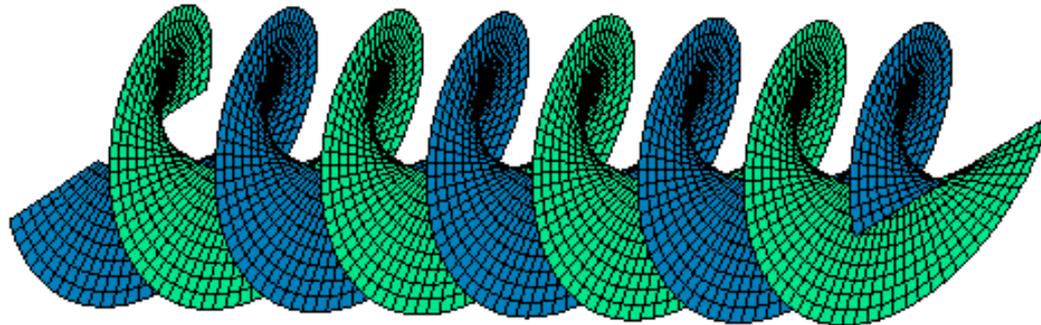




The Abdus Salam  
International Centre for Theoretical Physics

Workshop on Singular Optics and its Applications to Modern Physics



## Spin-to-orbital conversion of the angular momentum of light



Lorenzo Marrucci

*Università di Napoli "Federico II"*

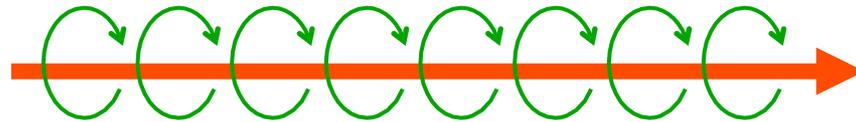
## Outline:

- ❑ Introduction: spin and orbital angular momentum of light
- ❑ Spin–orbital angular momentum conversion
- ❑ q-plate: the concept
- ❑ q-plates: the current technology
- ❑ Concept generalization: Pancharatnam-Berry phase optical elements (PBOE) for arbitrary wavefront shaping

# **Introduction: spin and orbital angular momentum of light**

# Spin and orbital angular momentum of light (SAM & OAM)

That is, different ways for a light “ray” to “rotate upon itself” while it propagates



Standard expression of field angular momentum:  $\mathbf{J} = \epsilon_0 \int d\mathbf{r} \mathbf{r} \times [\mathbf{E} \times \mathbf{B}]$

An equivalent expression (from Noether theorem):  $\mathbf{J}_{\text{rad}} = \mathbf{L}_{\text{rad}} + \mathbf{S}_{\text{rad}}$

SAM  $\mathbf{S}_{\text{rad}} = \epsilon_0 \int d\mathbf{r} \mathbf{E}_{\perp} \times \mathbf{A}_{\perp}$

OAM  $\mathbf{L}_{\text{rad}} = \epsilon_0 \sum_l \int d\mathbf{r} E_l^{\perp} (\mathbf{r} \times \nabla) A_l^{\perp}$

Only radiative terms  
(transverse fields)

Equivalent up to a  
surface term

# Spin and orbital angular momentum of light (SAM & OAM)

## Some old-standing problems with SAM and OAM definitions:

- ❑ Other definitions are also possible (particularly for the SAM and OAM density and/or fluxes): Is there a “most correct” one? What criteria should we use?
- ❑ Gauge invariance (but problem solved if we restrict to radiative transverse fields)
- ❑ Actual physical meaning of SAM and OAM terms (Independent measurability? Are they true angular momenta, i.e. generators of rotations with proper commutation rules? Etc.)
- ❑ Coupling with matter: local density or flux coupling? SAM and OAM coupling with different degrees of freedom of matter?

However, most problems go away in the paraxial limit (for the “z” component)



# Spin and orbital angular momentum of light (SAM & OAM)

SAM and OAM in the paraxial limit (monochromatic wave):

SAM: fully intrinsic, related with circular polarizations

$$\mathbf{S} = \frac{\epsilon_0}{2i\omega} \int d\mathbf{r} \mathbf{E}^* \times \nabla E \approx \frac{\epsilon_0 \hat{\mathbf{z}}}{2\omega} \int d\mathbf{r} S_3$$

External OAM, related with beam axis position and total momentum

$$\mathbf{L}_{ext} = \mathbf{r}_{cm} \times \mathbf{P}$$

Depends on the choice of origin of coordinate system

OAM: further splits into

Internal OAM, related with helical wavefront

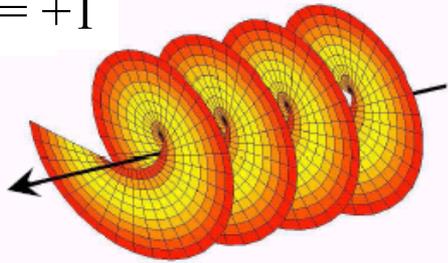
$$\mathbf{L}_{int} = \frac{\epsilon_0}{2i\omega} \int d\mathbf{r} \sum_{h=x,y,z} E_h^* \mathbf{r} \times \nabla E_h \approx \frac{\epsilon_0 \hat{\mathbf{z}}}{2i\omega} \int d\mathbf{r} \sum_{h=x,y,z} E_h^* \frac{\partial}{\partial \varphi} E_h$$

# Spin and orbital angular momentum of light (SAM & OAM)

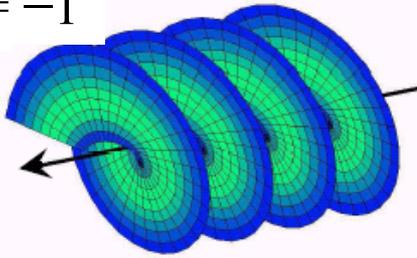
**Helical modes:**  
(using cylindrical  
coordinates  $r, \varphi, z$ )

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}_0(r, z) e^{im\varphi} e^{i(kz - \omega t)}$$

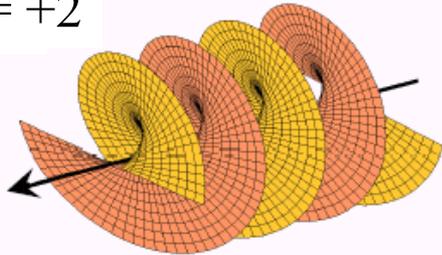
$m = +1$



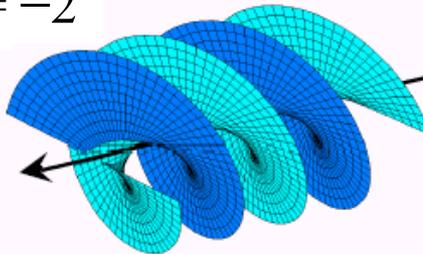
$m = -1$



$m = +2$



$m = -2$



helical phase factor:

$$e^{im\varphi}$$

$$m = 0, \pm 1, \pm 2, \pm 3, \dots$$

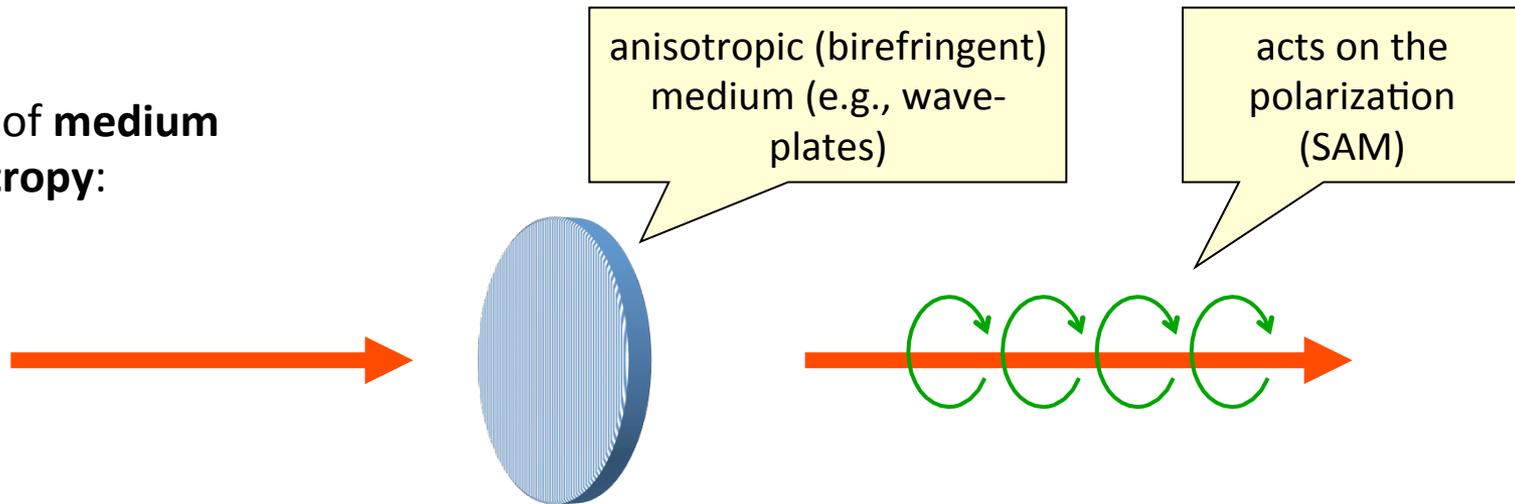
**Angular momentum (OAM):**

$$L_z = m\hbar \quad \text{per photon}$$

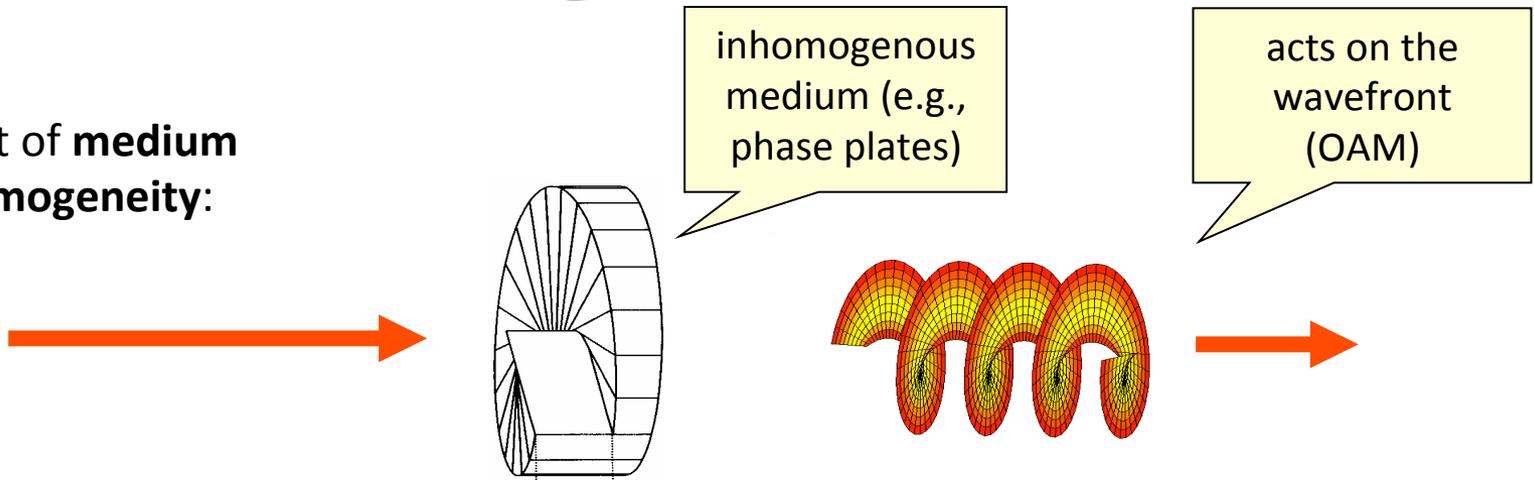
# SAM and OAM interaction with matter

**SAM and OAM are separately conserved** during propagation in vacuum and in isotropic homogeneous transparent media. What about other media?

Effect of **medium anisotropy**:

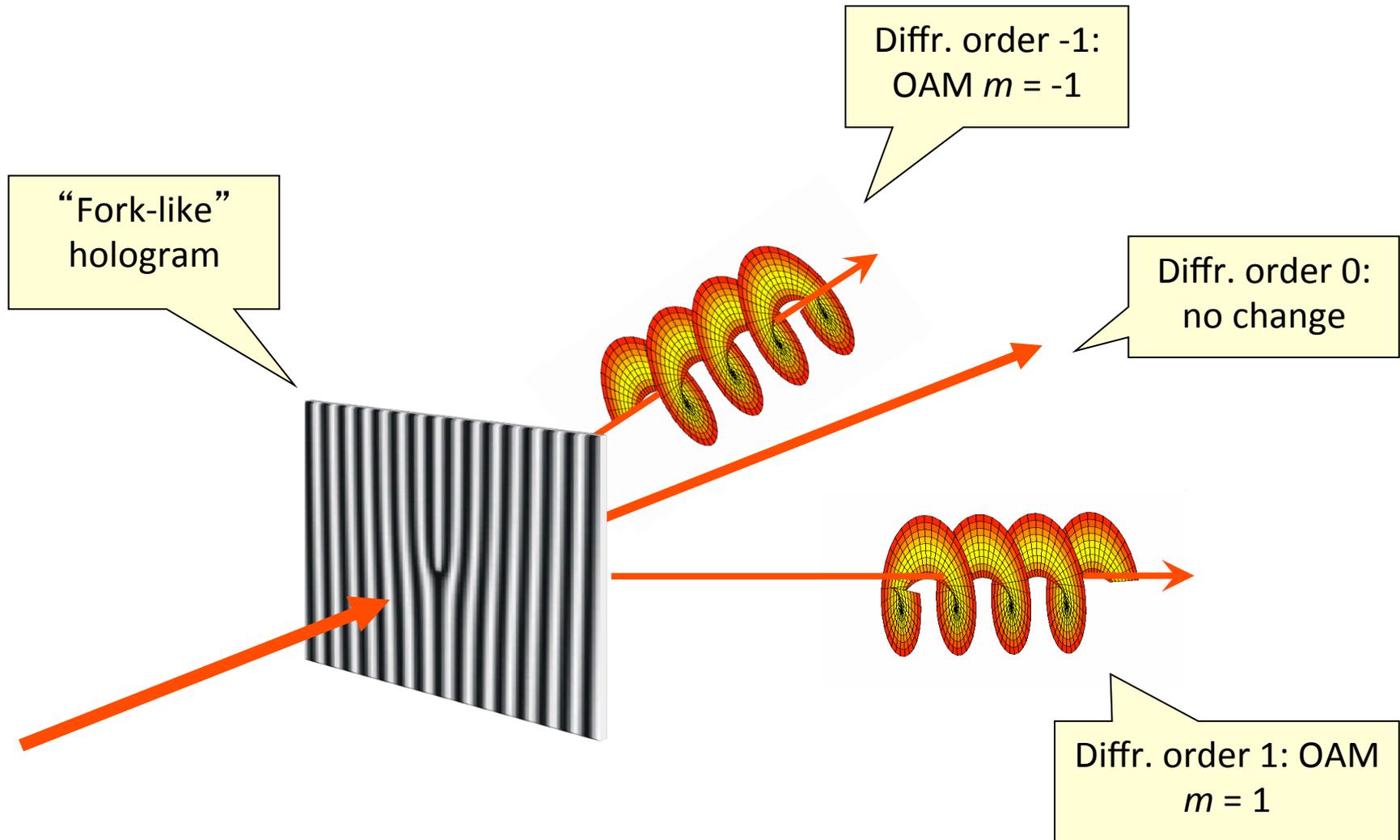


Effect of **medium inhomogeneity**:



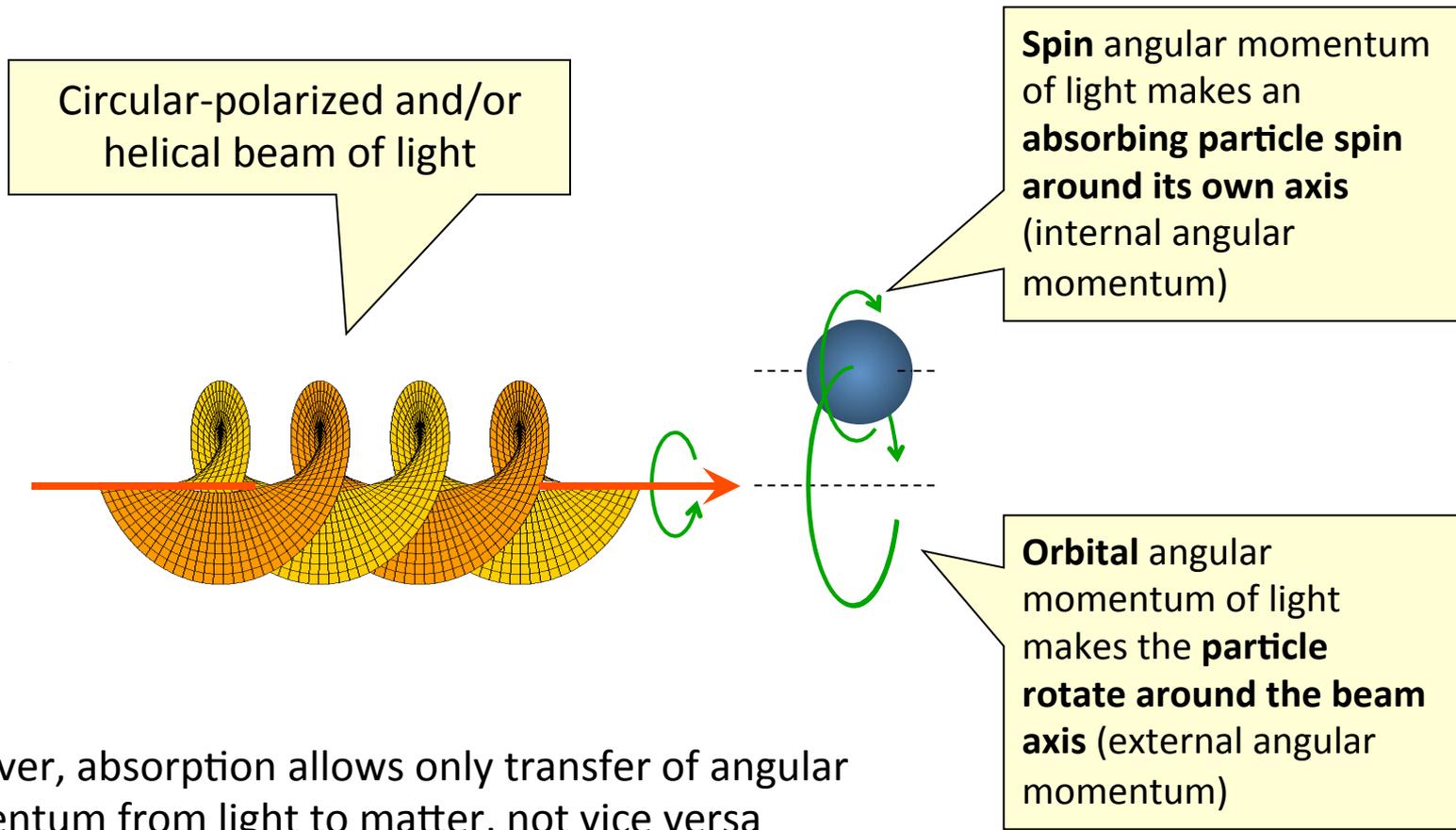
# SAM and OAM interaction with matter

**Holograms** can be considered as a special case of (strongly) inhomogeneous media:



# SAM and OAM interaction with matter

Absorbing media couple both with SAM and OAM, although not exactly in the same way:



# **Spin–orbital angular momentum conversion**

# SAM – OAM conversion (optical spin-orbit effects)

“SAM – OAM conversion” is here defined as an optical process in which SAM and OAM both vary during propagation but the total angular momentum is conserved, whatever the input state of light

More generally, optical spin-orbit coupling effects take place whenever SAM and OAM affect each other during propagation. SAM – OAM conversion is a special case of spin-orbit coupling effect

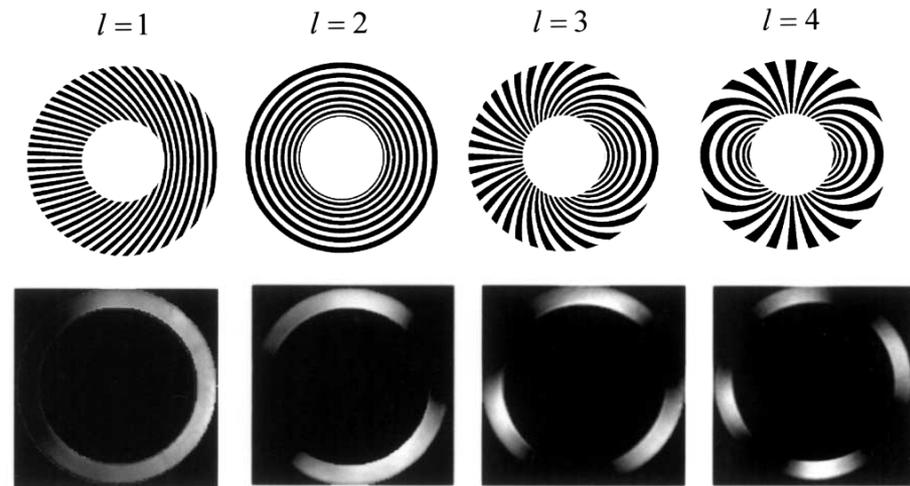
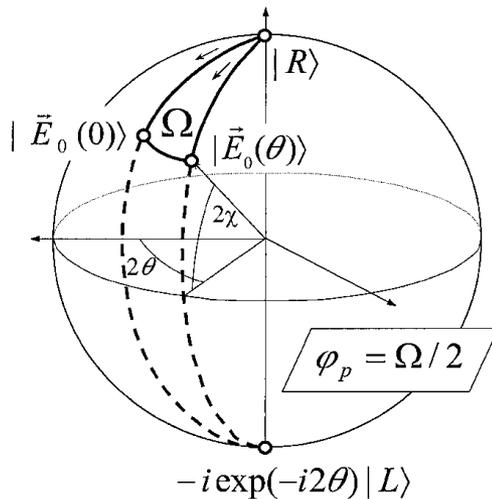
**Question:**  
**Under what conditions does SAM – OAM conversion take place?**

Before attempting an answer, let us **review the main findings reported so far**, following a chronological order (probably an incomplete list)

Notice: from 1992 (when the OAM research field actually started) to 2002, there was no prediction or observation of SAM – OAM conversion (except for external OAM effects)

## 2002: Space-variant sub-wavelength gratings

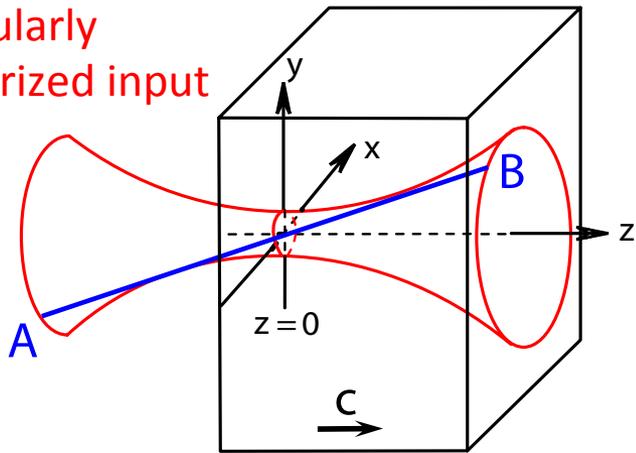
Hasman's group (after an idea of Rajendra Bhandari) demonstrates wavefront reshaping by exploiting the Pancharatnam-Berry phase arising in space-variant polarization manipulations. Among other examples, they demonstrate generation of helical modes (hence nonzero OAM) [Z. Bomzon et al., *OL* **27**, 1141 (2002); G. Biener et al., *OL* **27**, 1875 (2002)]



- this is actually the first reported observation of SAM – OAM conversion involving internal OAM (for mid-infrared light,  $\lambda \approx 10 \mu\text{m}$ )
- however, the authors **do not explicitly discuss the angular momentum of light** in the process
- the experiment was carried out only for a fixed input polarization and could not distinguish the output OAM sign (not a full test of SAM – OAM conversion)

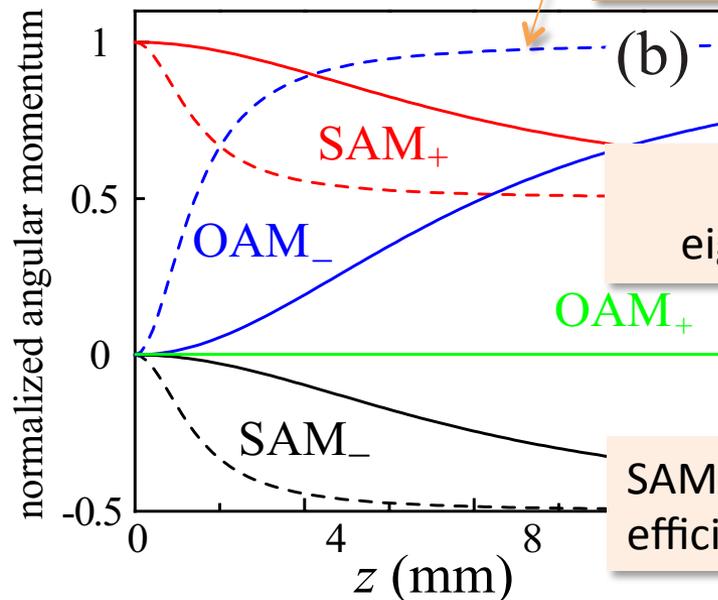
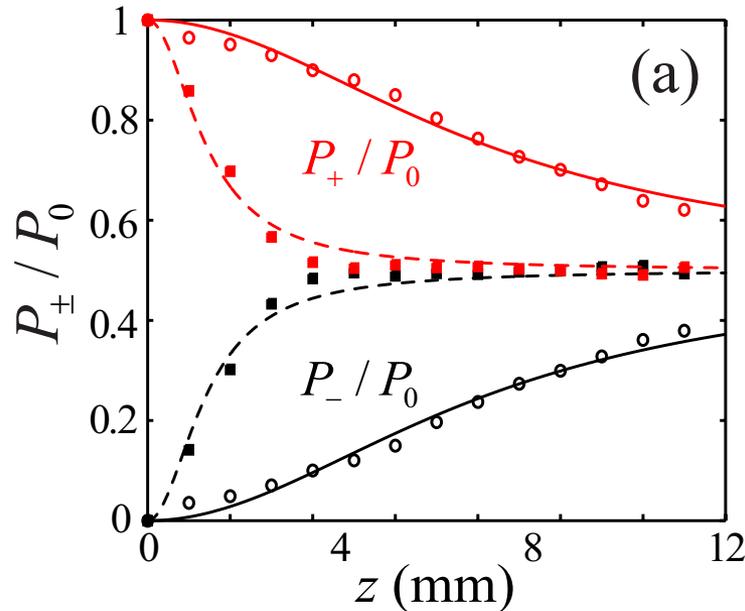
# 2003: Propagation in uniaxial birefringent crystals

Circularly polarized input



Ciattoni et al. predict theoretically that a gaussian beam travelling along the optical axis of a **uniaxial birefringent crystal** undergoes SAM – OAM conversion (although they do not use this expression) [JOSA A **20**, 163 and PRE **67**, 036618 (2003)]

First observed by Brasselet et al. in 2009 [Opt. Lett. **34**, 1021 (2009)]



Average output OAM = input SAM =  $\pm \hbar$

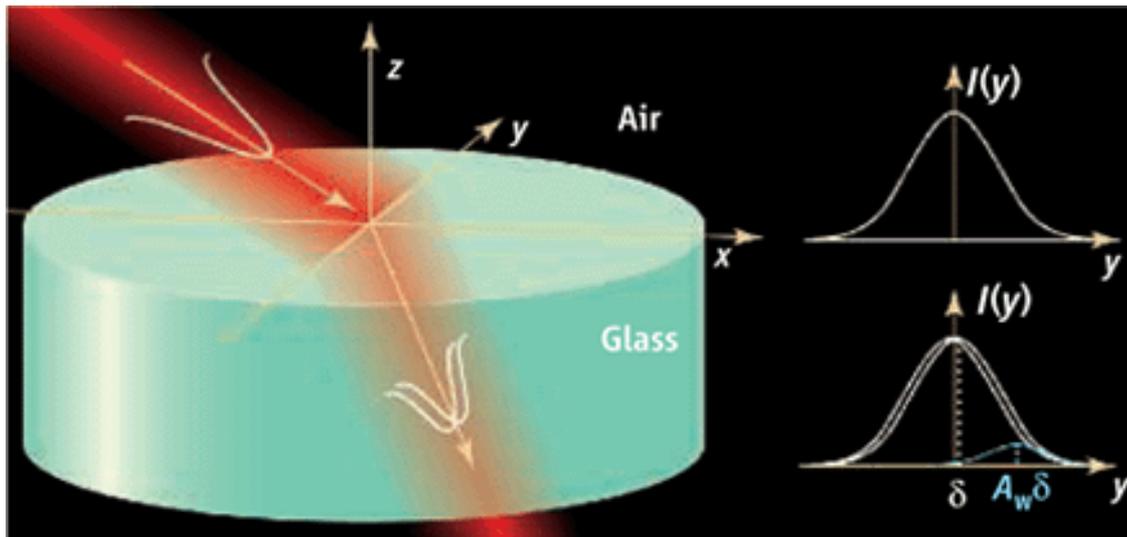
Output OAM eigenvalues =  $\pm 2\hbar$

SAM-OAM conversion efficiency  $\leq 50\%$

## 2004: Propagation in inhomogeneous media: spin Hall effect of light

M. Onoda et al. predict the occurrence of a “spin Hall effect of light”, a transverse shift of circularly polarized optical beams crossing gradients of dielectric properties, and explain it in terms of Berry phases (similar to Imbert-Fedorov shift of total internal reflection) [PRL **93**, 083901 (2004)]

A similar prediction is made also by Kostantin Yu Bliokh et al. [Phys. Lett. A **333**, 181 (2004)]. Later, he also extends this theory to a “OAM Hall effect” [K. Yu. Bliokh, PRL **97**, 0403901 (2006)]

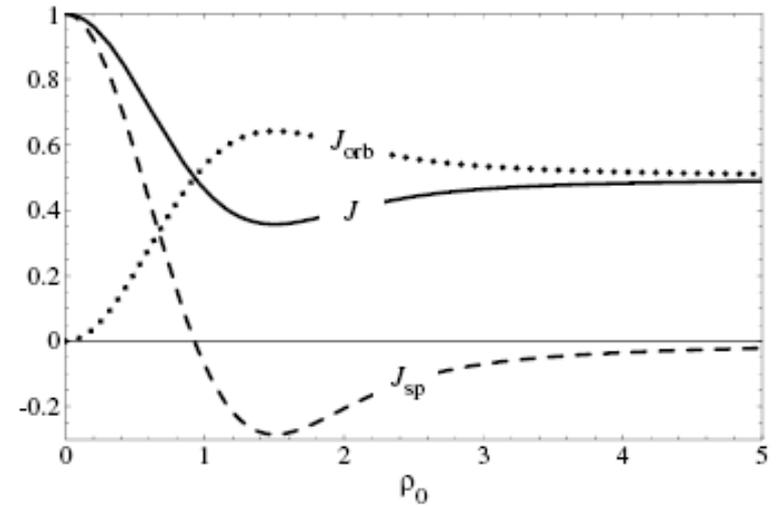
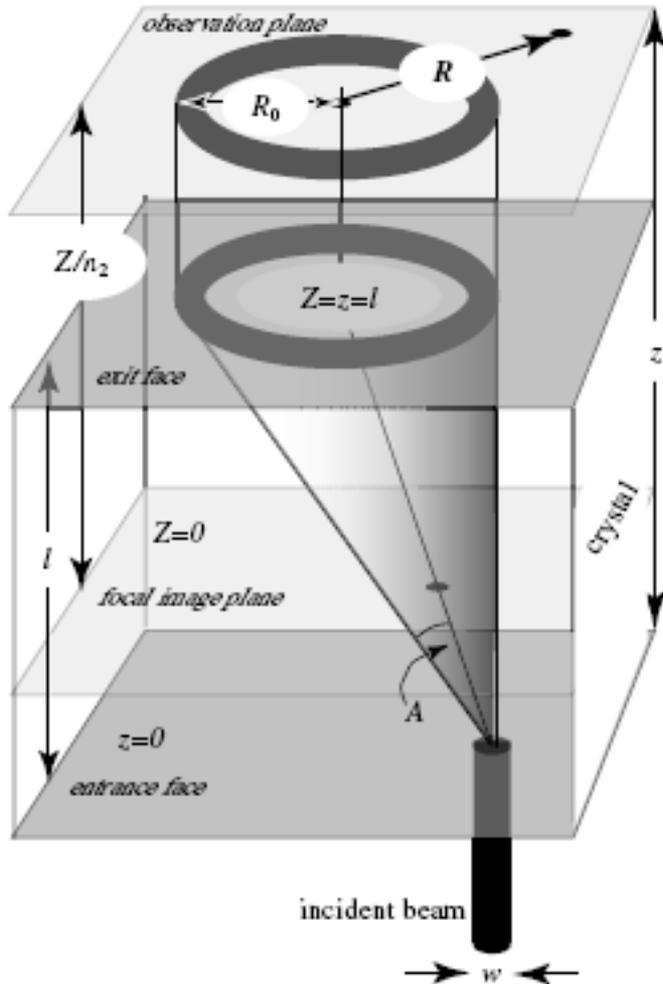


First observed in 2008 by Onur Ostein and Paul Kwiat [Science **319**, 787 (2008)]

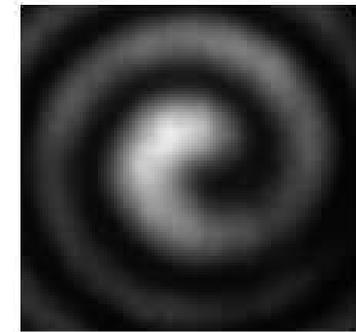
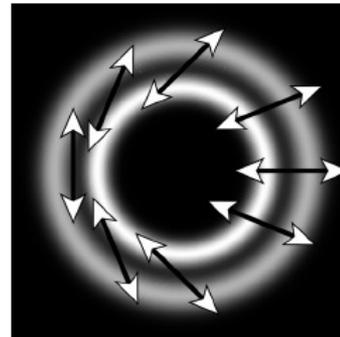
This effect (as well as the old Imbert-Fedorov shift) provide examples of **conversion of SAM into external OAM**

# 2005: Conical diffraction in biaxial birefringent crystals

Michael V. Berry et al. discuss a form of SAM-OAM conversion for light entering biaxial birefringent crystals [J. Opt. A: Pure Appl. Opt. **7**, 685 (2005)]



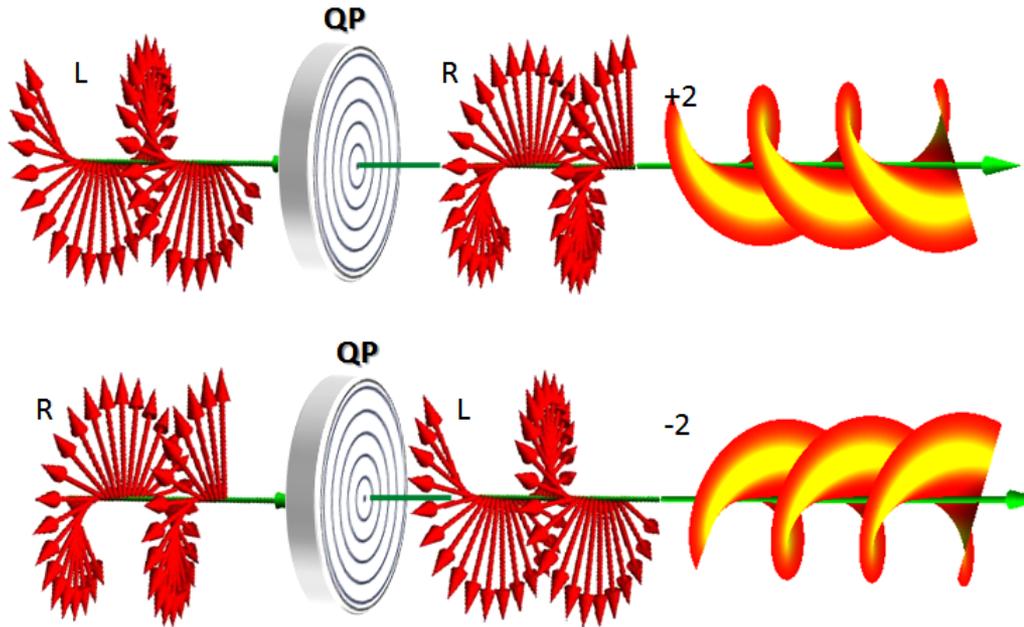
Output OAM eigenvalues =  $\pm \hbar$



Confirmed experimentally by D. P. O'Dwyer et al. in 2010 [Opt. Express **18**, 16480 (2010)]

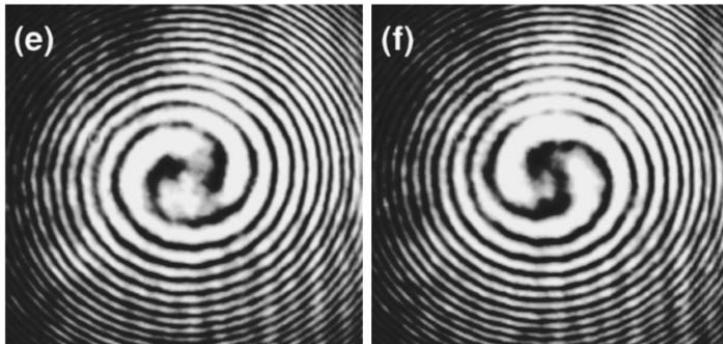
## 2006: Propagation in inhomogeneous anisotropic media

Marrucci et al. predict and observe SAM – OAM conversion in liquid crystal cells having a singular pattern with topological charge: the “**q-plates**” [PRL **96**, 163905 (2006); APL **88**, 221102 (2006)]



Output OAM eigenvalues =  $\pm 2\hbar$ , (but other values are also possible)

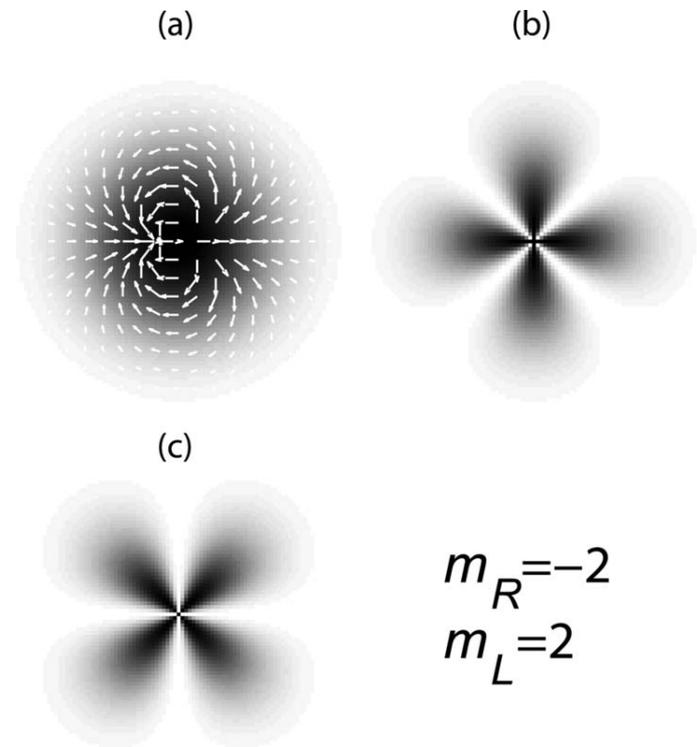
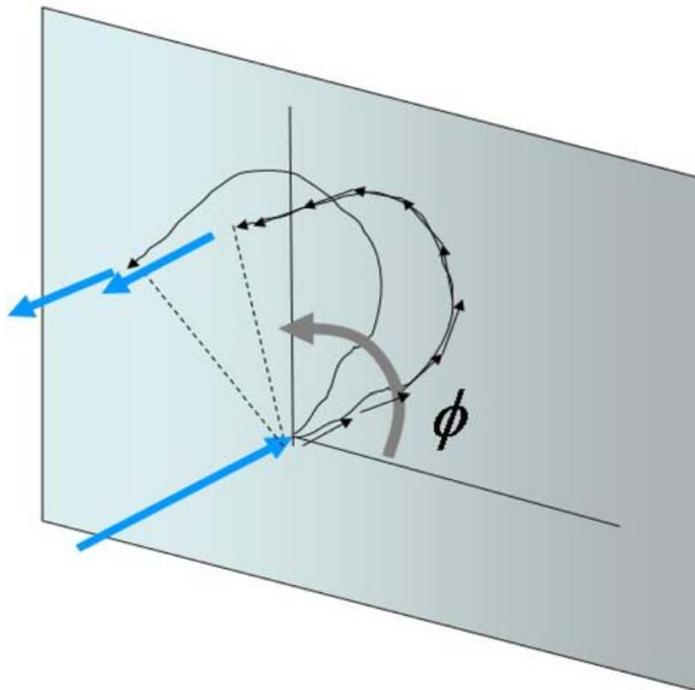
SAM-OAM conversion efficiency up to  $\approx 100\%$



This is the first paper explicitly reporting and fully demonstrating experimental optical SAM – OAM conversion!

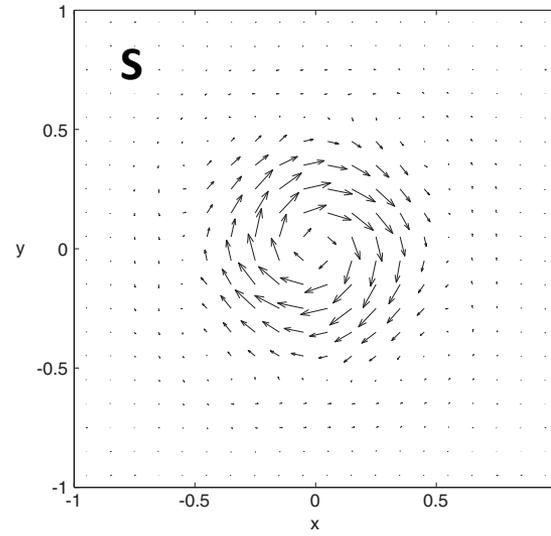
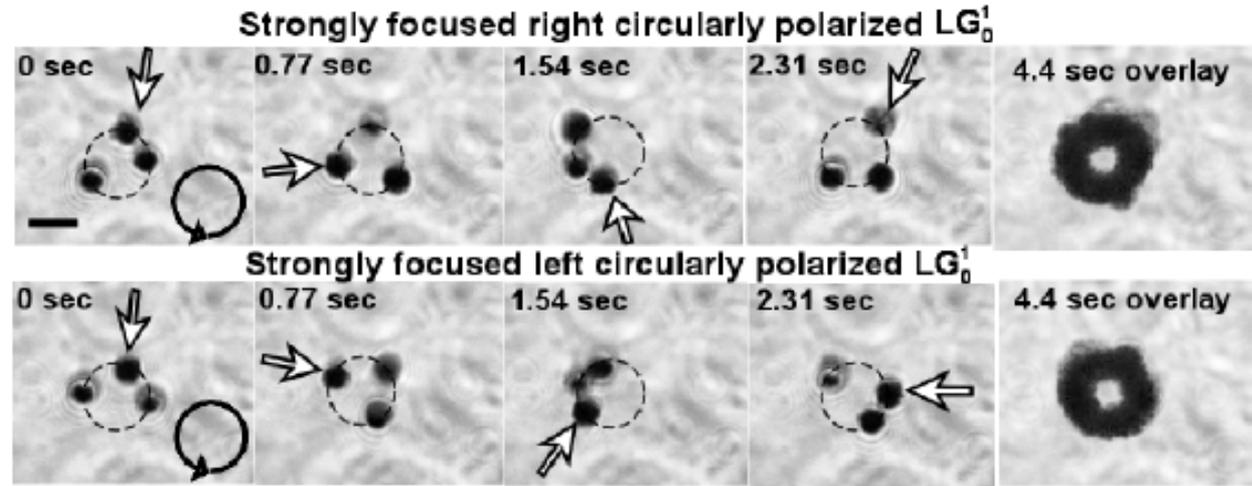
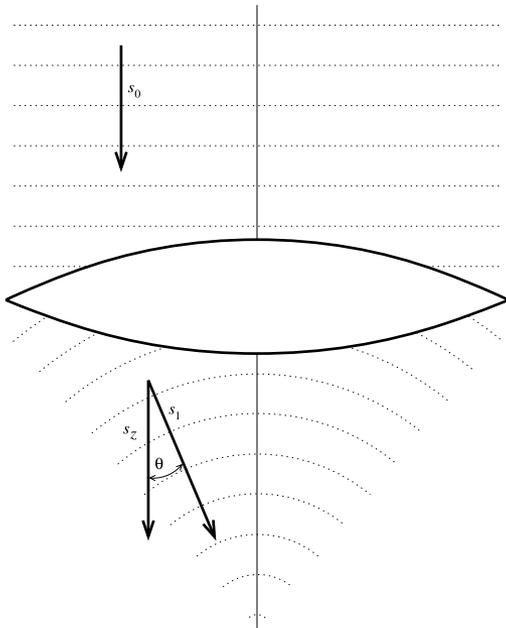
## 2006: Backscattering from disordered media

C. Schwartz explains the previously observed patterns of back-scattered light from disordered media in terms of Berry phases and SAM – OAM conversion [OL 31, 1121 (2006)]



# 2007: Strong focusing of circularly polarized light

Y. Zhao et al. [PRL **99**, 073901 (2007)] and H. Adachi et al. [PRA **75**, 063409 (2007)] report the experimental demonstration of a nonzero OAM content in strongly focused circularly polarized beams

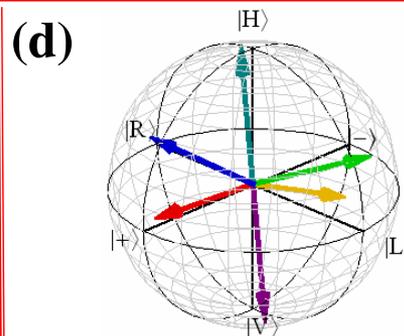
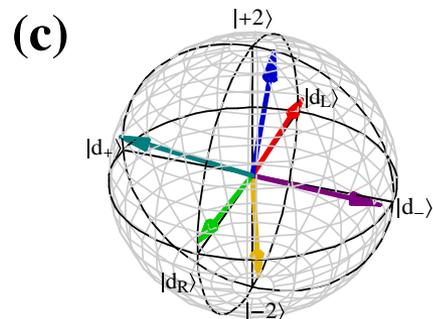
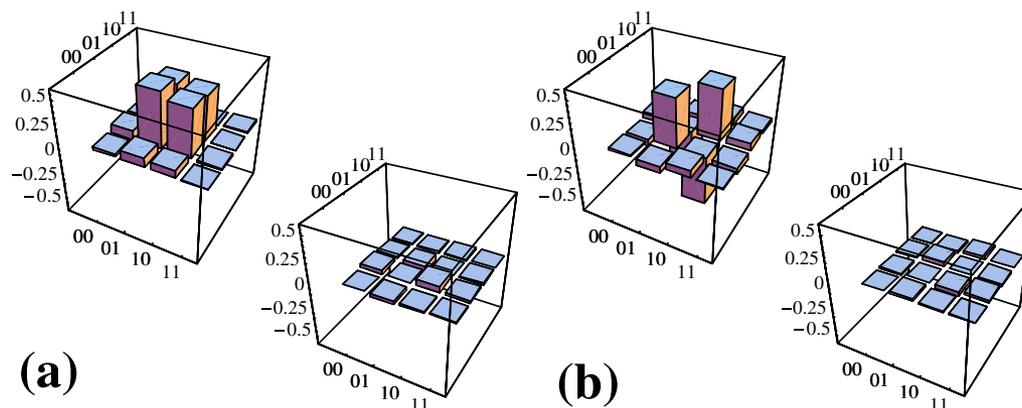


A first prediction of this effect was actually given by T. A. Nieminen et al. [Arxiv 2004; J. Opt. A **10**, 115005 (2008)]

(however, this OAM appears only in strongly non-paraxial regime, where the definition of SAM and OAM as distinct angular momenta is not so clear)

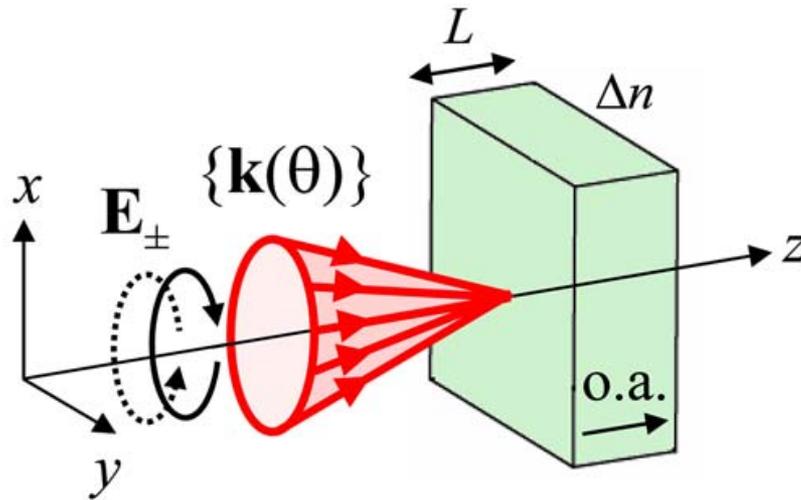
# 2009: SAM – OAM conversion for single photons (in q-plates)

E. Nagali et al. report the first experimental demonstration of SAM – OAM conversion in single photons and in correlated photon pairs [PRL **103**, 013601 (2009)]



## 2010: Conical beams in uniaxial birefringent crystals

Fadeyeva et al. [JOSA A **27**, 381 (2010), Opt. Express **18**, 10848 (2010)] and Loussert et al. [Loussert et al., Opt. Lett. **35**, 7 (2010)] demonstrate high efficiency SAM – OAM conversion of conical (Bessel) beams propagating through a uniaxial birefringent crystal



Output OAM =  $\pm 2\hbar$

Conversion efficiency  
can reach  $\approx 100\%$

# 2011: Propagation in curved space-time

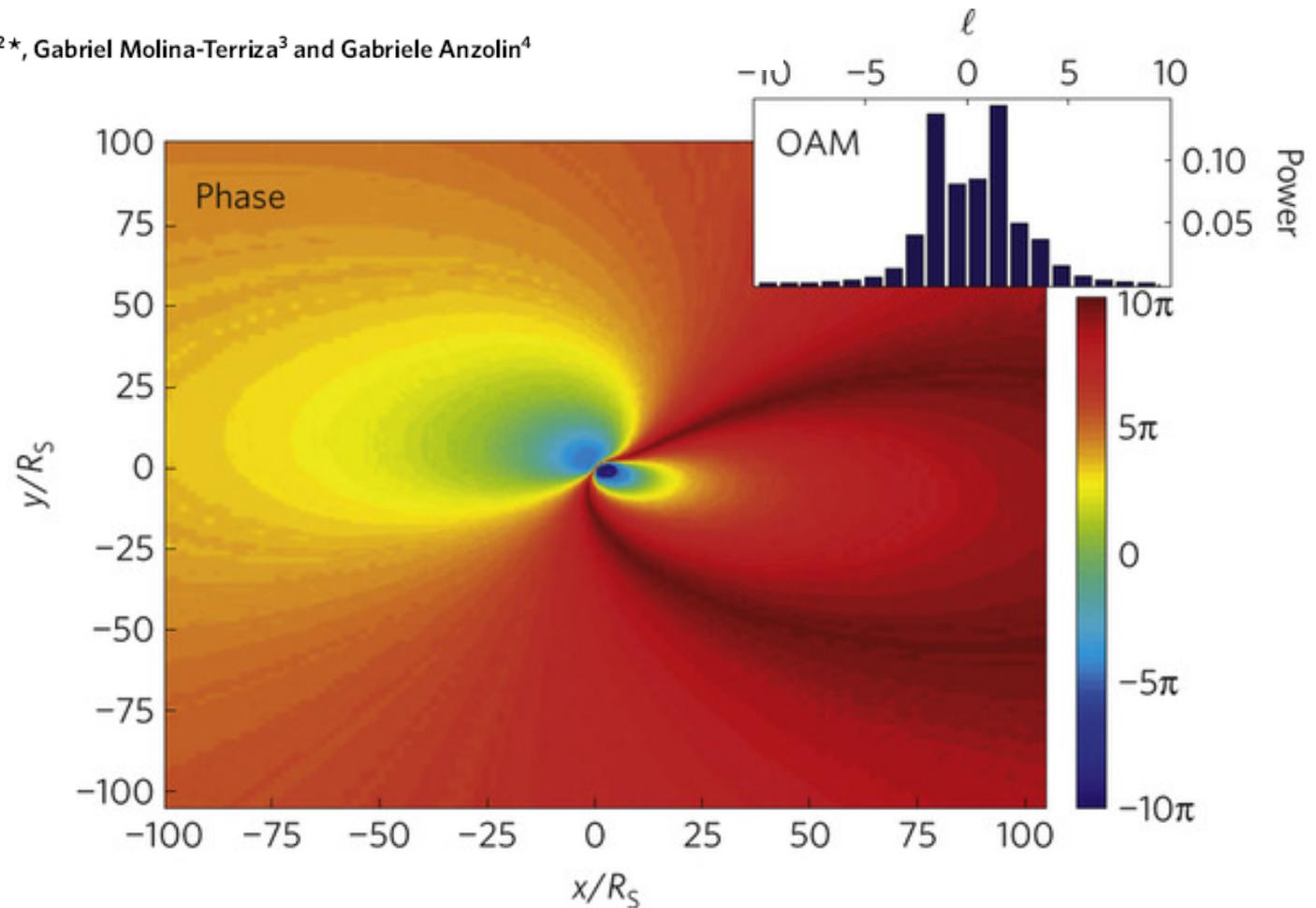
nature  
physics

LETTERS

PUBLISHED ONLINE: 13 FEBRUARY 2011 | DOI:10.1038/NPHYS1907

## Twisting of light around rotating black holes

Fabrizio Tamburini<sup>1</sup>, Bo Thidé<sup>2\*</sup>, Gabriel Molina-Terriza<sup>3</sup> and Gabriele Anzolin<sup>4</sup>



# SAM – OAM conversion: conditions for occurrence

Categories of SAM – OAM processes identified so far:

- ❑ Propagation in **homogeneous anisotropic media** + **deviation from full paraxiality** (uniaxial crystals, conical diffraction)
- ❑ Propagation in **inhomogeneous isotropic media** + **large variation of propagation direction** (spin Hall effect, strong focusing, back-scattering)
- ❑ Propagation in **inhomogeneous anisotropic media** (patterned liquid crystals, e.g. q-plates, sub-wavelength gratings, curved space-time)

In all cases, a **global rotational symmetry of medium** around the z-axis ensures exact conservation of the total angular momentum z-component:  $J_z = L_z + S_z$

Otherwise some exchange of angular momentum with the medium is involved (e.g., general q-plates, conical diffraction in biaxial crystals)

Notice: only the last case allows SAM – OAM conversion of undeflected fully paraxial beams

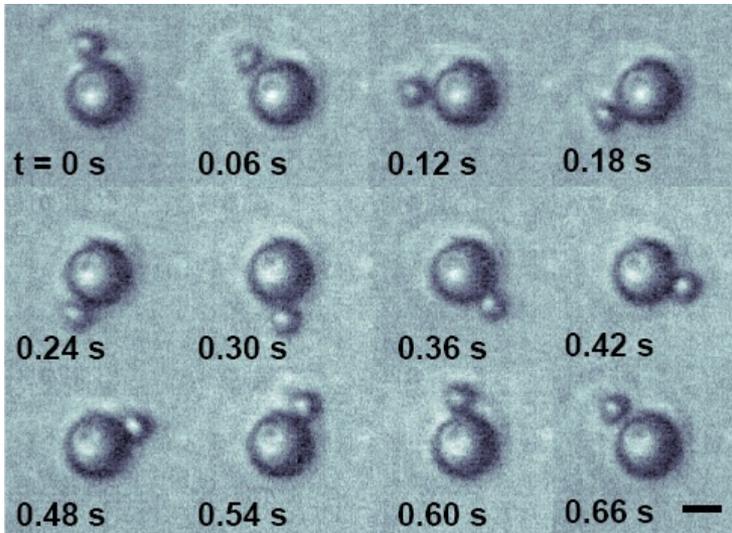
# q-plate: the concept

[L. Marrucci, C. Manzo, D. Paparo, PRL **96**, 163905 (2006); APL **88**, 221102 (2006)]

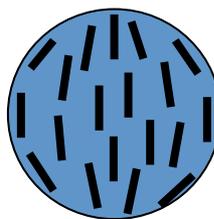
# q-plate: origin of the idea (2005)

Experiment on spinning liquid crystal droplets by circularly polarized light:

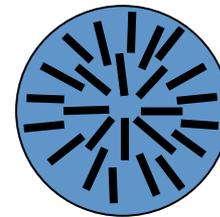
[C. Manzo, D. Paparo, L. Marrucci, I. Jánossy, *Phys. Rev. E* **73**, 051707 (2006)]



However, we found two kinds of droplets:



Bipolar droplets:  
Almost homogeneous  
birefringence. **They can  
be spun by light.**



Radial droplets:  
Inhomogeneous  
birefringence.  
**They do not spin!**

But **radial droplets** anyway modify the light polarization and therefore should **exchange (spin) angular momentum with light** [Istvan Jánossy, *private discussion*]

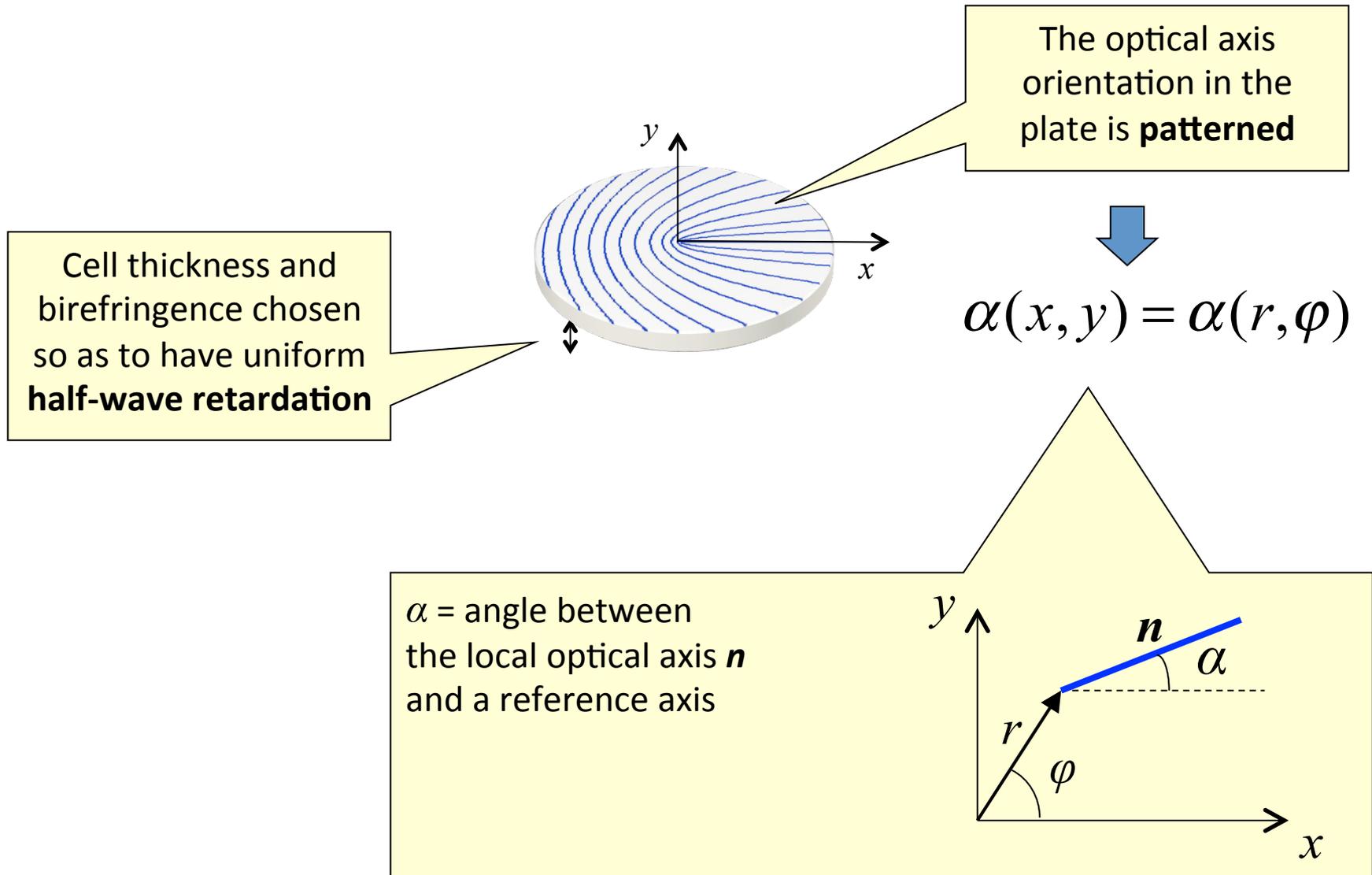
**So, why don't they rotate?**

The simple answer we found: SAM goes into OAM!



q-plate idea!!

# q-plate structure: patterned half-wave plates



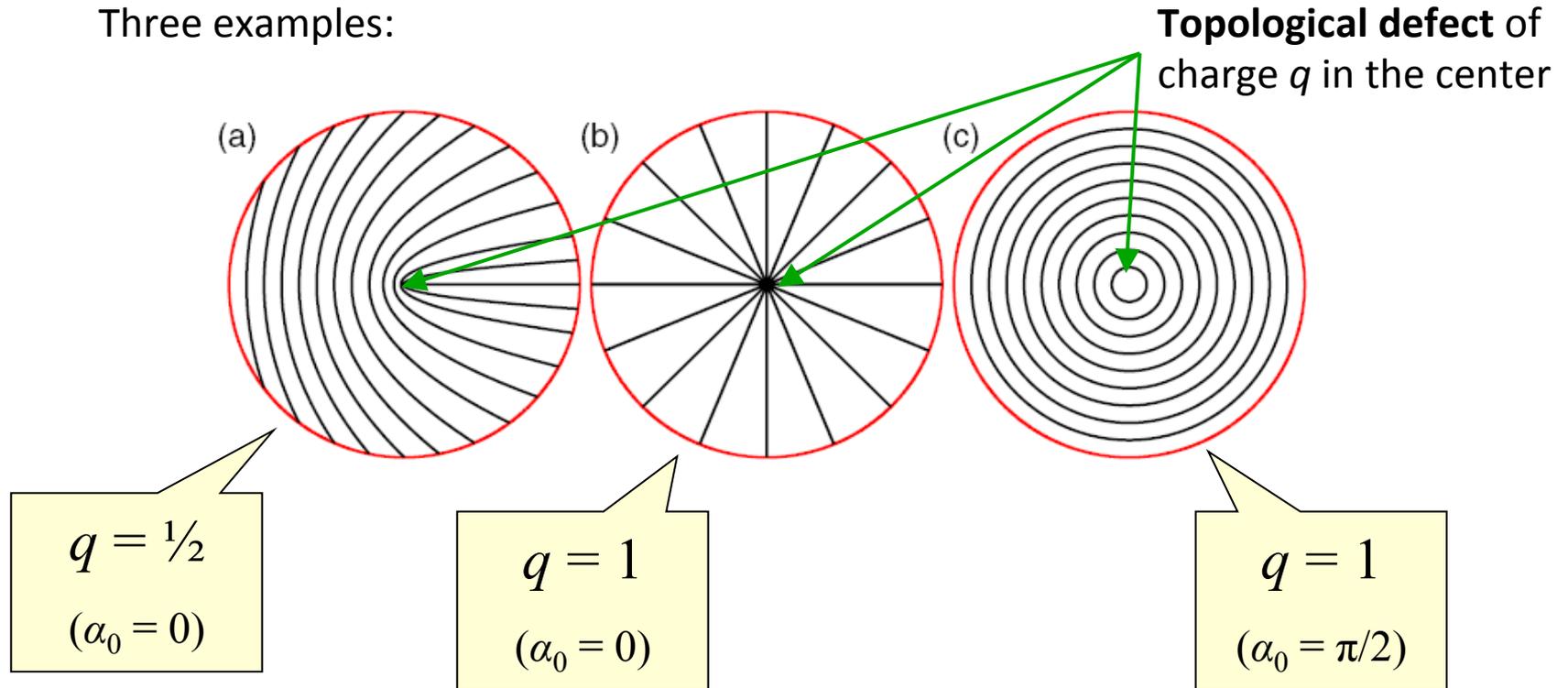
# q-plate structure: patterned half-wave plates

General  
pattern:

$$\alpha(x, y) = \alpha(r, \varphi) = q\varphi + \alpha_0$$

with  $q$  integer  
or half-integer

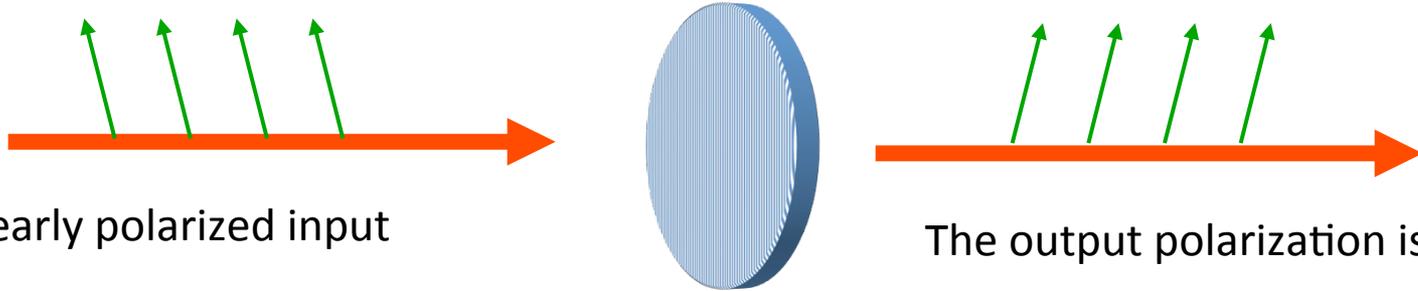
Three examples:



**Notice:  $q = 1$  yields rotational-symmetric patterns** (such as the radial droplets)

# q-plate optical effect

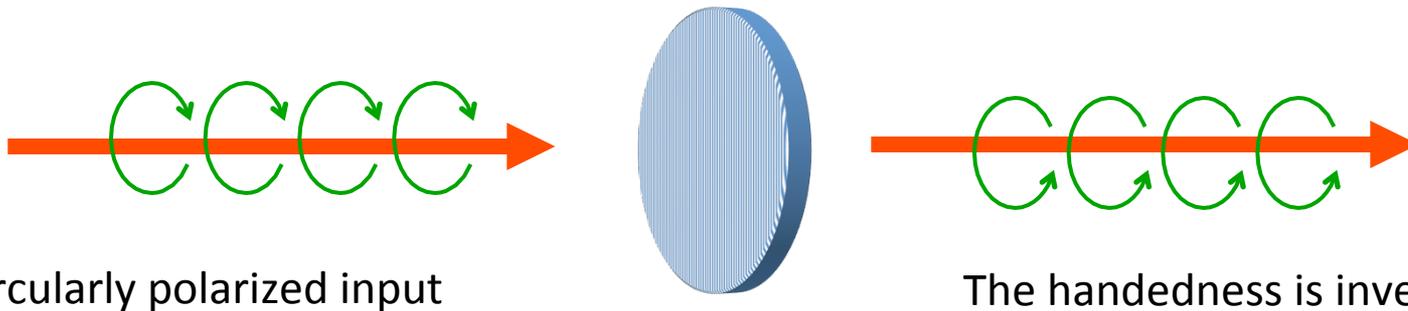
Consider first a normal (uniform) half-wave plate



For linearly polarized input

The output polarization is rotated

The extent of the polarization rotation depends on the optical axis orientation



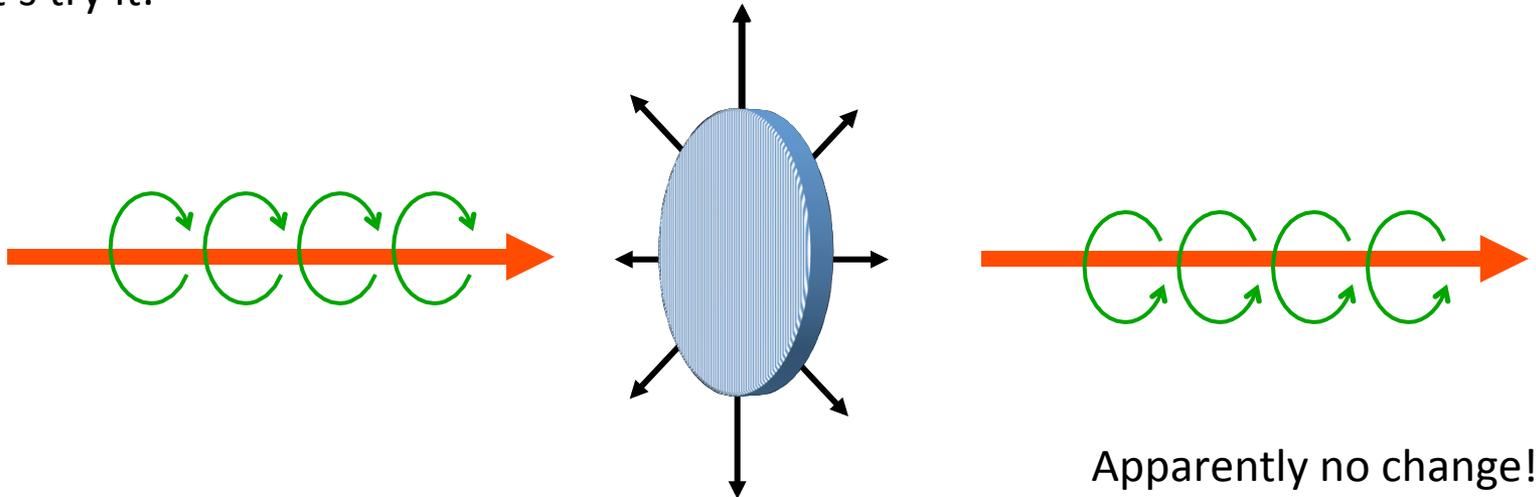
For circularly polarized input

The handedness is inverted

**But what is the effect of rotating the optical axis in this case?**

# q-plate optical effect

Let's try it:



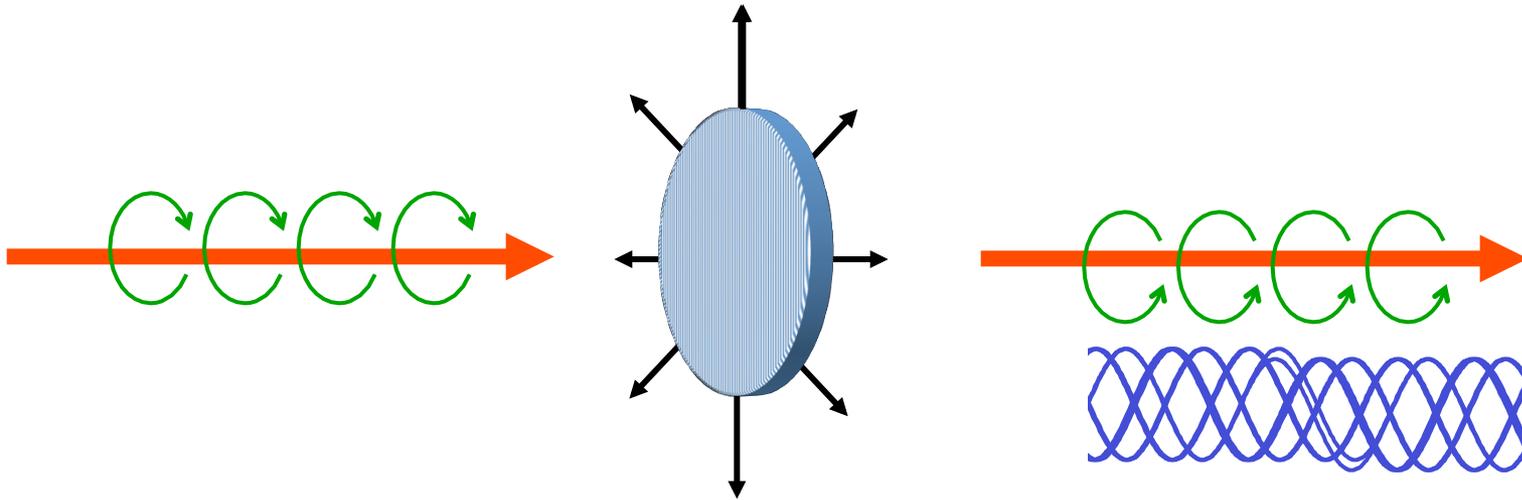
Apparently no change!

No change in the output polarization and optical intensity

**But let us check also the optical phase**

# q-plate optical effect

Phase-shift induced by rotated half-wave plate on circular-polarized light:



Phase-shift versus half-wave axis rotation:

$$\Delta\Phi = \pm 2\alpha$$

The  $\pm$  sign is determined by the input polarization handedness

# q-plate optical effect: Jones calculus

Jones matrix of an  $\alpha$ -oriented half-wave plate:  $\mathbf{M} = \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{bmatrix}$

Let us apply it to an **input left-circular** polarized plane wave:

$$\mathbf{M} \times \begin{bmatrix} 1 \\ i \end{bmatrix} E_0 = \begin{bmatrix} \cos 2\alpha + i \sin 2\alpha \\ -i \cos 2\alpha + \sin 2\alpha \end{bmatrix} E_0 = \begin{bmatrix} 1 \\ -i \end{bmatrix} e^{i2\alpha} E_0$$

The output polarization is **uniform right-handed circular**

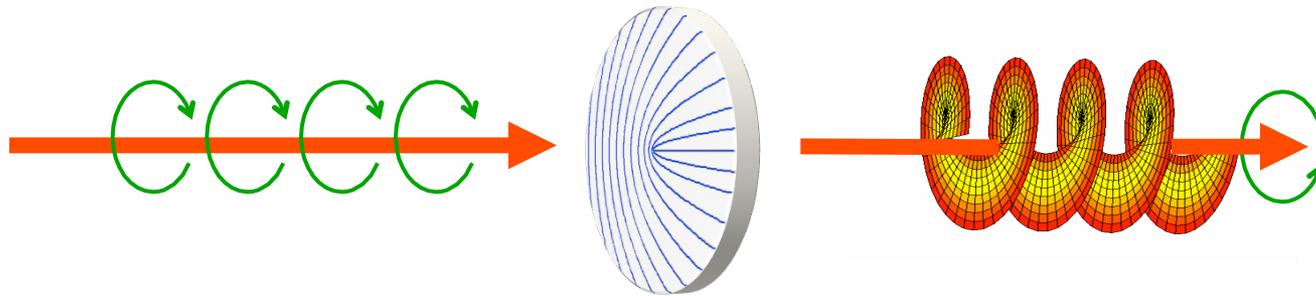
The wave has acquired a **phase retardation**

$$\Delta\Phi = 2\alpha$$

**Pancharatnam-Berry geometrical phase**  
(unrelated with optical path length)

# q-plate optical effect

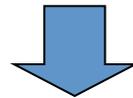
Now consider again a non-uniform half-wave plate:



The wavefront gets reshaped!

For the specific q-plate pattern:

$$\alpha(r, \varphi) = q\varphi + \alpha_0$$

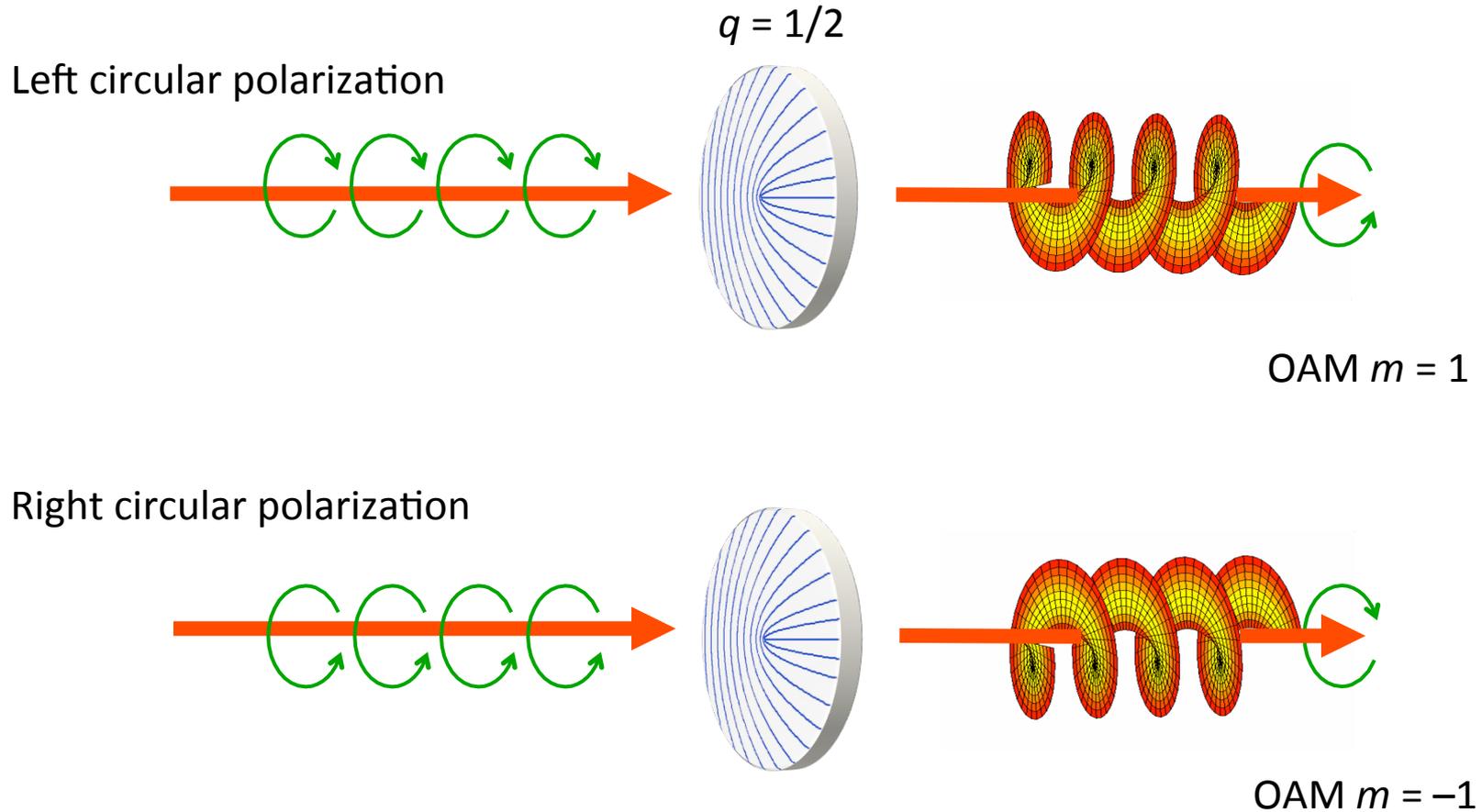


$$\Delta\Phi(x, y) = \pm 2\alpha = \pm 2q\varphi + (\pm 2\alpha_0) = m\varphi + \text{const.}$$

Helical phase with  $m = \pm 2q$  !

# q-plate optical effect

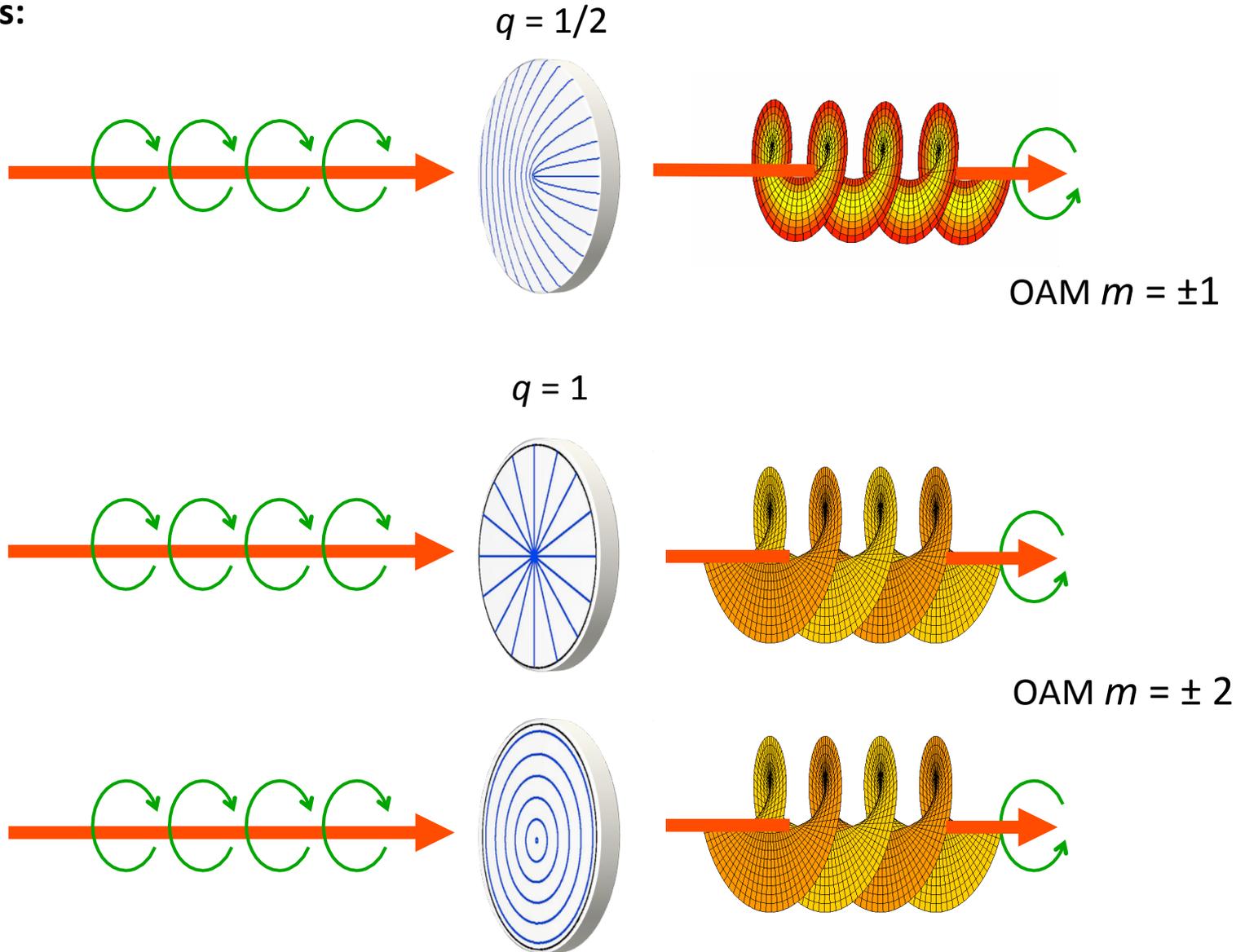
Examples:



**Polarization controlled OAM handedness!**

# q-plate optical effect

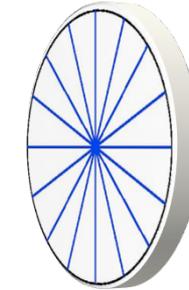
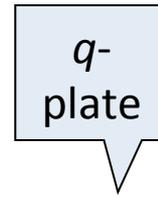
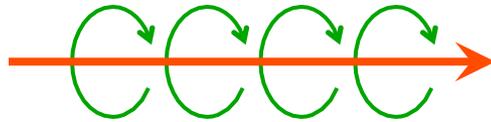
Examples:



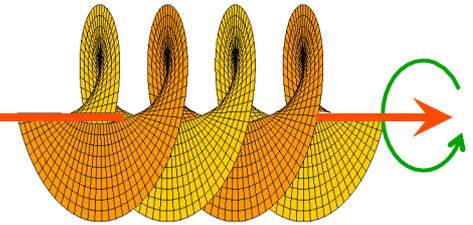
# Photon angular momentum balance: case $q = 1$

Left-circular  
input:

Spin:  $S_z = +\hbar$   
Orbital:  $L_z = 0$   
Total:  $J_z = +\hbar$

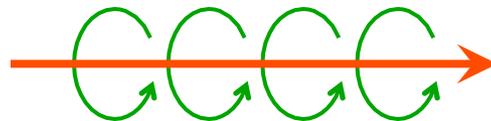


Spin:  $S_z = -\hbar$   
Orbital:  $L_z = +2\hbar$   
Total:  $J_z = +\hbar$

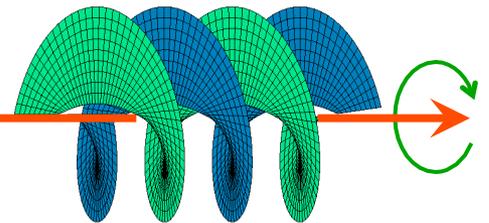


Right-circular  
input:

Spin:  $S_z = -\hbar$   
Orbital:  $L_z = 0$   
Total:  $J_z = -\hbar$



Spin:  $S_z = +\hbar$   
Orbital:  $L_z = -2\hbar$   
Total:  $J_z = -\hbar$



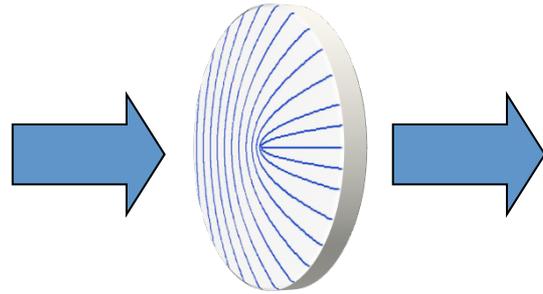
Spin-to-orbital conversion of optical angular momentum

# Photon angular momentum balance: general case

Spin:  $S_z = \pm\hbar$

Orbital:  $L_z = m\hbar$

Total:  $J_z = (m\pm 1)\hbar$

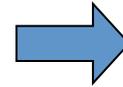


Spin:  $S_z = \mp\hbar$

Orbital:  $L_z = m\hbar \pm 2q\hbar$

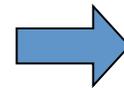
Total:  $J_z = [m\pm(2q-1)]\hbar$

For  $q \neq 1$ ,  $\Delta J_z = \pm 2(q-1)\hbar \neq 0$



**Torque on the  $q$ -plate**

For  $q = 1$ ,  $\Delta J_z = 0$



**No torque on the medium**

(medium is only a “coupler” between spin and orbital angular momentum of light)

This is why radial droplets don't rotate!

**But, what happens if the plate birefringent retardation is not just half-wave?**

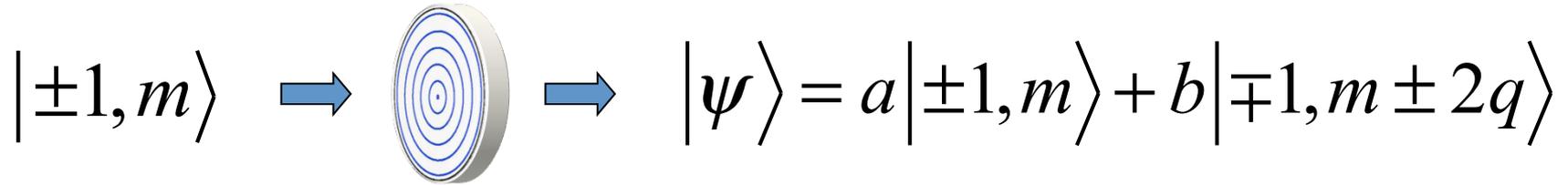
# q-plate optical effect: general birefringence retardation

We use a quantum notation:

Input photon  
 $|\text{spin,orbital}\rangle$

Birefringent  
retardation  $\delta$

Output photon: coherent superposition of  
“converted” and “unconverted” states



Notice: we call  $\delta = \pi$  the “optimal tuning” condition for the q-plate

Superposition coefficients: 
$$\begin{cases} a = \cos \frac{\delta}{2} \\ b = i \sin \frac{\delta}{2} e^{i\alpha_0} \end{cases}$$

The output photon state is **not an eigenstate of spin and orbital** angular momenta

Notice: in the  $q = 1$  case, still **all-optical conversion** (no torque on the medium)

# q-plates: the current technology

[L. Marrucci, C. Manzo, D. Paparo, PRL **96**, 163905 (2006); APL **88**, 221102 (2006)]

[E. Karimi, B. Piccirillo, E. Nagali, L. Marrucci, E. Santamato, APL **94**, 231124 (2009)]

[B. Piccirillo, V. D'Ambrosio, S. Slussarenko, L. Marrucci, E. Santamato, APL **97**, 241104 (2010)]

[S. Slussarenko, A. Murauski, T. Du, V. Chigrinov, L. Marrucci, E. Santamato, Opt. Express **19**, 4085-4090 (2011)]

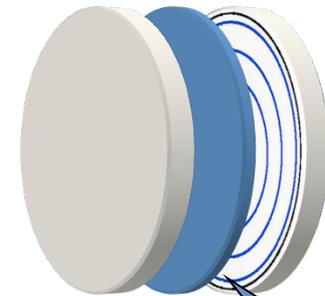
# Making a liquid crystal q-plate: the first method

- 1) **Circular rubbing** of one substrate (with **planar anchoring**)



$q = 1$   
geometry

- 2) Assemble the cell with **thickness chosen** for having **half-wave retardation** (only approximate)



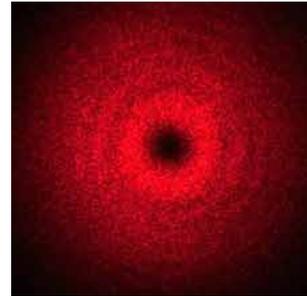
Nematic  
liquid  
crystal

The cell between  
crossed polarizers:

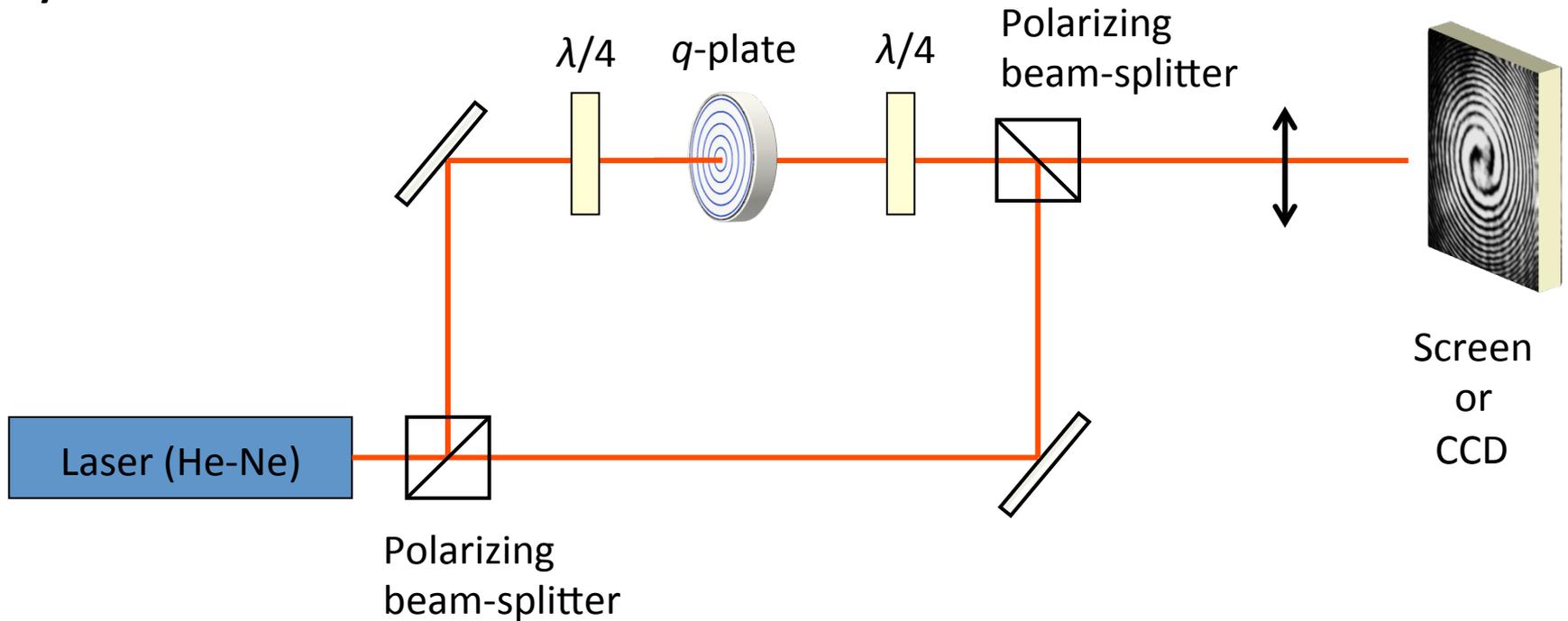


# Liquid crystal q-plate: testing the optical effect

As a first step, we check that a vortex appears in the outgoing beam:

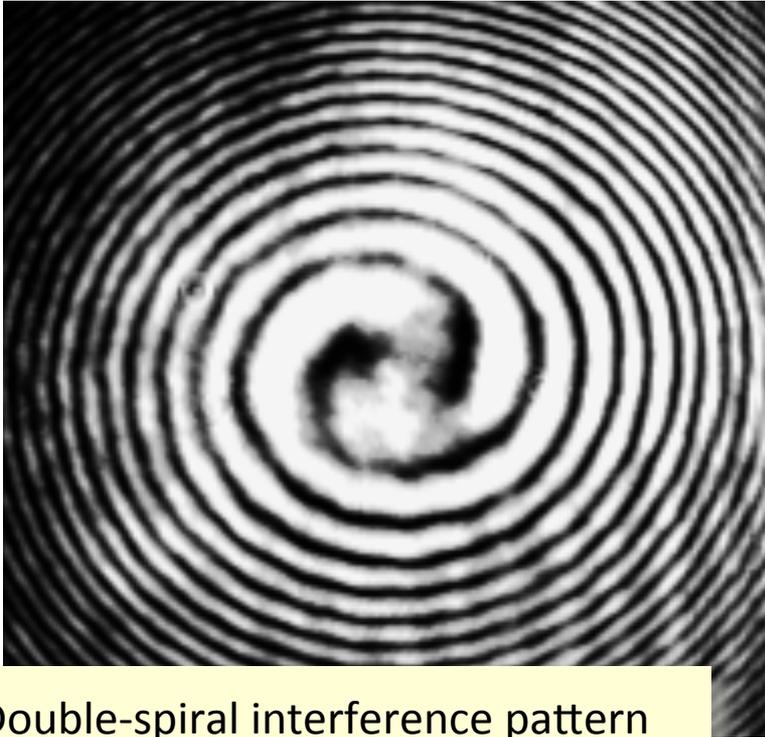


Wavefront measurement by interference:



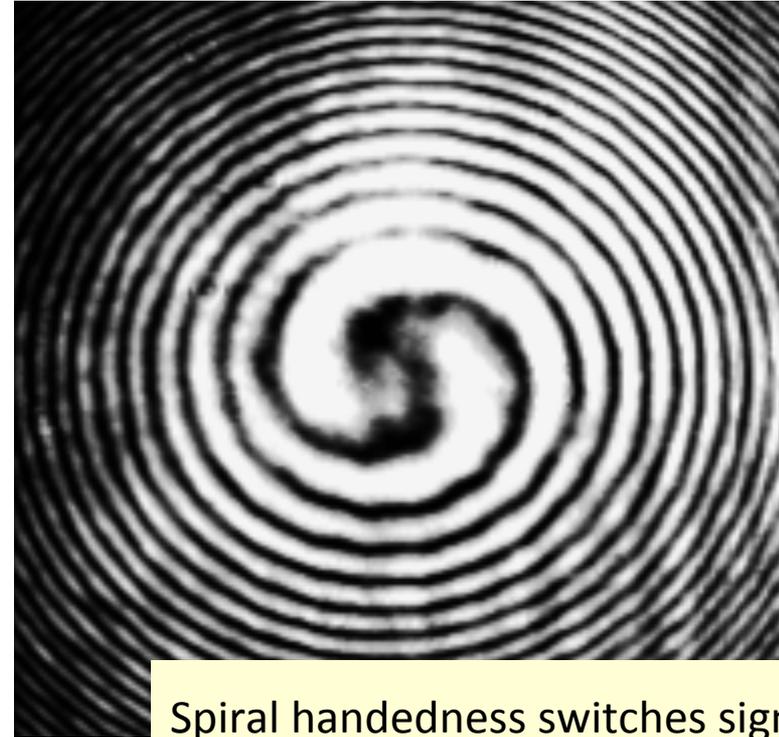
# Liquid crystal q-plate: testing the optical effect

Left-circular input



Double-spiral interference pattern  
→ Helical wavefront with  $m = \pm 2$

Right-circular input

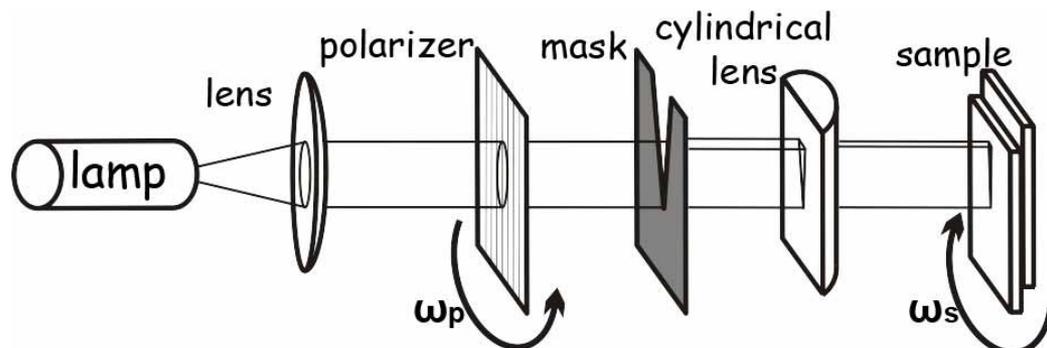


Spiral handedness switches sign  
with input polarization!

These simple observations confirm the occurrence of SAM – OAM conversion

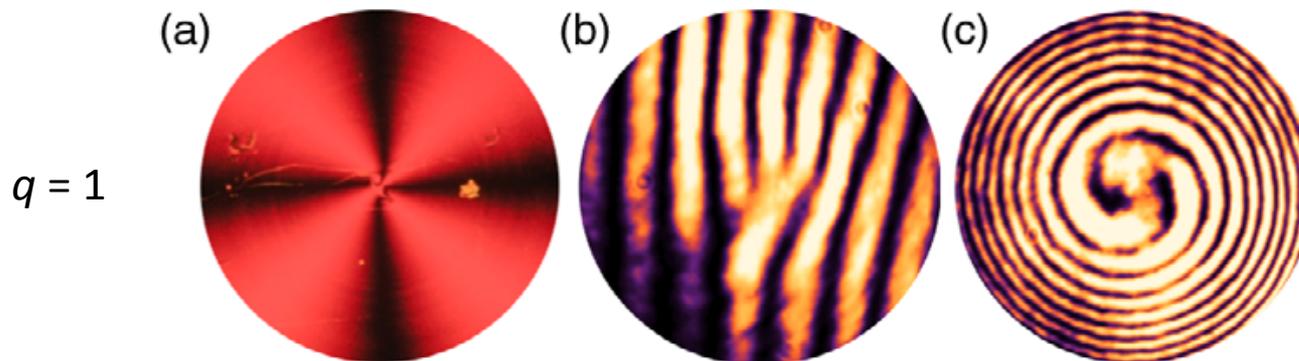
# Making a liquid crystal q-plate

A better method: optical writing of the liquid crystal pattern



Photosensitive surface layers (e.g., azo-polymers)

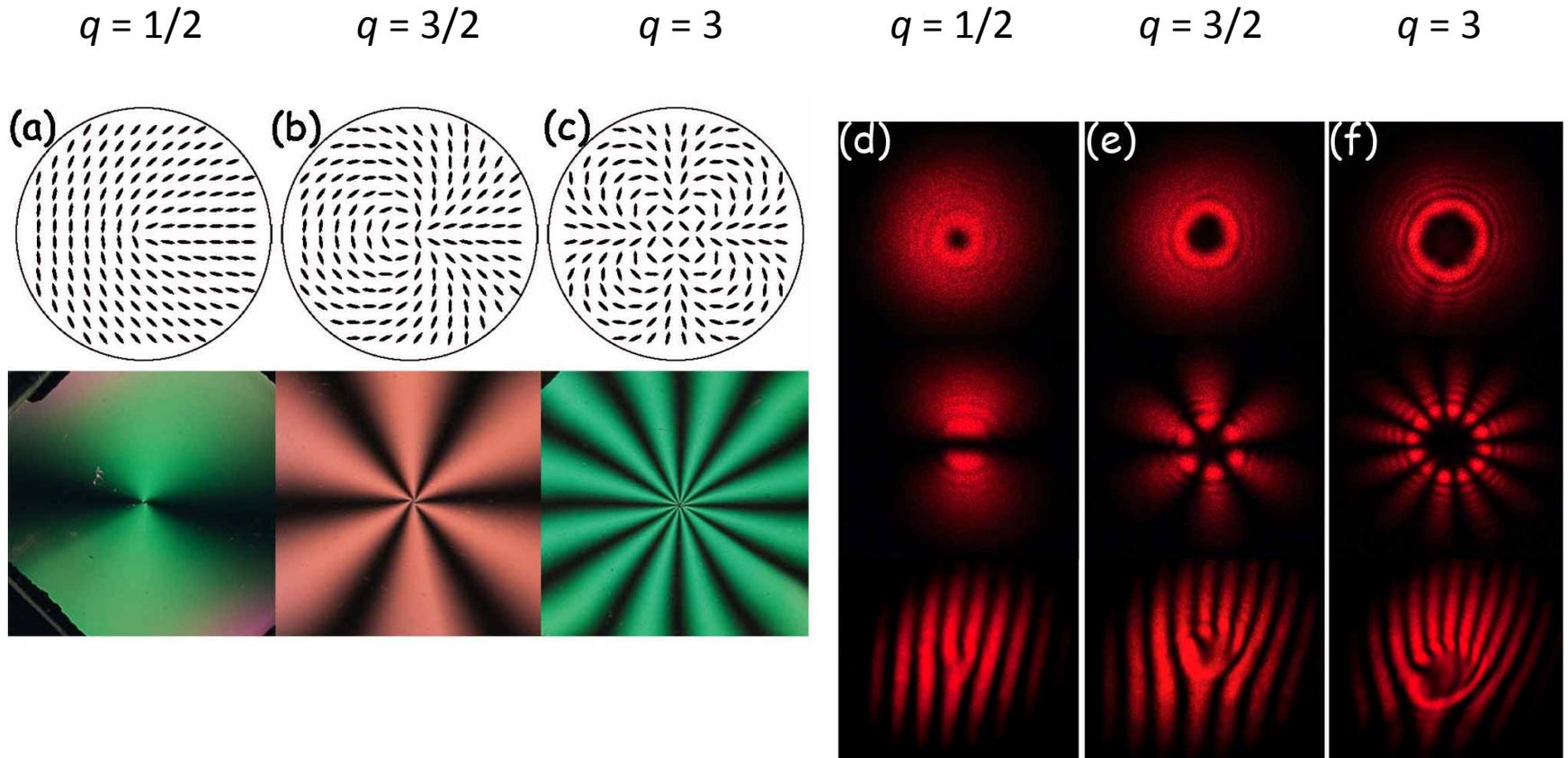
Resulting q-plates have better optical quality:



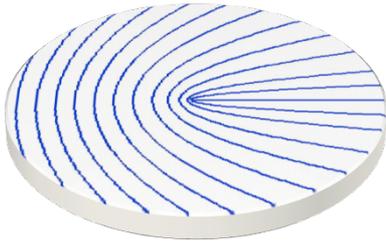
Moreover, by rotating both the polarizer and the sample...

# Making a liquid crystal q-plate

We can make q-plates with arbitrary  $q$ !

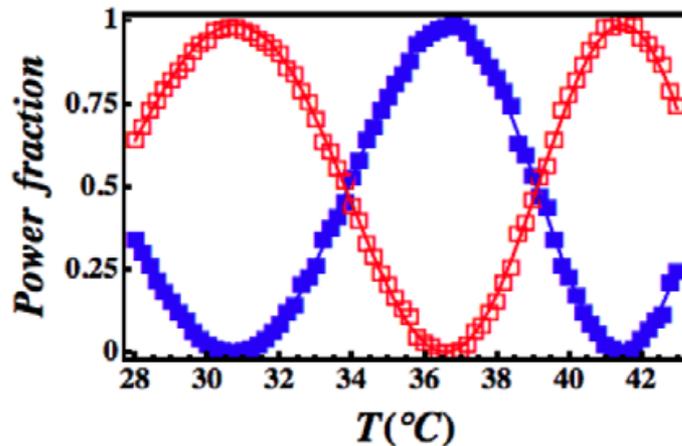
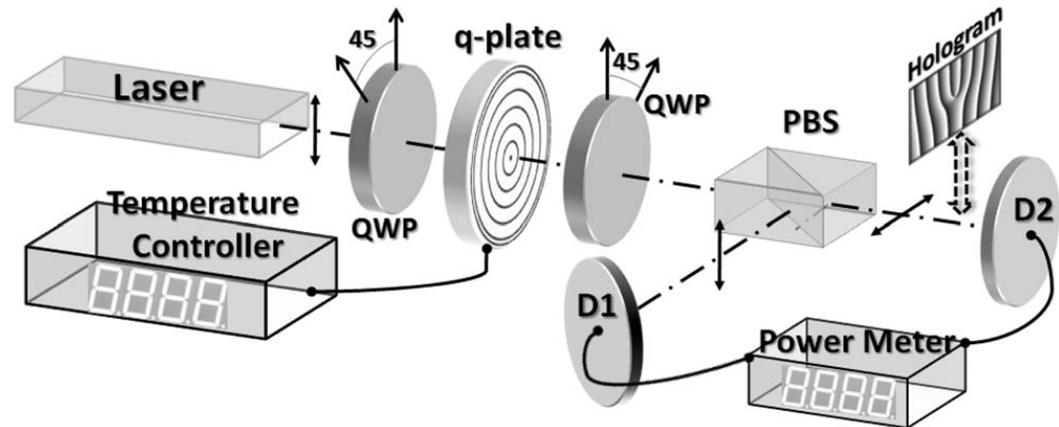


# “Tuning” a liquid crystal q-plate



We need a method for controlling and adjusting the birefringence retardation  $\delta$  of the q-plate

Our first demonstrated method was thermal: exploiting the material  $\Delta n(T)$

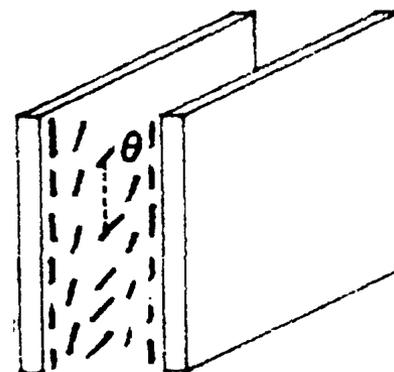


By changing  $\delta$ , the converted and unconverted components of the output wave oscillate

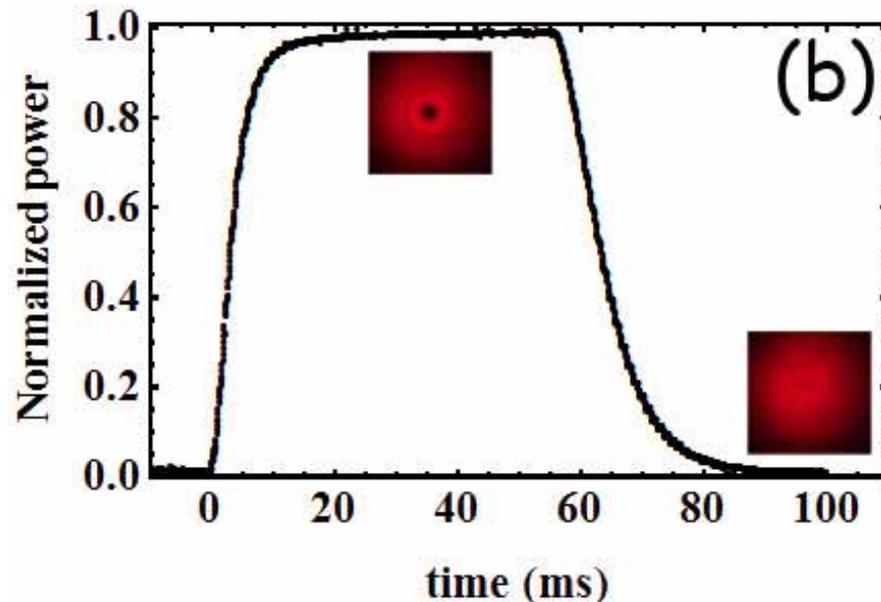
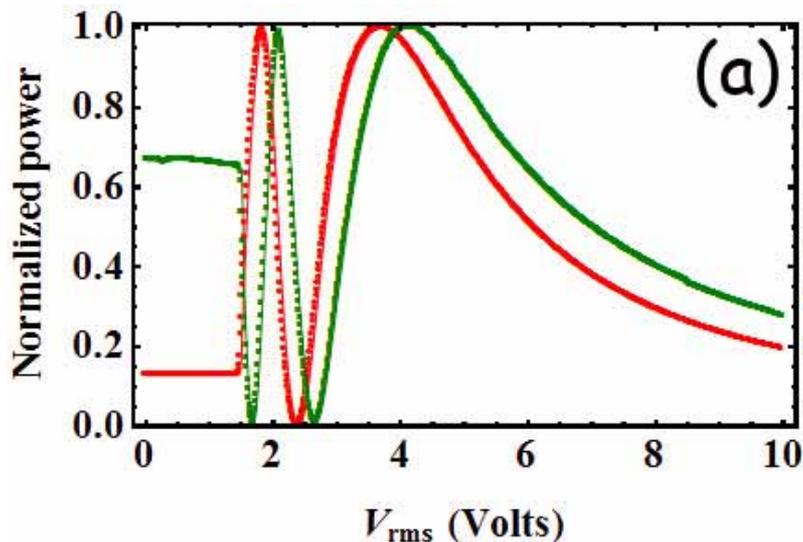
# “Tuning” a liquid crystal q-plate

A more convenient approach:  
electric tuning

The working principle is the electric-field induced reorientation of the liquid crystal molecular orientation

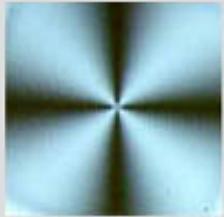


Electric q-plate: conversion efficiency and time response:

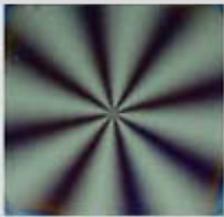


# q-plates: not only us

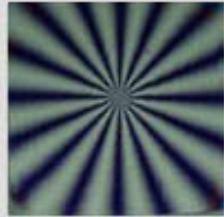
Liquid-crystal-polymer q-plates (diffractive waveplates) by Beam Co. (FL, USA):  
[Nelson V. Tabiryan et al.]



q=2



q=4



q = 8



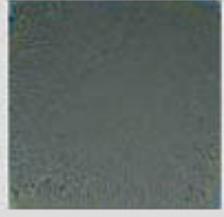
q = 16



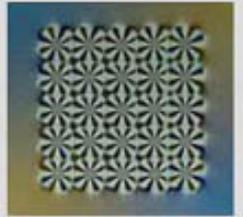
q = 32



q = 64



q = 128



A drawback of this polymer-only technology: not easily tunable

# Concept generalization:

## Pancharatnam-Berry phase optical elements (PBOE) for arbitrary wavefront shaping

[R. Bhandari, *Phys. Rep.* **281**, 1–64 (1997)]

[Z. Bomzon, G. Biener, V. Kleiner, and E. Hasman, *Opt. Lett.* **27**, 1141 (2002)]

[L. Marrucci, C. Manzo, D. Paparo, *Appl. Phys. Lett.* **88**, 221102 (2006)]

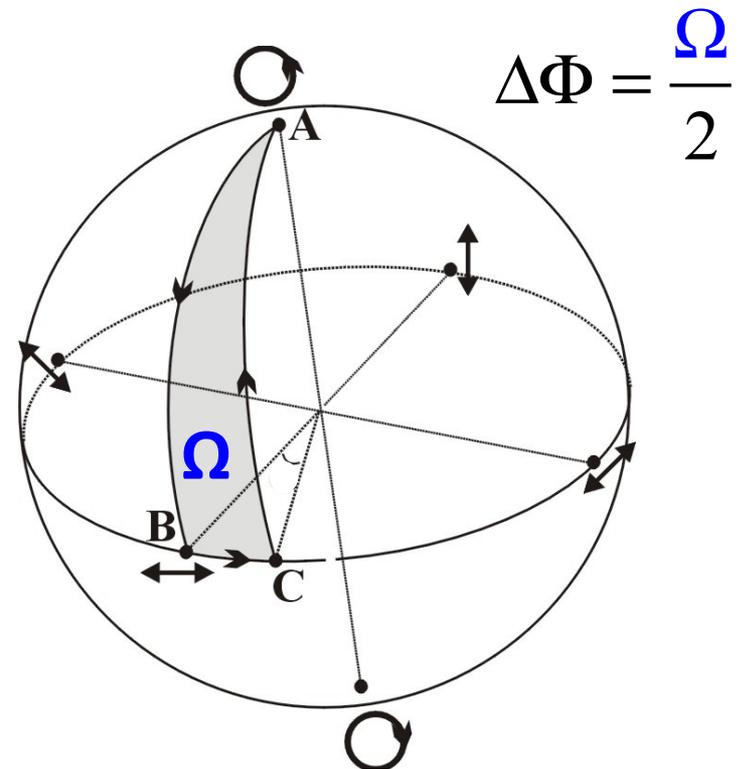
# Pancharatnam-Berry geometrical phase

It is an optical phase shift  $\Delta\Phi$  that arises due to a sequence of polarization transformations, independent of the optical path length

$\Delta\Phi$  is fixed by the geometry of the “path” in the polarization space (Poincaré sphere)

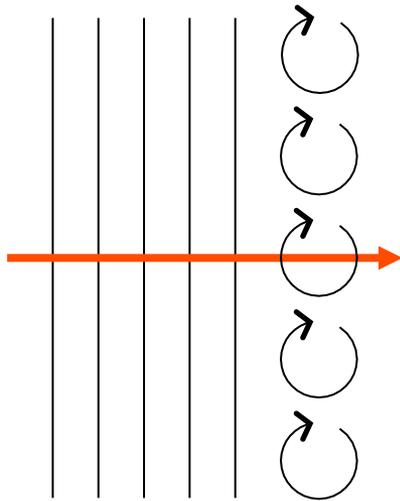
For a closed path,  $\Delta\Phi = \Omega / 2$ , where  $\Omega$  is the solid angle subtended by the enclosed area in the Poincaré sphere

For two different open paths sharing the same initial and final states,  $\Delta\Phi = \Omega / 2$  gives the difference in the acquired optical phase in the two transformations

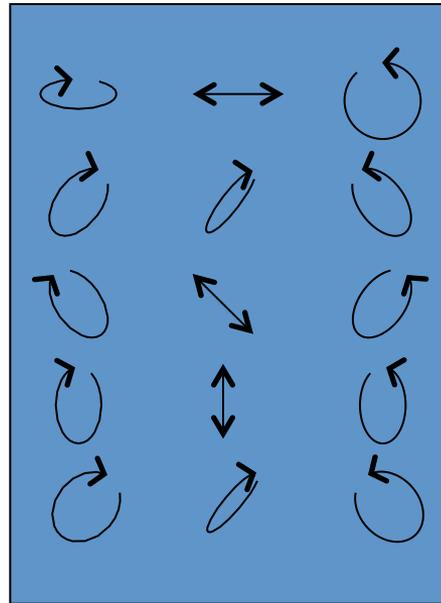


# Using Pancharatnam-Berry phase for wavefront shaping

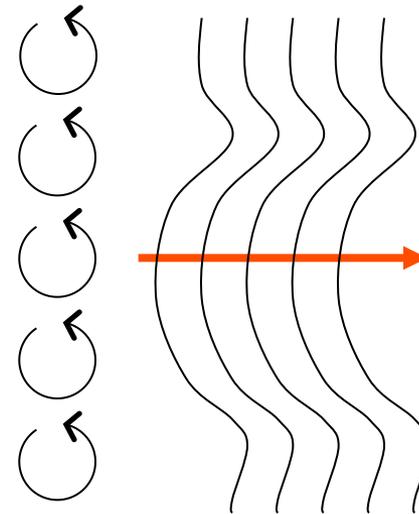
Input wavefront  
(with uniform initial polarization)



Optical system  
inducing non-uniform  
polarization  
transformations but a  
uniform final  
polarization



Reshaped output wavefront



**What kind of optical systems can be used?**

# Patterned half-wave plates (like “ $q$ -plates”)

Jones matrix:  $\mathbf{M}(x, y) = \begin{bmatrix} \cos 2\alpha(x, y) & \sin 2\alpha(x, y) \\ \sin 2\alpha(x, y) & -\cos 2\alpha(x, y) \end{bmatrix}$

Apply it to an **input left-circular** polarized plane wave:

$$\mathbf{M}(x, y) \times \begin{bmatrix} 1 \\ i \end{bmatrix} E_0(r, z) = \begin{bmatrix} \cos 2\alpha + i \sin 2\alpha \\ -i \cos 2\alpha + \sin 2\alpha \end{bmatrix} E_0(r, z) = \begin{bmatrix} 1 \\ -i \end{bmatrix} e^{i2\alpha(x, y)} E_0(r, z)$$

Pancharatnam-Berry geometrical phase

Wavefront acquires a **position-dependent phase retardation**

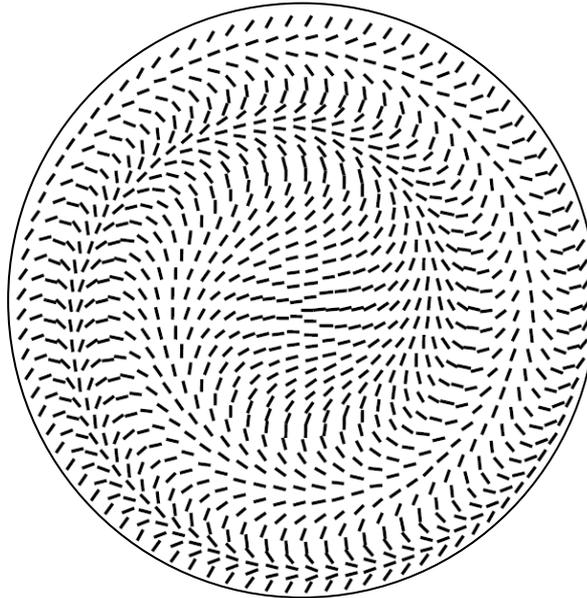
$$\Delta\Phi(x, y) = 2\alpha(x, y)$$

**With suitable patterning of the plate, we may generate wavefronts of any prescribed shape**

# Example: a PBOE lens

Optical axis pattern:

$$\left[ \alpha(r, \varphi) = cr^2 \right]$$



This pattern could be made with computer-controlled micro-rubbing or photo-alignment

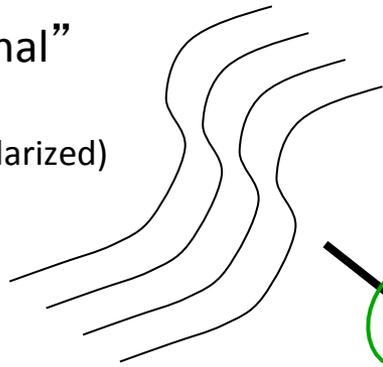
This lens will be focusing or defocusing depending on the input circular polarization handedness: **fast polarization multiplexing**



The lens **thickness will be uniform and very thin** (few microns). Similar to Fresnel lens, but without optical discontinuities

# PBOE and polarization holography

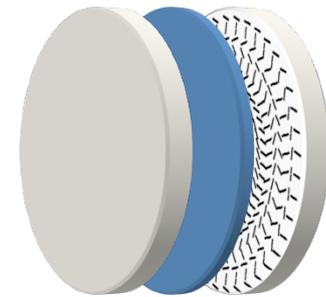
Input “signal”  
wavefront  
(circularly polarized)



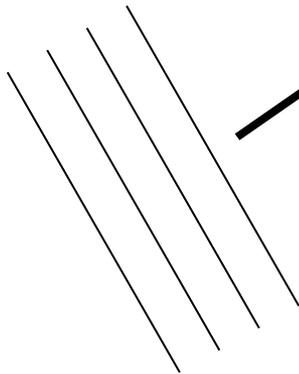
Polarization  
hologram



“Develop” it into a cell with  
half-wave retardation



Reference  
wavefront  
(with opposite  
circular  
polarization)



**PBOE which reconstructs the signal  
wavefront or its conjugate (with  
100% efficiency, single order  
output)**

# Acknowledgments

Coworkers of first works  
on SAM-OAM conversion:



Current coworkers:



Current sponsor:

