Electromagnetic vorticity in astronomy
Part I

Workshop on Singular Optics and its Applications to Modern Physics

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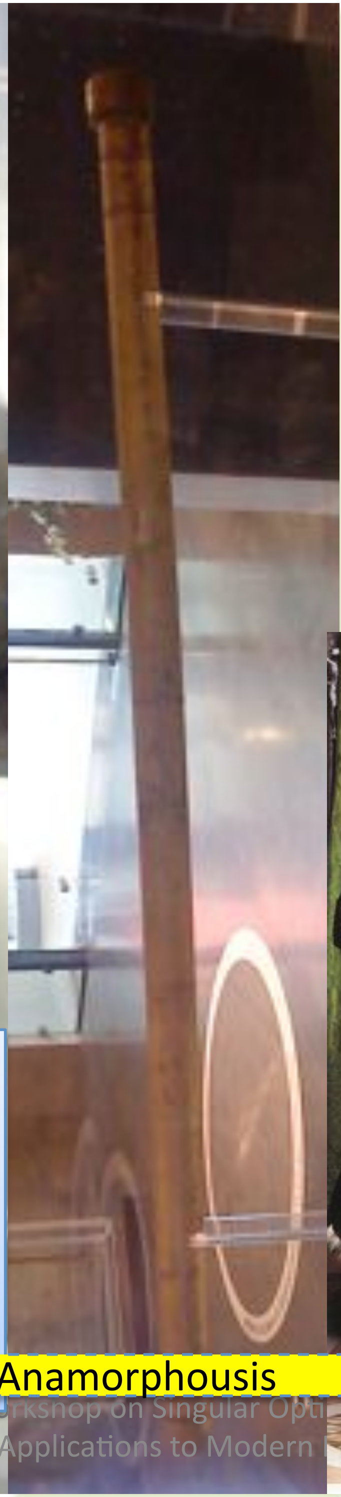
The Key point:
The importance of handling Quantum states & conserved quantities of light for astronomy

LIGHT is the **main** carrier of information for Astronomy.

LIGHT **is more complex than usually assumed by astronomers (intensity, frequency and polarization) and we can obtain much more information about astrophysical sources by analyzing also**

- Glauber's correlation functions of photons (e.g. for the laser iron lines in Eta Carinae).
- The use of quantum technology can help astronomical instrumentation, see e.g. the proposal of detecting gravitational waves with entangled states of light.
- Additional states of light must be used to get and transfer much more information or to control better the field behavior.
- In particular, among the properties of light still poorly exploited, **the Orbital Angular Momentum (OAM) and the associated Vorticity**, which instead are well known in other disciplines.



OAM a new tool for Astronomy?

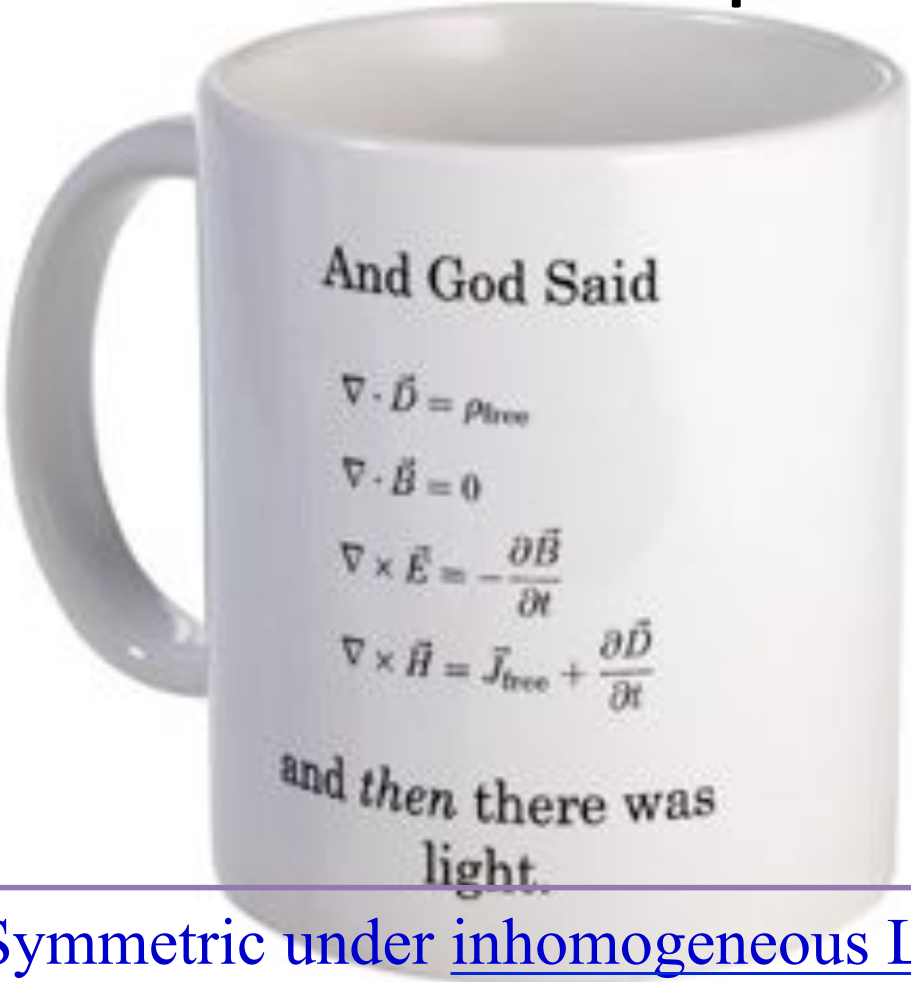
Look closer and see fainter objects:
- *intensity*

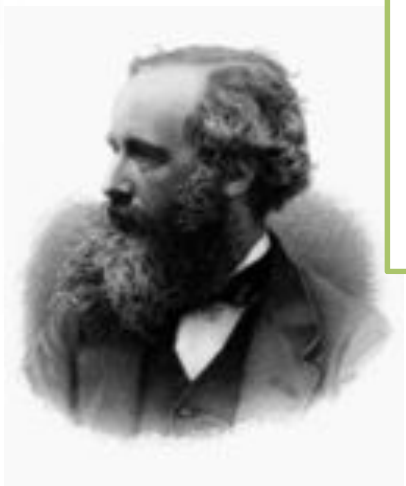
Observe and handle frequencies:
- *spectroscopy - polarization*

Observe "spatial" properties:
- *OAM spectroscopy*

Anamorphousis

Start: Maxwell equations for the EM field





It might sound trivial but this set of equations describe the propagation of light, from which we obtain most of the information from astrophysical sources

Symmetric under inhomogeneous Lorentz transformations. The concomitant Lie group is the 10-dimensional Poincaré group ***P(10)***. According to Noether's theorem there therefore exist 10 conserved EM quantities. In fact there exist **23** exact, plus an as yet unknown number of approximate conservation laws from other symmetries.

$$\mathbf{p}\times\Psi_1 = i\frac{\partial\Psi_2}{\partial t},\quad \mathbf{p}\times\Psi_2 = -i\frac{\partial\Psi_1}{\partial t},$$

$$\mathbf{p}\cdot\Psi_1 = \mathbf{p}\cdot\Psi_2 = 0;$$

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$$\Psi_1^* = \Psi_1,\quad \Psi_2^* = \Psi_2;$$

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$$\mathbf{p}\cdot\Psi_1 = \mathbf{p}\cdot\Psi_2 = 0,$$

Inequivalent Poincaré
EM invariant equations

$$i\frac{\partial}{\partial t}\Psi(t,\mathbf{x}) = \hat{\alpha}\cdot\mathbf{p}\Psi(t,\mathbf{x}),$$

$$\left[i\frac{\partial}{\partial t} - \hat{\alpha}\cdot\mathbf{p}\right]S_{\mu\nu}S^{\mu\nu}\Psi(t,\mathbf{x}) = 0$$

Maxwell equations with vectors

Ψ_1 and Ψ_2

 # Conserved quantities > 84

$$p(\Psi_1 - i\epsilon\Psi_2) + \epsilon'(i\,\mathbf{p}\times\Psi_1 + \epsilon\mathbf{p}\times\Psi_2) = 0;$$

$$\mathbf{p}\times\Psi_1 = i\frac{\partial\Psi_2}{\partial t},\quad \mathbf{p}\times\Psi_2 = -\frac{\partial\Psi_1}{\partial t},$$

$$\mathbf{p}\cdot\Psi_1 = \mathbf{p}\cdot\Psi_2 = 0,$$

$$\mathbf{p}\times\Psi_1 = i\epsilon p\,\Psi_1,\quad \mathbf{p}\times\Psi_2 = i\epsilon p\,\Psi_2;$$

$$\mathbf{p}\times\Psi_1 = i\frac{\partial\Psi_2}{\partial t},\quad \mathbf{p}\times\Psi_2 = -i\frac{\partial\Psi_1}{\partial t},$$

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$$\Psi_1^* = \Psi_1,\quad \Psi_2^* = \Psi_2;$$

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$$\mathbf{p}\cdot\Psi_1 = \mathbf{p}\cdot\Psi_2 = 0,\quad \Psi_1 = i\epsilon\Psi_2;$$

$$\mathbf{p}\times\Psi_1 = i\frac{\partial\Psi_2}{\partial t},\quad \mathbf{p}\times\Psi_2 = -i\frac{\partial\Psi_1}{\partial t},$$

$$\mathbf{p}\cdot\Psi_1 = \mathbf{p}\cdot\Psi_2 = 0,$$

$$\mathbf{p}\times\Psi_1 = \epsilon p\,\Psi_2,\quad \mathbf{p}\times\Psi_2 = -\epsilon p\,\Psi_1;$$

$$\mathbf{p}\times\Psi_1 = i\frac{\partial\Psi_2}{\partial t},\quad \mathbf{p}\times\Psi_2 = -i\frac{\partial\Psi_1}{\partial t},$$

$$\mathbf{p}\cdot\Psi_1 = \mathbf{p}\cdot\Psi_2 = 0,$$

$$\Psi_1 + i\epsilon\Psi_2 = 0,\quad \mathbf{p}\times\Psi_1 = -i\epsilon\epsilon' p\,\Psi_1,$$

where $p = (p_1^2 + p_2^2 + p_3^2)^{1/2}$, $\epsilon, \epsilon' = 1$.

Conserved quantities in a closed electromechanical system (matter + EM fields) [*Boyer*, 2005]

•Invariance in time translations => conservation of system energy (no *EMF,* no radiation: Poynting's theorem):

$$H = \sum_i m_i \gamma_i c^2 + \frac{\varepsilon_0}{2} \int \mathrm{d}^3x \left(|\mathbf{E}|^2 + c^2 |\mathbf{B}|^2 \right)$$

•Invariance in space translations => conservation of system linear momentum (EM Doppler shift!):

$$\mathbf{p} = \sum_i m_i \gamma_i \mathbf{v}_i + \varepsilon_0 \int \mathrm{d}^3x \left(\mathbf{E} \times \mathbf{B} \right)$$

•Invariance under proper Lorentz transformations => conservation of system centre of energy:

$$\mathbf{R} = \frac{1}{H} \sum_i (\mathbf{x}_i - \mathbf{x}_0) m_i \gamma_i c^2 + \frac{\varepsilon_0}{2H} \int \mathrm{d}^3x \left(\mathbf{x} - \mathbf{x}_0 \right) \left(\left| \mathbf{E}^2 \right| + c^2 \left| \mathbf{B}^2 \right| \right)$$

Time distribution

- Photon coherence statistics
- Time arrival of single photons from faint distant sources
- Glauber correlation functions characterise the emission process

VOLUME 10, NUMBER 3 PHYSICAL REVIEW LETTERS 1 FEBRUARY 1963

PHOTON CORRELATIONS*

Roy J. Glauber
Lyman Laboratory, Harvard University, Cambridge, Massachusetts

PHYSICAL REVIEW VOLUME 130, NUMBER 6 15 JUNE 1963

The Quantum Theory of Optical Coherence*

ROY J. GLAUBER
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts
(Received 11 February 1963)

The concept of coherence which has conventionally been used in optics is found to be inadequate to the needs of recently opened areas of experiment. To provide a fuller discussion of coherence, a succession of correlation functions for the complex field strengths is defined. The nth order function expresses the correlation of values of the fields at $2n$ different points of space and time. Certain values of these functions are measurable by means of n-fold delayed coincidence detection of photons. A fully coherent field is defined as one whose correlation functions satisfy an infinite succession of stated conditions. Various orders of incomplete coherence are distinguished, according to the number of coherence conditions actually satisfied. It is noted that the fields historically described as coherent in optics have only first-order coherence. On the other hand, the existence, in principle, of fields coherent to all orders is shown both in quantum theory and classical theory. The methods used in these discussions apply to fields of arbitrary time dependence. It is shown, as a result, that coherence does not require monochromaticity. Coherent fields can be generated with arbitrary spectra.

REVIEWS OF MODERN PHYSICS, VOLUME 78, OCTOBER–DECEMBER 2006

Nobel Lecture: One hundred years of light quanta*

Roy J. Glauber
Harvard University, Cambridge, Massachusetts 02138, USA
(Published 17 November 2006)

Quantum Astronomy means Correlation functions!

One-Photon Experiments
*E*E correspond to the intensity I*

ONE–PHOTON EXPERIMENTS

1:st order correlation function:

$G^{(1)}[r_1,t_1; r_2,t_2] = \langle E^*(r_1,t_1)\, E(r_2,t_2) \rangle$

Special case: $r_1 = r_2,\ t_1 = t_2$

$\langle E^*(0,0)\, E(0,0) \rangle$ – BOLOMETER

Special case: $r_1 \neq r_2,\ t_1 = t_2$

$\langle E^*(0,0)\, E(r,0) \rangle$ – [PHASE] INTERFEROMETER

Special case: $r_1 = r_2,\ t_1 \neq t_2$

$\langle E^*(0,0)\, E(0,t) \rangle$ – SPECTROMETER

All classical optical instruments measure properties of light that can be deduced from the first-order correlation function of light, $G^{(1)}$, for two coordinates in space r and time t (Glauber, 1970).

Two- and Multi-Photon Properties of Light

TWO-PHOTON EXPERIMENTS

2:nd order correlation function:

$$G^{(2)}[r_1,t_1;\ r_2,t_2] = \langle I(r_1,t_1)\ I(r_2,t_2) \rangle$$

$$g^{(2)} = \langle I(r_1,t_1)\ I(r_2,t_2)\rangle / \langle I(r_1,t_1)\rangle\ \langle I(r_2,t_2)\rangle$$

Special case: $r_1 = r_2,\ t_1 = t_2$

$\langle I(0,0)\ I(0,0) \rangle$ – "QUANTUM SPECTROMETER"

Special case: $r_1 \neq r_2,\ t_1 = t_2$

$\langle I(0,0)\ I(r,0) \rangle$ – INTENSITY INTERFEROMETER

Special case: $r_1 = r_2,\ t_1 \neq t_2$

$\langle I(0,0)\ I(0,t) \rangle$ – CORRELATION SPECTROMETER

Since a detection of a photon (measurement of I) enters twice, $G^{(2)}$ describes two-photon properties of light. $G^{(2)}$ is often normalized to the second-order coherence of light, $g^{(2)}$.

The description of collective multi-photon phenomena in a photon gas, in general requires a quantum-mechanical treatment since photons have integer spin (S = 1), and therefore constitute a boson fluid with properties different from a fluid of classical distinguishable particles.

Isotropy in space => conservation of system angular momentum

$$\mathbf{J} = \sum_i (\mathbf{x}_i - \mathbf{x}_0) \times m_i \gamma_i \mathbf{v}_i + \varepsilon_0 \int \mathrm{d}^3x \, (\mathbf{x} - \mathbf{x}_0) \times (\mathbf{E} \times \mathbf{B})$$

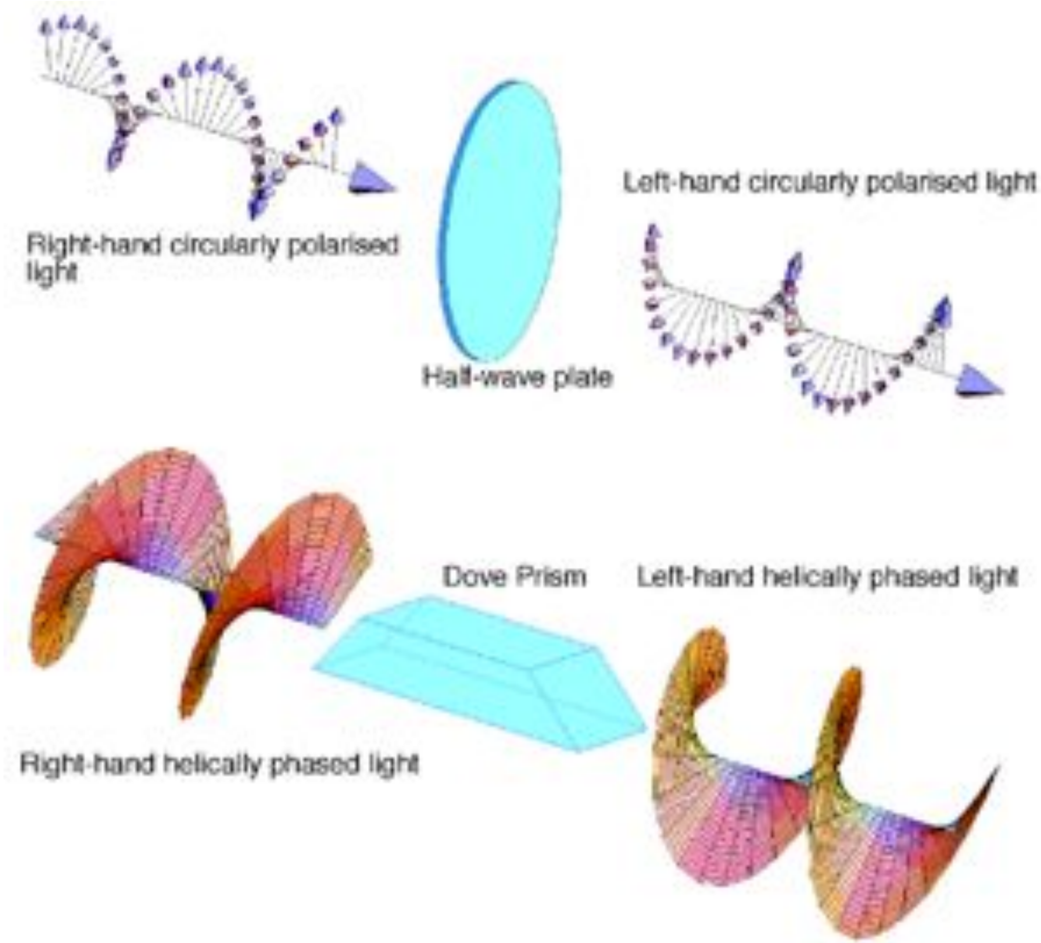
For radiation beams, the EM field angular momentum $\mathbf{J}^{\mathrm{EM}}$ can be separated into two parts [*van Enk & Nienhuis*, 1992]:

$$\mathbf{J}^{\mathrm{EM}} = \frac{\varepsilon_0}{2\mathrm{i}\omega} \int \mathrm{d}^3x \, E_i [(\mathbf{x} - \mathbf{x}_0)] \times \nabla) E_i + \frac{\varepsilon_0}{2\mathrm{i}\omega} \int \mathrm{d}^3x \, \mathbf{E}^* \times \mathbf{E}$$

•The first part is the EM *orbital angular momentum (OAM)* $\mathbf{L}^{\mathrm{EM}}$, and the second part is the EM *spin angular momentum (SAM)* $\mathbf{S}^{\mathrm{EM}}$*, wave polarisation*. $\mathbf{J}^{\mathrm{EM}} = \mathbf{L}^{\mathrm{EM}} + \mathbf{S}^{\mathrm{EM}}$ are radiated all the way out to the far zone.

Difference between polarisation (SAM) and orbital angular momentum (OAM)

Contemporary Physics, 2000, volume 41, number 5, pages 275–285



Light with a twist in its tail

MILES PADGETT and L. ALLEN

Second step - Faint sources:
Single photons can carry both spin angular momentum **S** *and* orbital angular momentum **L**

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Measuring the Orbital Angular Momentum of a Single Photon

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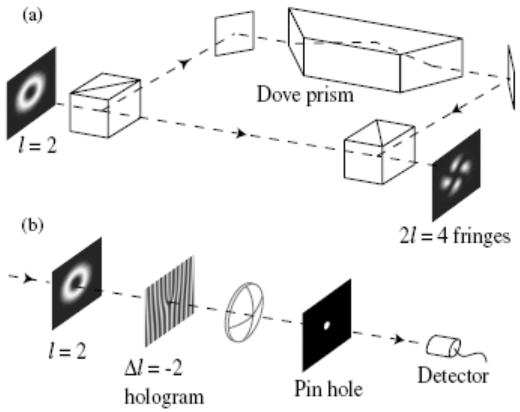

FIG. 1. Previous work on measuring the OAM of light. (a) A Mach-Zehnder interferometer with a Dove prism inserted into one arm interferes the incoming light beam with its own mirror image. In the case of light with l intertwined helical phase fronts, the interference pattern has $2l$ radial fringes. This setup is capable of distinguishing between an arbitrary number of states, but forming the required fringe pattern needs many photons. (b) A hologram can be used to "flatten" the phase fronts of light modes with specific values of l, which makes it possible to focus these modes (but no others) through a pinhole, behind which they can be detected. While this latter setup works with individual photons, it can only test for one particular OAM state.

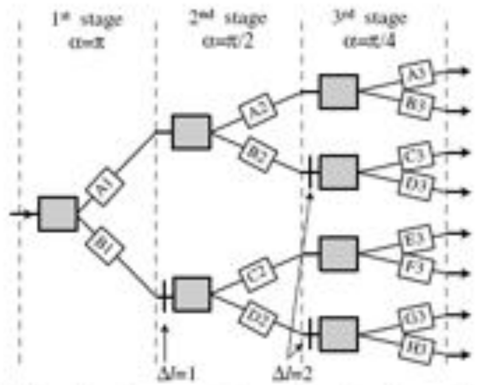

FIG. 4. First three stages of a general sorting scheme. The gray boxes each represent an interferometer of the form shown in Fig. 3 with different angles between the Dove prisms. The first stage introduces a phase shift of $\alpha = \pi$ and so sorts multiples of 2: even ls into Port A1 and odd ls into Port B1. The odd-l photons then pass though an $\Delta l = 1$ hologram so that they become even-l photons. The second stage introduces a phase shift of $\alpha = \pi/2$, so it sorts even-l photons into even and odd multiples of 2. The $\Delta l = 2$ hologram is required before the photons are sorted further in the third stage.

Tomorrow part II

- New degrees of freedom of the EM field for astronomy
- Time correlations – Polarization - OAM
- Space-time characterization of the beam

NEXT:

Experiencing OAM @ the telescope

Applications to astrophysics