Towards

Optical eigenmode applications

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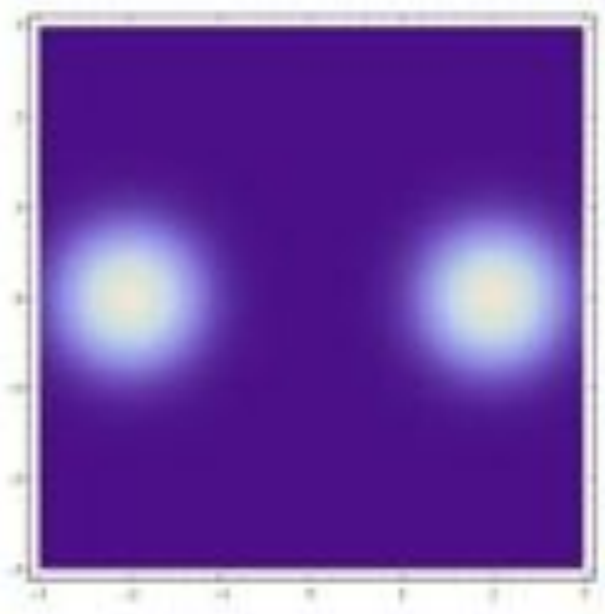
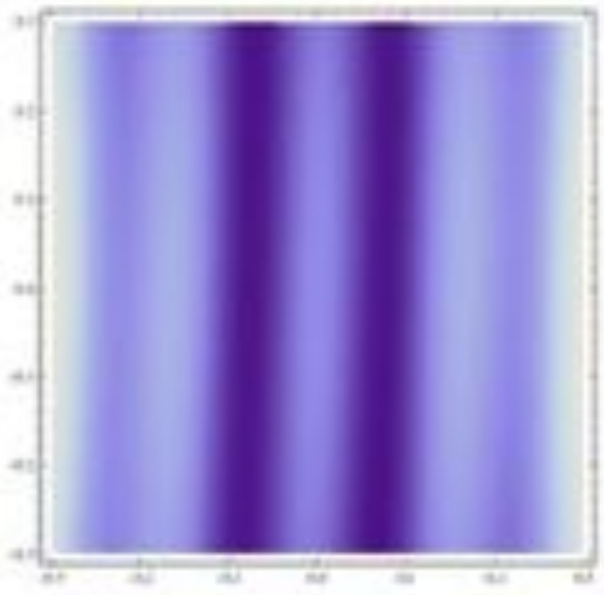
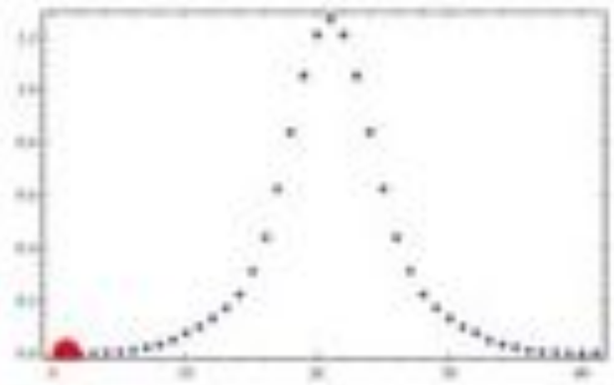
Outline

- Motivation/Challenges
- Finite size optical eigenmodes
  - Intensity, spot size,
  - Linear and angular momentum
- Optical eigenmode imaging
- Interactions to hard singularities

Interference

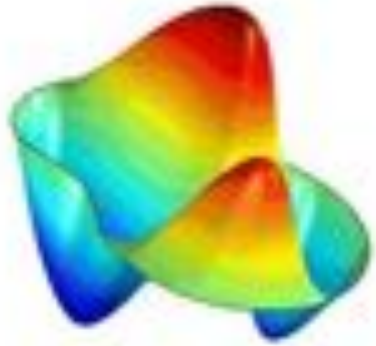


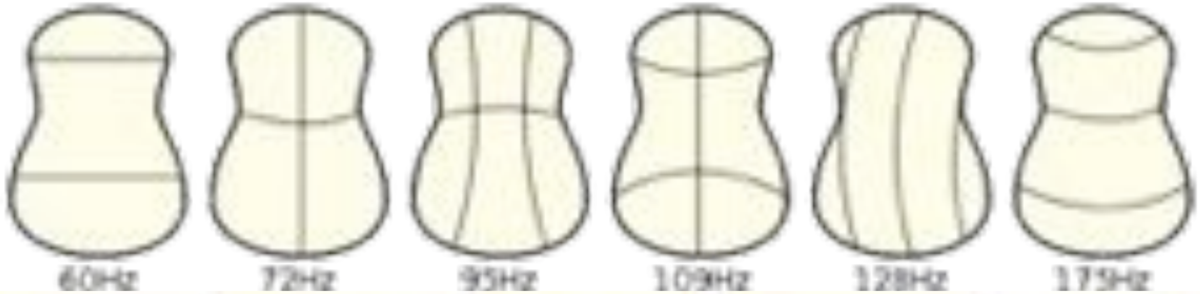




Superposition of beams does not conserve energy locally.

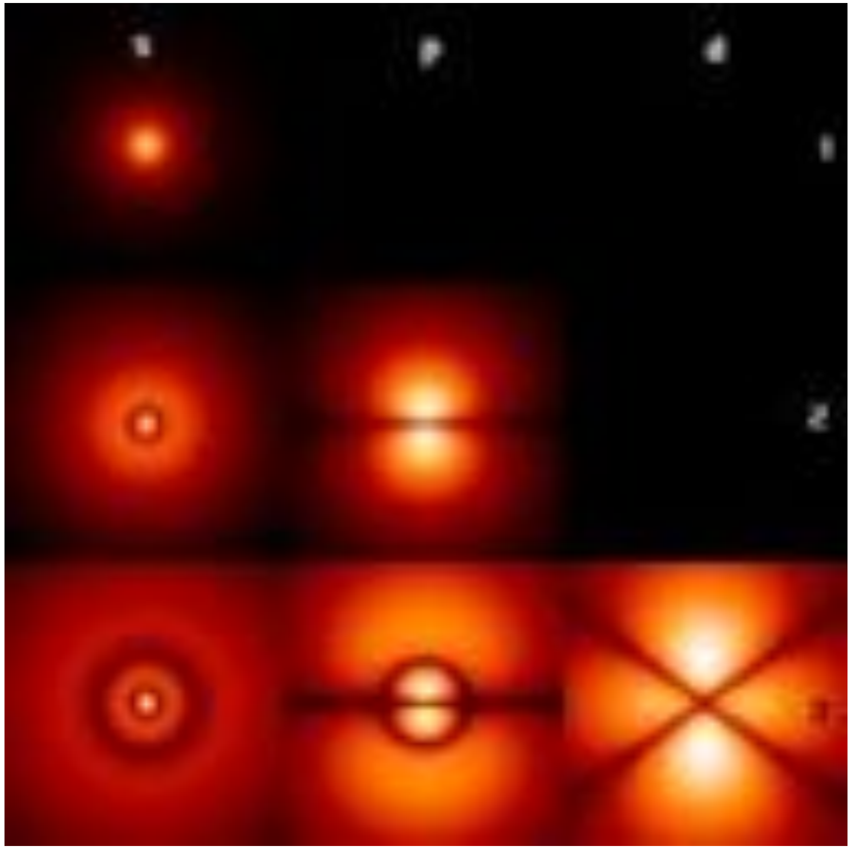
Chladni modes of a guitar plate





http://www.phys.unsw.edu.au/~jw/guitar/

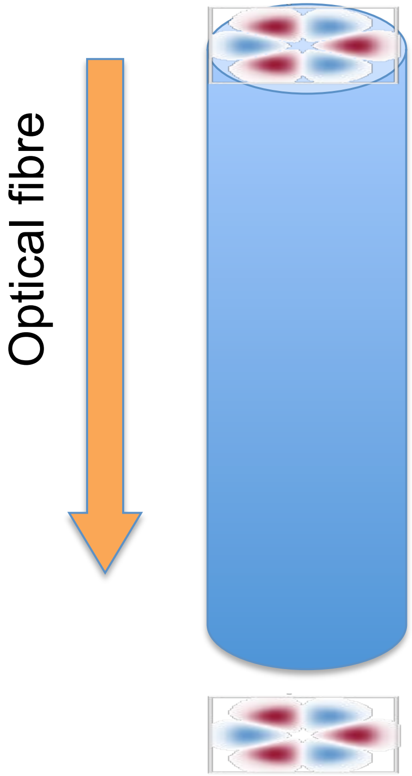
Hydrogen wavefunctions

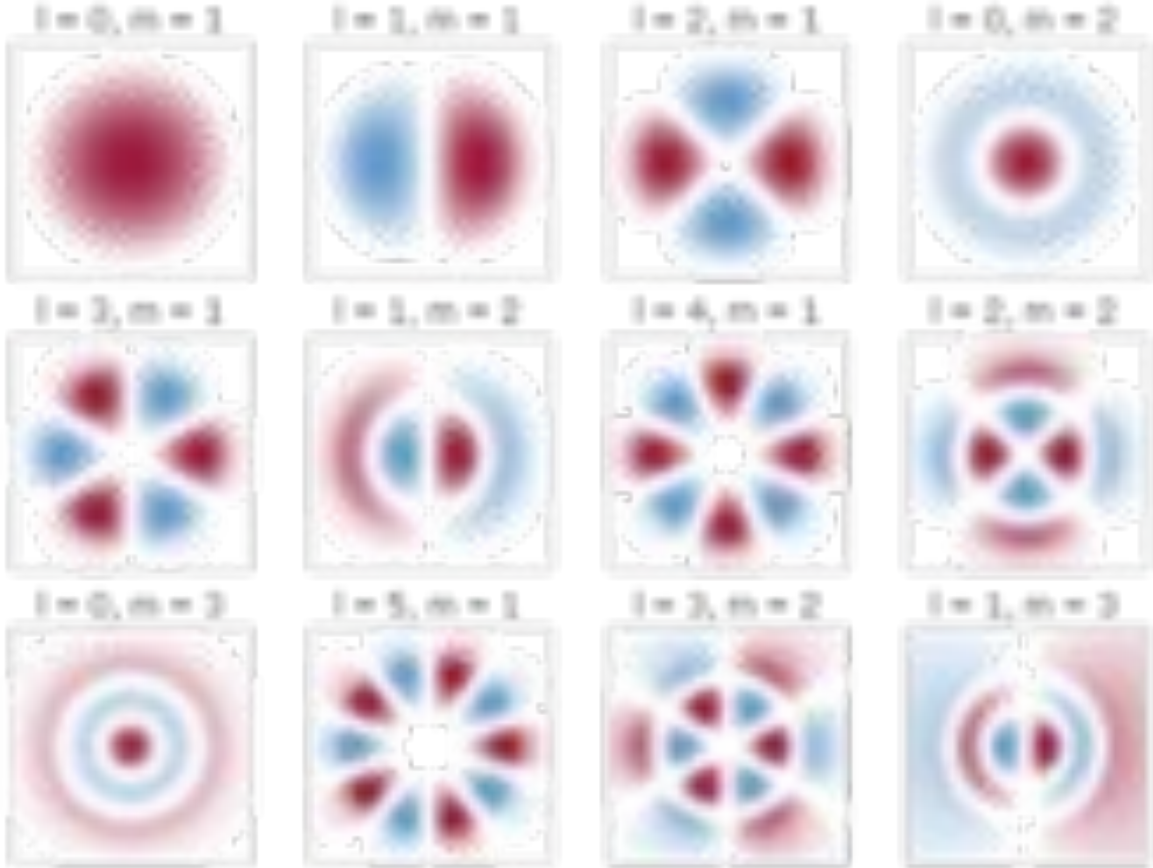


$$\psi_{4,3,1}(r,\vartheta,\varphi) = \sqrt{\left(\frac{1}{2\,a_0}\right)^3 \frac{1}{8\cdot 7!}} \cdot e^{-r/(4\,a_0)} \cdot \left(\frac{1\,r}{2\,a_0}\right)^3 \cdot Y_3^1(\vartheta,\varphi)$$

http://en.wikipedia.org/wiki/Hydrogen_atom

Waveguide modes





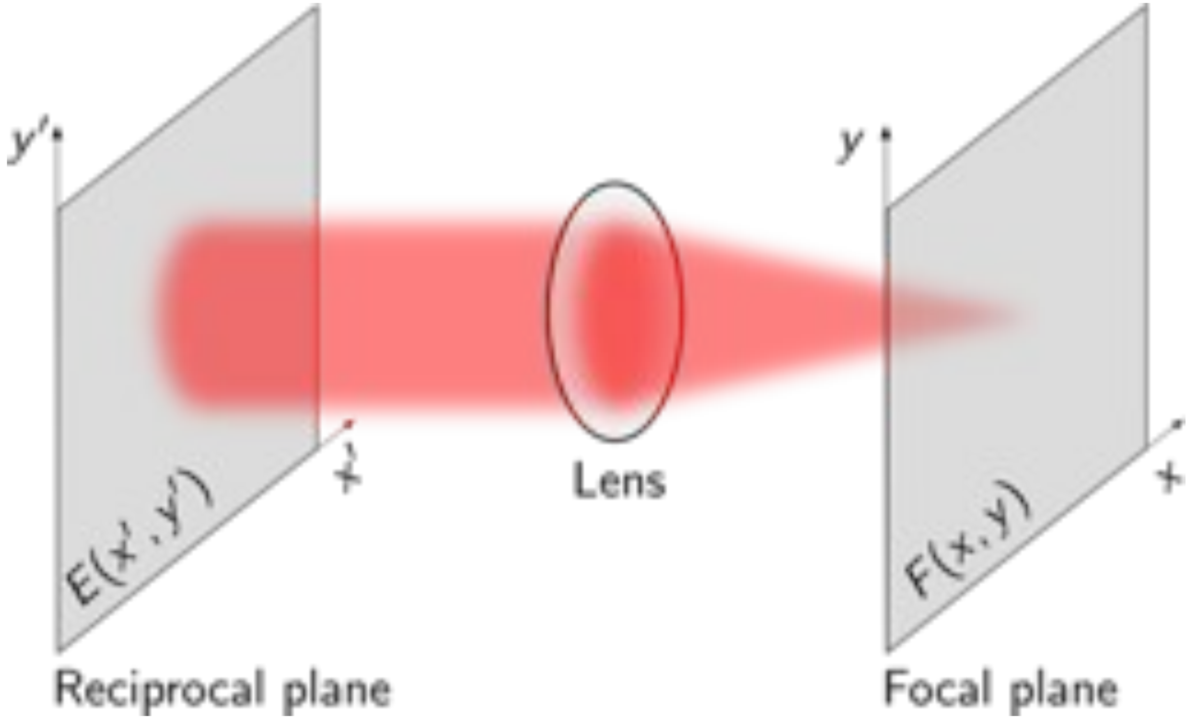
http://www.rp-photonics.com/modes.html

Optical Eigenmodes on SLM

- **Theory:** Decompose field in both planes into *N* modes:

$$E(x', y') = \sum_{i=1}^{N} a_i \, E_i(x', y')$$

$$F(x, y) = \sum_{i=1}^{N} a_i \, F_i(x, y)$$



- Probe system with N beams profiles e.g. Zernike polynomials, LG, HG, deflections, random phases

Optical Eigenmodes on SLM

Intensity $\int_{ROI} F^*(x,y)\,F(x,y)\,dxdy$

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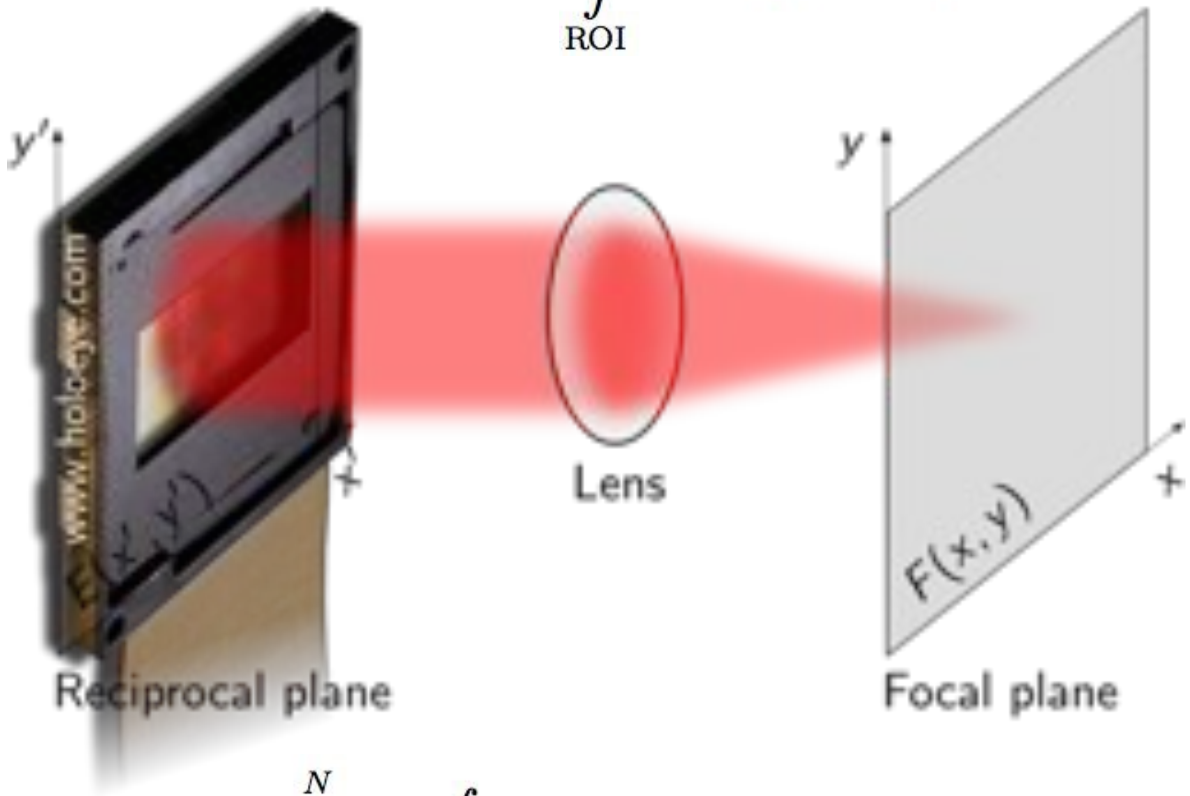

Intensity $\sum_{i,j=1}^{N} a_i^* \int_{ROI} F_i^*(x,y)\,F_j(x,y)\,dxdy\; a_j$

Operator M_{ij}

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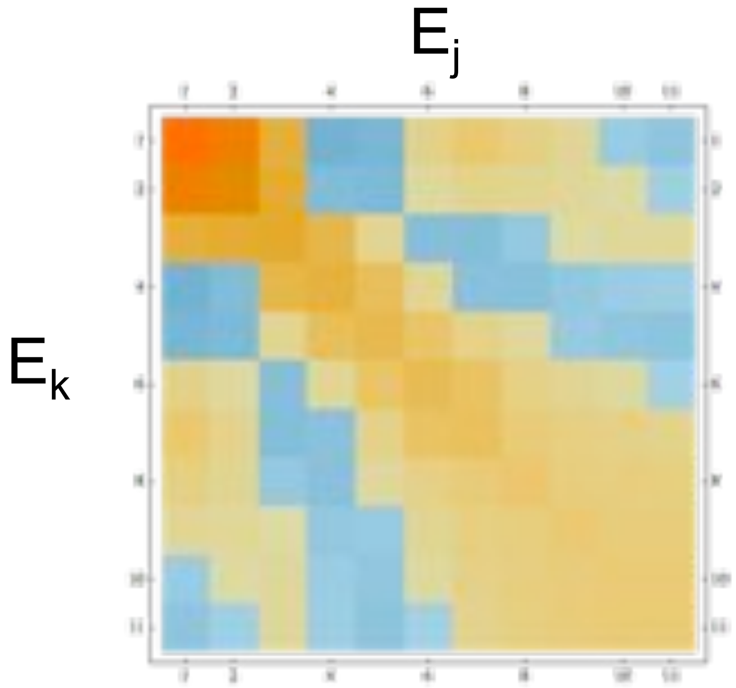
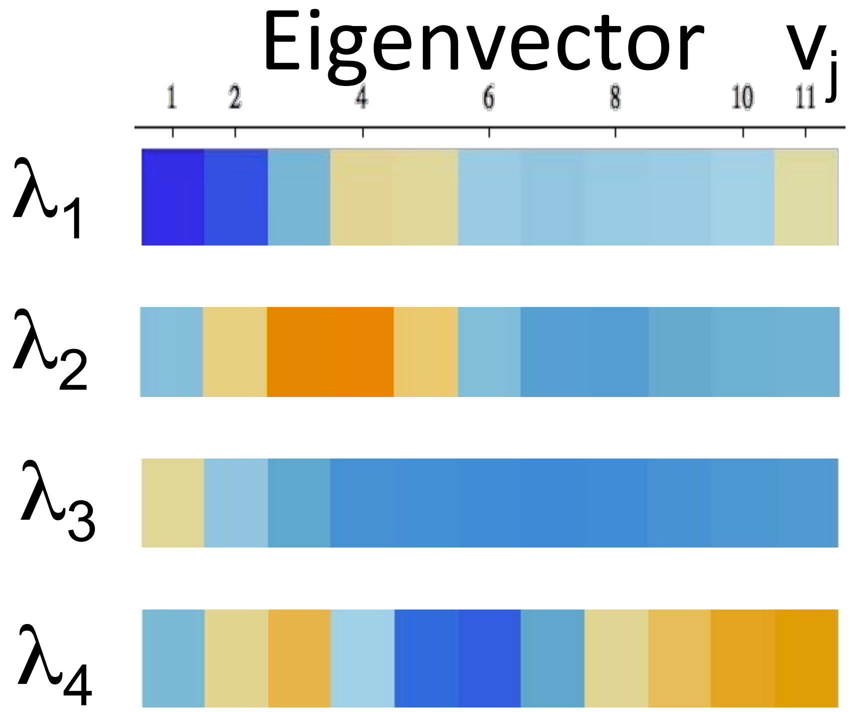
Optical eigenmode: Theory

Intensity matrix (operator)



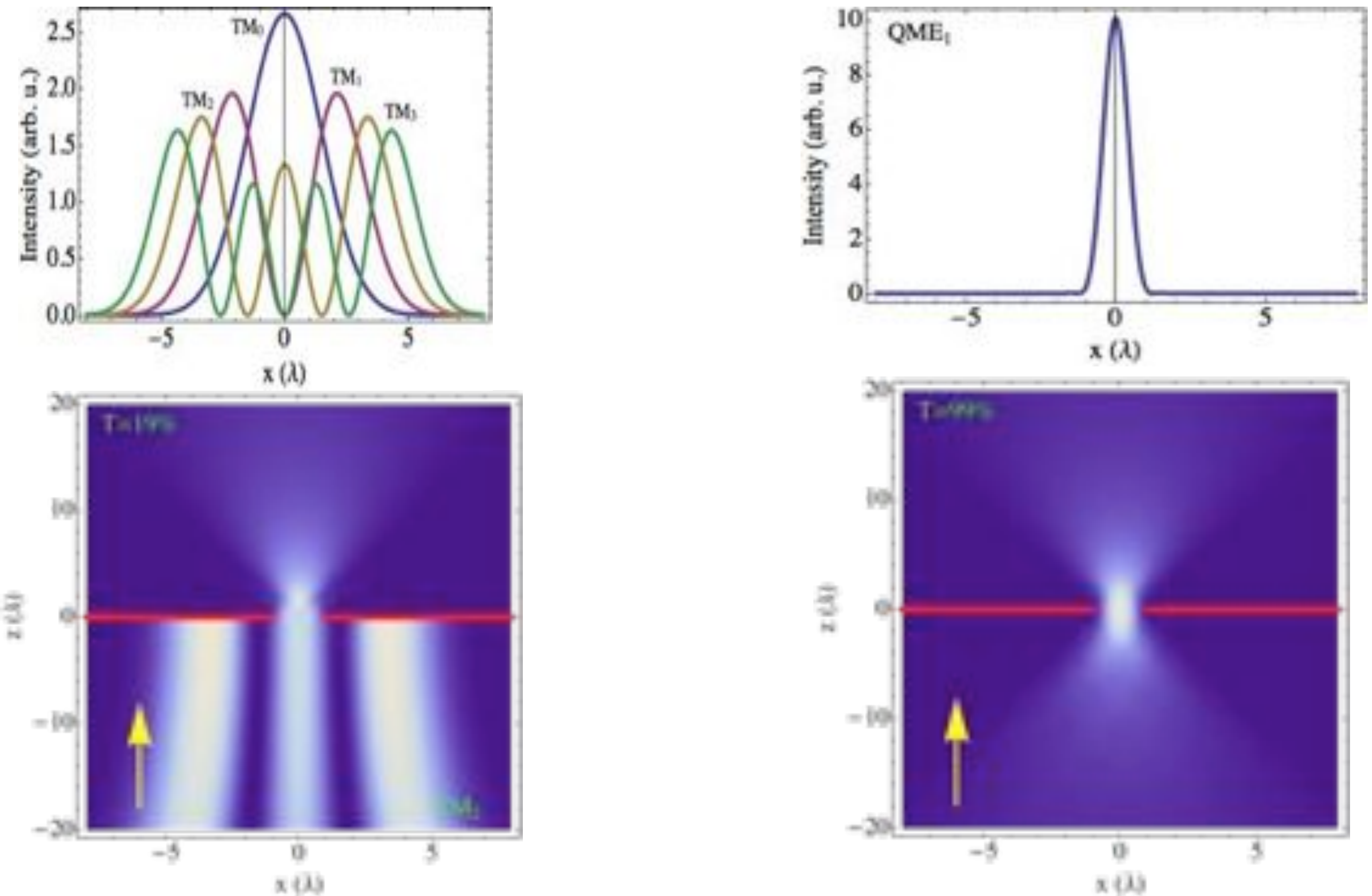


$$M_{jk} = \int_{\mathrm{ROI}} d\sigma \mathrm{E}_j \cdot \mathrm{E}_k^*$$

$M_{jk}\, v_k = \lambda\, v_j$

$\lambda_1 > \lambda_2 > \lambda_3 > \lambda_4$

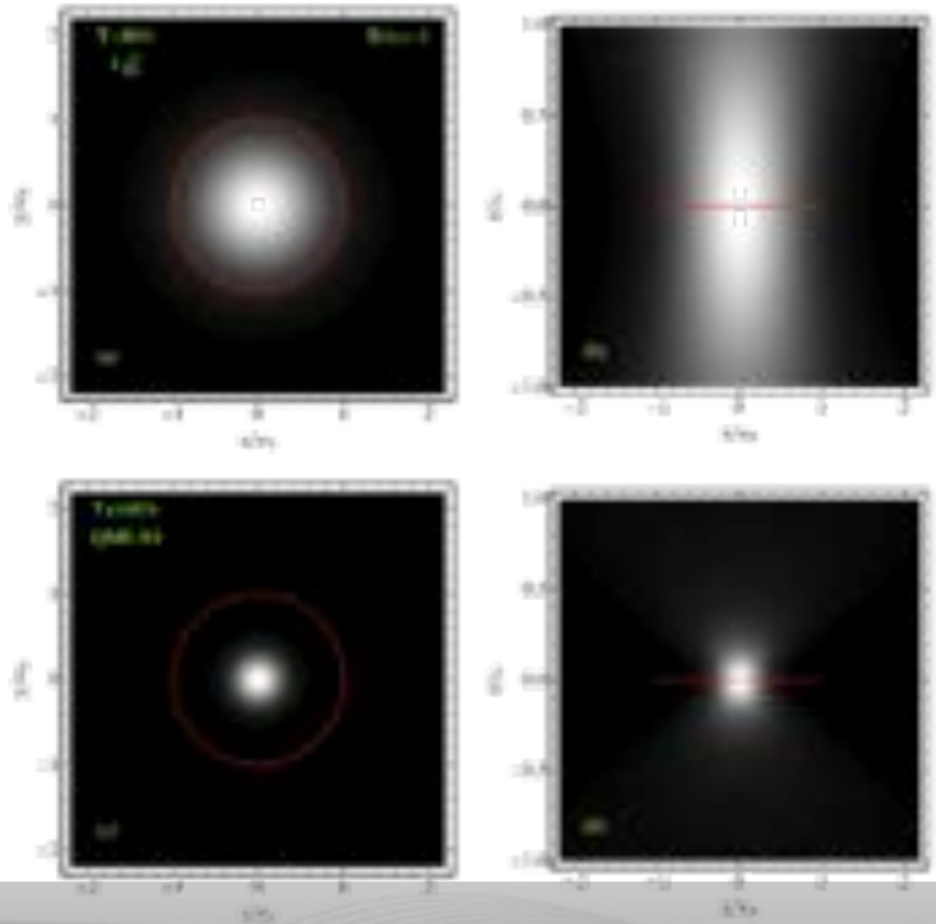
Superposition of Hermit Gaussian beams



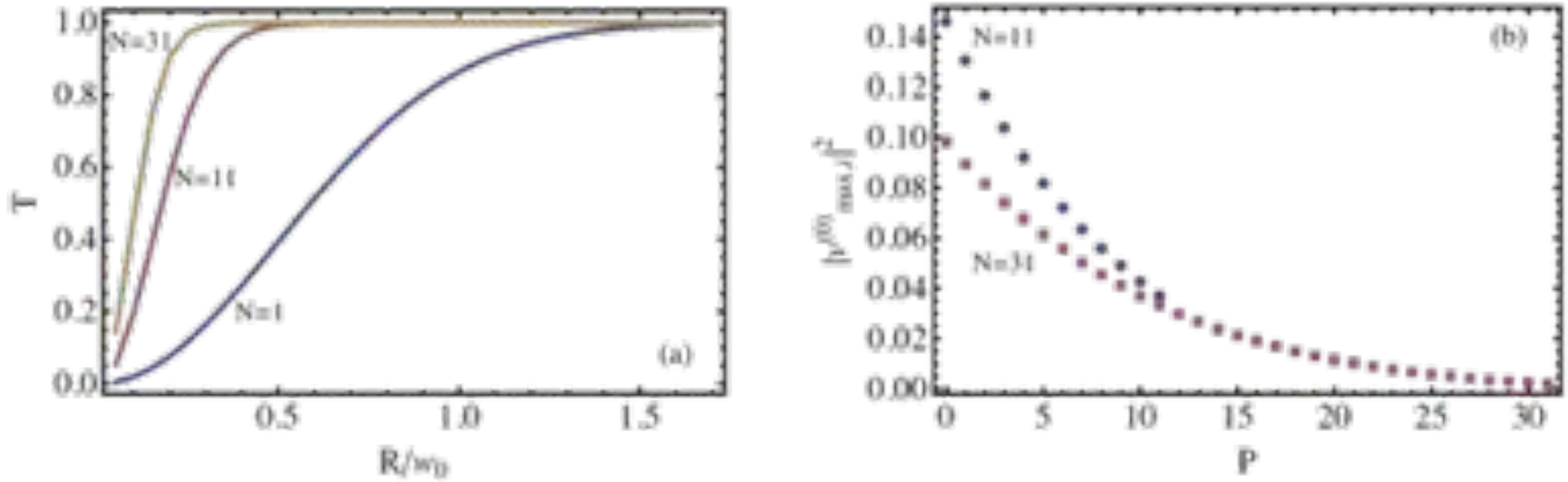
Superposition of Laguerre-Gaussian beams
First intensity eigenmode

LG : 0

OEi : 1



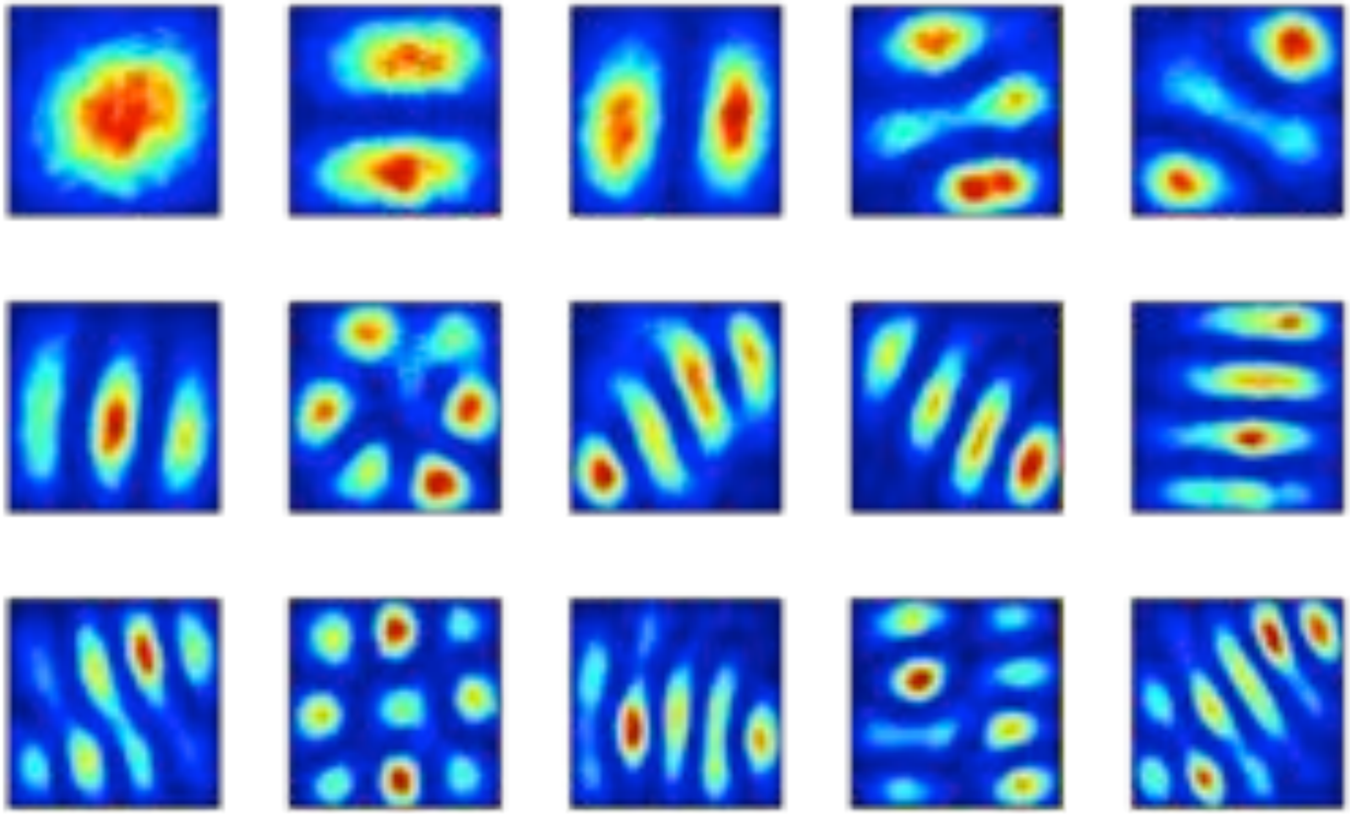
Intensity eigenmode: Efficiency vs ROI



(a) Total transmittance through the ROI for the intensity optimized beam as a function of the ROI relative radius *R*/*w*0 and for different numbers *N* of Laguerre Gaussian modes considered.

(b) Relative intensity $|v^{(0)}_{max}|^2$ of the Laguerre Gaussian modes (*P* = 0..*N*) decomposing the intensity optimized beam.

First 15 optical eigenmodes



Localized features in eigenfaces

Varimax rotation:

$$R_{\text{VARIMAX}} = \arg\max_R \left(\sum_{j=1}^{k} \sum_{i=1}^{p} (\Lambda R)_{ij}^4 - \frac{\gamma}{p} \sum_{j=1}^{k} \left(\sum_{i=1}^{p} (\Lambda R)_{ij}^2 \right)^2 \right)$$

M. Meyer and J. Anderson (PIXAR). *ACM SIGGRAPH* 74 (2007)

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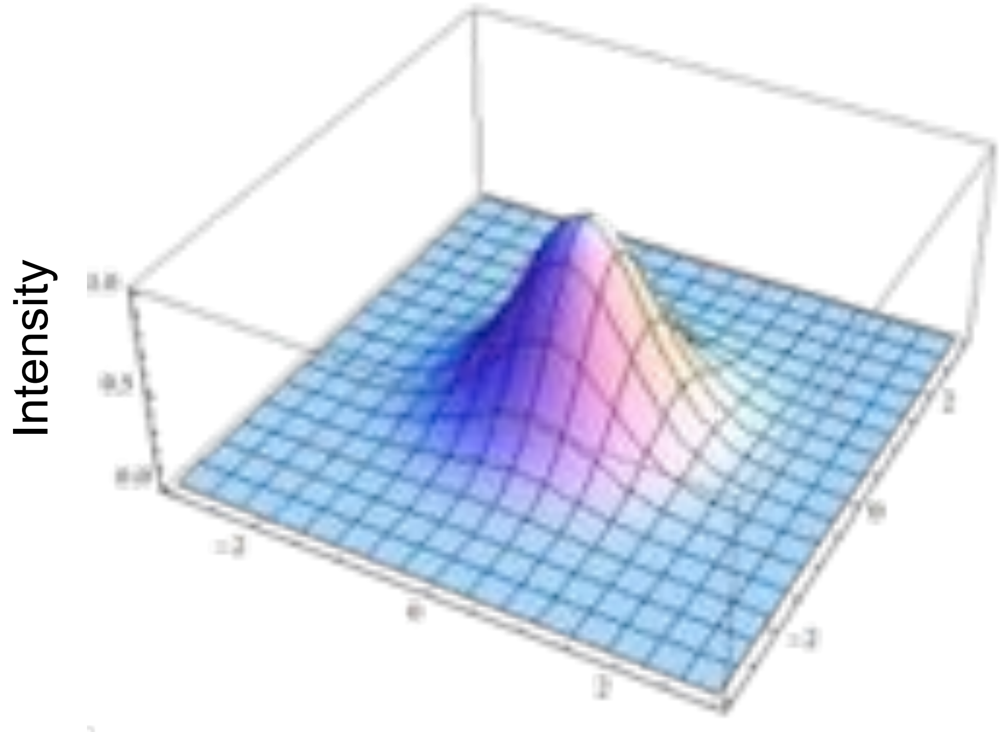
Spot size operator



Intensity operator:

$$M_{jk}^{(0)} = \int_S E_j^* E_k \, d\sigma$$

Second order momentum

$$M_{jk}^{(2)} = \int_S \mathbf{r}^2 E_j^* E_k \, d\sigma$$

Spot size:

$$w = 2\sqrt{\frac{m^{(2)}}{m^{(0)}}} = 2\sqrt{\frac{\mathbf{a}^\dagger \mathbf{M}^{(2)} \mathbf{a}}{\mathbf{b}^\dagger \mathbf{M}^{(0)} \mathbf{b}}}$$

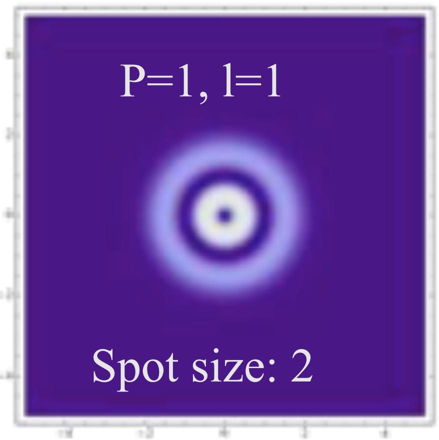
First eigenmode ensures the smallest achievable spot:

⟺

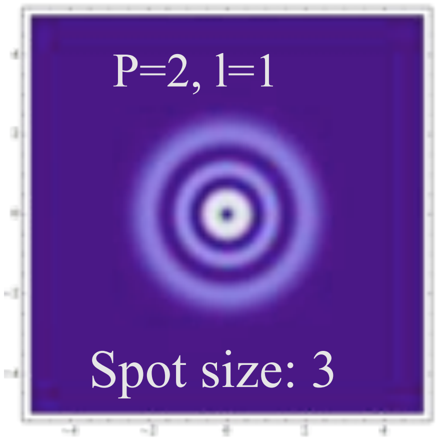
Highest mode purity with respect to a point decomposition.

Finding the tightest focused spot

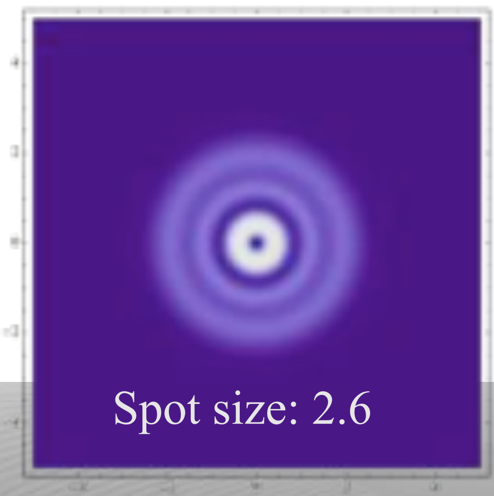
E_1



E_2



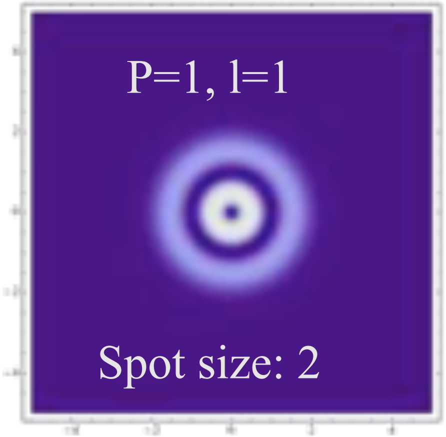
Direct superposition



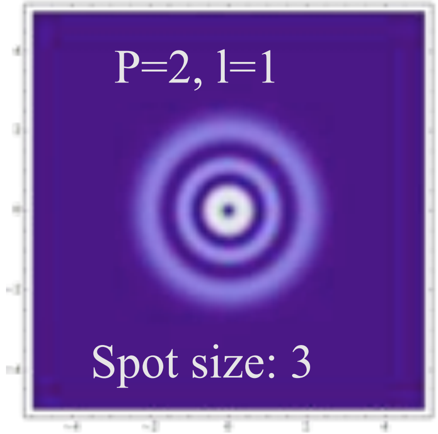
$$E_{superposition}=E_1+E_2$$

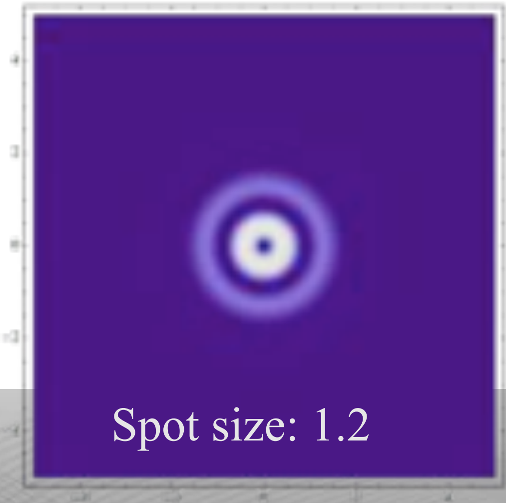
Spot size optical eigenmodes

E_1



E_2





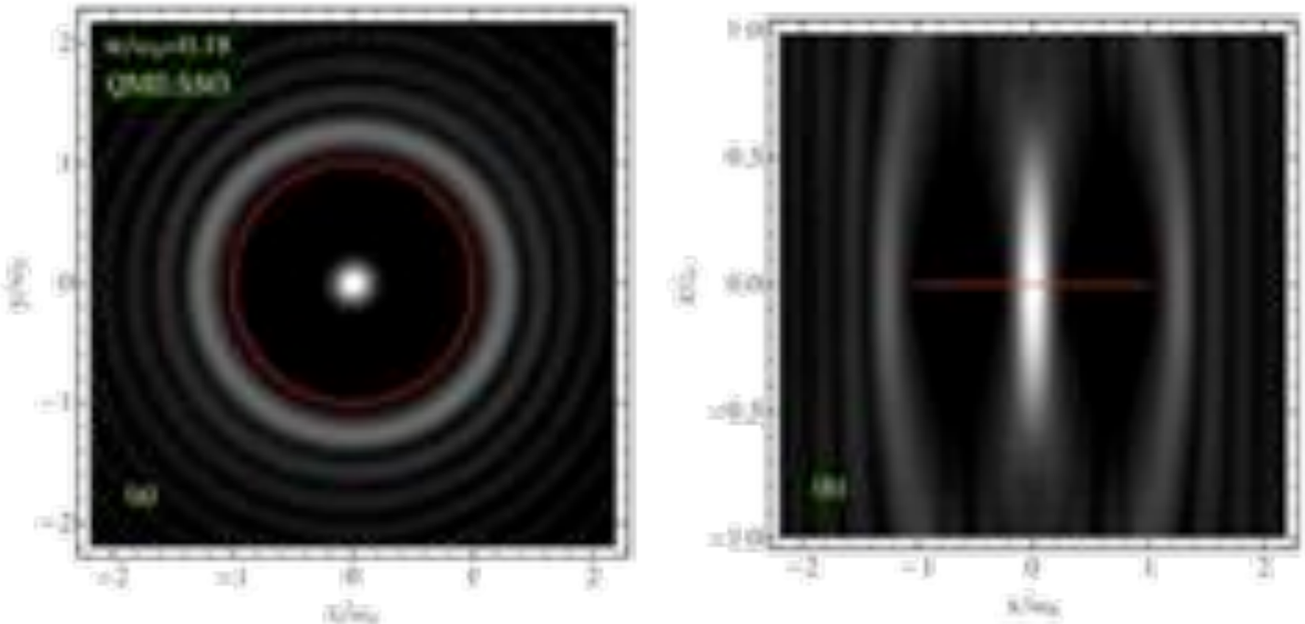
eigenmode

$E_{eigenmode} = a_1\ E_1 + a_2\ E_2$

Spot size operator

$$\begin{pmatrix} \frac{\pi}{4} & -\frac{\pi}{4} & 0 & 0 & 0 & 0 \\ -\frac{\pi}{4} & \pi & -\frac{3\pi}{4} & 0 & 0 & 0 \\ 0 & -\frac{3\pi}{4} & \frac{9\pi}{4} & -\frac{3\pi}{2} & 0 & 0 \\ 0 & 0 & -\frac{3\pi}{2} & 4\pi & -\frac{5\pi}{2} & 0 \\ 0 & 0 & 0 & -\frac{5\pi}{2} & \frac{25\pi}{4} & -\frac{15\pi}{4} \\ 0 & 0 & 0 & 0 & -\frac{15\pi}{4} & 9\pi \end{pmatrix}$$

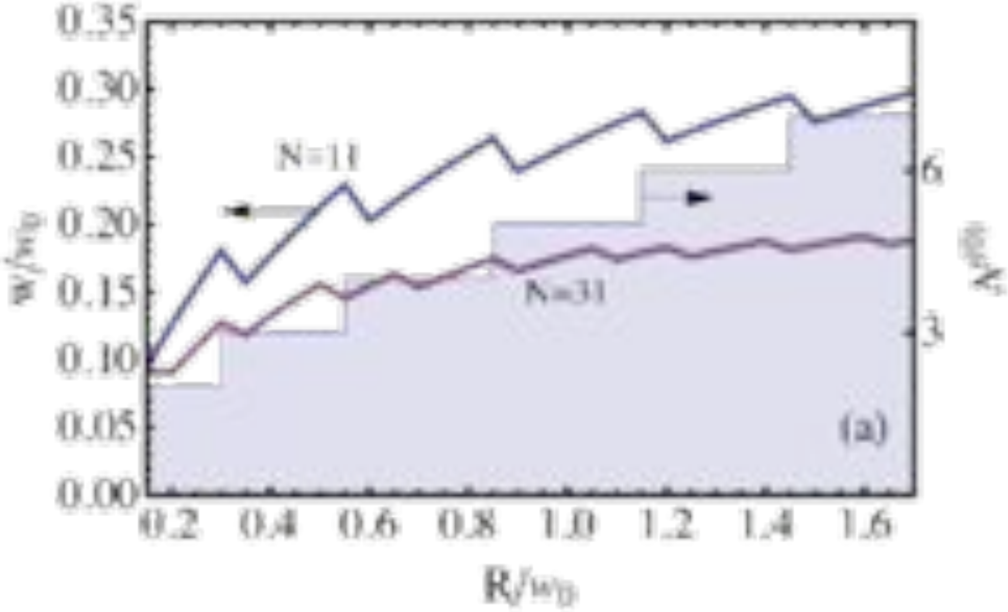
Smallest spot



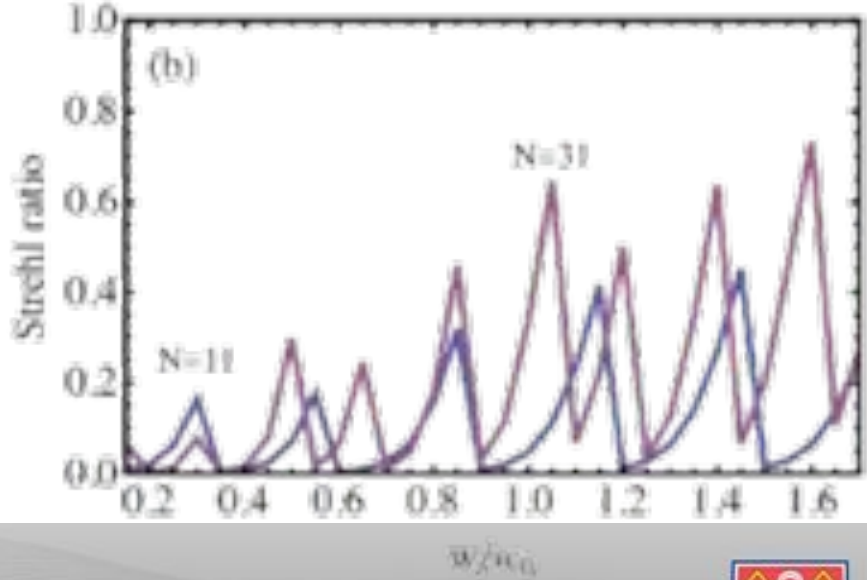
(a) Transversal and (b) longitudinal 2D intensity cross sections of the QME superposition delivering the smallest focal spot in the ROI ($R = \lambda$) considering 25 LG modes. $w/w0$ is the relative spot size. The Strehl ratio in (a) is 4.5%.

Mazilu et al. Opt Express **19**. 933 (2011)

Efficiency



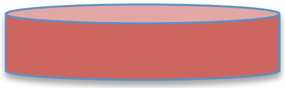
Spot size as a function of the radius of the ROI for different number of LG modes considered. The right hand scale and filled curve indicate the numbers of intensity eigenmodes *N*(0) fulfilling the intensity criteria for the *N* = 11 case. The arrows indicate the corresponding scales.

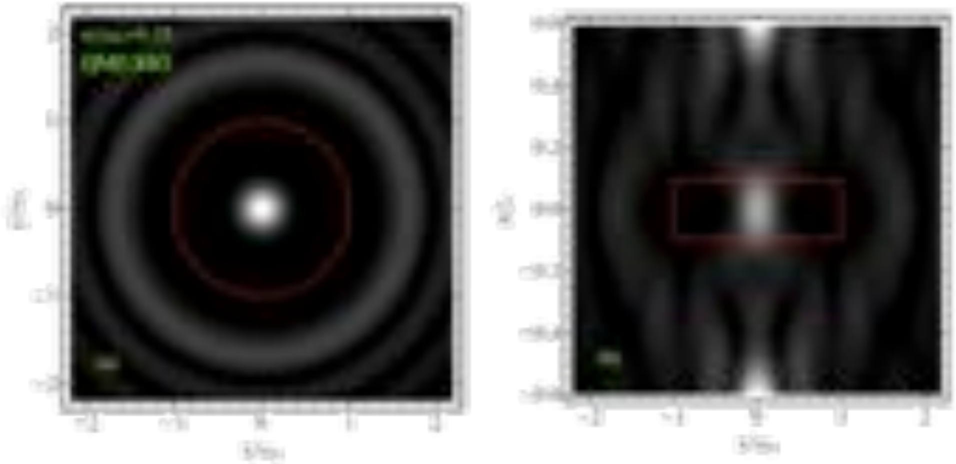


Ratio between the ROI intensity of the smallest spot size eigenmode and the largest intensity achievable in the ROI (Strehl ratio).

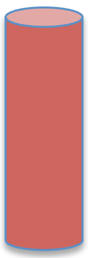
Mazilu et al. Opt Express **19**. 933 (2011)

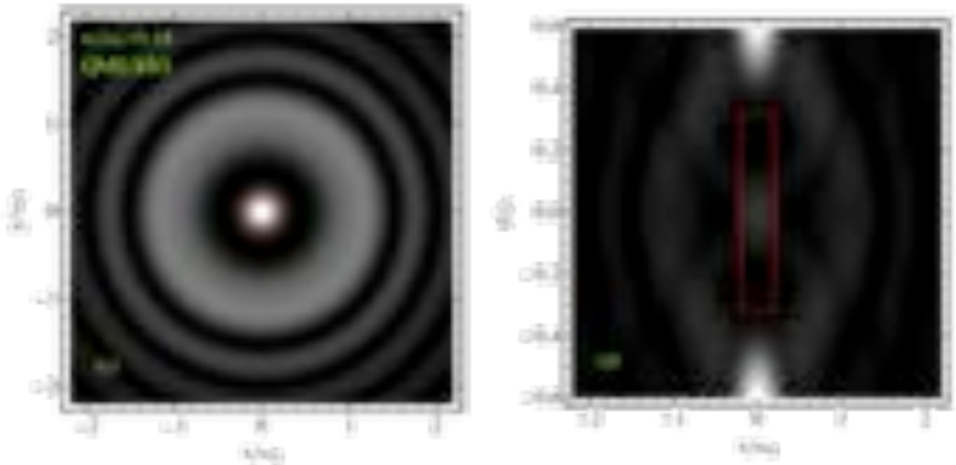
Volume focusing





Smallest spot inside a volume





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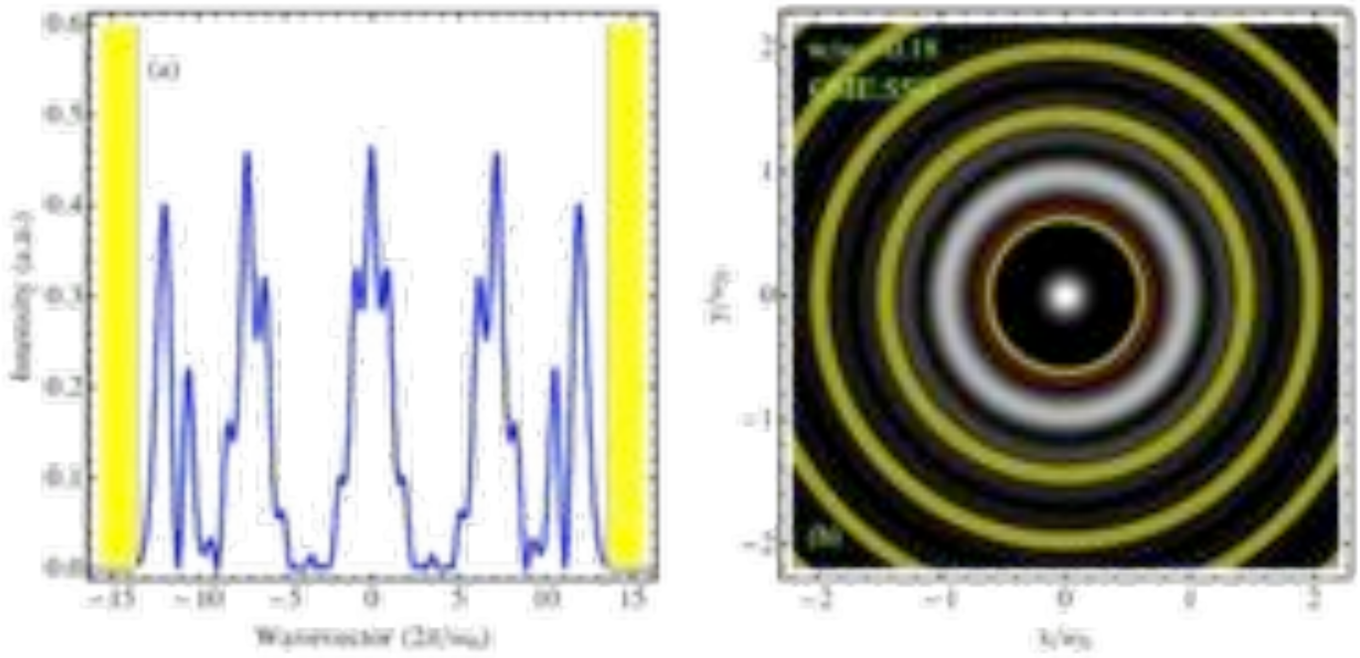
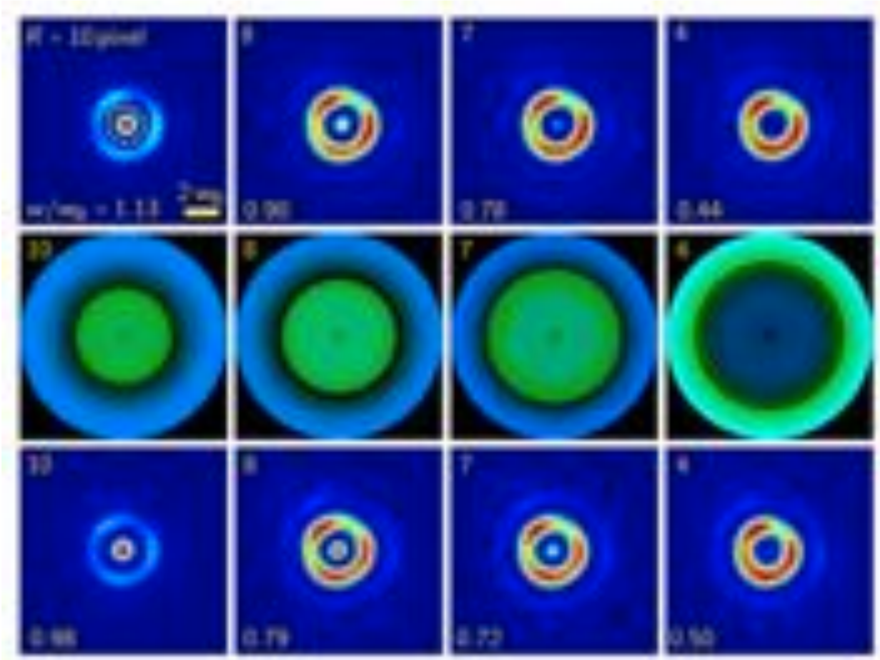
Bessel beam superposition & Super-oscillating regions

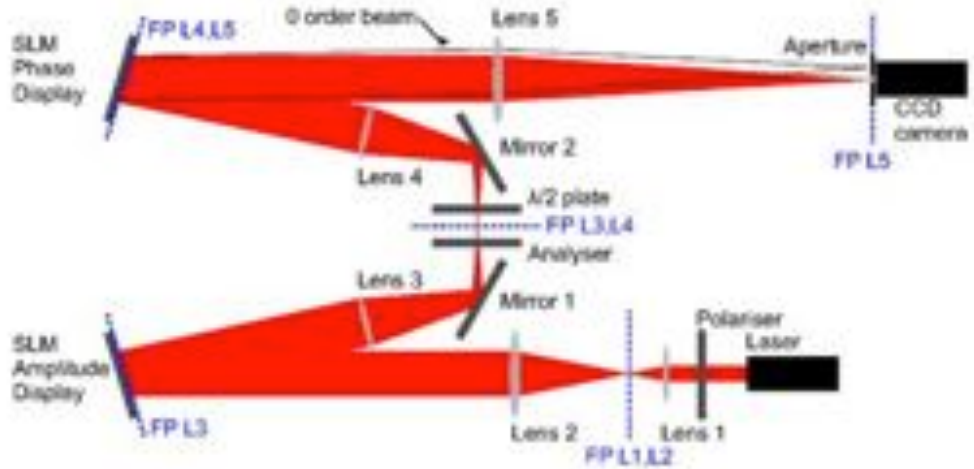


(a) Radial wavevector spectral density. Yellow highlights regions outside the spectral bandwidth. (b) Transversal cross section of the eigenmode spot size optimized field intensity with yellow showing super-oscillating regions.

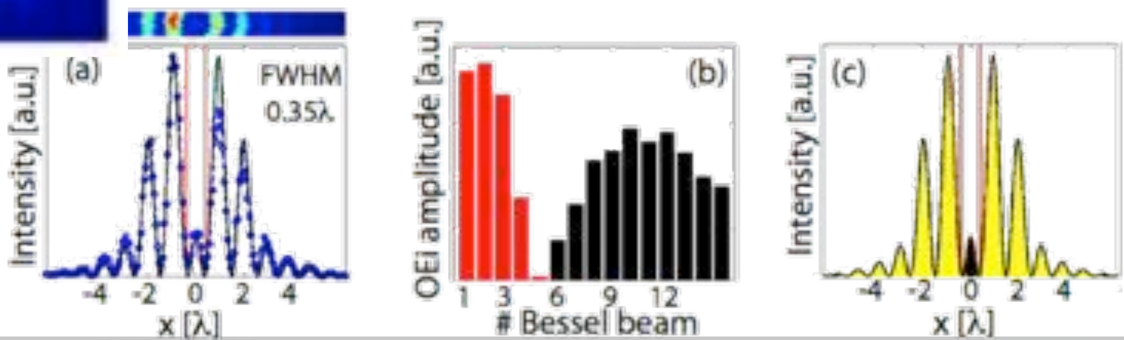
Experimental evidence:



Sub diffraction [1]



Sub wavelength [2]



[1] Mazilu et al. Opt Express **19**. 933 (2011)
[2] Baumgartl et al. Appl. Phys. Lett. **98**, 181109 (2011)

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Optical eigenmodes: Wider applicability

Any quadratic measure of the electromagnetic field leads to a linear operator whose eigenvector optimize the measure.

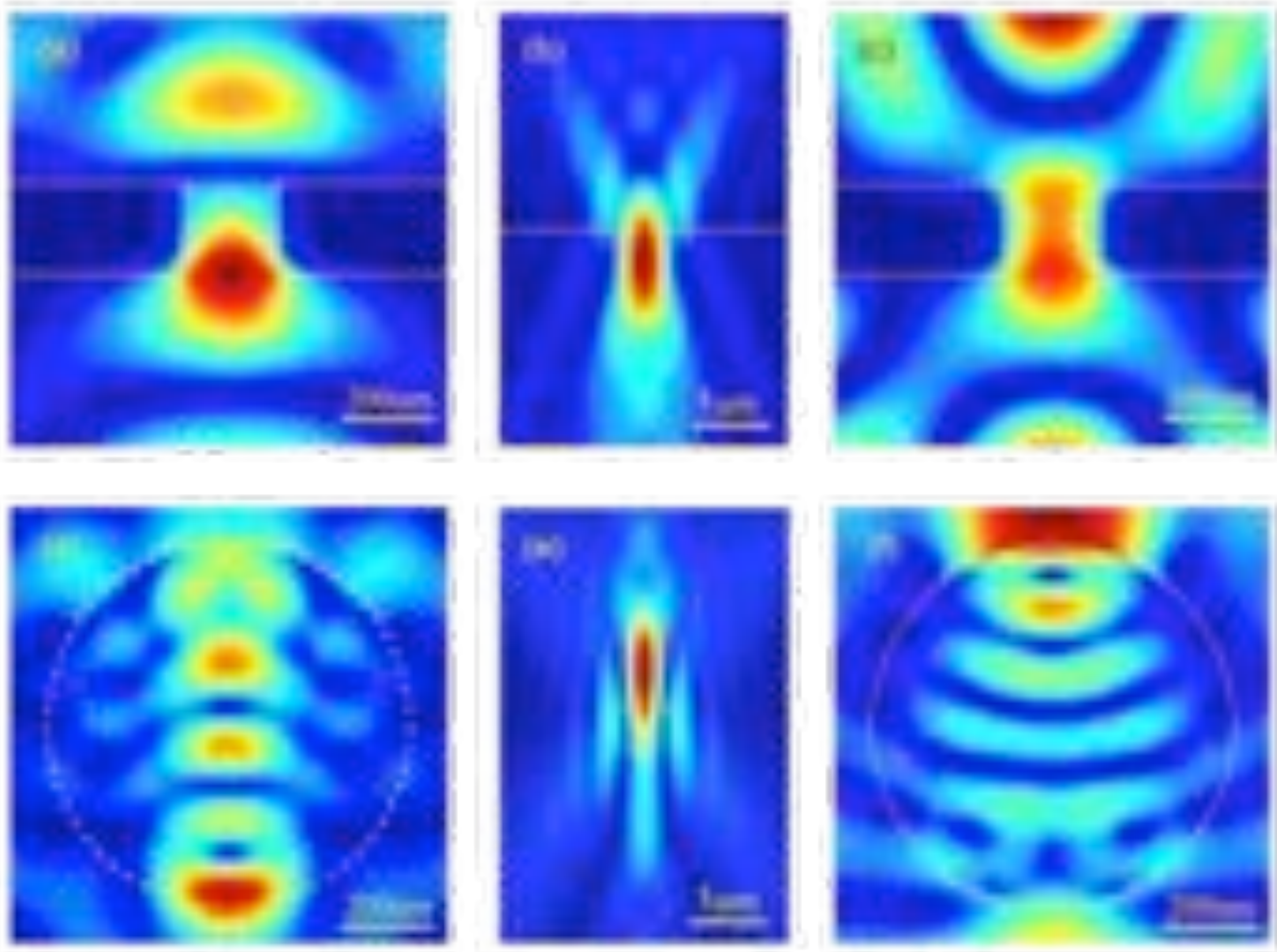
Energy	$m^{(\mathscr{E})}(\mathbf{E},\mathbf{H}) = \frac{1}{2}\int_V \mathscr{E}\, dv$
Intensity	$m^{(0)}(\mathbf{E},\mathbf{H}) = \frac{1}{4}\int_S (\mathbf{E}^* \times \mathbf{H}) \cdot \mathbf{n}\, d\sigma + \text{c.c.}$
Spot size	$m^{(2)}(\mathbf{E},\mathbf{H}) = \frac{1}{4}\int_S \mathbf{r}^2 (\mathbf{E}^* \times \mathbf{H}) \cdot \mathbf{n}\, d\sigma + \text{c.c.}$
Momentum	$m^{(\mathbf{F}\cdot\mathbf{u})}(\mathbf{E},\mathbf{H}) = \frac{1}{4}\int_S (\varepsilon_0(\mathbf{E}^* \cdot \mathbf{n})\mathbf{E} + \mu_0(\mathbf{H}^* \cdot \mathbf{n})\mathbf{H} - \frac{1}{2}\mathscr{E}\mathbf{n}) \cdot \mathbf{u}\, d\sigma + \text{c.c.}$

Time averaged quadratic measures *m* of common light-matter interactions. The integration either over a volume *V* or a surface *S* which in general corresponds to the Range of interest = ROI of the measure.

$$\mathscr{E} = 1/2(\varepsilon_0 \mathbf{E} \cdot \mathbf{E}^* + \mu_0 \mathbf{H} \cdot \mathbf{H}^*)$$

Sub-wavelength transmission & Optical forces

Plane wave illumination



2x larger transmission

Eigenmode illumination

50x larger scattering force

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Mazilu et al. Opt Express **19**. 933 (2011)

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Finite angular momentum eigenmode 1

Test fields, angular spectral decomposition

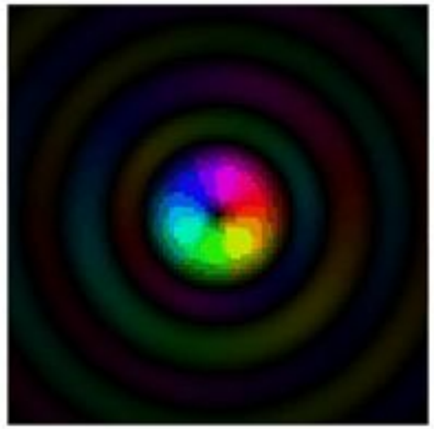
Region of interest: single point

four points

Operator

$$i\hbar r \times \nabla$$

2 eigenvalues



8 eigenvalues

Finite angular momentum eigenmode, 2

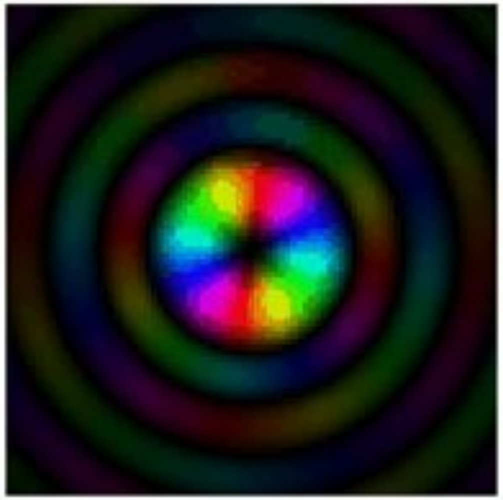
Test fields,
angular spectral
decomposition

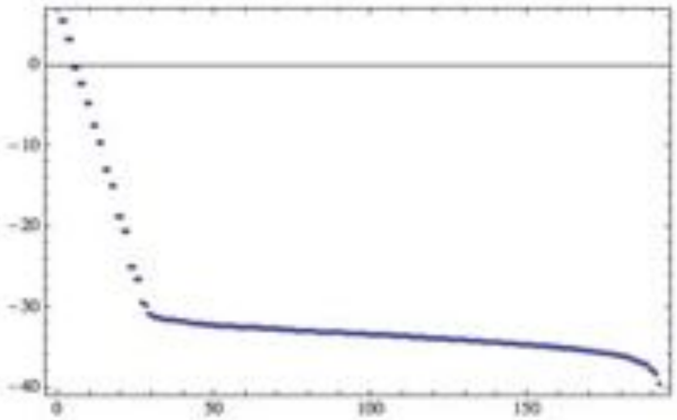
Region of
interest

Operator

$$i\hbar r \times \nabla$$

Eigenvalue distribution
(semi-log scale):





Orbital angular momentum transfer

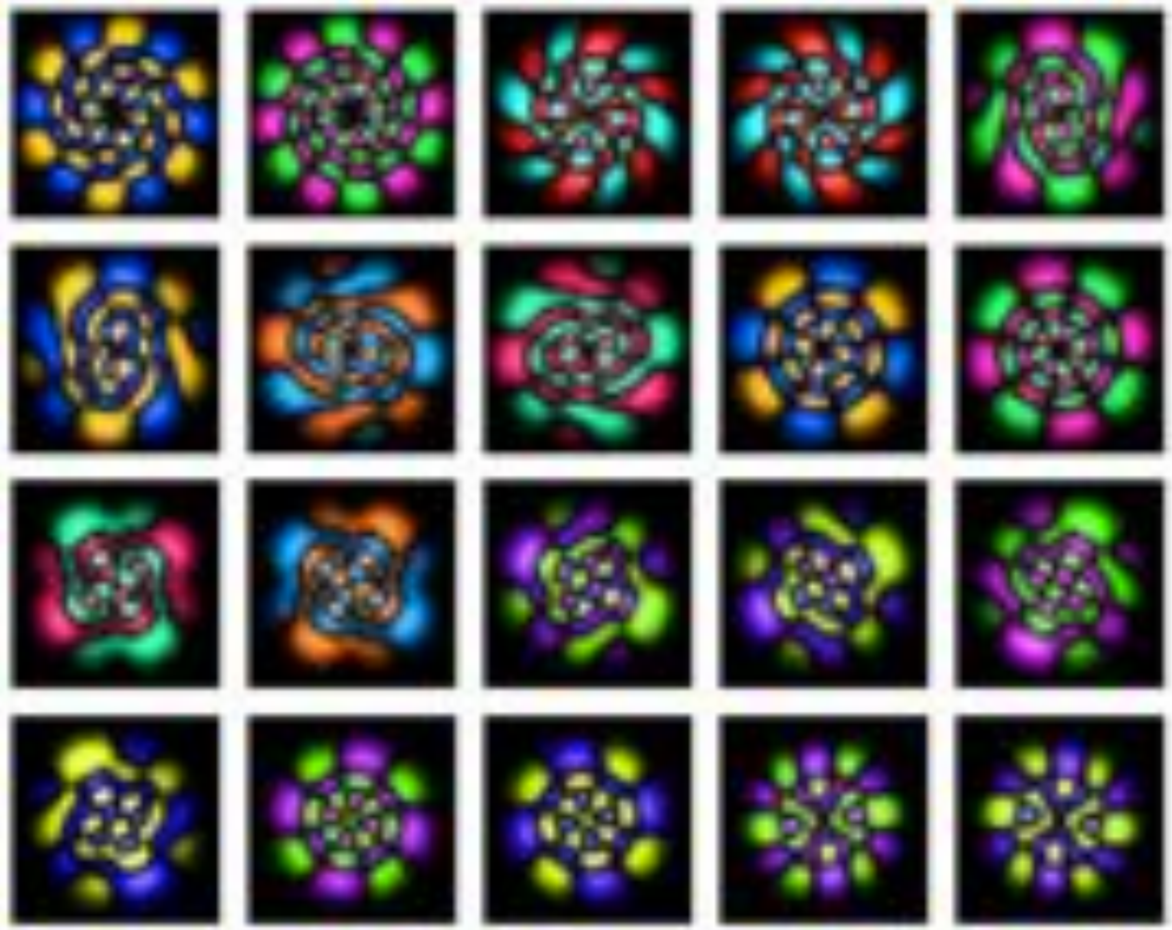
Optical force on nanoparticles α=1

$$< F_i >_{Rayleigh} = \frac{\epsilon_0 \epsilon_h}{2} \Re(\alpha E_j \partial_i E_j^*)$$

Optical Momentum:

$$\mathbf{M} = \mathbf{r} \wedge \mathbf{F}$$

Operator



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Orbital angular momentum transfer

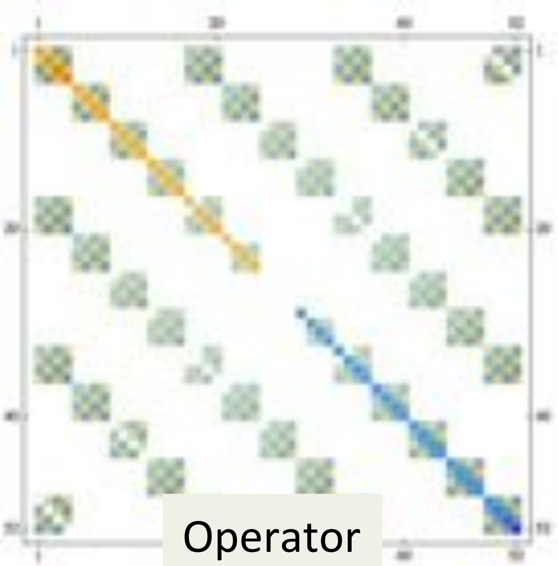
Optical force on nanoparticles

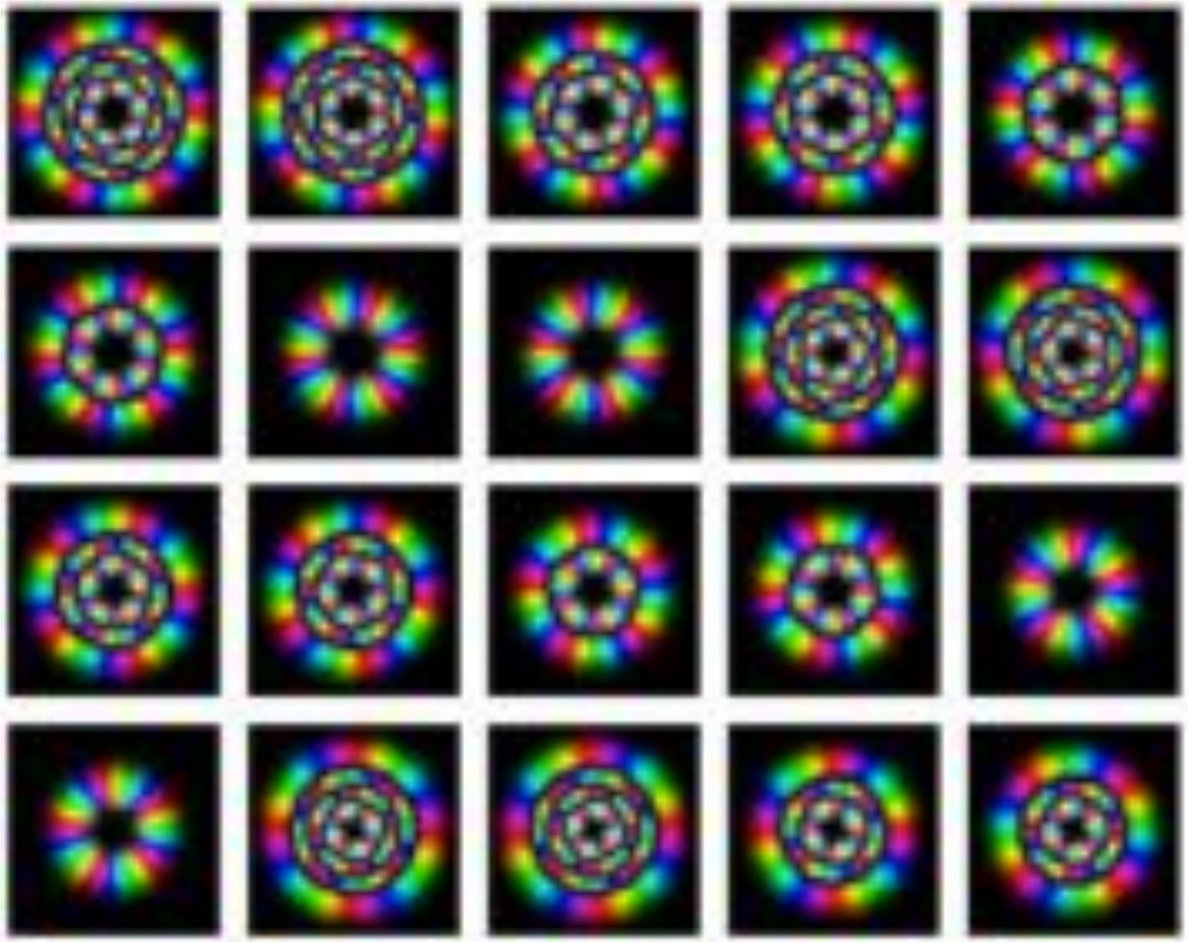
α=1+i

$$< F_i >_{Rayleigh} \;=\; \frac{\epsilon_0 \epsilon_h}{2} \Re(\alpha E_j \partial_i E_j^*)$$

Optical Momentum:

$$\mathbf{M} = \mathbf{r} \wedge \mathbf{F}$$





Spin angular momentum transfer

Optical force on nanoparticles including polarisation spin

$$\boldsymbol{\tau}\cdot\hat{\mathbf{z}} \quad \propto \quad (\mathbf{r}\times <\mathbf{E}\times\mathbf{H}>)\cdot\hat{\mathbf{z}}$$
$$\propto \quad \frac{\omega l}{\mu_0}|u(LG_l^p)|^2 - \frac{\omega r(a_x a_y^* - a_x^* a_y)}{2\mu_0}\partial_r|u(LG_l^p)|^2$$

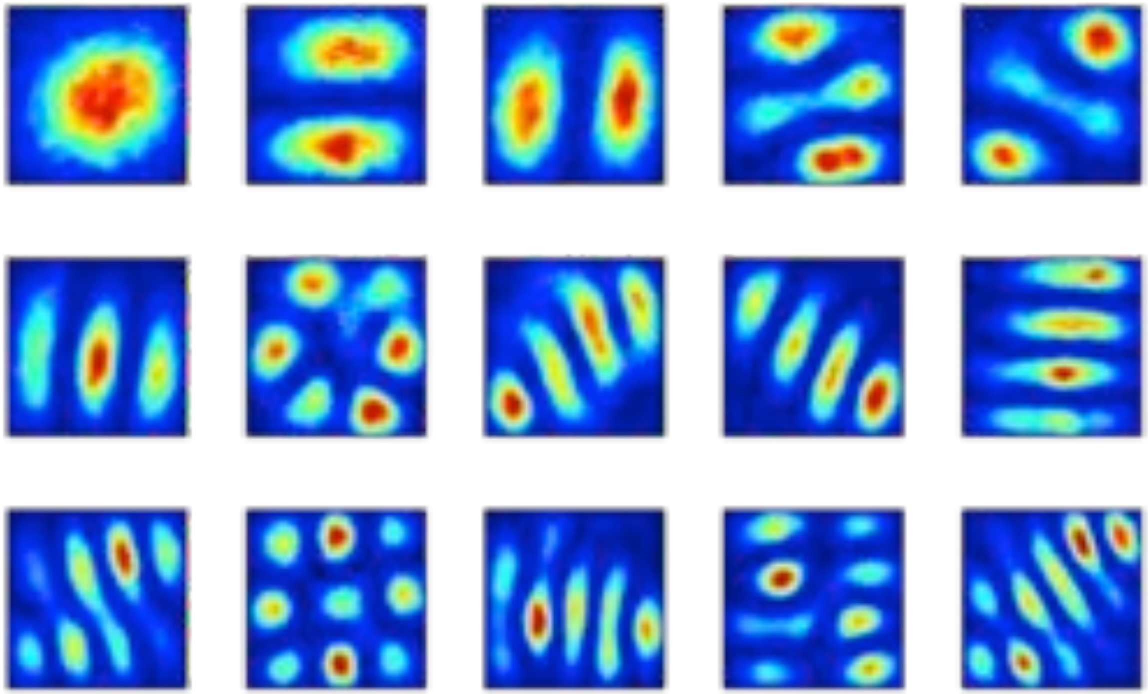
Polarisation spin

Eigenmodes $(a_x, a_y) = (1, -i)$ $(a_x, a_y) = (1, i)$

Circular polarisation

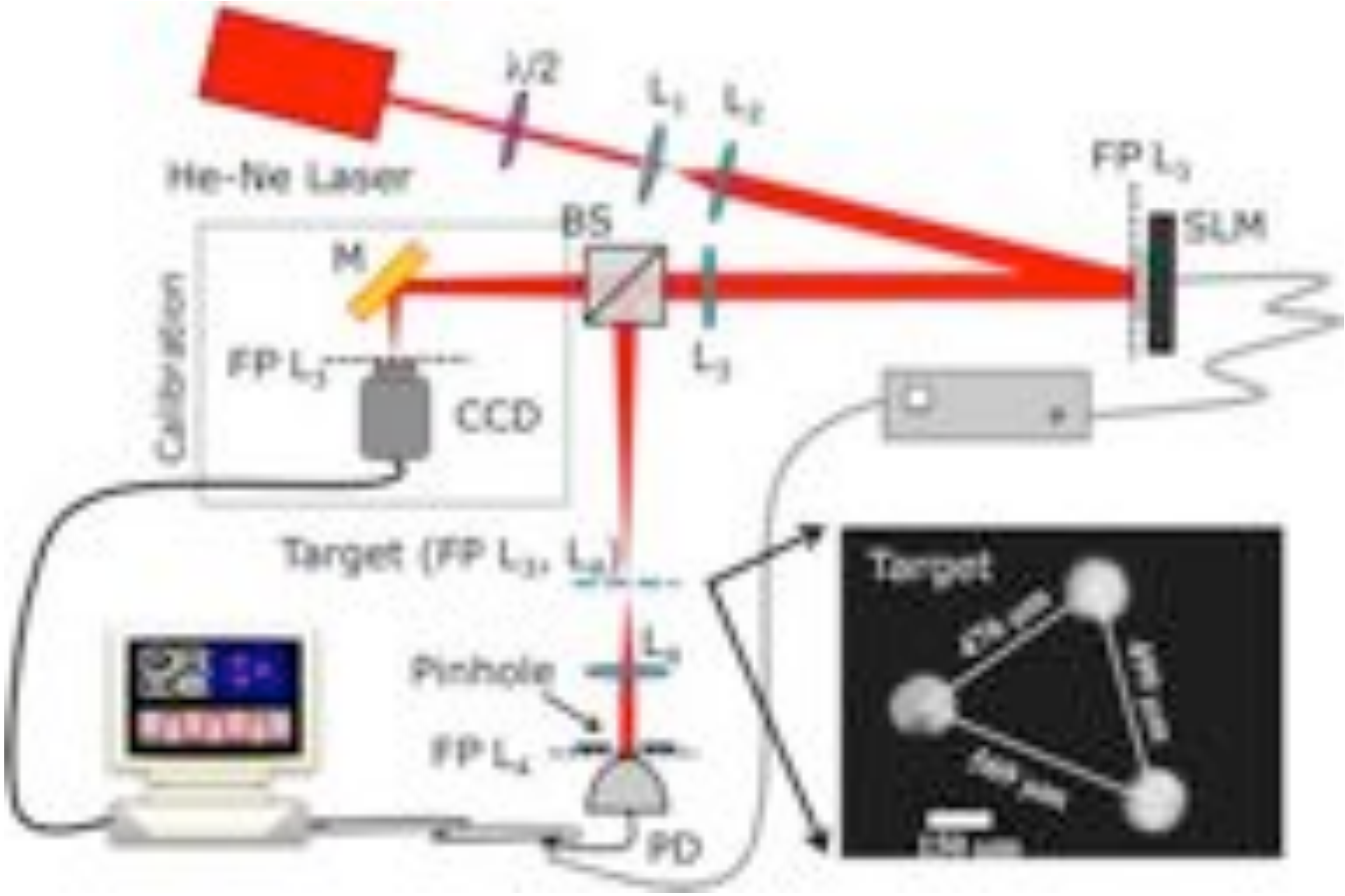
Optical eigenmode imaging & vortex creation

These modes represent a decomposition of the field into orthogonal fields with respect to one or multiple “quadratic” measures.



Orthogonal eigenmode

Experimental setup



De Luca et al., arXiv:1105.5949 [physics.optics]

Eigemode decomposition

Experimental measure of the intensity operator

$$M_{jk} = \int_{\mathrm{ROI}} d\sigma \mathrm{E}_j \cdot \mathrm{E}_k^*.$$

Normalization of the eigenmodes:

$$\mathbb{E}_\ell = \frac{1}{\sqrt{\lambda^\ell}} \sum_j v_{\ell j}^* \mathrm{E}_j$$

with:

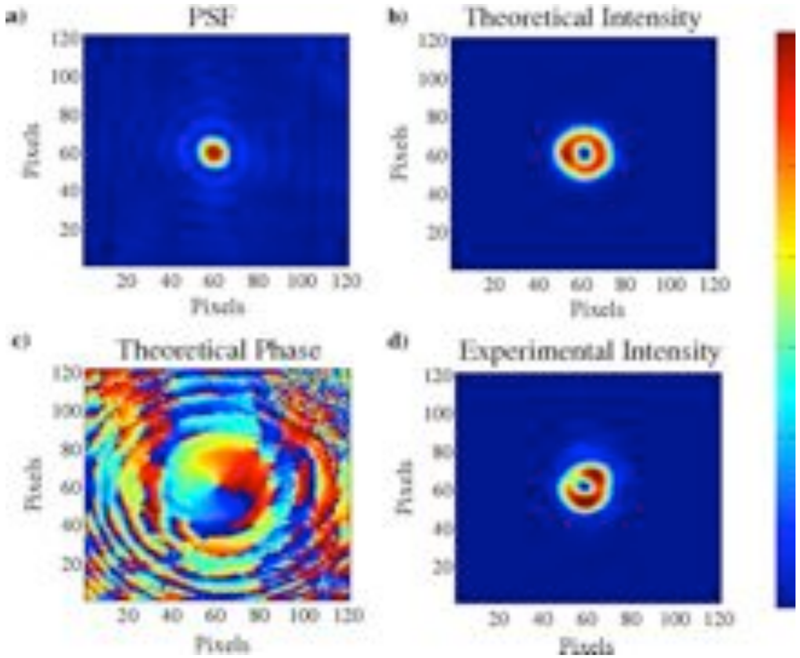
$$\sum_j M_{jk} v_{\ell j} = \lambda^\ell v_{\ell k}$$

Orthonormal:

$$\int_{\mathrm{ROI}} d\sigma\ \mathbb{E}_j \cdot \mathbb{E}_k^* = \delta_{jk}$$

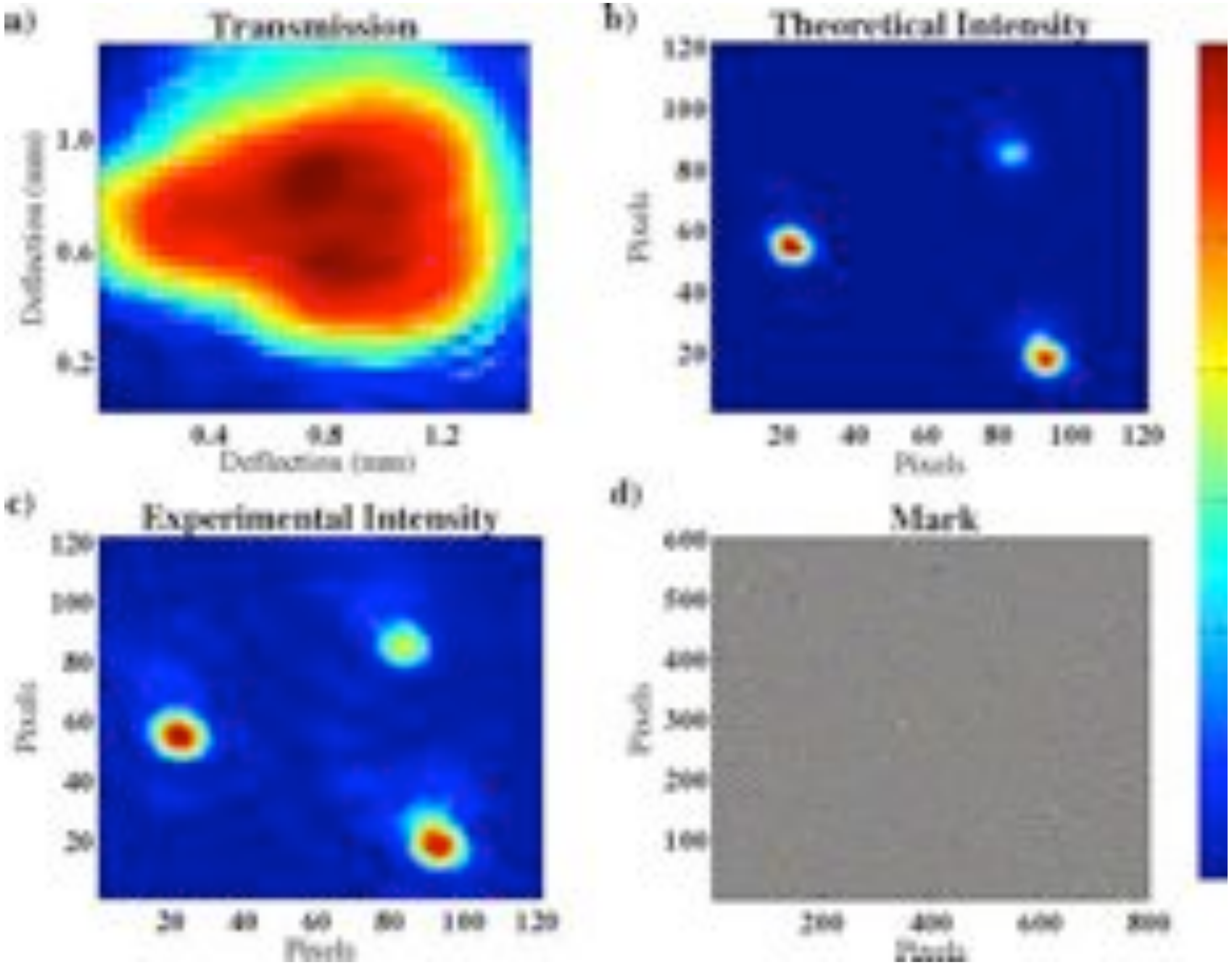
Decomposition of the target field T:

$$c_\ell^* = \int_{\mathrm{ROI}} d\sigma\ \mathrm{T} \cdot \mathbb{E}_\ell^*.$$



Experimental indirect imaging

(a) Conventional transmission image of the target reconstructed from the intensity signal collected by the PD as a function of the beam displacement in the target plane.
(b) Corresponding numerical indirect intensity Image of the target;
(c) Experimental indirect optical eigenmode image;
(d) Final mask encoded on the SLM.



First order correlation function

Experimental measure of the intensity operator

$$M_{jk} = \int_{\mathrm{ROI}} d\sigma \mathrm{E}_j \cdot \mathrm{E}_k^*.$$

Normalization of the eigenmodes:

$$\mathbb{E}_\ell = \frac{1}{\sqrt{\lambda^\ell}} \sum_j v_{\ell j}^* \mathrm{E}_j$$

with:

$$\sum_j M_{jk} v_{\ell j} = \lambda^\ell v_{\ell k}$$

$$G^{(1)}(\tau) = \int_{\mathrm{ROI}} \langle \mathrm{E}(t)\mathrm{E}^*(t+\tau)\rangle \, d\sigma$$
$$= e^{-i\omega\tau} \left\langle \sum_{j,k} a_j^* M_{jk} a_k \right\rangle = \sum_{jk} G_{jk}^{(1)}(\tau) \quad (12)$$

with

$$G_{jk}^{(1)}(\tau) = \int_{\mathrm{ROI}} \langle \mathrm{E}_j(t)\mathrm{E}_k^*(t+\tau)\rangle \, d\sigma = e^{-i\omega\tau}\delta_{jk} \quad (13)$$

Physical meaning of optical eigenmodes

$$\hat{m}^{(I)} = \int_{ROI} d\sigma \; \hat{E}^{(-)}(\mathbf{r})\hat{E}^{(+)}(\mathbf{r})$$

where $\hat{E}^{(-)}(\mathbf{r}) = \hat{a}_j^{\dagger} E_j(\mathbf{r}) e^{i\omega t}$ and $\hat{E}^{(+)}(\mathbf{r}) = \hat{a}_k E_k^{*}(\mathbf{r}) e^{-i\omega t}$

are the field operators

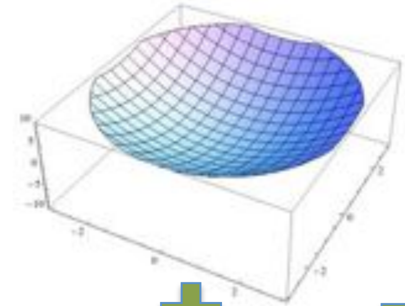
$$\hat{\mathbb{A}}^{\ell} = \frac{1}{\sqrt{\lambda^{\ell}}} \left(v_j^{\ell}\right)^{*} \hat{a}^{j} \quad ; \quad \hat{\mathbb{A}}_{\ell}^{\dagger} = \frac{1}{\sqrt{\lambda_{\ell}}} v_{\ell}^{j} \hat{a}_j^{\dagger}.$$

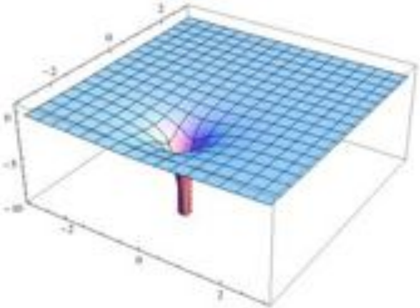
$$\hat{m}^{(I)} = \hat{\mathbb{A}}_j^{\dagger} \hat{\mathbb{A}}_j$$

Probing fields with singularities

External field

U_0

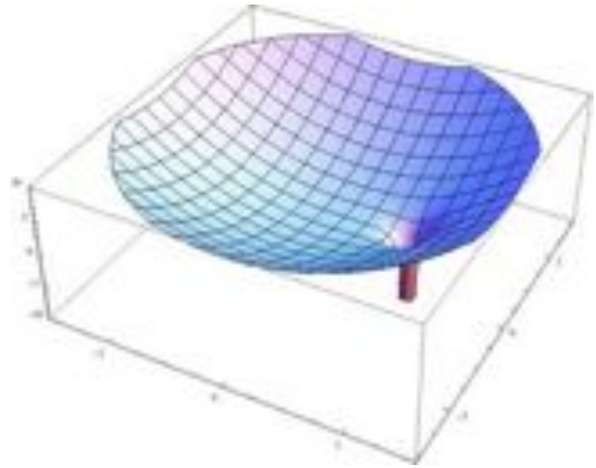




U_s

1/r field singularity

Interaction minimising the total action of the system.



$$\mathcal{S}_{U_0+U_s} = \int d^3x\, dt\, \mathcal{L}(U_0 + U_s)$$

Action

Example: Scalar field

$$\mathcal{S}_U = \int \mathcal{L}(U)\, d^3x\, dt = \frac{1}{2}\int \frac{1}{c^2}(\partial_t U)^2 - (\boldsymbol{\nabla} U)^2\, d^3x\, dt$$

$$\boldsymbol{\nabla}^2 U - \frac{1}{c^2}\partial_t^2 U = 0$$

+

$$\boldsymbol{\nabla}^2 U_s - \frac{1}{c^2}\partial_t^2 U_s = m_s \delta(\mathbf{r})$$

Lorentz transformation

$$\boldsymbol{\nabla}^2 U_s - \frac{1}{c^2}\partial_t^2 U_s = m_s \sqrt{1 - \mathbf{v}^2/c^2}\,\delta(\mathbf{r} - \mathbf{r}_s)$$

Euler equations

$$\frac{d}{dt} m_i \mathbf{v} = -m_g \boldsymbol{\nabla} U_0$$

with:

$$m_i \propto m_s$$

$$m_g \propto m_s$$

Lorentz force

$$\mathcal{S}_{em} = \int \left(\frac{\epsilon_0}{2} \mathbf{E} \cdot \mathbf{E} - \frac{1}{2\mu_0} \mathbf{B} \cdot \mathbf{B} \right) \, d^3x \, dt$$

$$\begin{aligned} \mathbf{E} &= \mathbf{E_q} + \mathbf{E_0} \\ \mathbf{B} &= \mathbf{B_q} + \mathbf{B_0} \end{aligned}$$

$$\begin{aligned} \boldsymbol{\nabla}' \times \mathbf{E}'_{\mathbf{q}} + \partial_{t'} \mathbf{B}'_{\mathbf{q}} &= 0 \\ \frac{1}{\mu_0} \boldsymbol{\nabla}' \times \mathbf{B}'_{\mathbf{q}} - \epsilon_0 \partial_{t'} \mathbf{E}'_{\mathbf{q}} &= q \mathbf{v} \delta(\mathbf{r}' - \mathbf{r}'_q) \\ \boldsymbol{\nabla}' \cdot \epsilon_0 \mathbf{E}'_{\mathbf{q}} &= q \delta(\mathbf{r}' - \mathbf{r}'_q) \\ \boldsymbol{\nabla}' \cdot \mathbf{B}'_{\mathbf{q}} &= 0 \end{aligned}$$

Euler equations
including inertial terms

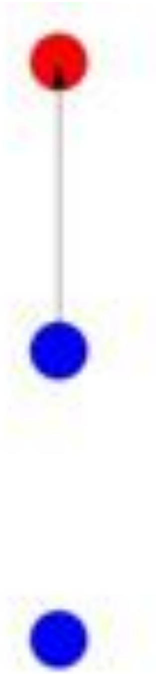
$$\frac{d}{dt} m_i \mathbf{v} = - m_g \boldsymbol{\nabla} U_0 + q \mathbf{E_0} + q \mathbf{v} \times \mathbf{B_0}$$

Lorentz force

(using real fields)

Possible spin probe

Two dipoles out of phase at the same optical frequency as the incident plane wave.



$$E_{12}(\mathcal{F}, \mathbb{A}_s\mathcal{F}) = \frac{\mathrm{i}}{2c}(\mathbf{E}^* \cdot \mathbf{H} - \mathbf{H}^* \cdot \mathbf{E}),$$

$$\mathbf{S}_{12}(\mathcal{F}, \mathbb{A}_s\mathcal{F}) = \frac{\mathrm{i}c}{2}(\epsilon_0 \mathbf{E}^* \times \mathbf{E} + \mu_0 \mathbf{H} \times \mathbf{H}^*),$$

$$\tilde{\sigma}_{12}(\mathcal{F}, \mathbb{A}_s\mathcal{F}) = \frac{\mathrm{i}c}{2}((\mathbf{E}^* \cdot \mathbf{H} - \mathbf{H}^* \cdot \mathbf{E})\tilde{I} - \mathbf{E}^* \otimes \mathbf{H} - \mathbf{H} \otimes \mathbf{E}^* + \mathbf{H}^* \otimes \mathbf{E} + \mathbf{E} \otimes \mathbf{H}^*).$$

Spin symmetry

$$\mathbb{A}_s = \begin{pmatrix} 0 & \mathrm{i}Z_0 \\ -\mathrm{i}/Z_0 & 0 \end{pmatrix}$$

Field spin loss :

$$(-\mathbf{E}^* \wedge \mathbf{p} - \mathbf{E} \wedge \mathbf{p}^*) \cdot \hat{\mathbf{k}}$$

Torque:

$$(\mathbf{E}^* \wedge \mathbf{p} + \mathbf{E} \wedge \mathbf{p}^*) \cdot \hat{\mathbf{k}}$$

Summary

We applied the concept of finite optical eigenmode to:

- optimise superposition of beams
- decompose light fields in orthogonal modes
- image and create vortex fields
- introduced probes for the conserved currents

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