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Advanced School on Scaling Laws in Geophysics: Mechanical and Thermal Processes in Geodynamics

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Simple problems of isostasy

P. Molnar

Univ. of Colorado at Boulder USA

Isostasy

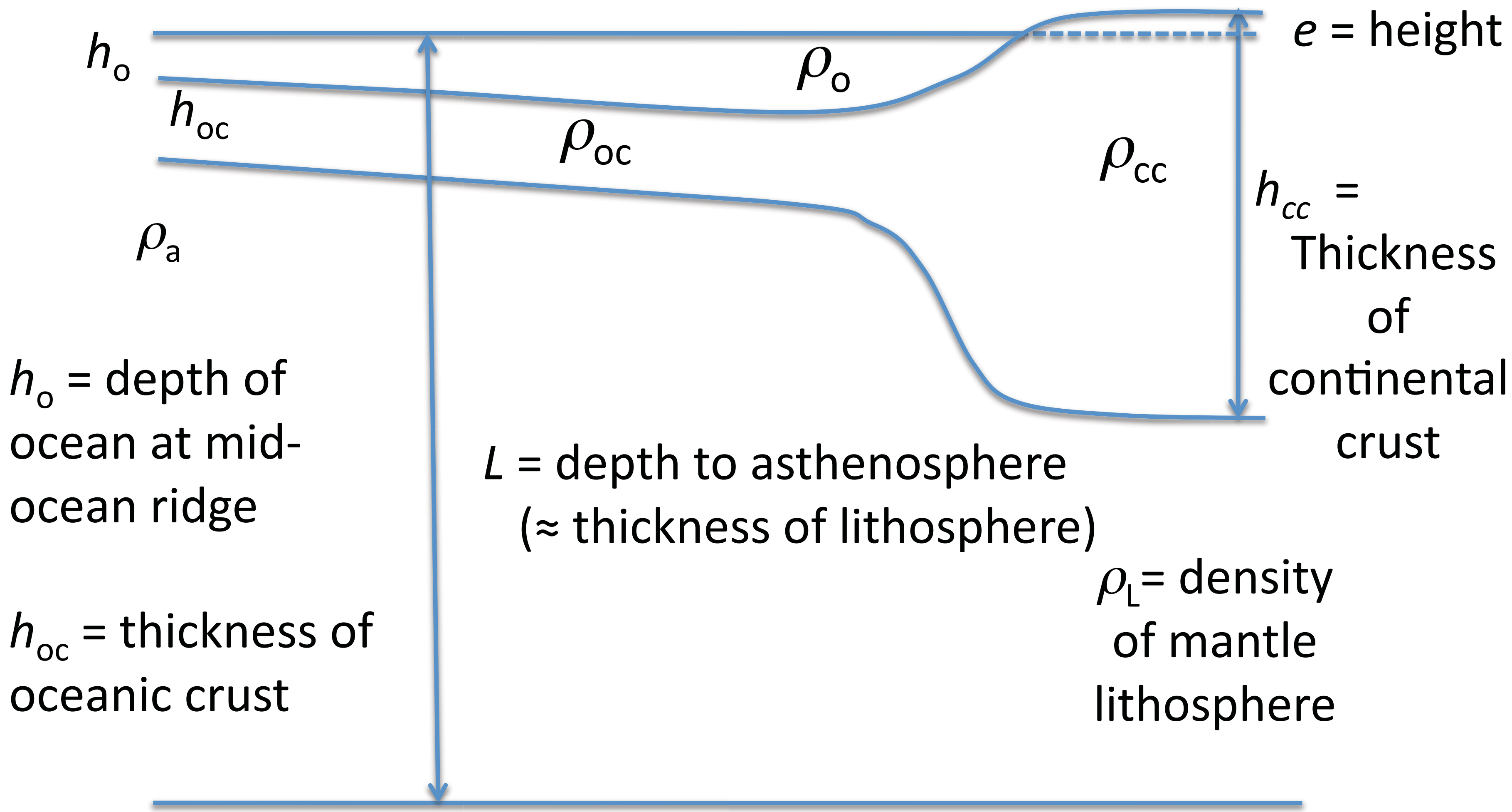
Pressure below some depth ("depth of compensation") is the same everywhere.

$$P = \int_{\text{Surface}}^{\text{Depth of Compensation}} \rho(x,y,z)\,g\,dz$$

As g is (essentially) constant in the crust and mantle, a column of mass (per unit area) above the depth of compensation should be the same everywhere:

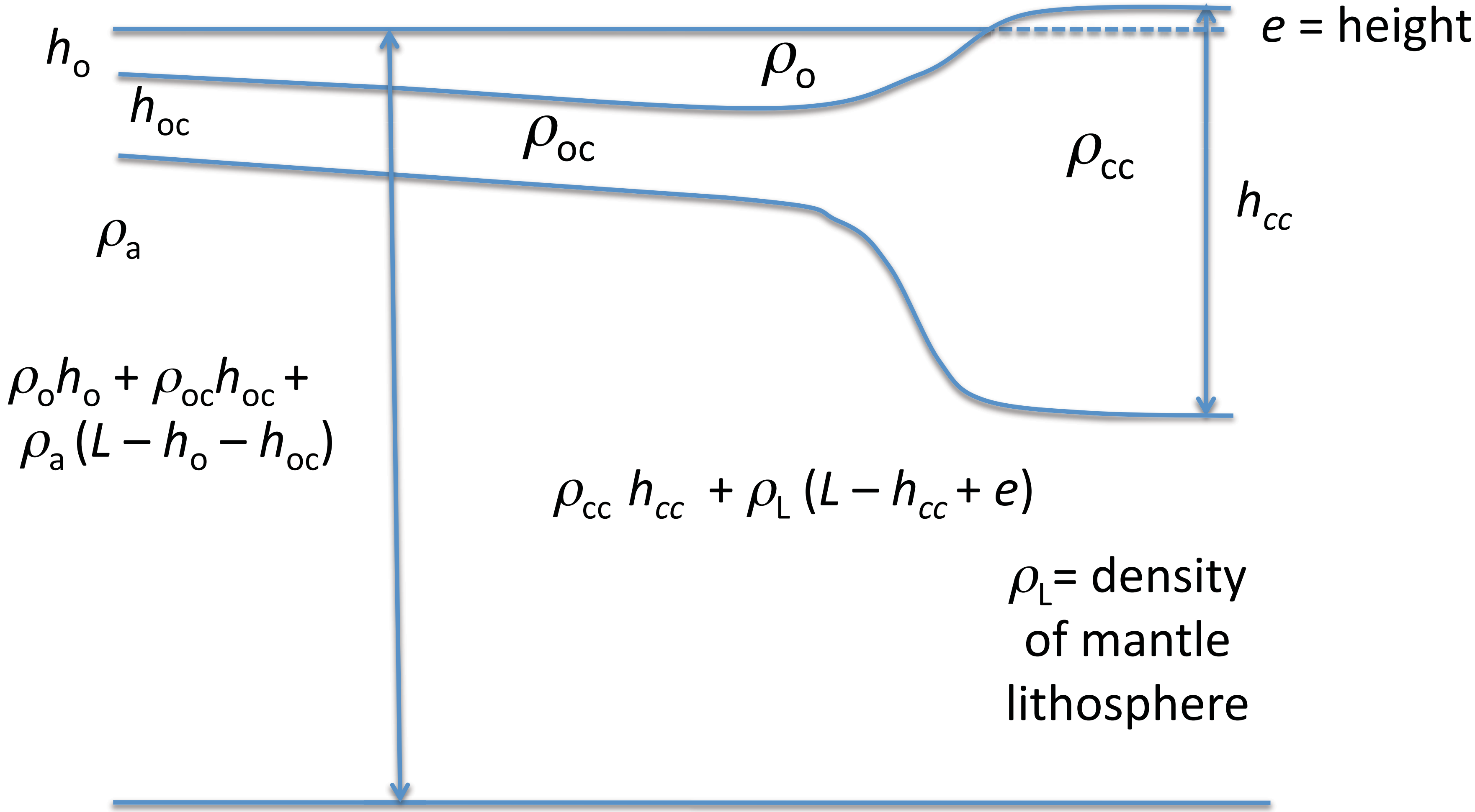
$$\int_{\text{Surface}}^{\text{Depth of Compensation}} \rho(x,y,z)\,dz = \text{Constant}$$

Basic structure



ρ_a = density of mantle at mid-ocean ridge

Mid-Ocean Ridge defines the reference structure



ρ_a = density of mantle at mid-ocean ridge

Equilibrium between Mid-ocean ridge and continental lithosphere

$$\rho_o h_o + \rho_{oc} h_{oc} + \rho_a (L - h_o - h_{oc})$$
$$= \rho_{cc} h_{cc} + \rho_L (L - h_{cc} + e)$$

Collect terms:

$$e\rho_L = (\rho_L - \rho_{cc})\, h_{cc}$$
$$- (\rho_L - \rho_a)\, L$$
$$- (\rho_a - \rho_o)\, h_o$$
$$- (\rho_a - \rho_{oc})\, h_{oc}$$

Magnitudes of terms

$e\rho_L$: e ~ 0.5 – 5 km; ρ_L = 3.3 x 10^3 kg/m^3

1.65 – 16.5 x 10^6 kg/m^2

= $(\rho_L - \rho_{cc})\,h_{cc}$: $\rho_L - \rho_{cc}$ = 0.45 x 10^3 kg/m^3;

h_{cc} ~ 30-70 km; **13.5 – 31.5 x 10^6 kg/m^2**

– $(\rho_L - \rho_a)\,L$: $\rho_L - \rho_a = \rho_a\,\alpha\,\Delta T$; ΔT ~ 100 - 500°C;

$\rho_L - \rho_a$ = 10 – 50 kg/m^3; L = 100 – 200 km;

1 – 10 x 10^6 kg/m^2

– $(\rho_a - \rho_o)\,h_o$: $\rho_a - \rho_o$ = 2.2 x 10^3 kg/m^3;

h_o = 2.5 km **5.5 x 10^6 kg/m^2**

– $(\rho_a - \rho_{oc})\,h_{oc}$: $\rho_a - \rho_{oc}$ = 0.3 x 10^3 kg/m^3;

h_o = 7 km **2.1 x 10^6 kg/m^2**

Magnitudes of terms

$e\rho_L =$	**1.65 – 16.5 x 10^6 kg/m^2**
$(\rho_L - \rho_{cc})\, h_{cc}$	**13.5 – 31.5 x 10^6 kg/m^2**
$-\, (\rho_L - \rho_a)\, L$	**1 – 10 x 10^6 kg/m^2**
$-\, (\rho_a - \rho_o)\, h_o$	**5.5 x 10^6 kg/m^2**
$-\, (\rho_a - \rho_{oc})\, h_{oc}$	**2.1 x 10^6 kg/m^2**

Airy Isostasy (first term) dominates, and can account for 5 km of surface elevation.

Pratt Isostasy (second term) can contribute ~ 1 km, or at most 2 km, of elevation (unless both L and ΔT are large: $\Delta\rho = \rho_L - \rho_a = \rho\alpha\Delta T$).