Dipartimento di Scienze Fisiche
Università degli Studi di Napoli “Federico II”

Manipulation of the OAM of a paraxial optical beam with linear optics devices and OAM decoherence

Enrico Santamato

Outline

- Part 1
  - – Light OAM eigenstates properties
  - – Light OAM eigenstates generation
  - – Light OAM eigenstates manipulation

- Part 2
  - – The q-plate as archetype of Zurek's model for quantum decoherence

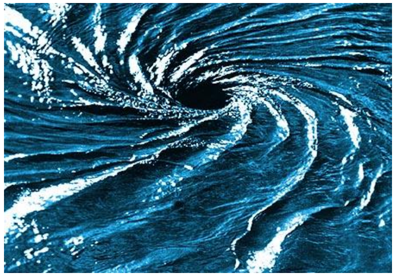
Part 1

Optical vortex

What are optical
OAM eigenstates?

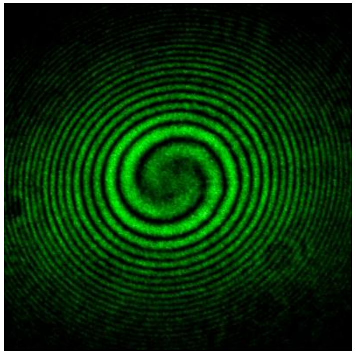
Optical vortices

hydrodynamics



$$\oint \mathbf{v} \cdot d\mathbf{r} = \text{const.}$$

optics



$$\oint \nabla \psi \cdot d\mathbf{r} = \text{const.}$$

Optical phase

Optical Angular Momentum (OAM)

- We are interested in paraxial Gaussian beams carrying definite orbital angular momentum along the propagation direction.

- These optical beams are vortex beam with

$$\oint \nabla \psi \cdot d\mathbf{r} = 2m\pi$$

where *m* is an integer.

Light OAM eigenstates

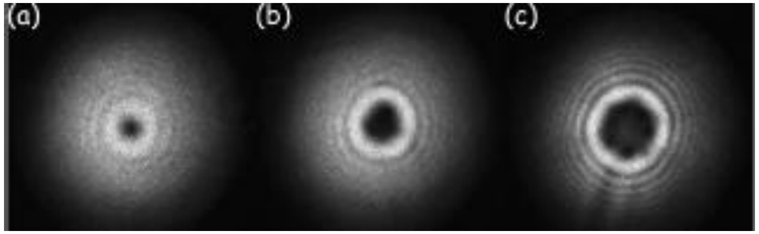
- As for any particle, the OAM eigenstates are eigenstates of the o[...]rat[...]

- The opt[...] $/w^2$

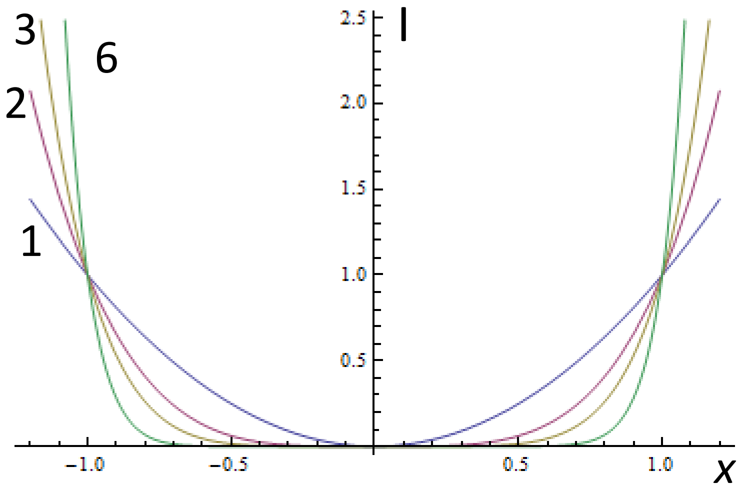
The OAM factor does not depend on wavelength

Gaussian factor

Intensity profile of OAM eigenstates



$m = 1$ $m = 3$ $m = 6$



Method:
CCD camera

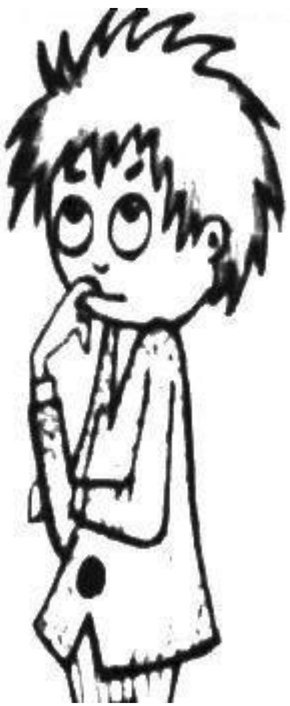
- There is a hole at the beam center (doughnut beam)
- At the hole the intensity vanishes as r^{2m}
- The hole is a geometric subwavelength structure

Phase fronts of OAM eigenstates

$m = 1$ $m = 2$ $m = 3$ $m = 6$

The phase fronts of eigenstates with OAM value *m* are *m* interleaved spirals with common pitch λ

Making optical beam in OAM eigenstates



How to make beams
with given OAM?

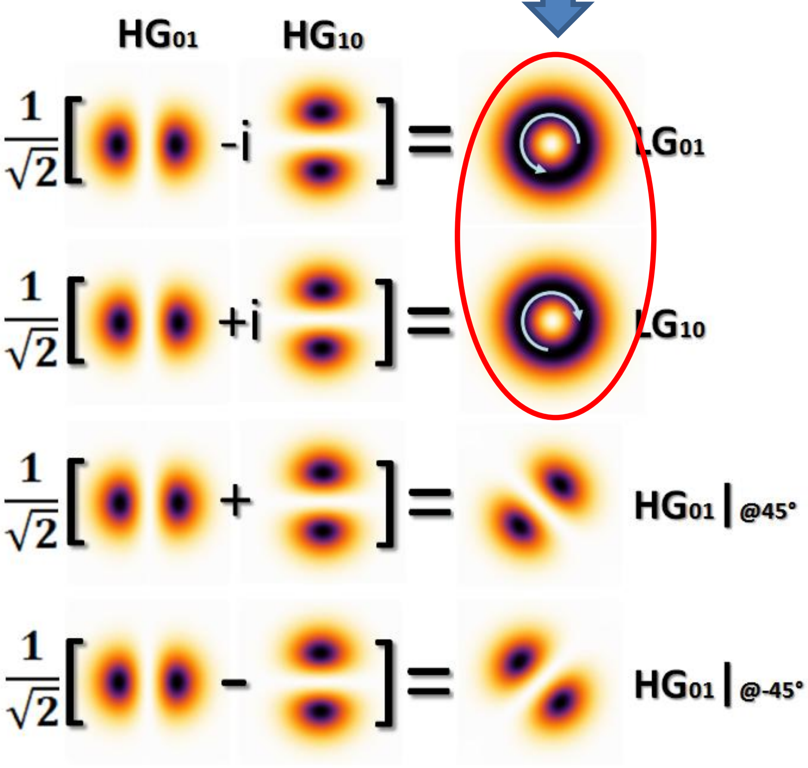
Method 1

Photon OAM eigenstates and HG modes

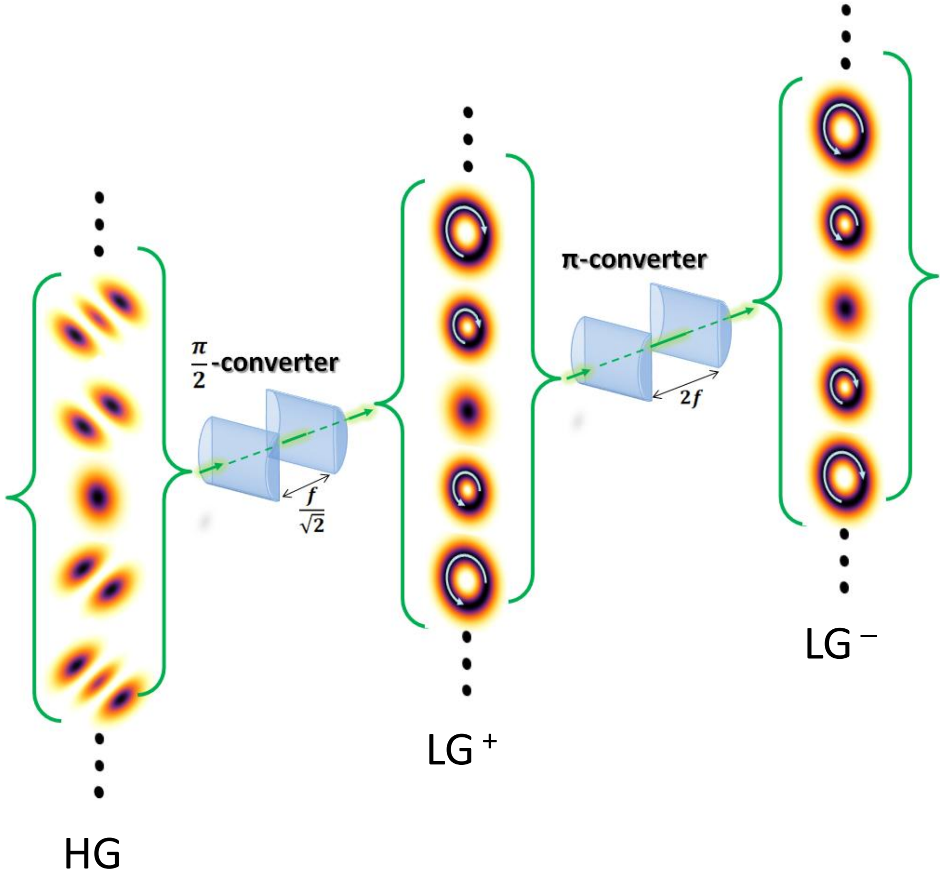
$$E \propto e^{im\varphi} \quad (m = 0, \pm 1, \pm 2, \ldots)$$

Complete sets of modes of the paraxial wave equation

1. HG modes
2. LG modes



Mode converters

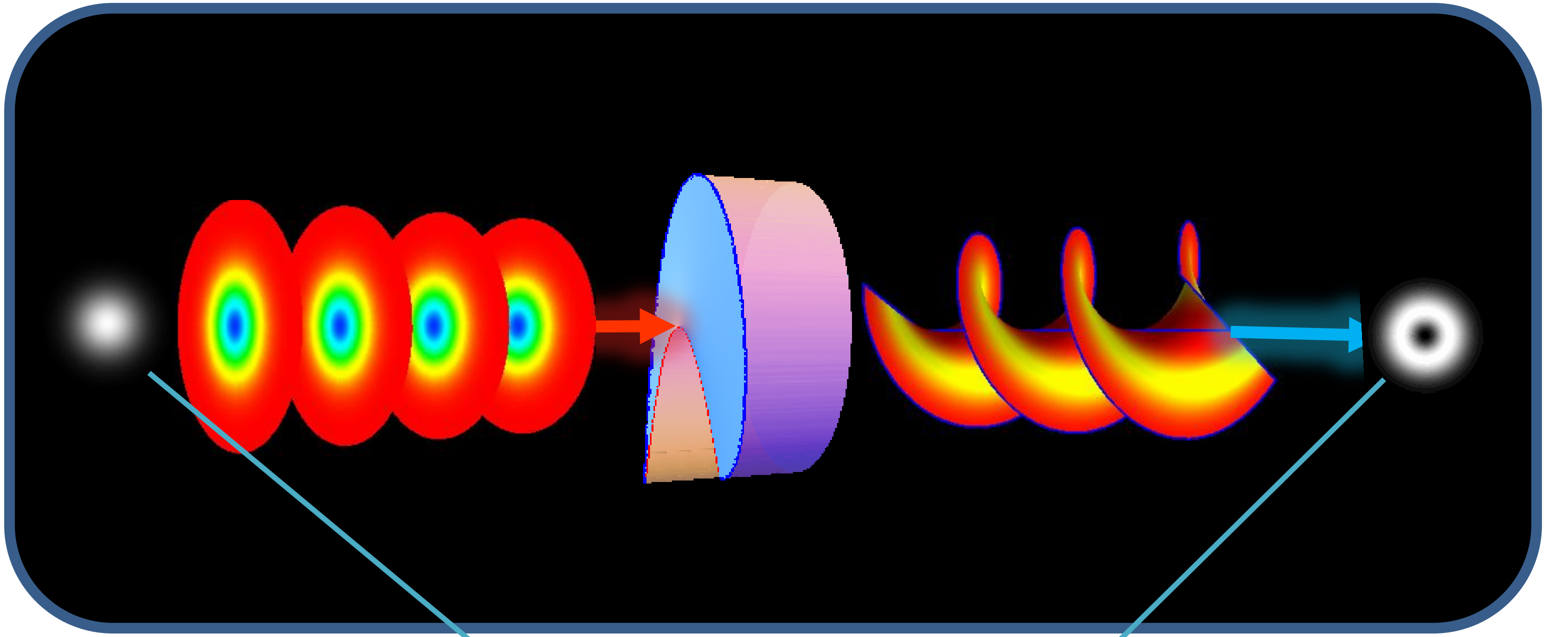


To obtain LG modes higher-order HG input modes are required

Impossible to convert TEM_{00} mode into LG modes

Method 2

The spiral plate

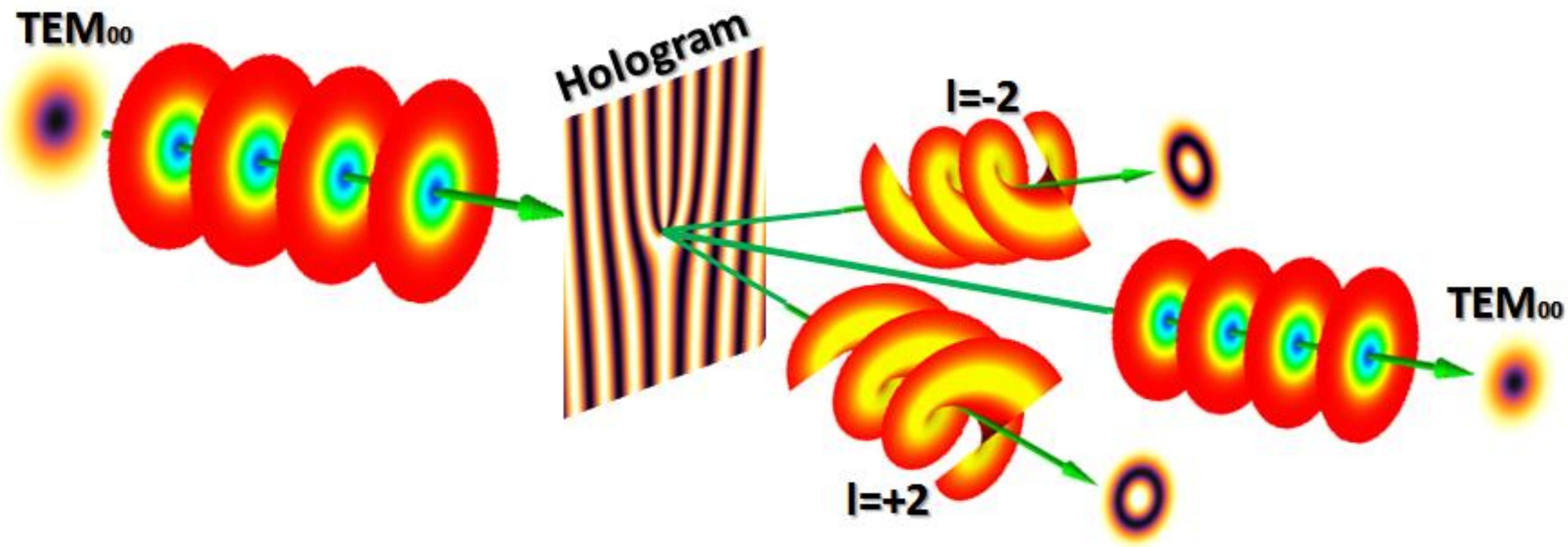


TEM$_{00}$ laser beam

OAM laser beam

Method 3

The fork hologram

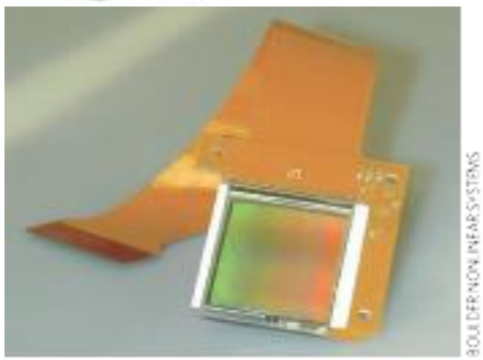


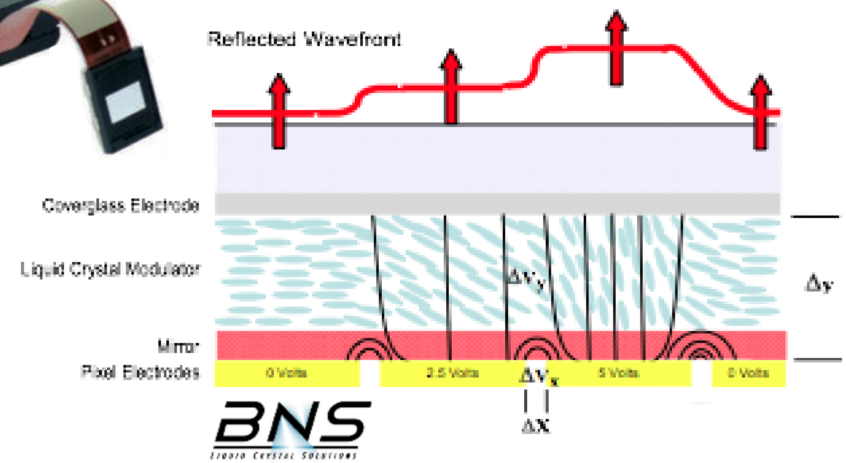
The fork hologram has the same profile as the interference of OAM beam with plane wave at small angle

The fork hologram

- When blazed, can transfer more than 80% incident power into the OAM carrying diffracted mode
- Can be calculated by computer and sent to Spatial Light Modulator (SLM)
- With SLM the hologram switching rate is of the order of 50 Hz
- The beam polarization is not affected

Space Light Modulator

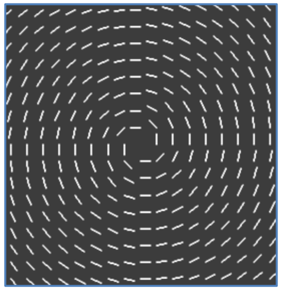




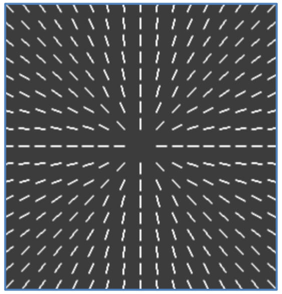
Method 4

The q-plate

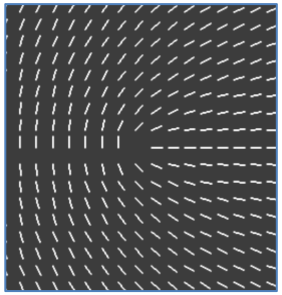
The q-Plate is a liquid crystal-based optical device which imprints a singularity into the phase of the optical field.



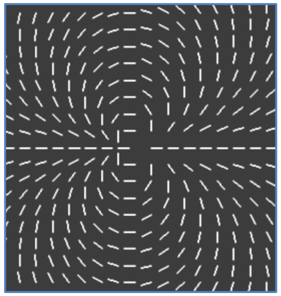
$\alpha = \varphi + \pi / 2$



$\alpha = \varphi$



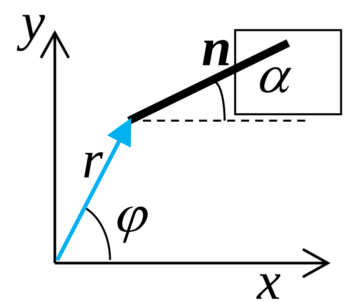
$\alpha = \varphi / 2$



$\alpha = 2\varphi$

L. Marrucci et al., PRL **96**, 163905 (2006)

$$\alpha = q\varphi + \alpha_0$$



OAM value created by q-plate

$$m = 2q$$

To have odd *m* we need half-integer *q*

The q-plate

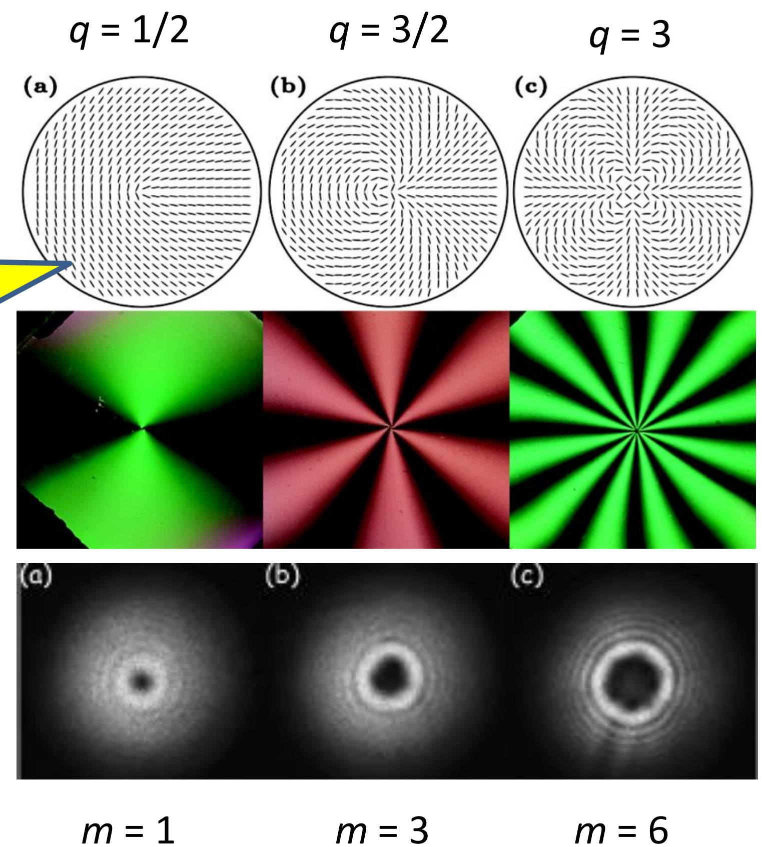
Our q-plates

ment

cope

rtex beam

Tanks to HKUST (prof.Chigrinov)!

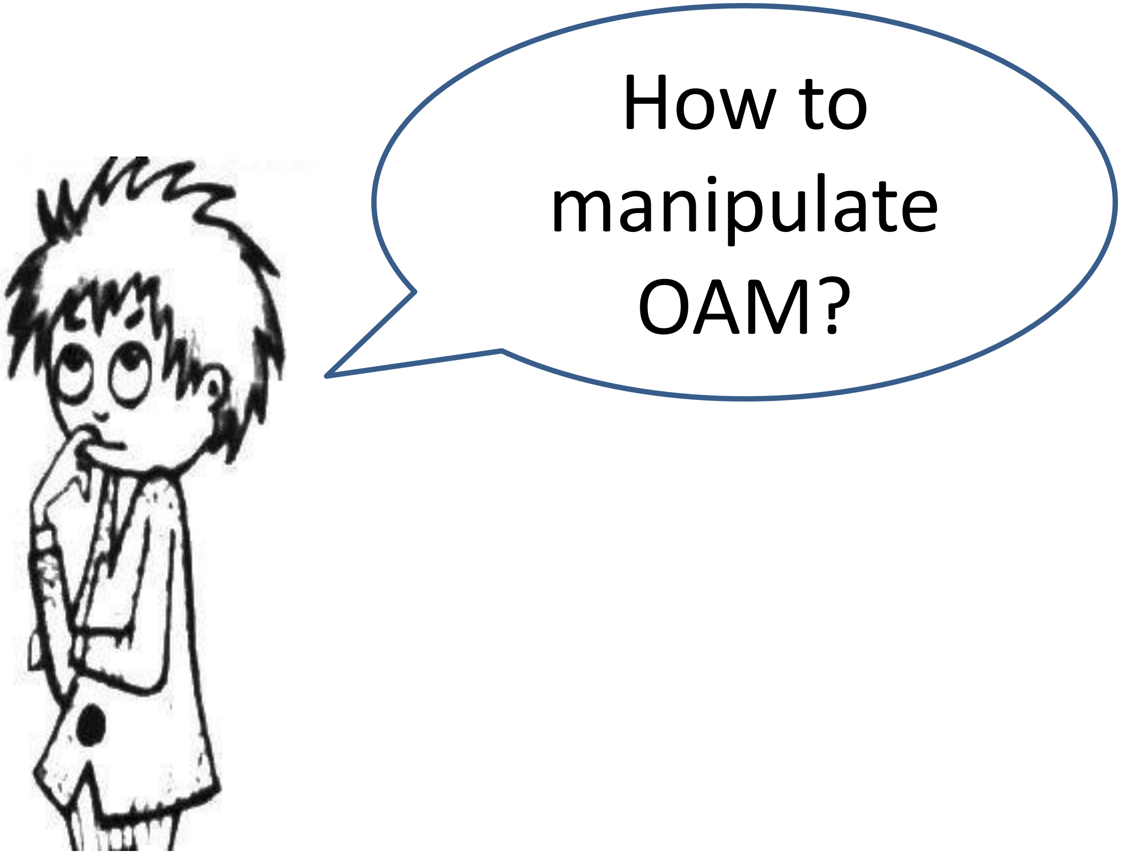


S. Slussarenko et al., *Opt. Express* **19**, 4085 (2011)

q-plates

- Can be tuned by electric field for maximum OAM conversion efficiency [B. Piccirillo et al., *APL.* **97**, 241104 (2010)]
- Conversion efficiency is larger than 95% (when tuned)
- Purity of the OAM generated mode is larger than 92% and not affected by input wavelength
- The beam is not deflected
- Can be switched on and off by electric field
- Work with single photons
- Produce entanglement between the photon spin and OAM
- May produce coherent superposition of OAM states

Manipulating OAM

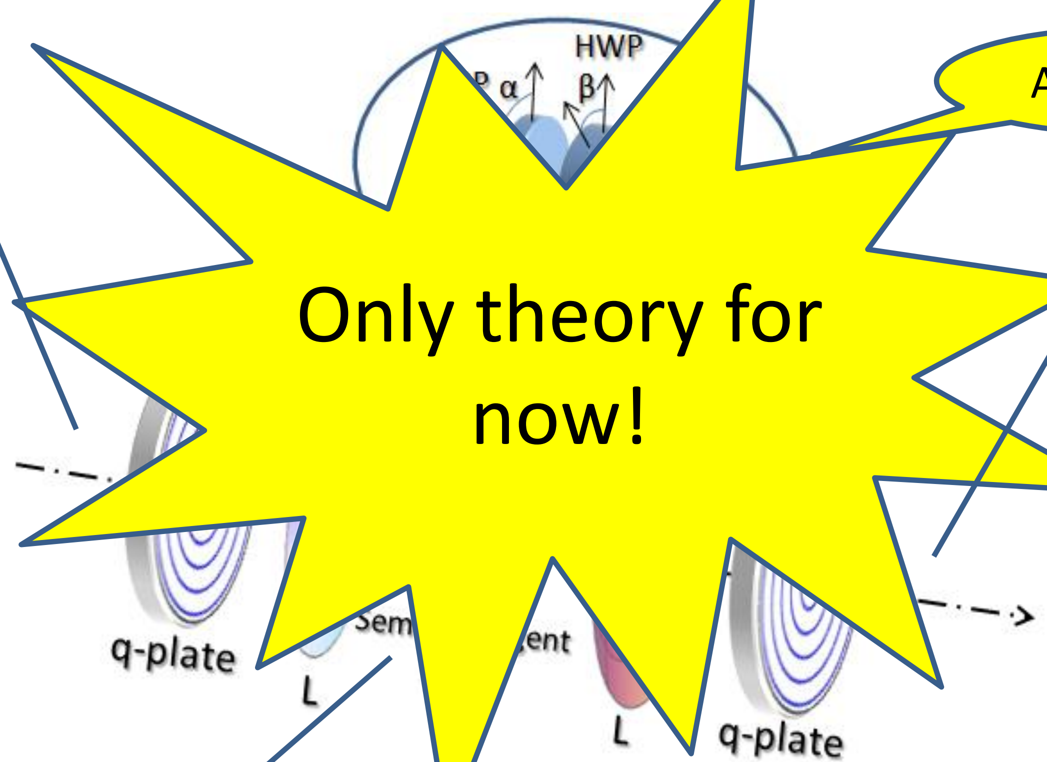


The Universal Unitary Gate

S. Slussarenko et al., *PRA* **80**, 022326 (2009

$|2q\rangle, |-2q\rangle$

Acts on $|0\rangle$



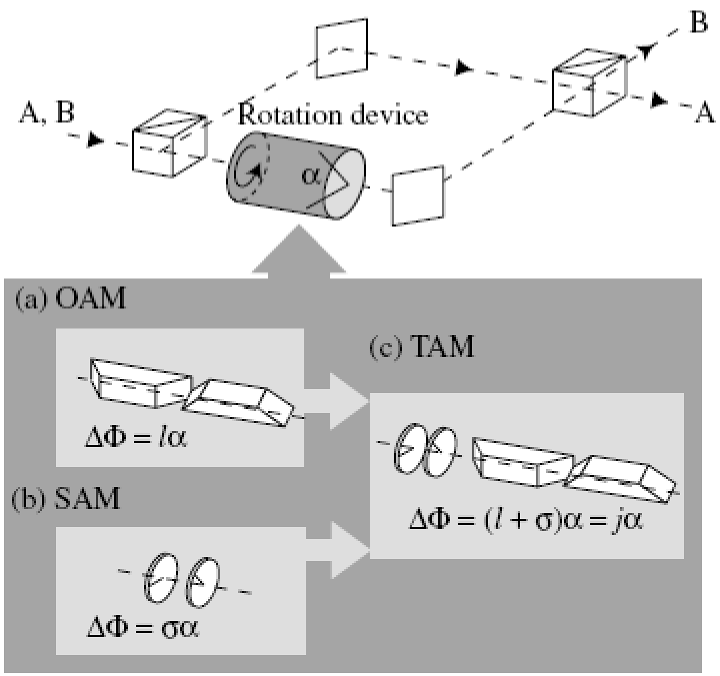
Only theory for now!

$|2q\rangle, |-2q\rangle$

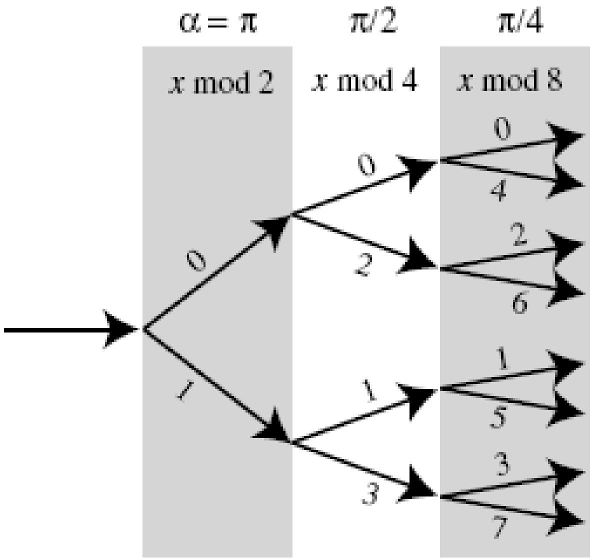
$|0\rangle, |-4q\rangle$

The Leach interferometer

Leach et al., *PRL* **92**, 013601 (2004)

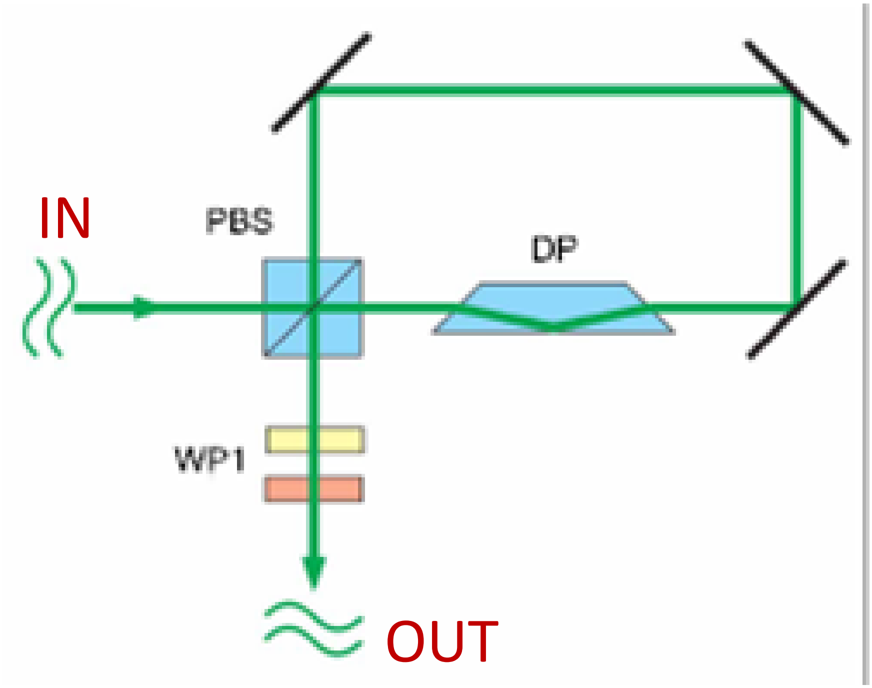


OAM sorting



The polarizing Sagnac Interferometer

Slussarenko et al., *Opt. Expr.* **92**, 27205 (2010)



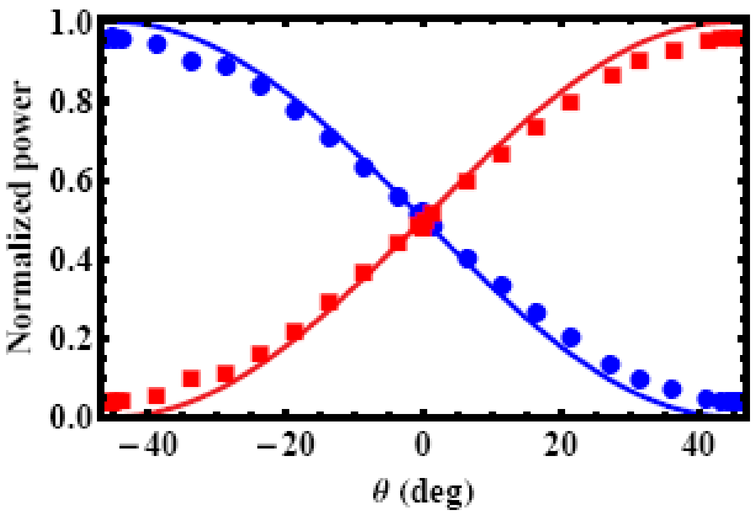
Very stable and easy to align

Uses of polarizing Sagnac interferometer

- Opposite OAM sorter: any two values of OAM (with common polarization) can be sent into opposite polarizations

$$\alpha = \frac{(2k+1)\pi}{4(m_2 - m_1)}$$

$$|A\rangle\left(\cos 2\theta|+2\rangle + \sin 2\theta|-2\rangle\right) \xrightarrow{\text{PSIDP}} \cos 2\theta|+2\rangle|H\rangle + \sin 2\theta|-2\rangle|V\rangle$$



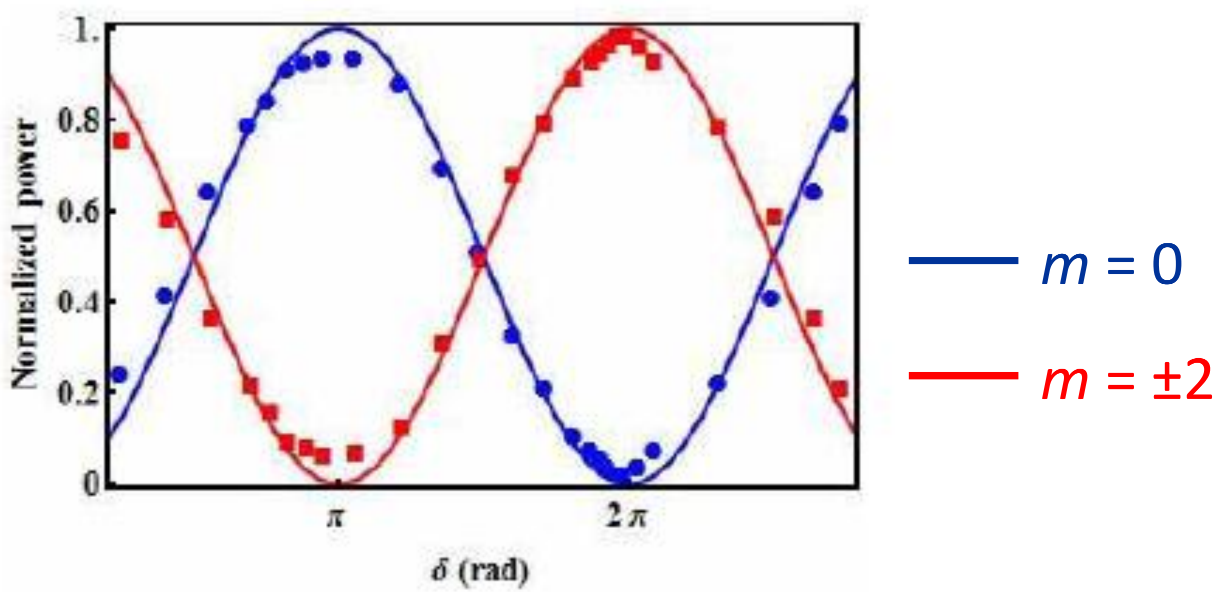
$m = +2$

$m = -2$

Uses of polarizing Sagnac interferometer

- OAM *m*=0 cleaner $\alpha = \frac{\pi}{8}$

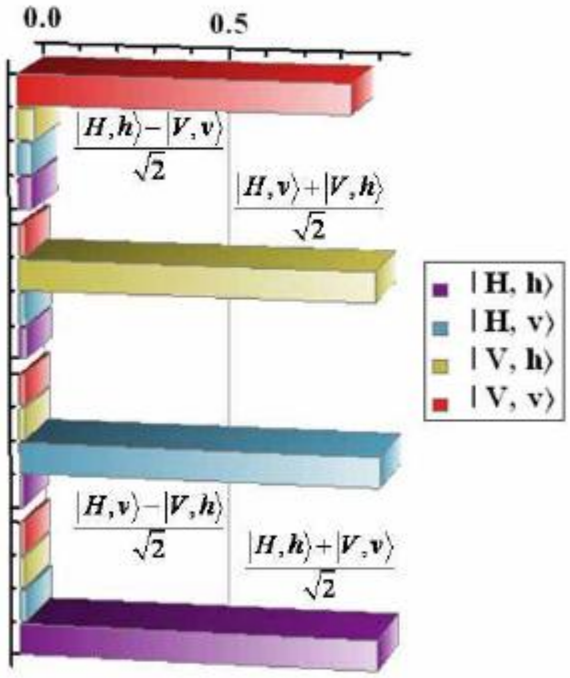
$$|A\rangle\left[\cos\frac{\delta}{2}|0\rangle + \sin\frac{\delta}{2}(|2\rangle + |-2\rangle)\right] \xrightarrow{\text{PSIDP}} \cos\frac{\delta}{2}|0\rangle|H\rangle + \sin\frac{\delta}{2}(|2\rangle + |-2\rangle)|V\rangle$$



Uses of polarizing Sagnac interferometer

- Spin-orbit Bell measure: the PSIDP behaves as a SAM-controlled c-NOT spinorbit gate

$$\alpha = \frac{\pi}{8m}$$



Base states

Conclusions

- We revised some practical devices to generate light OAM eigenstates
- We revised some practical devices to manipulate the light OAM eigenstates
- We introduced the Sagnac polarizing interferometer with Dove prism as a multifunction device to manipulate both the light SAM and OAM

Part 2

Light OAM for telecommnications

Can we use OAM for telecom?

OAM for telecommunications

- With OAM we may build up qu-*d*-its so to encode more information per single photon

$$|\psi\rangle = \sum_m c_m |m\rangle$$

- The qu-*d*-it stream can be used for high capacity long distance transmission

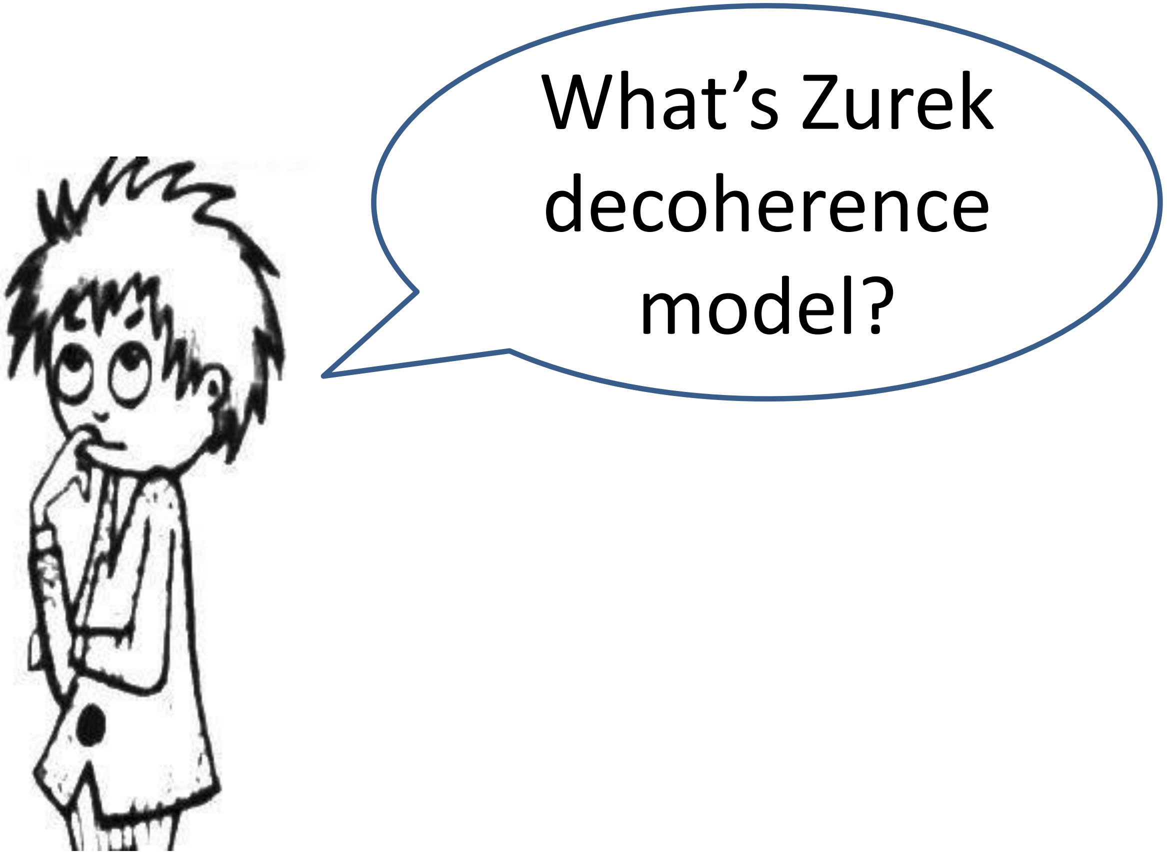
OAM qu-*d*-it decoherece

- Unlike SAM qubits, the OAM qu-*d*-its are affected by spontaneous decoherence
- This may be a serious problem for
  - long distance telecommunication
  - saving qu-*d*-it of correlated photon pair in optical cavities or light circulators for a long time
- The spontaneous decoherence is central to quantum mechanics and it was first studied by Zurek in the framework of measure theory

  W. H. Zurek, Phys. Rev. D, **26**, 1862 (1982);

  W. H. Zurek, Rev. Mod. Phys., **75**, 715 (2003)

Zurek's approach to decoherence



Zurek's model for quantum decoherence and measure

[W. H. Zurek, Phys. Rev. D, **26**, 1862 (1982); W. H. Zurek, Rev. Mod. Phys., **75**, 715 (2003)]

- Zurek's model is made of
  a) A quantum system prepared in the initial state

$$|\psi_S(0)\rangle = \sum_i c_m |m\rangle; \quad \langle m|n\rangle = \delta_{mn}$$

  b) A measure apparatus prepared in the initial state $|A(0)\rangle$
  c) An environment in some initial state $|\varepsilon(0)\rangle$
  d) The quantum "universe" (system + apparatus + environment) is initially in the not entangled state

$$|\psi_U(0)\rangle = |\psi_S(0)\rangle |A(0)\rangle |\varepsilon(0)\rangle = \sum_m c_m |m\rangle |A(0)\rangle |\varepsilon(0)\rangle$$

Zurek’s model for quantum decoherence and measure

- The apparatus is able to measure the occurrence of the state $|m\rangle$ of the system.
- Just after the measurement the apparatus changes in one of its pointer states $|A_m\rangle$ depending on $|m\rangle$.
- Thus the system must disturb the apparatus.
- The converse is not true, in general: two extreme cases can be envisaged
  a) After the measurement the system is destroyed by the apparatus (example: a photon is absorbed)
  b) The apparatus does not disturb the system (QND cases)
- In the QND case the apparatus remains entangled with the system
- But the environment is still not entangled

$$|\psi_U(0)\rangle = \sum_m c_m |m\rangle |A(0)\rangle |\varepsilon(0)\rangle \xrightarrow{premeasure} \sum_m c_m |m\rangle |A_m\rangle |\varepsilon(0)\rangle$$

Zurek’s model for quantum decoherence and measure

- The system+ apparatus state after the pre-measurement is an EPR state and hence it is ambiguous
- A change of basis in the system Hilbert space changes the quantity to be measured
- Moreover the system and apparatus entanglement is symmetric: the question arise, what is measuring what?
- For example, a c-NOT can entangle two initially not entangled qubits. Here the control qubit is the system and the target qubit is the apparatus
- But a change of the logical basis exchanges the control and target; so what is measuring what?

Zurek's model for quantum decoherence and measure

- The EPR ambiguity is resolved by the entanglement between the apparatus the environment under free evolution of the universe

$$|\psi_U(0)\rangle = \sum_m c_m |m\rangle |A(0)\rangle |\varepsilon(0)\rangle \xrightarrow{premeasure} \sum_m c_m |m\rangle |A_m\rangle |\varepsilon(0)\rangle$$

$$\xrightarrow{U} \sum_m c_m |m\rangle |A_m\rangle |\varepsilon_m\rangle$$

- This triply entangled state has no EPR ambiguity
- The crucial point is that, due to the large number of degrees of freedom of the ambient, one can suppose that the final environment states $|\varepsilon_m\rangle$ are orthogonal

$$\langle \varepsilon_m | \varepsilon_n \rangle = \delta_{mn}$$

- Then the correlations between the apparatus pointer states $|A_m\rangle$ and the system states $|m\rangle$ become classical.

Zurek's model for quantum decoherence and measure

- This happens only for a particular set of apparatus pointer states $|A_m\rangle$
- This set of pointer states is named by Zurek *einselected* (environment-selected)
- Only for this set of states we have the einselection condition $\langle \varepsilon_m | \varepsilon_n \rangle = \delta_{mn}$
- A set of pointer states $|A_m\rangle$ are einselected only if the apparatus operator Â commutes with the apparatus-environment Hamiltonian

Environment -induced decoherence and entropy

- Reduced density matrix of the system+apparatus at *z*

$$\rho_{SA}(z) = \cos^2\frac{\delta}{2}\left|L,m\right\rangle\left\langle L,m\right| + \sin^2\frac{\delta}{2}\left|R,m+2q\right\rangle\left\langle R,m+2q\right| +$$

$$+i\left(\sin\frac{\delta}{2}\cos\frac{\delta}{2}\left\langle r_{m+2q}(z)\middle|r_m(z)\right\rangle\left|L,m\right\rangle\left\langle R,m+2q\right| + h.c.\right)$$

- System + apparatus entropy at *z*

The initial S-A state is a pure state

$$S_{SA}(z) = -\left\langle \log_2 \rho_{SA}(t)\right\rangle = -\mathrm{Tr}[\rho_{SA}(z)\log_2 \rho_{SA}(z)] =$$

$$= -\sum_i \lambda_i(z)\log_2\lambda_i(z) > S_{SA}(0) = 0$$

A note about average values

- Let $\hat{M}$ the op… rator corr… he qu… tity *m* to be measured: …
- The average … “uni…

$$\langle \hat{M}^k \rangle = \sum \ldots \sum_{n} |c_m|^2 \, m^k \langle \varepsilon_m | \varepsilon_m \rangle$$

whe…

- In our … environment states is unit… $|\varepsilon_m$ … = 1, and we obtain the incoherent s…

$$\langle \hat{M}^k \rangle = \sum_{m,n} |c_m|^2 \, m^k = \text{const.}$$

Entanglement with environment produces decoherence in the system and entropy increase

Zurek's decoherence of photon OAM

How apply Zurek's approach to photon OAM?

The "environment" of the photon OAM eigenstates

- When the q-plate (or hologram) is used to create or measure OAM eigenstates we have no control on the photon radial state
- The radial state of a photon with given OAM $m = m_0$ is the superposition of many radial modes
- These radial modes are the "environment" of the OAM state $|\, m_0 \rangle$

OAM eigenstates generated by q-plate and fork hologram for TEM_{00} input

- Phase plates, fork holograms, and q-plates may generate OAM eigenstates but not LG modes
- They generate Hypergeometric Gaussian modes $|\mathrm{HyGG}\rangle_{pm}$ with $p = -|m|$

$$
\begin{aligned}
|HyGG\rangle_{pm} \;=\; & u_{pm}(\rho,\phi;\zeta) = C_{pm}\frac{\Gamma(1+|m|+\frac{p}{2})}{\Gamma(|m|+1)} \\
& \times i^{|m|+1}\zeta^{\frac{p}{2}}(\zeta+i)^{-(1+|m|+\frac{p}{2})} \\
& \times \rho^{|m|}e^{-\frac{i\rho^2}{(\zeta+i)}}e^{im\phi} \\
& \times {}_1F_1\left(-\frac{p}{2},|m|+1;\frac{\rho^2}{\zeta(\zeta+i)}\right)
\end{aligned}
$$

E. Karimi et al., Opt. Lett. **32** (2007) 3053

Action of the q-plate on a photon

$$\begin{cases} |L\rangle|m\rangle|r_0\rangle \xrightarrow{QP} \left(\cos\left(\frac{\delta}{2}\right)|L\rangle|m\rangle - i\sin\left(\frac{\delta}{2}\right)|R\rangle|m+2q\rangle\right)|r_0\rangle \\ |R\rangle|m\rangle|r_0\rangle \xrightarrow{QP} \left(\cos\left(\frac{\delta}{2}\right)|R\rangle|m\rangle - i\sin\left(\frac{\delta}{2}\right)|L\rangle|m-2q\rangle\right)|r_0\rangle \end{cases}$$

premeasurement

$|L\rangle, |R\rangle$ Left, Right circular SAM states

$|m\rangle$ OAM eigenstates $(m = 0, \pm1, \pm2, \ldots)$

$|r_0\rangle$ Radial state

Radial state not entangled

The photon radial state is not changed by a thin q-plate

SAM and OAM entangled

Action of the free propagation on a photon

$$\begin{cases} |L\rangle|m\rangle|r_0\rangle \xrightarrow{QP} \left(\cos\left(\frac{\delta}{2}\right)|L\rangle|m\rangle - i\sin\left(\frac{\delta}{2}\right)|R\rangle|m+2q\rangle\right)|r_0\rangle \xrightarrow{free\ prop} \\ \xrightarrow{free\ prop} \cos\left(\frac{\delta}{2}\right)|L\rangle|m\rangle|r_m\rangle - i\sin\left(\frac{\delta}{2}\right)|R\rangle|m+2q\rangle|r_{m+2q}\rangle \end{cases}$$

einselected measurement

$$|r_0\rangle \xrightarrow{free\ prop} |r_m\rangle = U_m|r_0\rangle$$

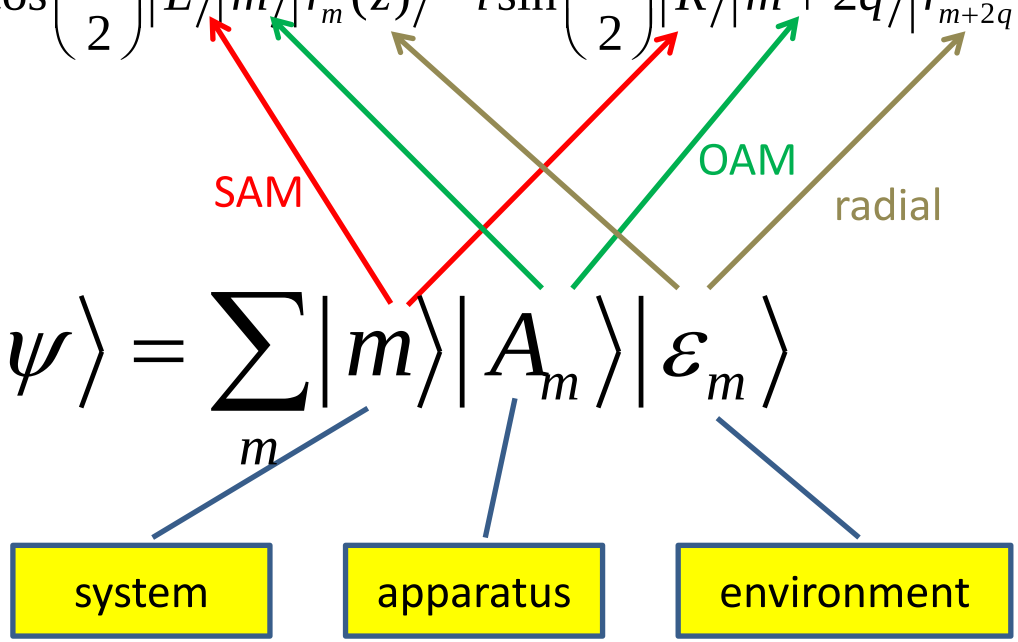
Radial states have OAM dependent unitary free evolution

SAM, OAM and radial states become all entangled during the free evolution

Identification with Zurek's model for quantum decoherence

$$\begin{cases} |L\rangle|m\rangle|r_0\rangle \xrightarrow{QP} \left(\cos\left(\frac{\delta}{2}\right)|L\rangle|m\rangle - i\sin\left(\frac{\delta}{2}\right)|R\rangle|m+2q\rangle\right)|r_0\rangle \xrightarrow{free\ prop} \\ \xrightarrow{free\ prop} \cos\left(\frac{\delta}{2}\right)|L\rangle|m\rangle|r_m(z)\rangle - i\sin\left(\frac{\delta}{2}\right)|R\rangle|m+2q\rangle|r_{m+2q}(z)\rangle \end{cases}$$

$$|\psi\rangle = \sum_m |m\rangle|A_m\rangle|\varepsilon_m\rangle$$



W. H. Zurek, Rev. Mod. Phys., **75**, 715 (2003)

Identification with Zurek's model for quantum decoherence

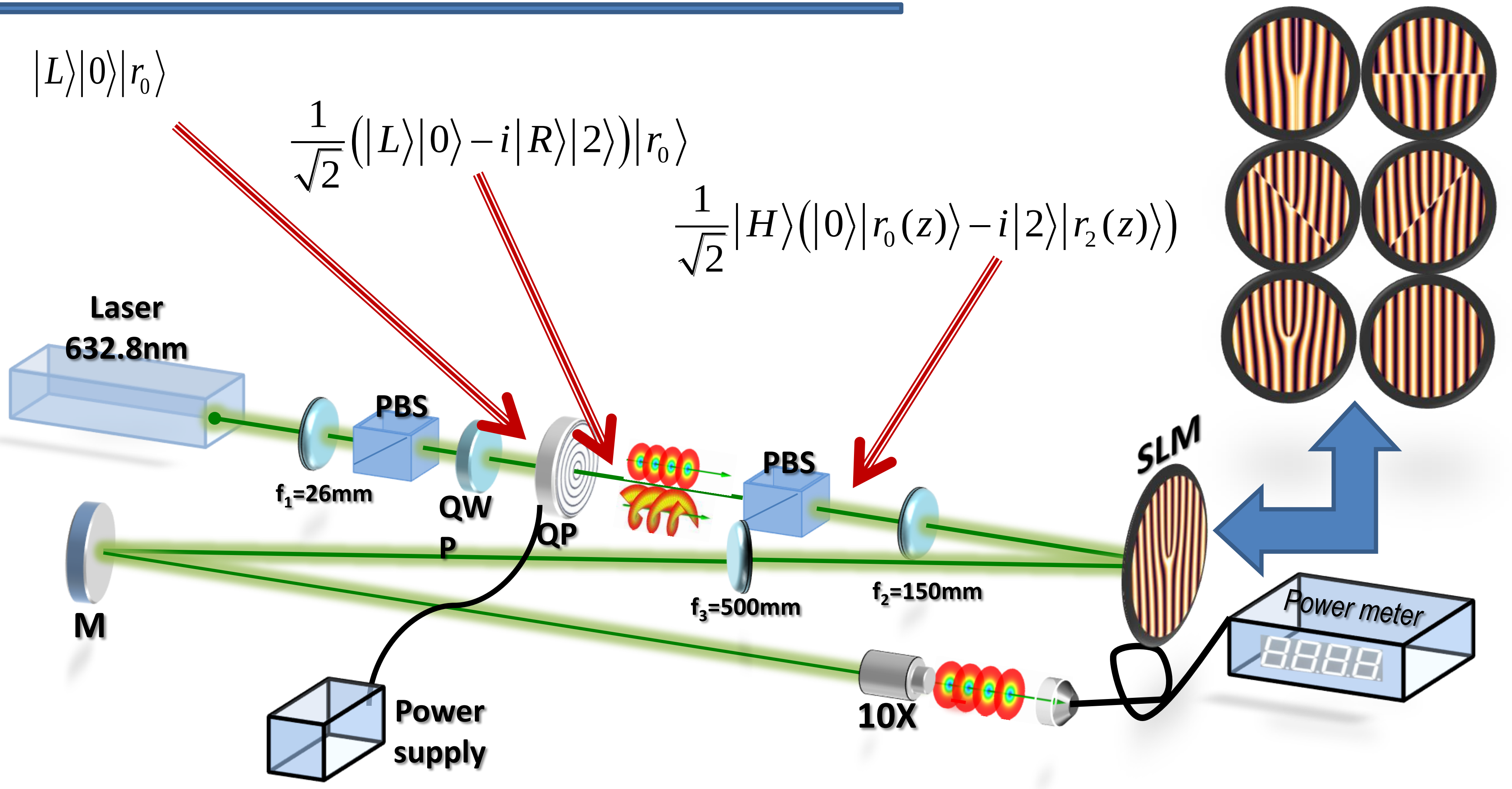
- The SAM (system) states are entangled with the OAM (apparatus) states and are not affected by propagation
- The OAM (apparatus) states are not disturbed by the radial (environment) states and not affected by propagation
- The radial (environment) states are disturbed by the OAM (apparatus states) and change during propagation
- The radial (environment) states obey

$$\langle r_m(z) | r_m(z) \rangle = 1$$

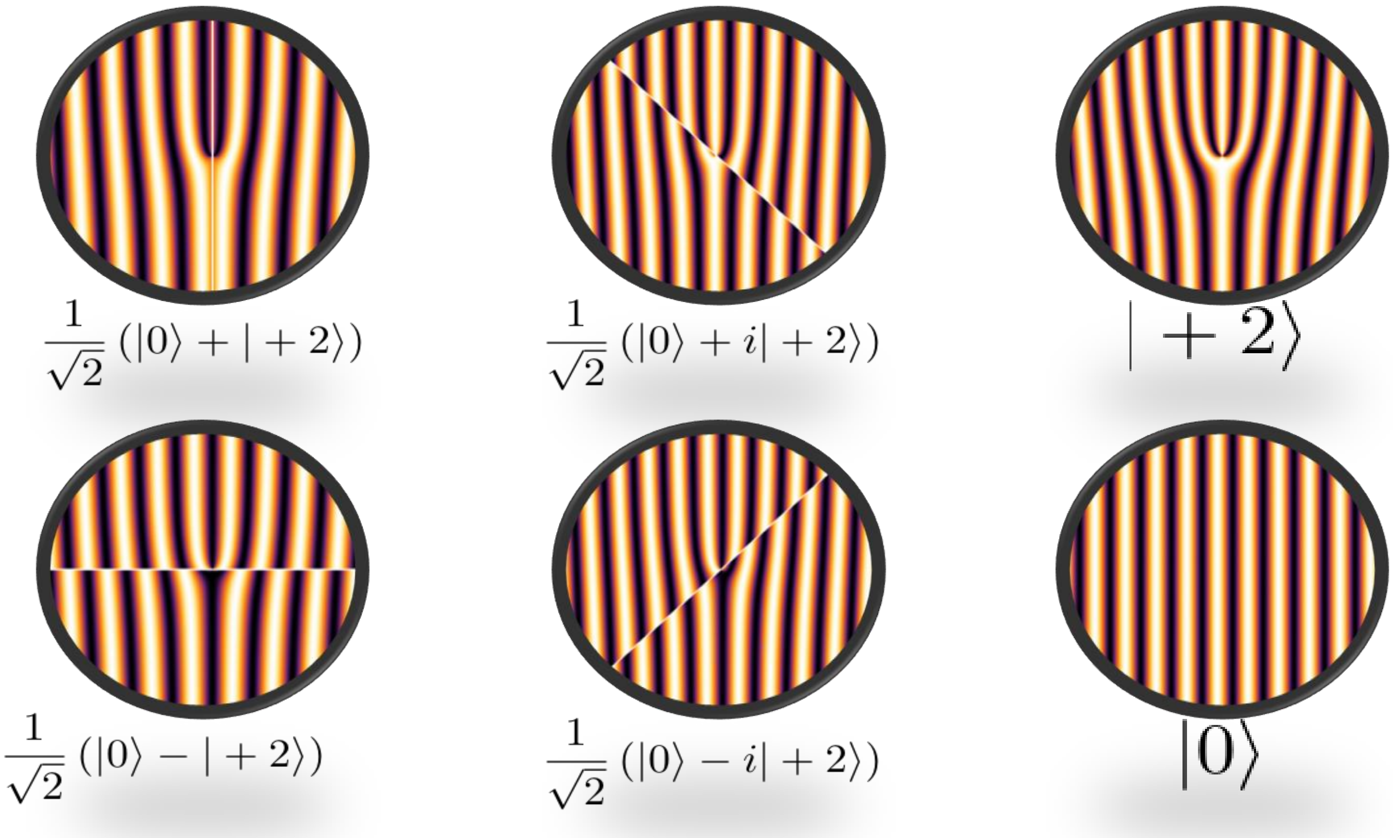
$$\langle r_m(z) | r_n(z) \rangle = f_{mn}(z) \rightarrow \delta_{mn}$$

Einselection condition

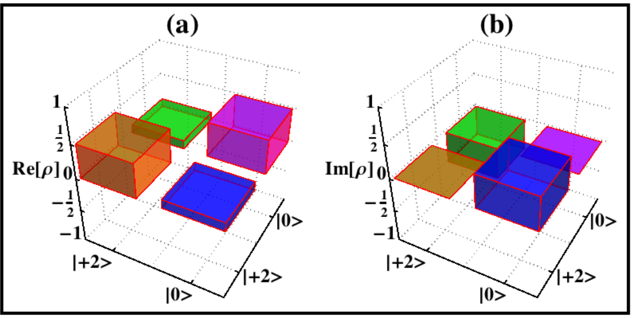
Experimental setup



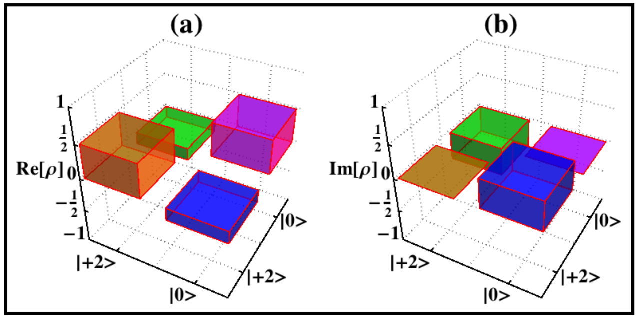
Hologram patterns for OAM-state tomography



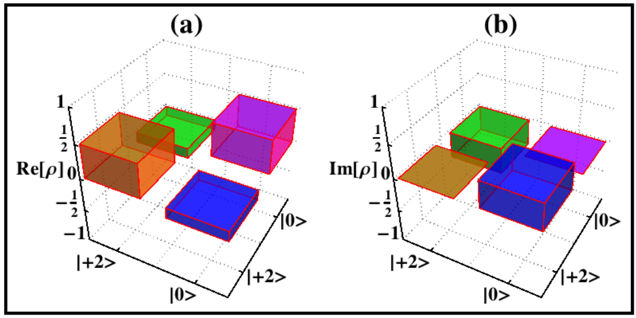
Experimental OAM tomography data



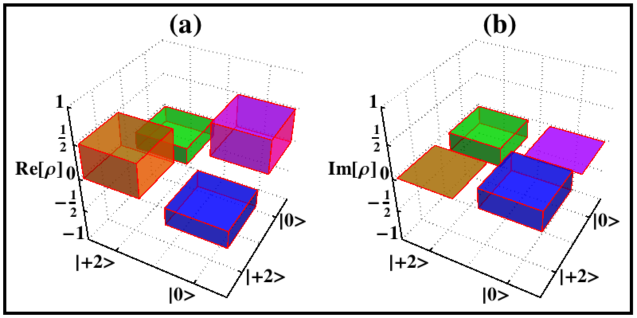
z=0



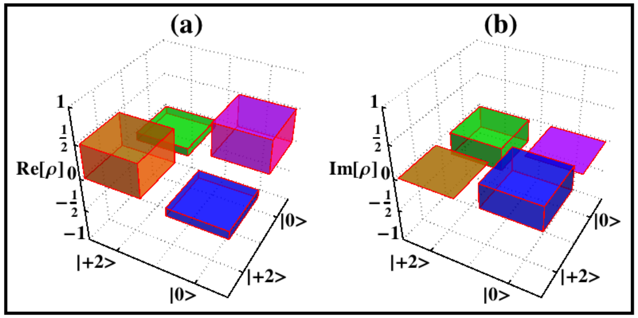
z=0.2 z_R



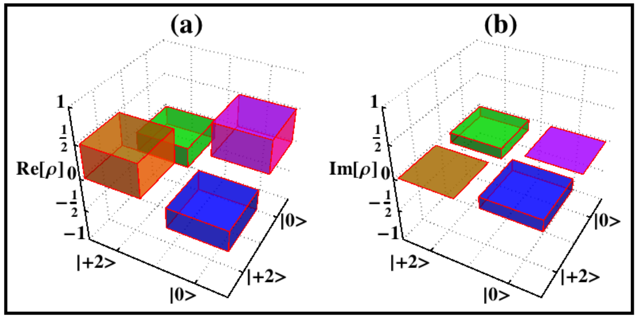
z=0.4 z_R



z=0.6 z_R



z=0.8 z_R



Z=1 z_R

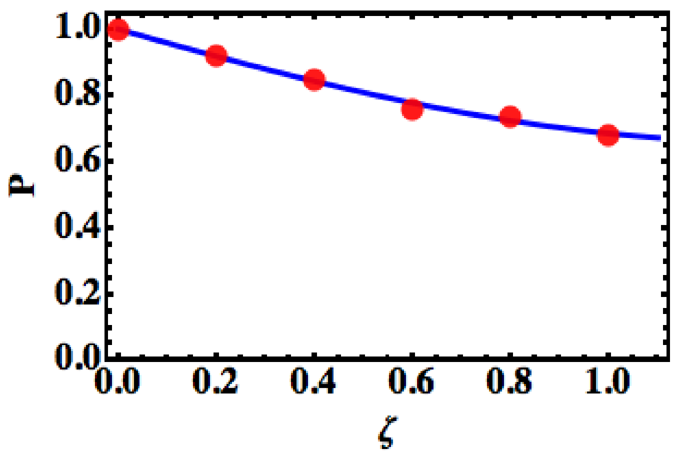
(a) and (b) are the real and imaginary parts of the beam density matrix at different plane. As you can see, by propagation the imaginary parts of beam decreases. The data has been cleaned by maximum likelihood estimator technique.

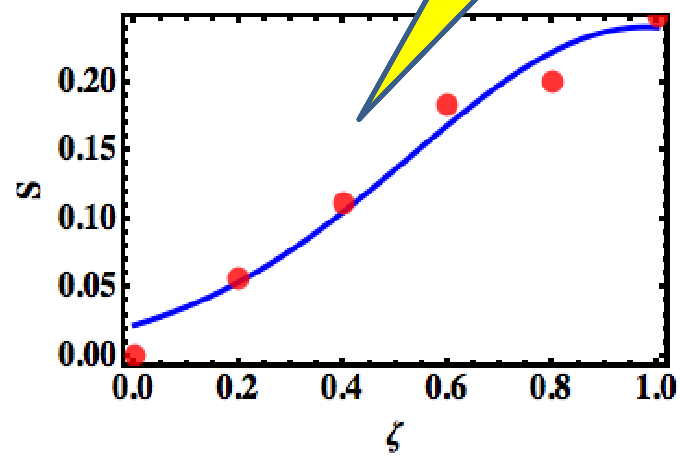
Very preliminary experimental data

Entropy increases spontaneously

$$P = \sqrt{1 - \frac{4\mathrm{Det}(\rho_{OAM})}{\mathrm{Tr}(\rho_{OAM})}}$$

$$S = -\mathrm{Tr}(\rho_{OAM}\ \mathrm{lo} \ldots _{OAM})$$





(Left) Decoherence degrees, (Right) Entropy of information. Blue curves are only interpolations for better viewing

Conclusions

- We presented an optics experiment to test Zanek's decoherence and measure theory
- We proved that a photon OAM qubit suffer decoherence due to the entanglement with radial states acting as "environment"
- OAM decoherence and entropy increase spontaneously during free propagation
- OAM spontaneous decoherence may be a problem and must be corrected in long distance telecommunications (e.g. among satellites)
- OAM spontaneous decoherence may be a problem and must be corrected when one wants to conserve OAM entangled photons in a cavity

Thank you for your attention.

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