Mechanical effects of light spin and orbital angular momentum in liquid crystals

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Outline

I. Experiments on the mechanical effects induced by the light OAM in nematic liquid crystals

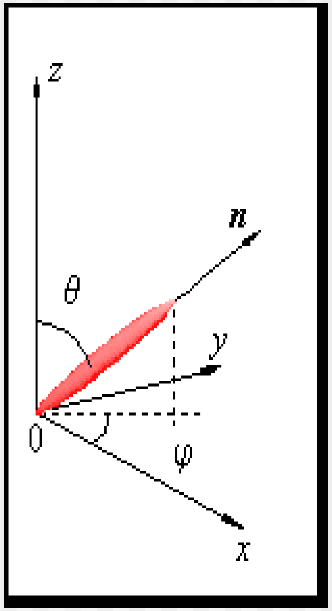
II. A discussion on the different mechanical effects induced by the light SAM and OAM fluxes in liquid crystal

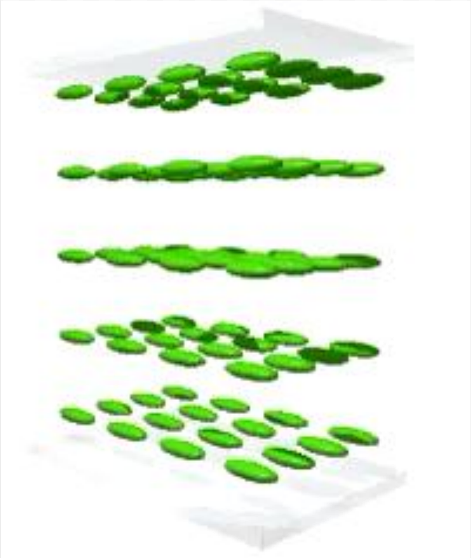
Part I

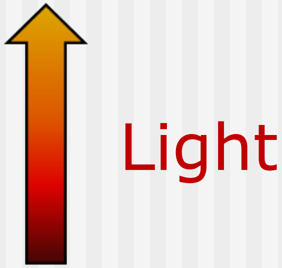
Experiments on the mechanical effects induced by the light OAM in nematic liquid crystals

Liquid Crystals

- Liquid crystals have **internal** orientational degrees of freedom
- External fields (electric, magnetic and **optical**) may induce the LC reorientation
- The reorientation is described by the molecular director ***n***







Liquid crystal reorientation

- An electrostatic field produces a torque

$$\vec{\boldsymbol{\tau}}_E = \frac{\varepsilon_a}{4\pi}(\hat{\boldsymbol{n}} \cdot \vec{\boldsymbol{E}})(\hat{\boldsymbol{n}} \times \vec{\boldsymbol{E}})$$

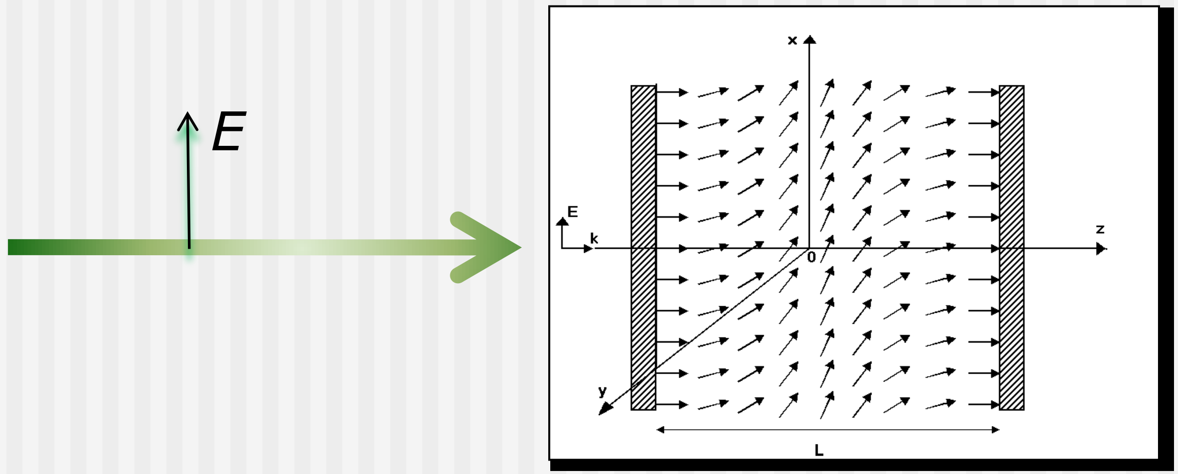
- An optical field produces a torque

$$\vec{\boldsymbol{\tau}}_o = \frac{\varepsilon_a}{8\pi}\mathrm{Re}\left[(\hat{\mathbf{n}} \cdot \vec{\mathbf{E}}^*)(\hat{\mathbf{n}} \times \vec{\mathbf{E}})\right] = \frac{I}{\omega}\Delta s_3$$

- The torque is zero when ***E*** is either parallel or perpendicular to ***n***

The Experimental Geometry

- *Homeotropic anchoring at the walls*
- *Normal incidence of the laser beam*



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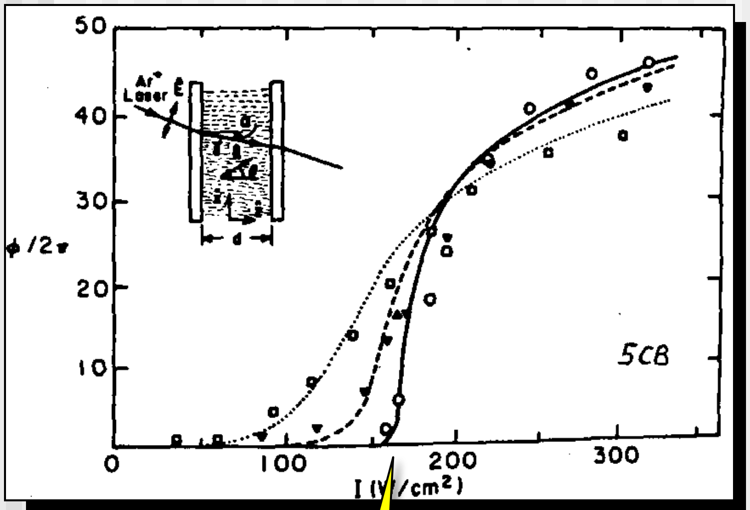
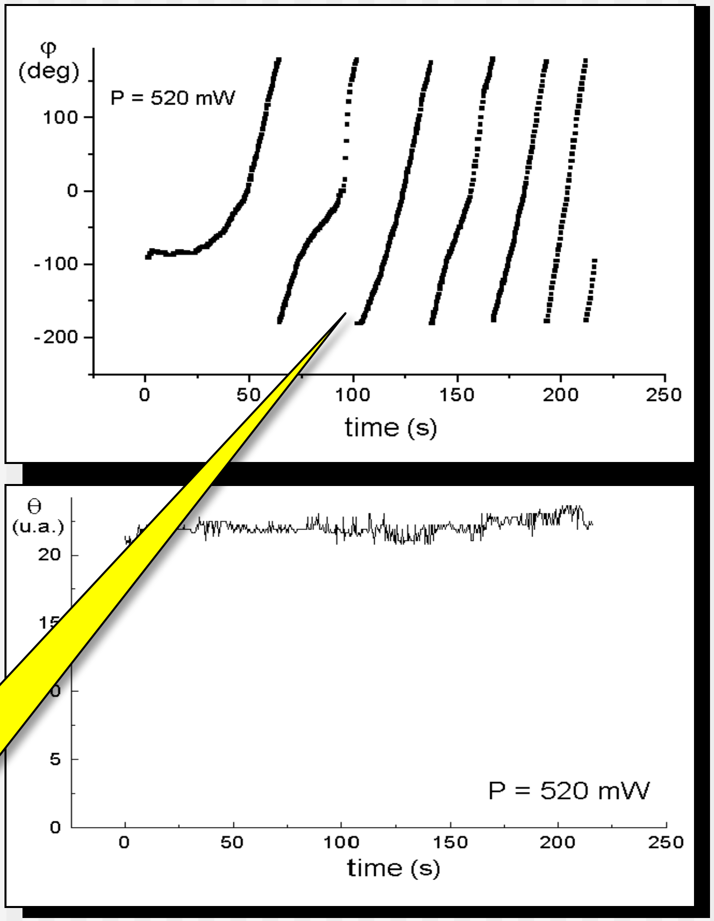
Liquid crystal are reoriented by the light polarization (SAM)

(b) Linear polarization



(b) Circular polarization



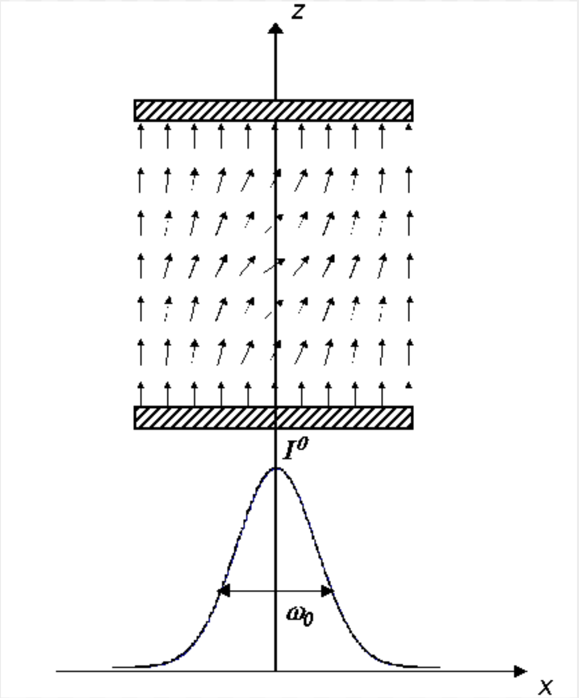
*(a) S. D. Durbin et al., Phys. Rev. Lett., **47**, 1411 (1981)*

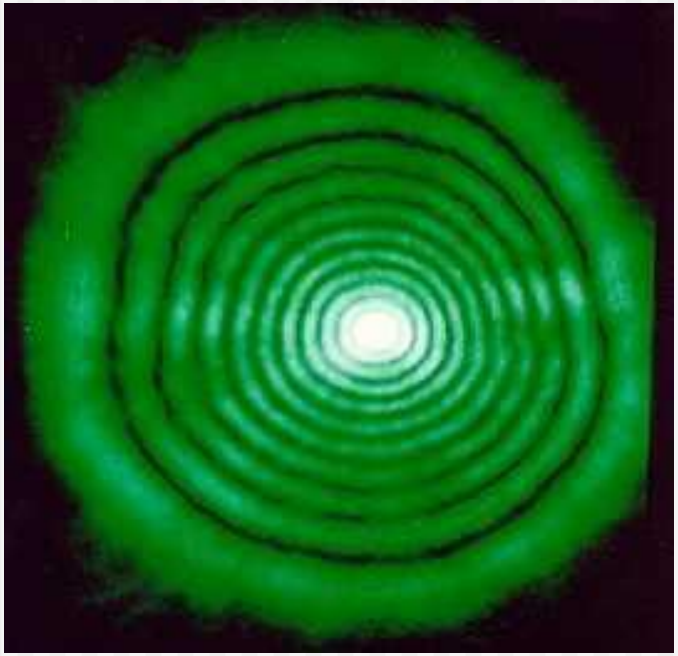
*(b) E. Santamato et al., Phys. Rev. Lett., **57**, 19 (1986)*

Threshold

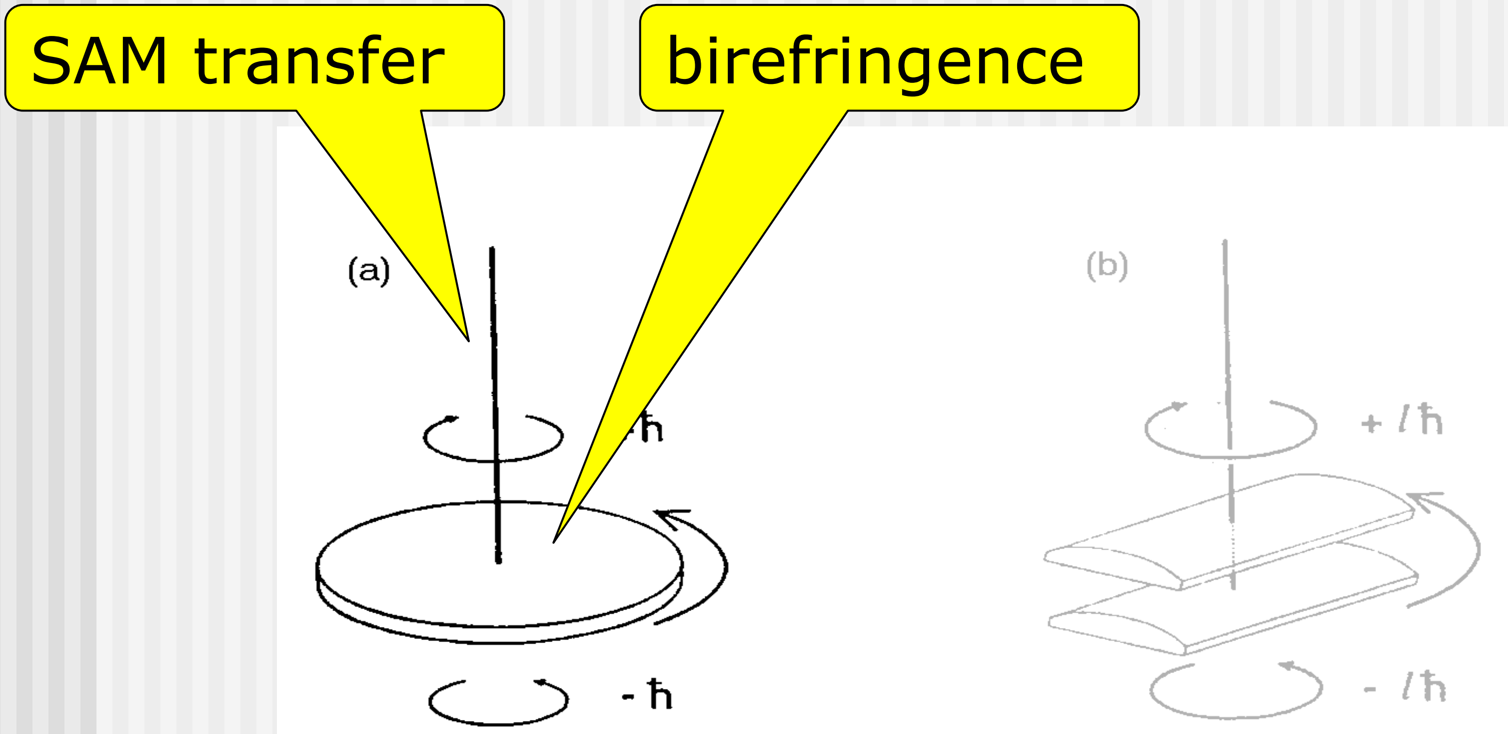
Rotation

Self-phase modulation





Reorientation mechanism: light Angular Momentum transfer



R. A. Beth, Phys. Rev. **50**, 115 (1936) L. Allen *et al.*, Phys. Rev. **A45**, 8185 (1992)

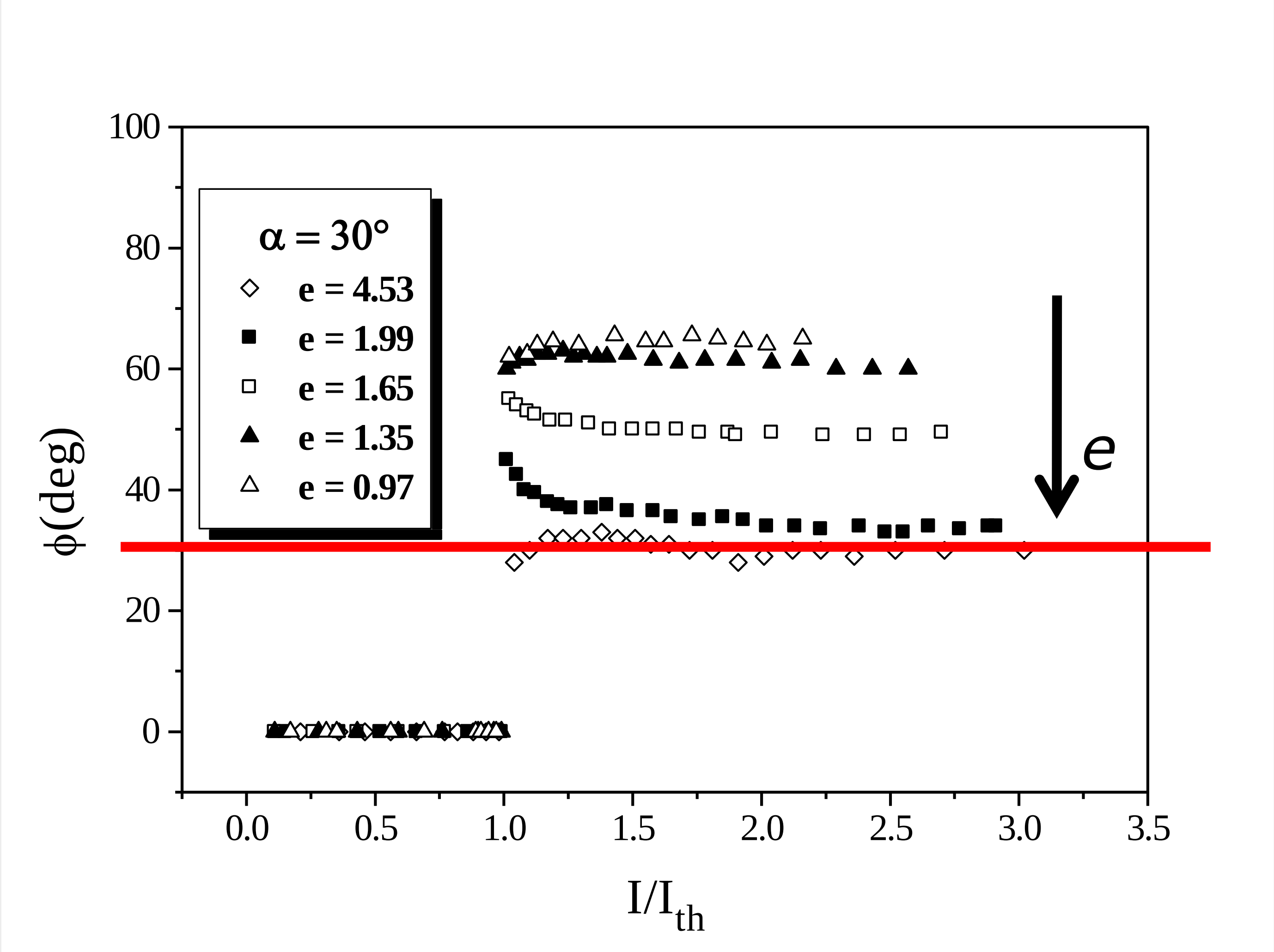
What happens with unpolarized light?

- LC are sensitive to second-order correlations of zero-average polarization fluctuations
  [L. Marrucci er al., PRE, **57**, 3033 (1998)]

- But LC are much more sensitive to the <u>beam shape</u>
  [L. Marrucci er a Opt mmun., **171**, 131 (1999)]

What's the source of this torque?

Director azimuthal angle vs incident laser intensity



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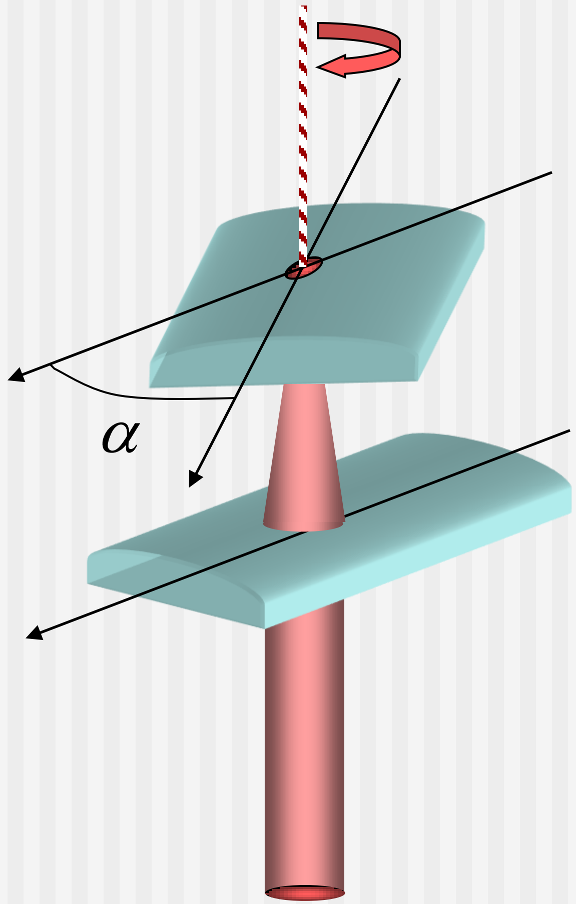
A simple model for the effect



- A torque is generated on the upper cylindrical lens
- The two lenses tens to align

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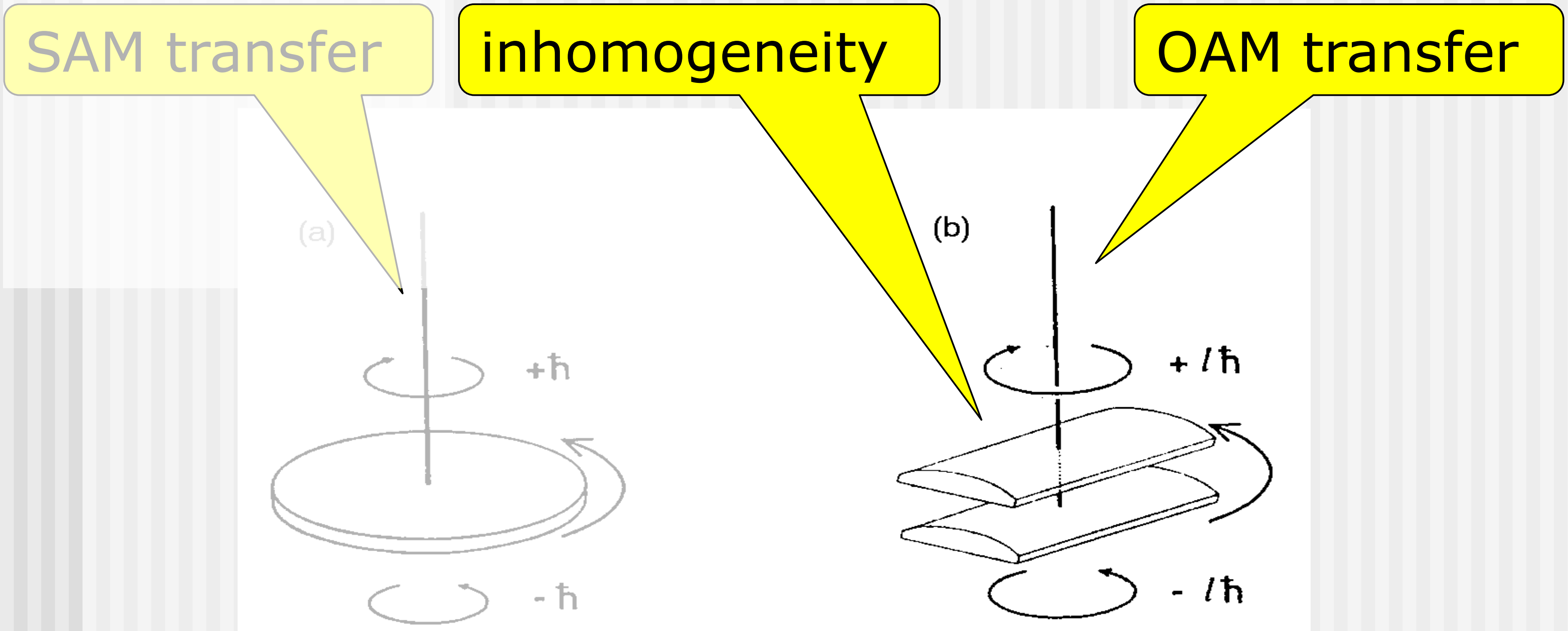
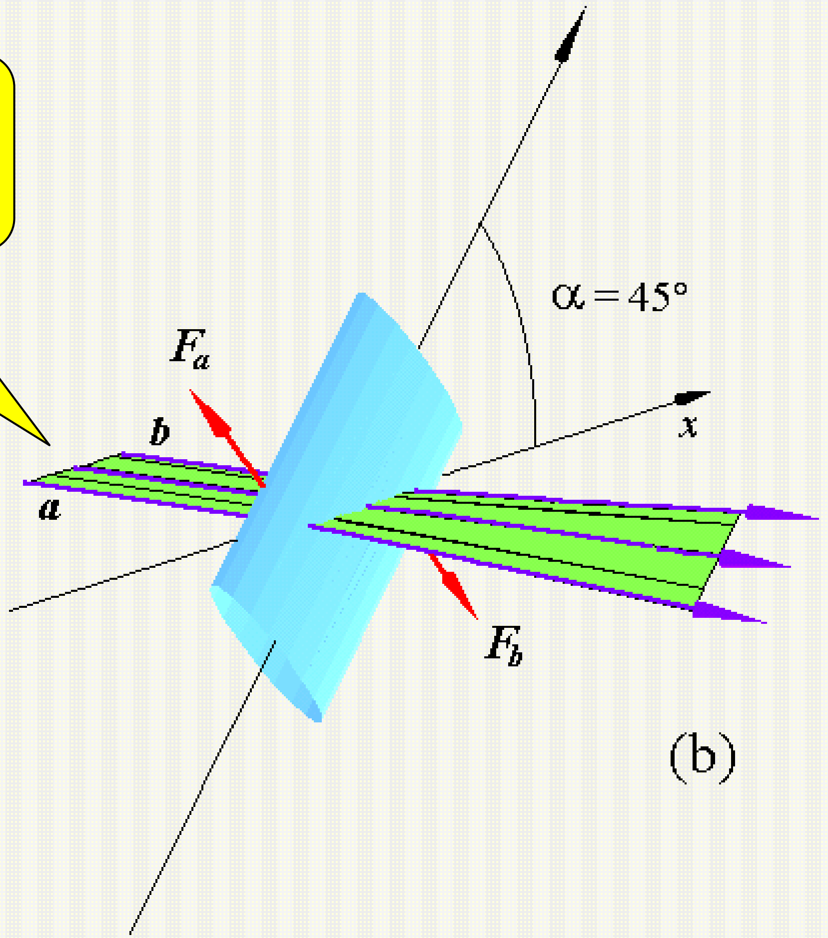
Reorientation mechanism: light Angular Momentum transfer



R. A. Beth, Phys. Rev. **50**, 115 (1936) L. Allen *et al*., Phys. Rev. **A45**, 8185 (1992)

The origin of the OAM torque

Thin sheet of light along *x*



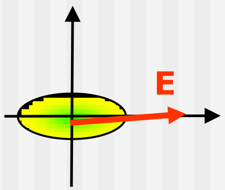
The transfer of angular momentum of light to LC

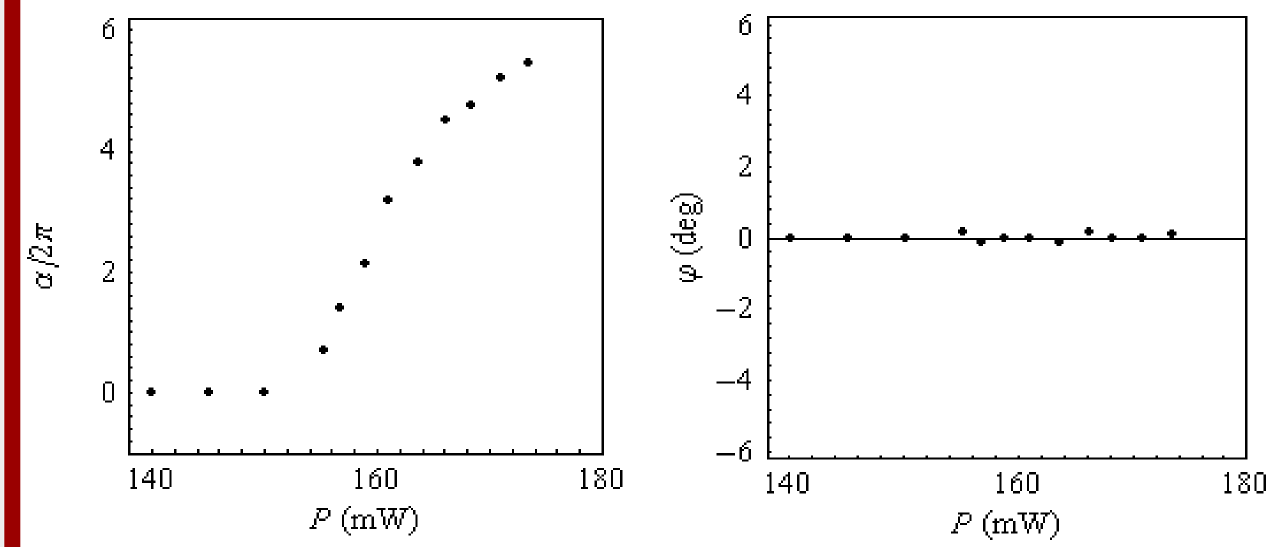
- LC are sensitive to the photon spin because they are birefringent and change the light polarization

- LC are sensitive to the photon orbital angular momentum because they are inhomogeneous and change the ray direction

What happens if the light SAM and OAM compete?

Linear polarization ($\alpha = 0°$)





- Standard OFT

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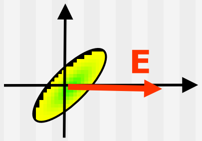
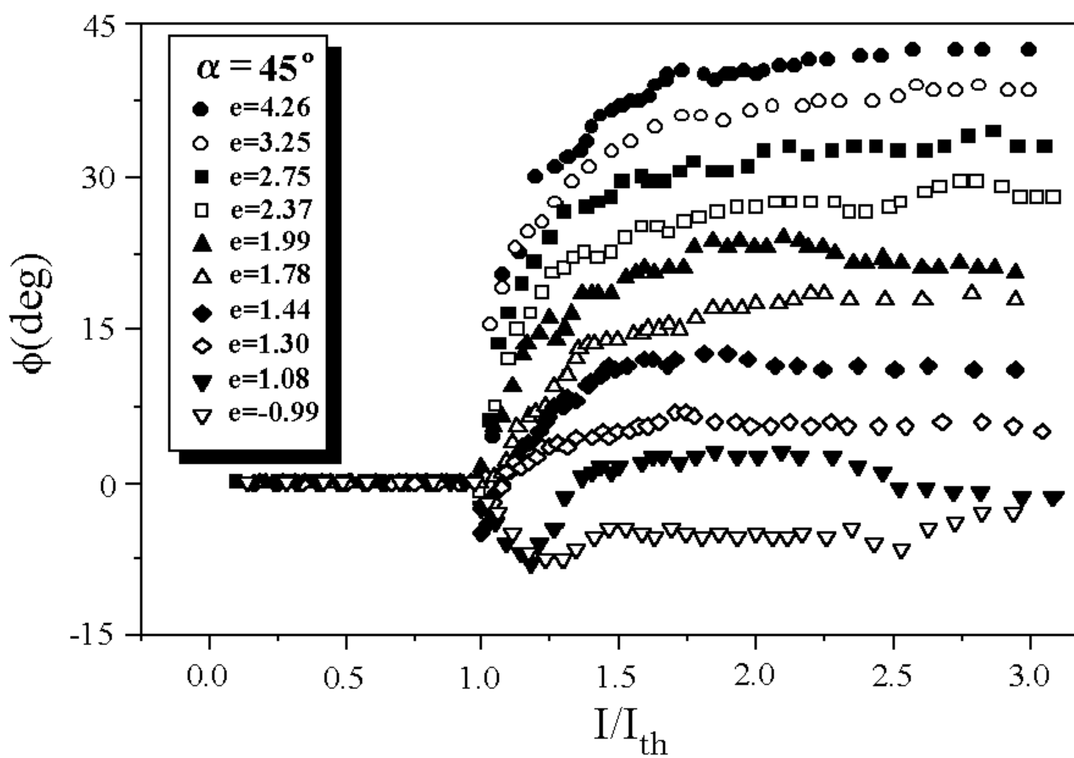
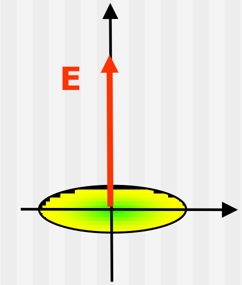
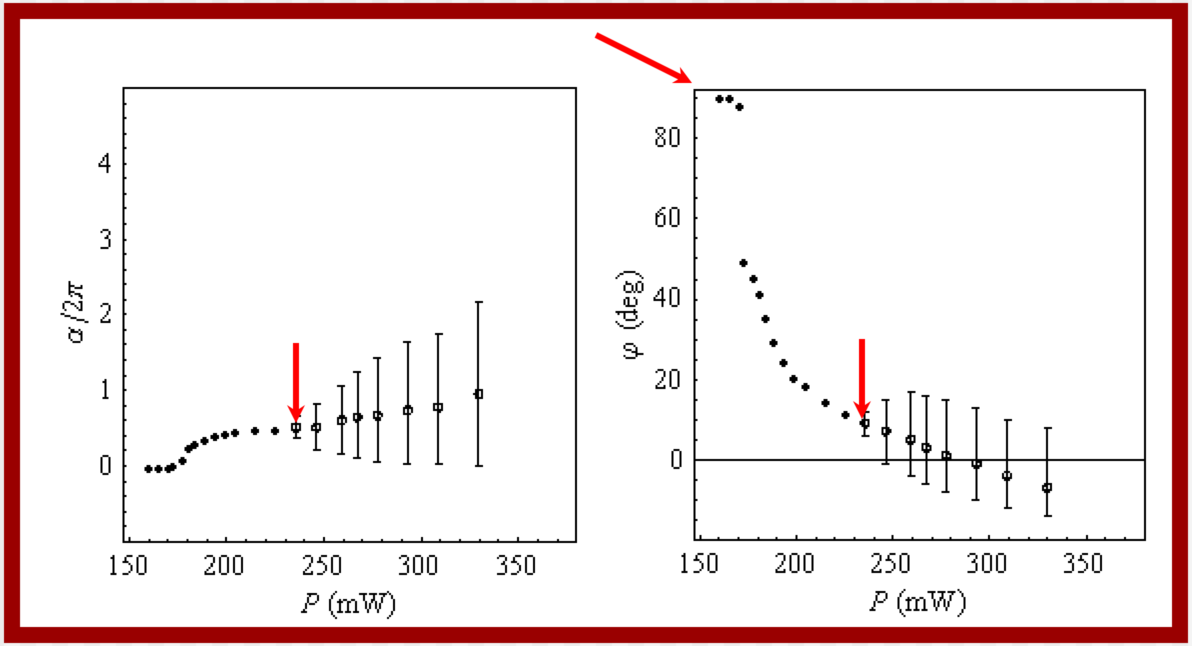
Linear polarization (α = 45°)





Linear polarization (α = 90°)

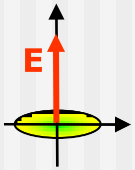


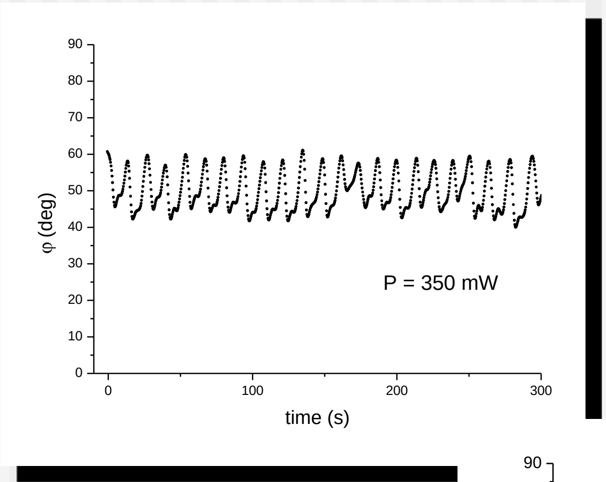


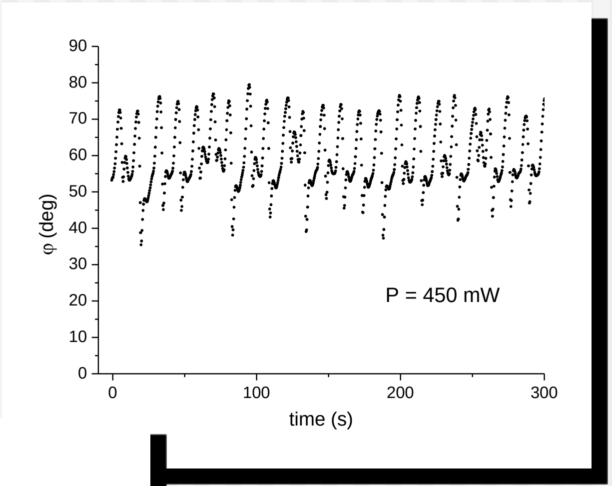
- At low incident power, the reorientation is in the polarization plane
- Above a critical power threshold <u>nonlinear oscillations </u>start up

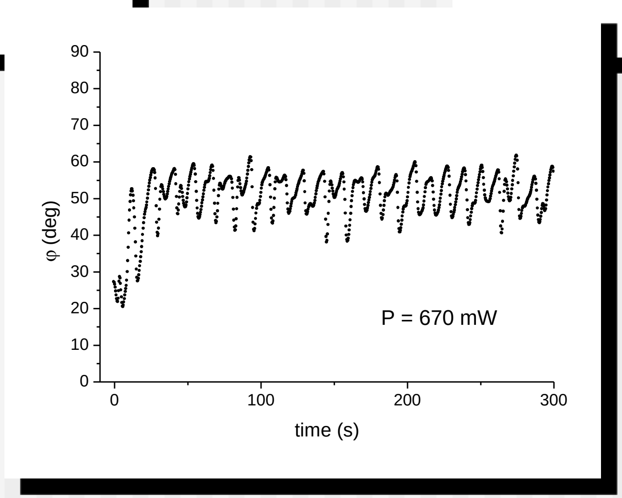
B.Piccirillo et al., PRL, **86**, 2285 (2001)

Linear polarization (α = 90°)



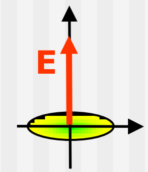




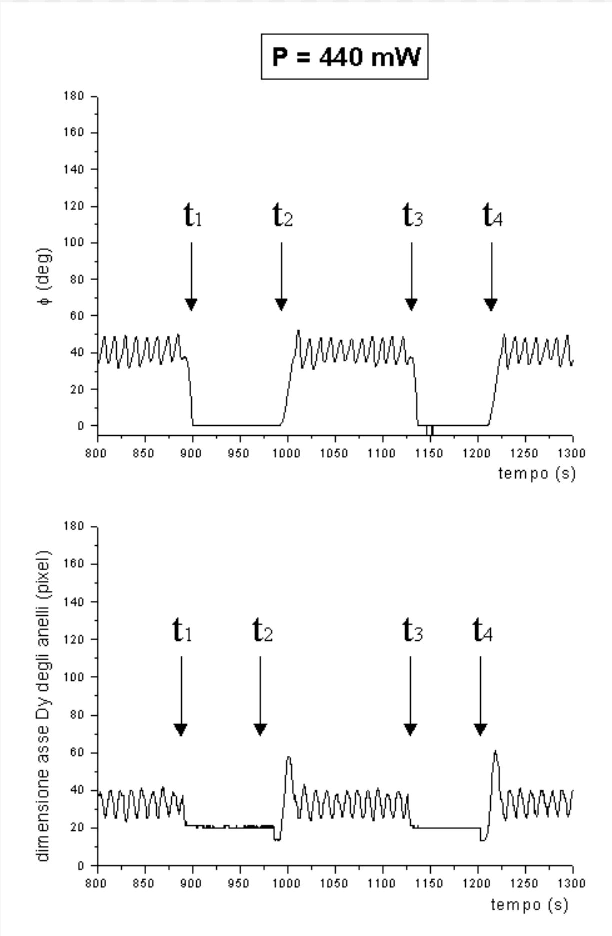


B.Piccirillo et al., PRL, **86**, 2285 (2001)

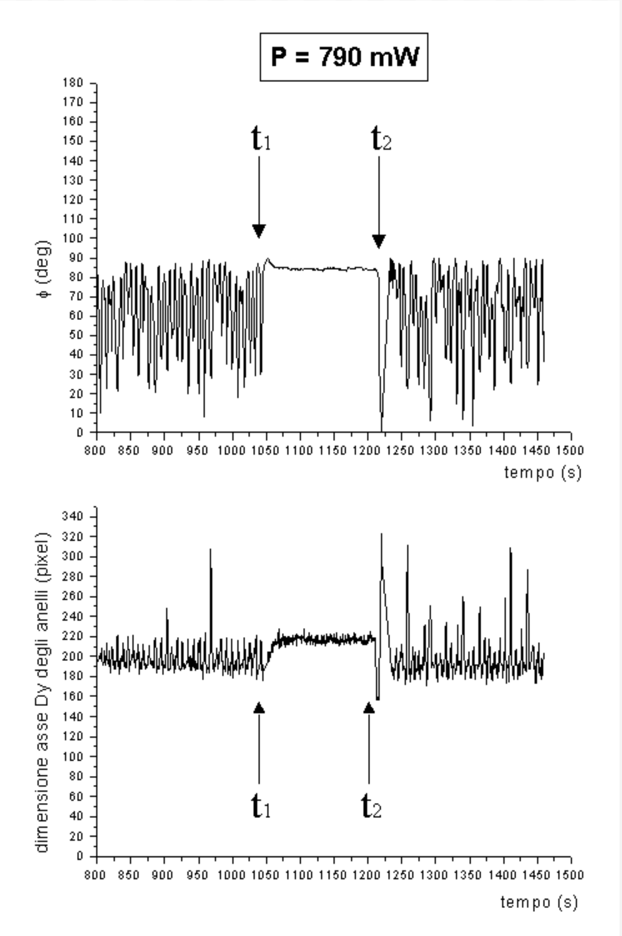
Switchin on-off the polarization



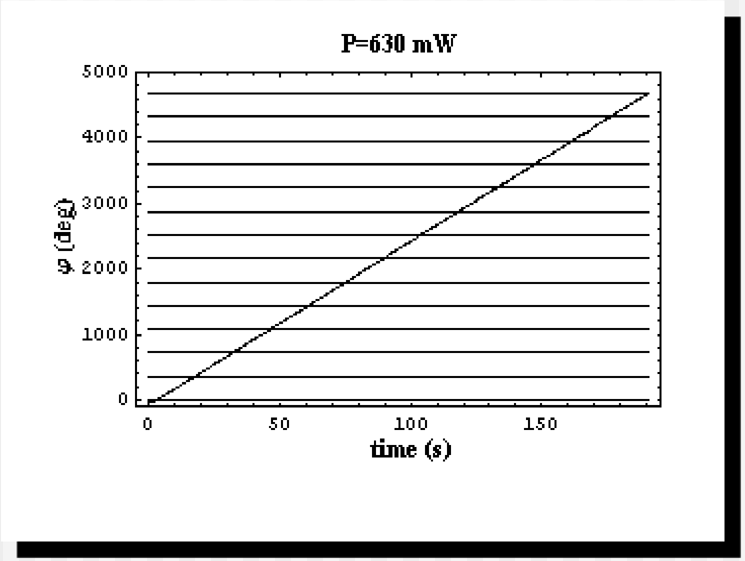
Low power

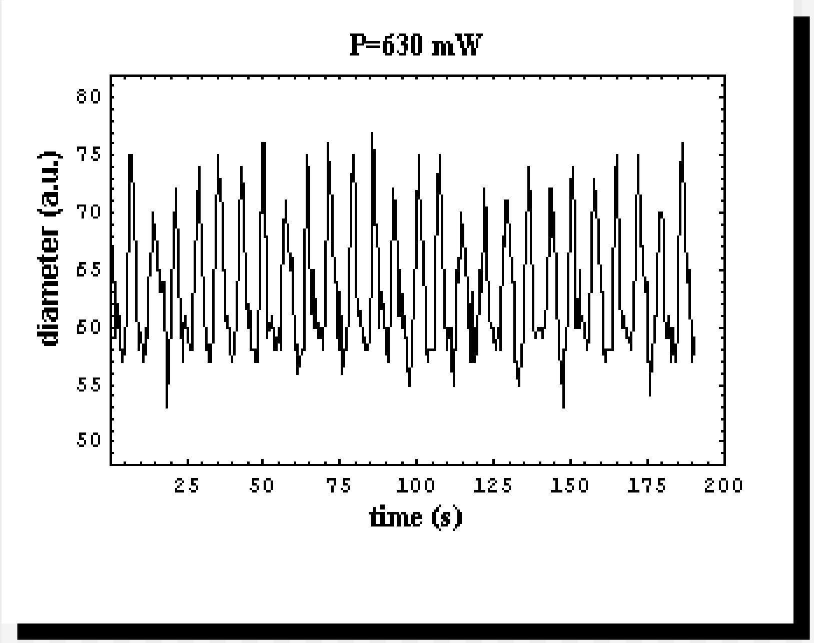


High power



Circular polarization, low power





Azimuthal angle

The azimuthal angle increases linearly in time (precession).

Ring diameter

The ring diameter in the far field oscillates in time.

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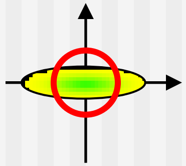
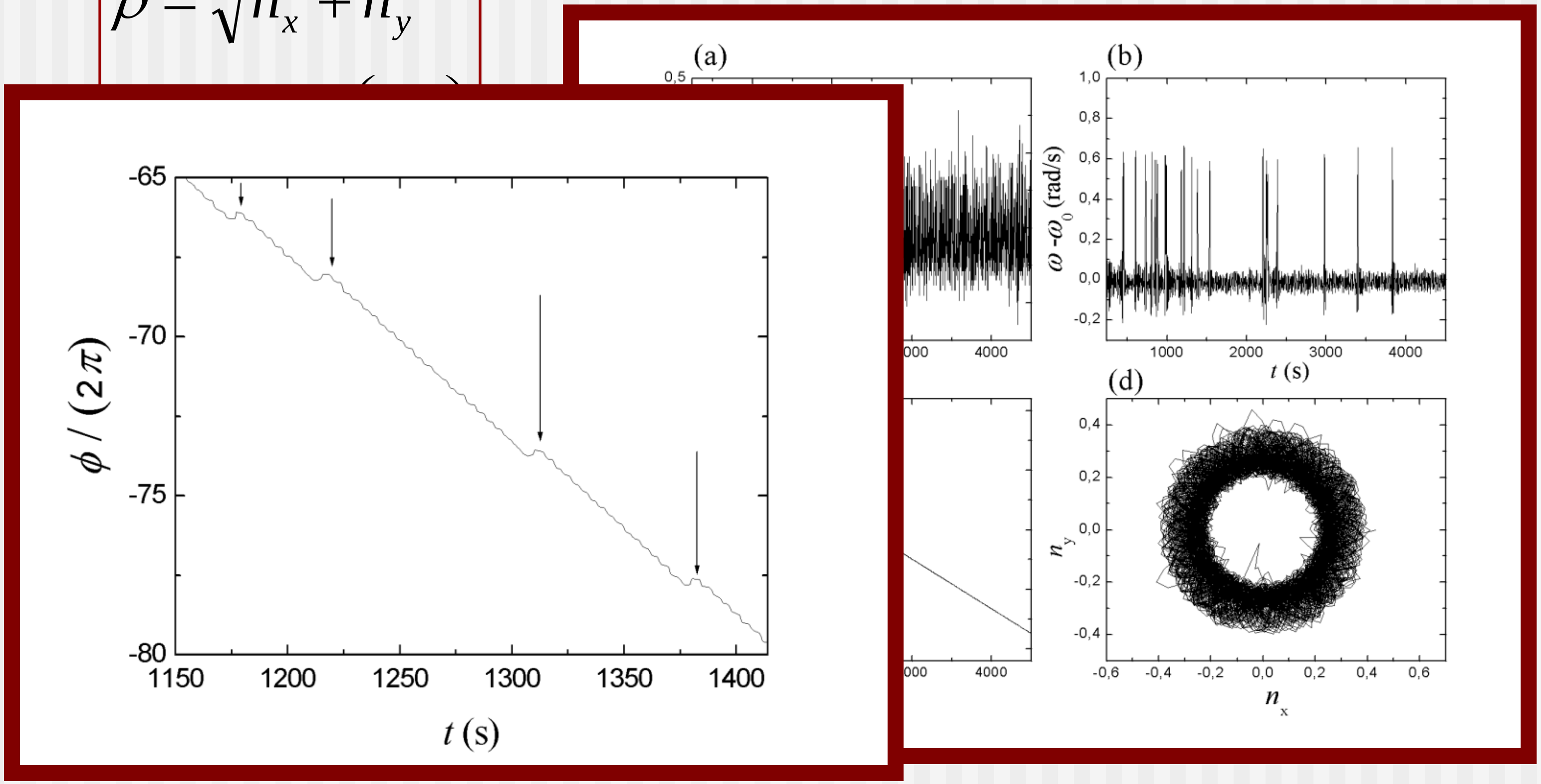
Circular polarization, high power



$$\rho = \sqrt{n_x^2 + n_y^2}$$



A. Vella, A. Setaro, B. Piccirillo, and E. Santamato, *Phys. Rev. E*, **67**, 051704 (2003)

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Conclusions

- It was experimentally shown that liquid crystals are sensitive to the light SAM and OAM
- When SAM and OAM are competing in a LC complex nonlinear dynamics is excited
  - Steady states
  - Rotating states
  - Oscillating states
  - Intermittency states

Part II

A discussion on the mechanical effects induced by the light SAM and OAM fluxes in liquid crystal

The problem

- The main problem is connecting the forces and torques acting in the bulk of the LC sample with the **fluxes** of linear and angular momentum coming from outside.
- In particular, we are interested in the angular momentum coming from an incident light beam.

Difference between SAM and OAM in liquid crystals

SAM ***s***

OAM ***l***

Hydrodynamics of liquid crystals

$$\dot{\boldsymbol{p}} = \boldsymbol{f} = -\mathrm{div}\,\hat{\sigma}$$

$$\dot{\boldsymbol{l}} = \boldsymbol{r} \times \boldsymbol{f} = -\mathrm{div}\hat{L} + \boldsymbol{\omega}$$

$$\dot{\boldsymbol{s}} = \boldsymbol{\tau} = -\mathrm{div}\hat{S} - \boldsymbol{\omega}$$

Force density on the fluid

Linear momentum density flux (stress)

Orbital angular momentum density flux

Intrinsic angular momentum density flux

Torque density on ***n***

l-s coupling torque

$$\dot{\boldsymbol{l}} + \dot{\boldsymbol{s}} = -\mathrm{div}(\hat{L} + \hat{S}) = -\mathrm{div}\hat{J}$$

$$\boldsymbol{p} = \rho \boldsymbol{v}$$

$$\boldsymbol{l} = \boldsymbol{r} \times \boldsymbol{p}$$

$$\boldsymbol{s} = I(\boldsymbol{n} \times \dot{\boldsymbol{n}})$$

Orbital angular momentum density

Intrinsic angular momentum density

Definitions

$$(\mathrm{div}\hat{T})_\alpha = \partial_\rho T_{\rho\alpha}$$

$$L_{\rho\alpha} = \varepsilon_{\alpha\beta\gamma} x_\beta \sigma_{\rho\gamma}$$

$$\omega_\alpha = \varepsilon_{\alpha\beta\gamma} \sigma_{\beta\gamma} = \text{skewsymmetric part of } \hat{\sigma}$$

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The elastic free energy

$$F_e = \frac{1}{2}[k_1 \operatorname{div}\mathbf{n} + k_2(\mathbf{n}\cdot\operatorname{rot}\mathbf{n})^2 + k_3(\mathbf{n}\times\operatorname{rot}\mathbf{n})^2]$$

Splay

Twist

Bend

$$\mathbf{h} = \operatorname{div}\hat{\pi} - \frac{\partial F_e}{\partial \mathbf{n}} = \lambda(\mathbf{r})\mathbf{n} \qquad \pi_{\rho\gamma} = \frac{\partial F_e}{\partial(\partial_\rho n_\gamma)}$$

molecular field

field equations

The electromagnetic free energy

$$F^{em} = \frac{1}{16\pi}\left(\boldsymbol{B}^* \cdot \boldsymbol{H} - \boldsymbol{D}^* \cdot \boldsymbol{E}\right)$$

Field equations

$$\boldsymbol{B} = \boldsymbol{H} = -\frac{ic}{\omega}\,\mathrm{rot}\boldsymbol{E}$$

$$\mathrm{rot}\,\mathrm{rot}\boldsymbol{E} = k_0^2\hat{\varepsilon}\boldsymbol{E}$$

- Monochromatic field at frequency ω
- Average over the optical period
- Nonmagnetic optical medium

Electromagnetic force and torque in the medium

$$f_{\alpha}^{em} = -\partial_{\rho}\sigma_{\rho\alpha}^{em} = -\frac{1}{16\pi}E_{\gamma}E_{\rho}^{*}\partial_{\alpha}\varepsilon_{\gamma\rho}$$

on the fluid

$$\boldsymbol{\tau}^{em} = -\boldsymbol{n}\times\frac{\partial F^{em}}{\partial \boldsymbol{n}} = \frac{1}{16\pi}(\boldsymbol{D}^{*}\times\boldsymbol{E} + c.c.)$$

on the director ***n***

- The presence of the optical field do not change the viscous forces and torques, because no source of entropy is associated with light.

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The splitting of SAM and OAM

- The stress tensor is not unique
- We may add a divergence free tensor to $\hat{\sigma}$ and change $\hat{L}$ and $\hat{S}$ so to leave the hydro-dynamics equations invariant
- For example, we may choose $\hat{\sigma}$ so to have $\hat{S} = 0$

The splitting $\hat{J} = \hat{L} + \hat{S}$

may be dictated by physics?

The choice $\hat{S} = 0$ (spinless gauge)

$$\hat{S} = 0$$
$$\hat{L} = \hat{J}$$
$$\dot{\boldsymbol{l}} = -\mathrm{div}\hat{J} + \boldsymbol{\omega}$$
$$\dot{\boldsymbol{s}} = -\boldsymbol{\omega}$$

$\hat{\sigma}^{em}$ = Maxwell's e.m. tensor

D.Forster et al., Phys. Rev. Lett., ***26***, 1016, (1971) [Harvard group].

$$\dot{\boldsymbol{s}} = 0 \Rightarrow \boldsymbol{\omega} = 0 \Rightarrow \hat{\sigma} = \hat{\sigma}^T$$

- Dynamical constraints not always satisfied
- True if the inertia I is neglected (overdamped motion of $\boldsymbol{n}$)
- True in the limit of isotropic fluids

Stress tensors and SAM fluxes

B. PICCIRILLO AND E. SANTAMATO PHYSICAL REVIEW E 69, 056613 (2004)

TABLE I. Stress tensor and intrinsic angular momentum flux in different gauges. The flux of the orbital angular momentum is given by $L_{\alpha\beta}=\epsilon_{\beta\mu\nu}x_{\mu}\sigma_{\alpha\nu}$. The several contributions, for each block, are labeled as S for splay, T for twist, and B for bend, corresponding to the fundamental elastic distortion in nematic liquid crystals. The label E represents the elastic contribution as a whole and em is for the electromagnetic contribution. Finally, we posed $A=\boldsymbol{n}\cdot\text{rot}\,\boldsymbol{n}$; $\boldsymbol{B}=\boldsymbol{n}\times\text{rot}\,\boldsymbol{n}$.

		$\sigma_{\alpha\beta}$	w	$S_{\alpha\beta}$
I	S	$n_\alpha h^S_\beta-\delta_{\alpha\beta}(F^S+\boldsymbol{n}\cdot\boldsymbol{h}^S)$	$\boldsymbol{n}\times\boldsymbol{h}^S$	0
	T	$-n_\beta h^T_\alpha-\delta_{\alpha\beta}F^T$	$\boldsymbol{n}\times\boldsymbol{h}^T$	0
	B	$-n_\beta h^B_\alpha-k_3B_\alpha B_\beta$ $-(n_\alpha\partial F^B/\partial n_\beta+n_\beta\partial F^B/\partial n_\alpha)+\delta_{\alpha\beta}F^B$	$\boldsymbol{n}\times\boldsymbol{h}^B$	0
	em	$1/16\pi[(D^*_\alpha E_\beta+B^*_\alpha H_\beta+\text{c.c.})$ $-\delta_{\alpha\beta}(\boldsymbol{D}^*\cdot\boldsymbol{E}+\boldsymbol{B}^*\cdot\boldsymbol{H})]$	$1/16\pi(\boldsymbol{D}^*\times\boldsymbol{E}+\boldsymbol{B}^*\times\boldsymbol{H}+\text{c.c.})$	0
II	S	$-k_1\partial_\rho n_\rho\partial_\beta n_\alpha+\delta_{\alpha\beta}F^S$	$k_1\text{rot}\,\boldsymbol{n}\,\text{div}\,\boldsymbol{n}$	$k_1\partial_\rho n_\rho\epsilon_{\alpha\beta\gamma}n_\gamma$
	T	$-k_2A\epsilon_{\alpha\gamma\rho}n_\rho\partial_\beta n_\gamma+\delta_{\alpha\beta}F^T$	$-k_2A(\boldsymbol{B}+\boldsymbol{n}\,\text{div}\,\boldsymbol{n})$	$k_2A(n_\alpha n_\beta-\delta_{\alpha\beta})$
	B	$-k_3(n_\gamma B_\alpha-n_\alpha B_\gamma)\partial_\beta n_\gamma+\delta_{\alpha\beta}F^B$	$-k_3\{[(\boldsymbol{n}\times\boldsymbol{B})\cdot\nabla]\boldsymbol{n}-(\boldsymbol{n}\times\boldsymbol{B})\text{div}\,\boldsymbol{n}]\}$	$k_3n_\alpha\epsilon_{\beta\gamma\rho}B_\gamma n_\rho$
	em	$i/16\pi k_0\,\epsilon_{\alpha\gamma\rho}(H^*_\rho\partial_\beta E_\gamma-\text{c.c.})+\delta_{\alpha\beta}F^{em}$	$i/16\pi k_0[\boldsymbol{H}^*\text{div}\,\boldsymbol{E}-(\boldsymbol{H}^*\cdot\nabla)\boldsymbol{E}-\text{c.c.}]]$	$i/16\pi k_0(-H^*_\beta E_\alpha+\delta_{\alpha\beta}\boldsymbol{H}^*\cdot\boldsymbol{E}-\text{c.c.})$
III	E	$-K(\partial_\alpha n_\gamma\partial_\beta n_\gamma-\frac{1}{2}\delta_{\alpha\beta}\partial_\gamma n_\rho\partial_\gamma n_\rho)$ $+\delta k_1\sigma^S_{\alpha\beta}+\delta k_2\sigma^T_{\alpha\beta}+\delta k_3\sigma^B_{\alpha\beta}$	$\delta k_1\omega^S+\delta k_2\omega^T+\delta k_3\omega^B$	$K\epsilon_{\beta\gamma\rho}n_\gamma\partial_\alpha n_\rho$ $+\delta k_1S^S_{\alpha\beta}+\delta k_2S^T_{\alpha\beta}+\delta k_3S^B_{\alpha\beta}$
	em	$-1/16\pi k_0^2\{\eta_o(\partial_\alpha E_\gamma\partial_\beta E^*_\gamma-\partial_\rho E_\rho\partial_\beta E^*_\alpha+\text{c.c.})$ $-\delta_{\alpha\beta}[\eta_o(\partial_\gamma E_\rho\partial_\gamma E^*_\rho-\partial_\rho E_\rho\partial_\gamma E^*_\gamma)-k_0^2\boldsymbol{D}^*\cdot\boldsymbol{E}]\}$	$-\eta_o/16\pi k_0^2\text{rot}\,\boldsymbol{E}^*\text{div}\,\boldsymbol{E}+\text{c.c.}$	$\eta_o/16\pi k_0^2(\epsilon_{\beta\gamma\rho}E^*_\gamma\partial_\alpha E_\rho$ $-\epsilon_{\alpha\beta\gamma}E^*_\gamma\partial_\rho E_\rho)+\text{c.c.}$

The limit of the isotropic medium (vacuum)

- One elastic constant ($k_1 = k_2 = k_3$)
- No internal torque ($\omega = 0$)
- Symmetric elastic stress tensor

- No optical birefringence ($\boldsymbol{D} = \varepsilon_0 \boldsymbol{E}$)
- Symmetric electromagnetic stress tensor

Properties of the three groups of stress tensors

- Block I (Harvard group)
  - ✓ Elastic and e.m. stress tensors are symmetric
  - ✗ Unrecognizable SAM flux
  - ✗ The total elastic torque is identically zero (no LC inertia)
  - ✗ The e.m. flux densities L^{em} and S^{em} are not divergence free in the isotropic limit
- Block II (Ericksen)
  - ✗ Elastic and e.m. stress tensors are not symmetric even in the isotropic limit
  - ✓ SAM and OAM can be formally recognized
  - ✗ The e.m. flux densities L^{em} and S^{em} are not divergence free in the isotropic limit
- Block III
  - ✓ Elastic and e.m. stress tensors become symmetric in the isotropic limit
  - ✓ SAM and OAM can be formally recognized
  - ✓ The e.m. flux densities L^{em} and S^{em} are <u>divergence free</u> in the isotropic limit

Example: the rotating dipole radiation

- Block III

$$\mathbf{S} = \oint_{\partial V} \hat{S}^{em}\mathbf{u}d\sigma = \mathbf{L} = \oint_{\partial V} \hat{L}^{em}\mathbf{u}d\sigma = \frac{ik_0^{\ 3}}{6}(\mathbf{p}\times\mathbf{p}^*)$$

J.H.Crichton and P. Martson, J. Diff. Eqs., **4**, 37 (2000)

- Block II

$$L_z = ik_0^{\ 3}\left[\frac{1}{4e^2} - \frac{1}{8e^3}(1-e^2)\ln\frac{1+e}{1-e}\right](\mathbf{p}\times\mathbf{p}^*)_z$$

$$S_z = \frac{ik_0^{\ 3}}{3}(\mathbf{p}\times\mathbf{p}^*) - L_z$$

e = surface ellipticity

S. M. Barnett, J. Opt. B: Quantum Semiclassiical Opt. **4**, S7 (2002)

Conclusions

- The equation governing the electro-optic hydrodynamic of LC has been entirely worked out
- Explicit expression of energy, SAM and OAM flux densities have been calculated in different gauges
- It was proved that one gauge exist where
  a) The e.m. stress tensor becomes symmetric in the zero birefringence limit
  b) The e.m. SAM and OAM flux density become independently conserved quantities (divergence free) in the zero birefringence limit
  c) The LC stress tensor becomes symmetric in the zero elastic anisotropy limit
  d) The LC SAM and OAM flux density become independently conserved quantities (divergence free) in the zero in the zero elastic anisotropy limit
  e) The SAM and OAM LC degrees of freedom become decoupled and couple separately each other and are coupled, respectively, with the SAM and OAM fluxes of the e.m. field


Thanks for your
attention