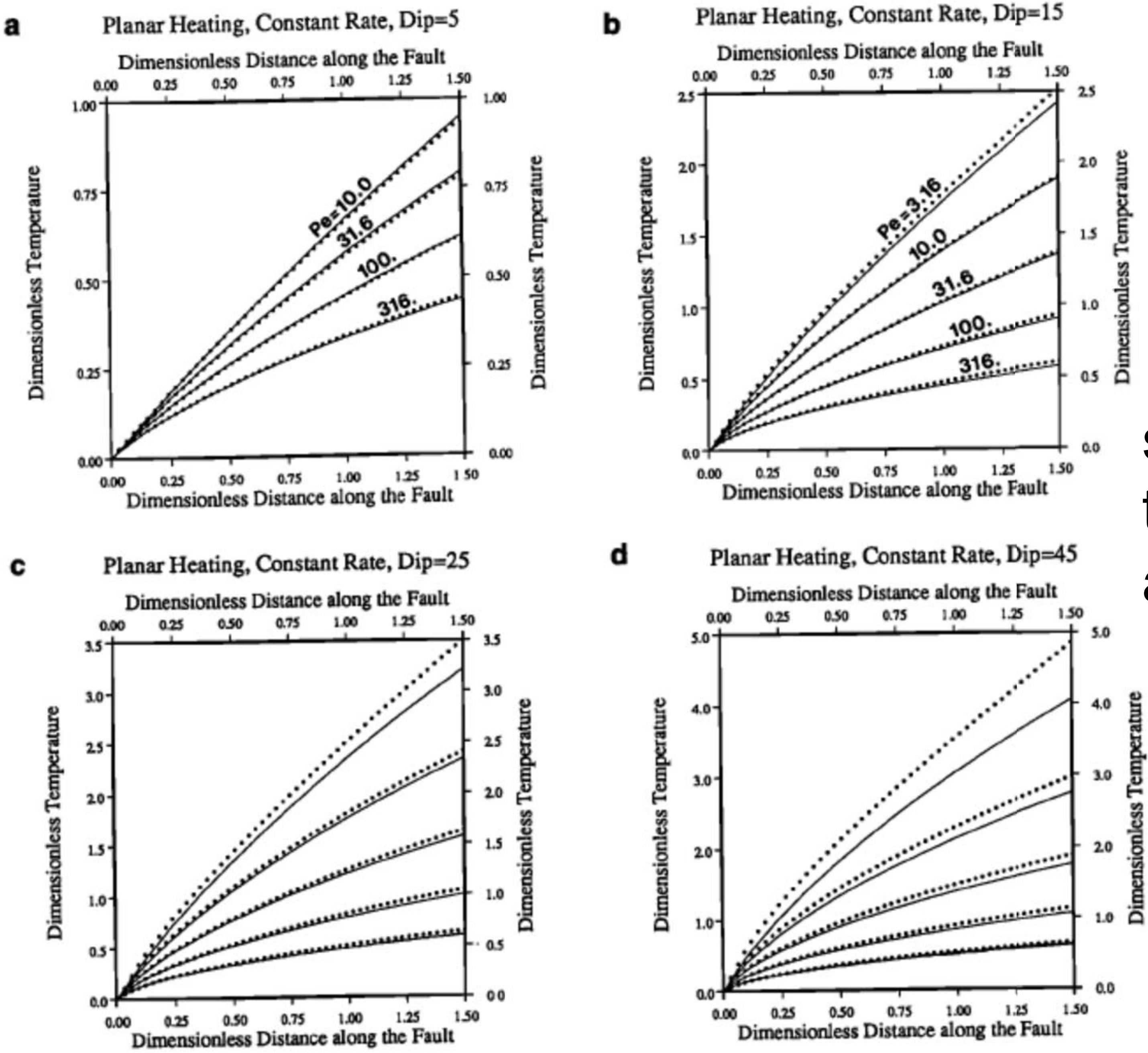
$$T_f = \frac{q_0}{k} \frac{z_f}{1 + b\sqrt{z_f V \sin\delta / \kappa}}$$

To appreciate the solution, note that if $V = 0$, $T_f = q_0 z_f / k$, which is the temperature at depth z_f with a flux from below of q_0.

Thus, thrust faulting **reduces** the temperature below what it would be by a divisor:

$$S = 1 + b\sqrt{z_f V \sin\delta / \kappa}$$

Numerical experiments show that $b \approx 1$.



Numerical solutions for temperature along thrust faults

Effect of thrust faulting

Temperatures are reduced from what they would be in the absence of thrust faulting by dividing those values by S:

$$T = \frac{T(\text{without thrust faulting})}{S}$$

$$S = 1 + b\sqrt{\frac{z_f V \sin\delta}{\kappa}}$$

Friction as a heat source

The rate of heat generation per unit area of a fault surface is simply the product of shear stress times slip rate:

$$q_f = \tau V$$

Some numbers: $\tau =$ 100 MPa (1 kbar)

V = 30 mm/yr (≈ 1 x 10^{-9} m/s); thus,

q_f = 100 mW/m^2

Typical heat flux from the earth is ~60 mW/m^2

Frictional heating could be important!

Realistic frictional heating

OK: 100 MPa may be a large shear stress.

A slip rate of 30 mm/yr also is fast for a fault within a continental region.

Still, a reasonable shear stress of 40 MPa and slip rates of 15 mm/yr yield heat production of q_f = 20 mW/m^2, which is a significant fraction of typical heat fluxes through the crust.

Heat flux above a thrust fault

The same algebraic rearrangement gives

$$q = \frac{q_0 + \tau V}{S}, \quad S = 1 + b\sqrt{z_f V \sin\delta / \kappa}$$

In principle, we can measure all of the relevant parameters, and then deduce the shear stress on the fault from the measured heat flux.

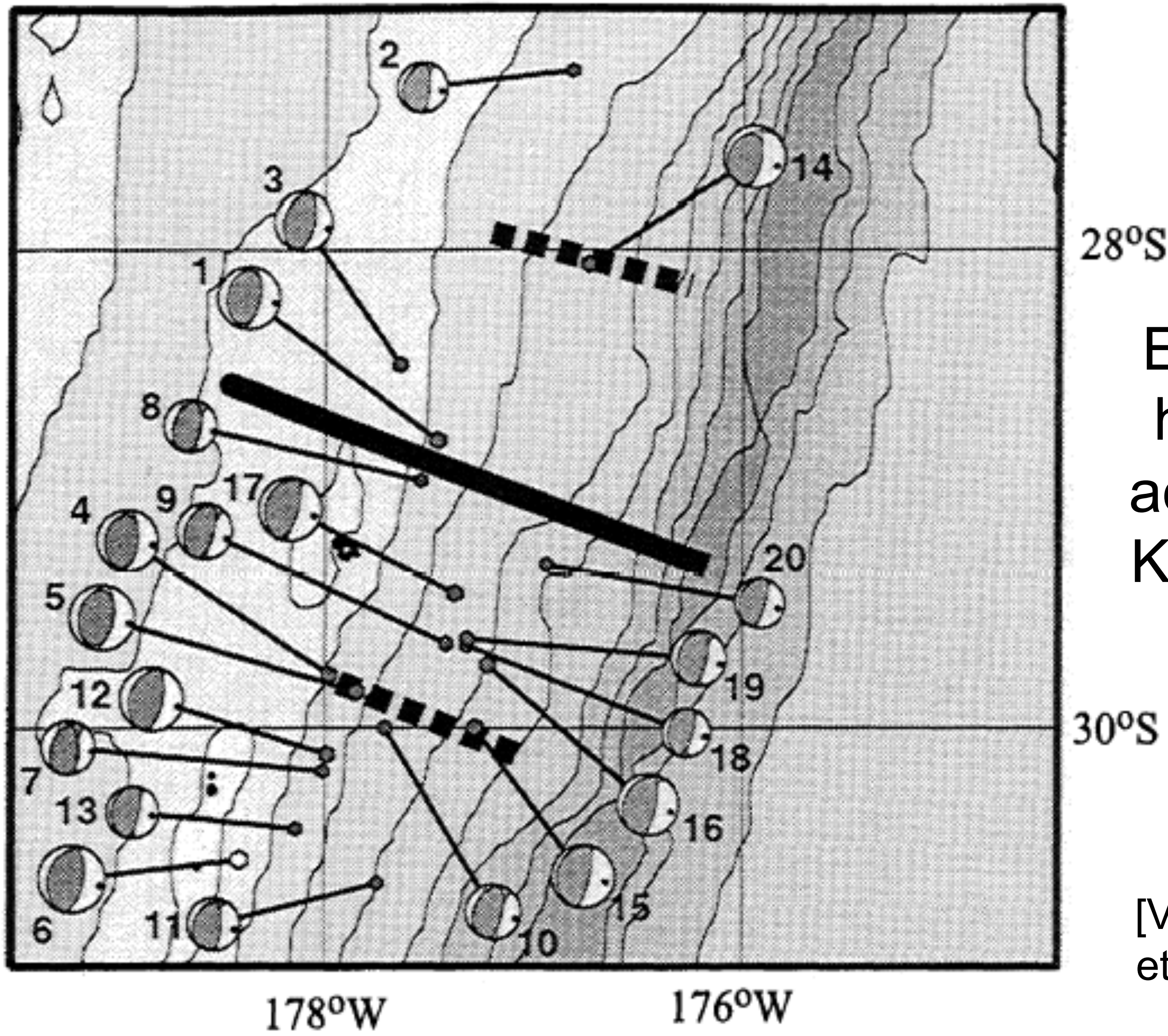
How big is S? For a depth of 20 km, a slip rate of 20 mm/yr, and a dip of 15°, $S = 2.7$.

Magnitudes of frictional heating, and resulting temperature changes

From the discussion above for slip at 20 mm/yr on a fault dipping 15°, at a depth of 20 km, $S = 2.7$.

Similarly from above, for 40 MPa, friction generates heat q_f = 27 mW/m^2 on the fault surface.

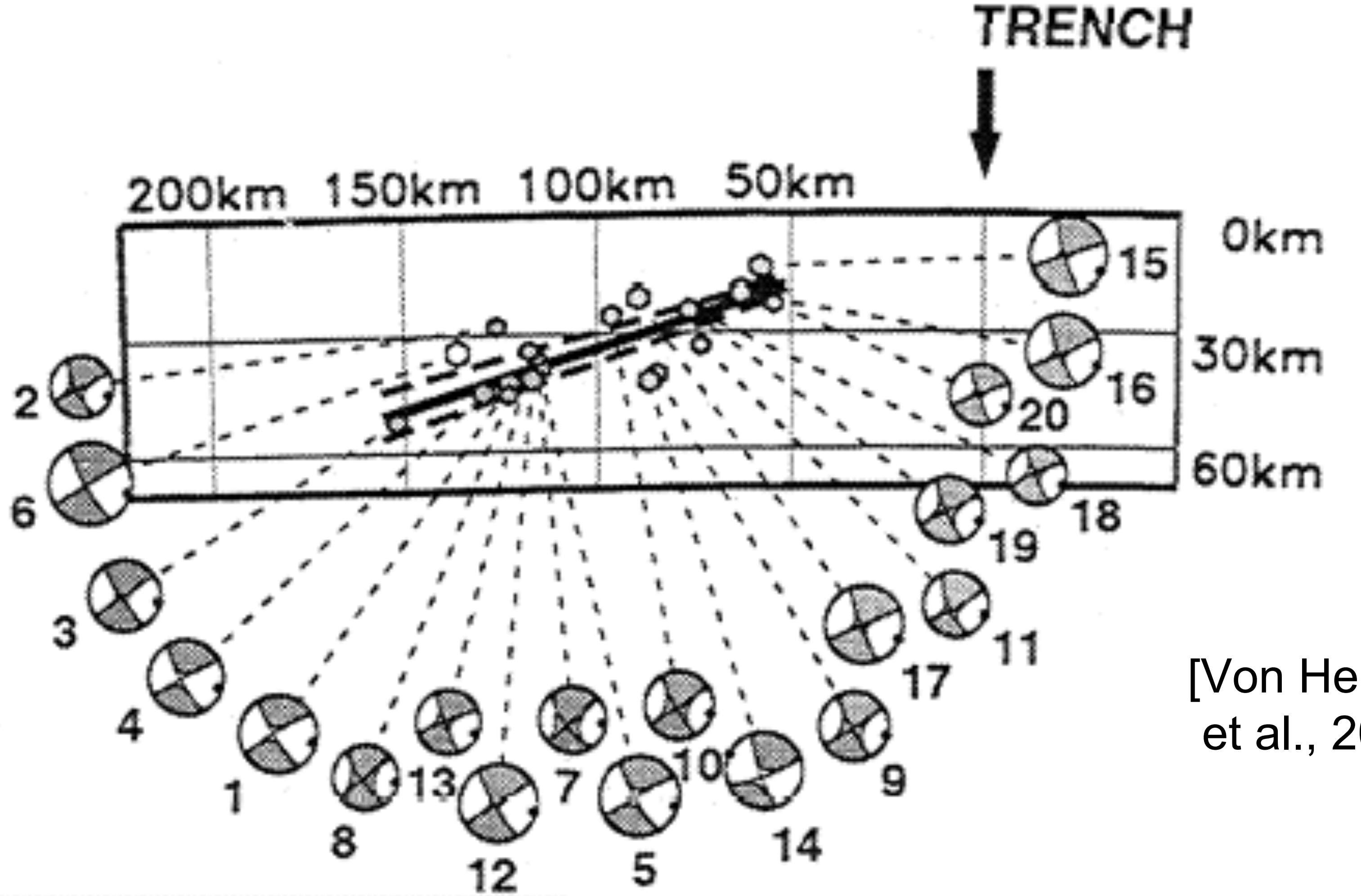
The additional temperature at 20 km should be $q_f z_f / kS = 80$°C.



An Example: heat flux across the Kermadec arc

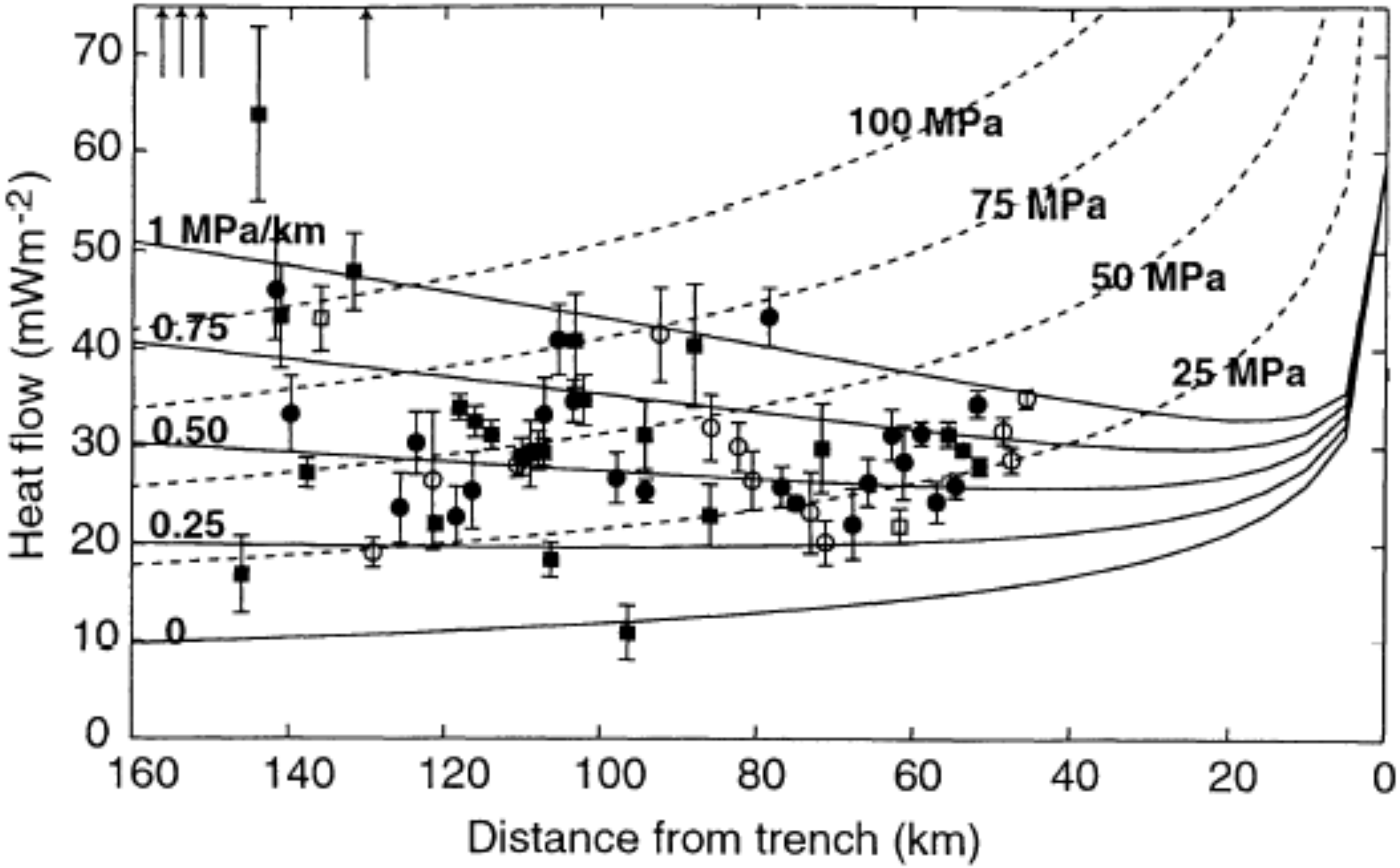
[Von Herzen et al., 2001]

Profile of earthquakes



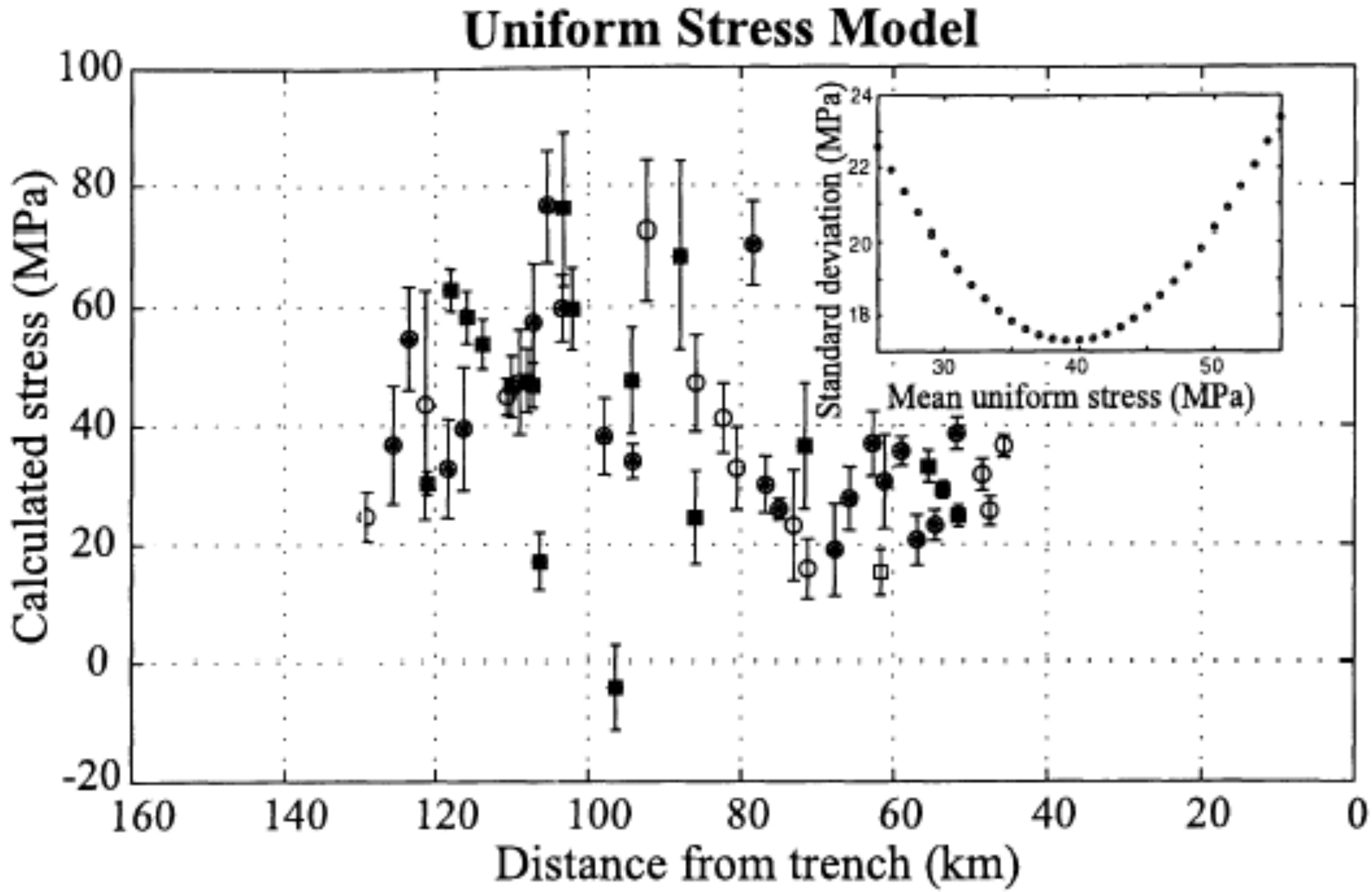
[Von Herzen
et al., 2001

Heat flux across the Kermadec Trench



[Von Herzen et al., 2001]

Inferred stress



[Von Herzen et al., 2001

Summary

1. Steady-state temperatures will be those that would exist in the absence of thrust faulting divided by *S*:

$$S = 1 + b\sqrt{z_f V \sin\delta / \kappa}$$

2. Thrust faults cool the overlying rock, because they draw heat from it. ($S > 1$)
3. Shear stresses on faults seem to be tens of MPa (< 1 kbar = 100 MPa).
4. Heating due to friction on a thrust fault is small, <100°C, again because of *S*.

Molnar, P., and P. England (1990), Temperatures, heat flux, and frictional stress near major thrust faults, *J. Geophys. Res., 95*, 4833-4856.

Molnar, P., and P. England (1995), Temperatures in zones of steady-state underthrusting of young oceanic lithosphere, *Earth Planet. Sci. Lett, 131*, 57-70.

Von Herzen, R., C. Ruppel, P. Molnar, M. Nettles, S. Nagihara, and G. Ekström (2001), A Constraint on the shear stress at the Pacific-Australia plate boundary from heat flow and seismicity at the Kermadec forearc, *J. Geophys. Res., 106*, 6817-6833.

Add frictional heating to thrust

Heat is generated on the fault at a rate: τV

Suppose in steady state it creates an additional temperature on the fault of ΔT_f.

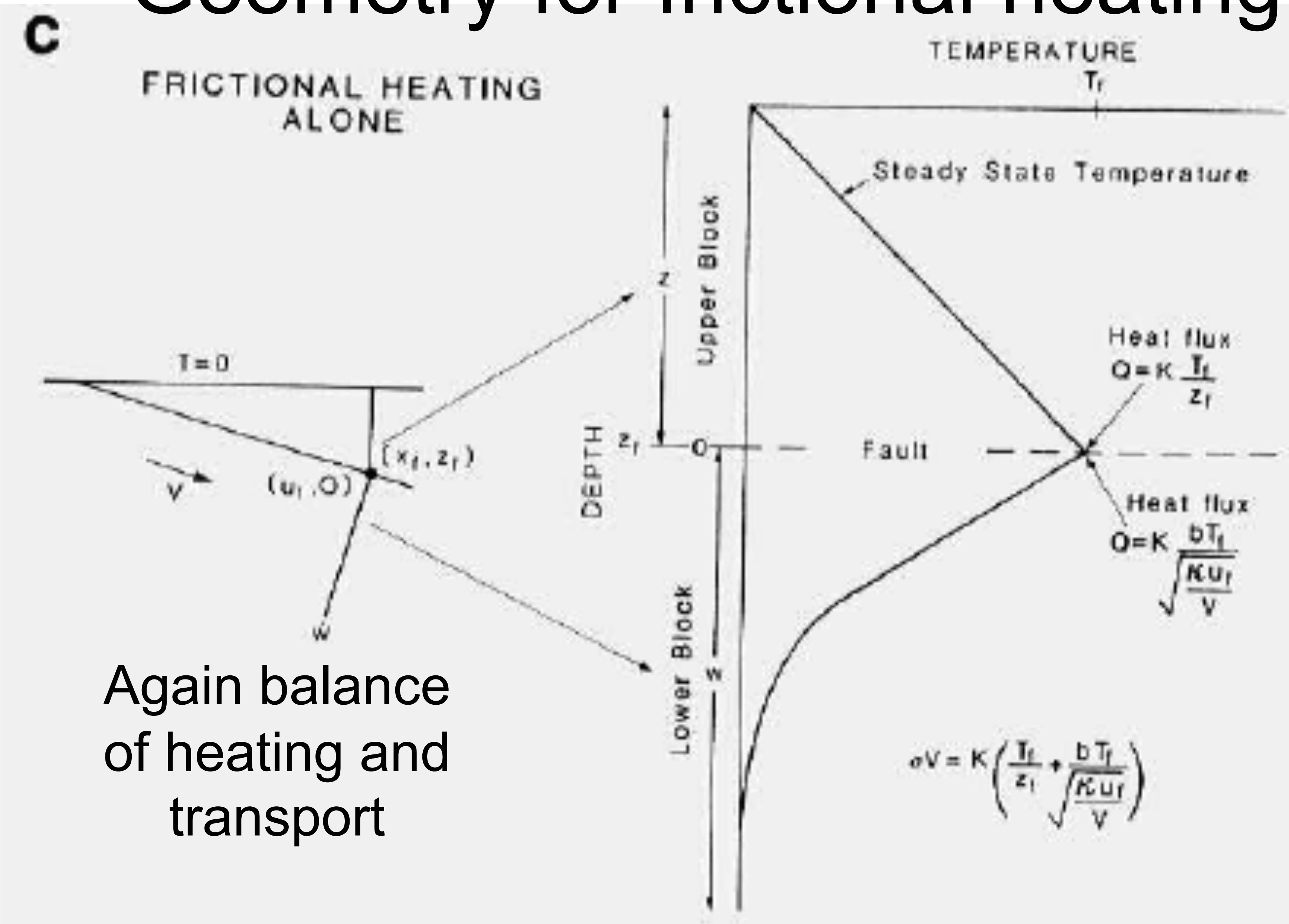
Part of the heat generated is conducted upward through the upper plate: $k \Delta T_f / z_f$

Part is conducted downward into the subducting plate: $-bk\Delta T_f / \sqrt{\kappa t}$

Combining them gives:

$$\tau V = k \frac{\Delta T_f}{z_f} + bk \frac{\Delta T_f}{\sqrt{\kappa t}}$$

Geometry for frictional heating



Again balance of heating and transport