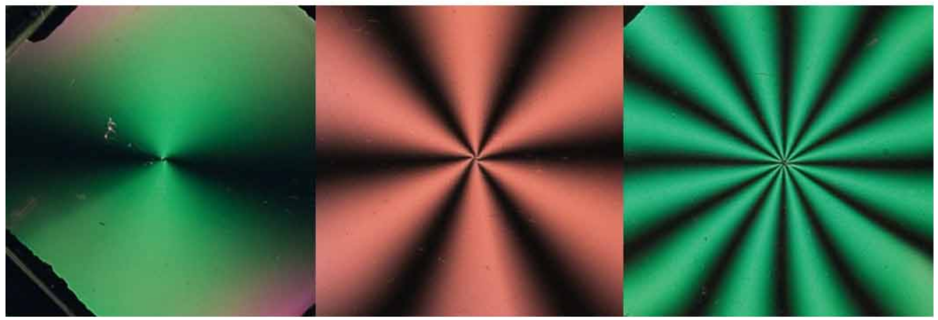
The Abdus Salam
International Centre for Theoretical Physics

Workshop on Singular Optics and its Applications to Modern Physics



q-plates: some classical and quantum applications

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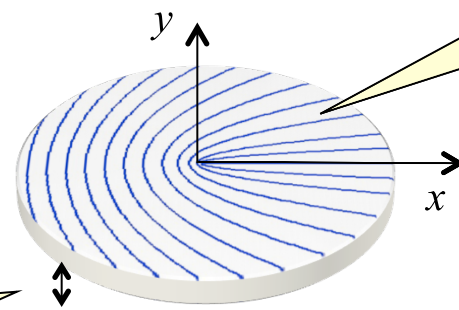
Outline:

- A brief reminder on the q-plate concept and operation principles
- Let’s “go quantum”: single photons with OAM
- q-plate effect on single photons
- A “quantum interface”: Quantum information transfer SAM ↔ OAM
- Coherent unitary mapping SAM ↔ OAM
- Generating a 2-photon quantum state with OAM correlations
- Quantum information transfer: some examples of applications
- Moving further up in the photon space dimensionality

A brief reminder on the q-plate concept and operation principles

[L. Marrucci, C. Manzo, D. Paparo, PRL **96**, 163905 (2006); APL **88**, 221102 (2006)]

q-plate structure: patterned half-wave plates

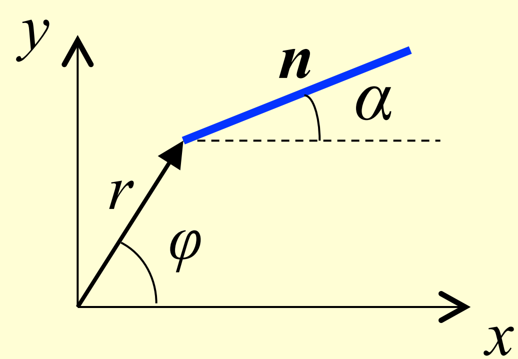


The optical axis orientation in the plate is **patterned**

$$\alpha(x,y) = \alpha(r,\varphi)$$

Cell thickness and birefringence chosen so as to have uniform **half-wave retardation**

α = angle between the local optical axis ***n*** and a reference axis



q-plate structure: patterned half-wave plates

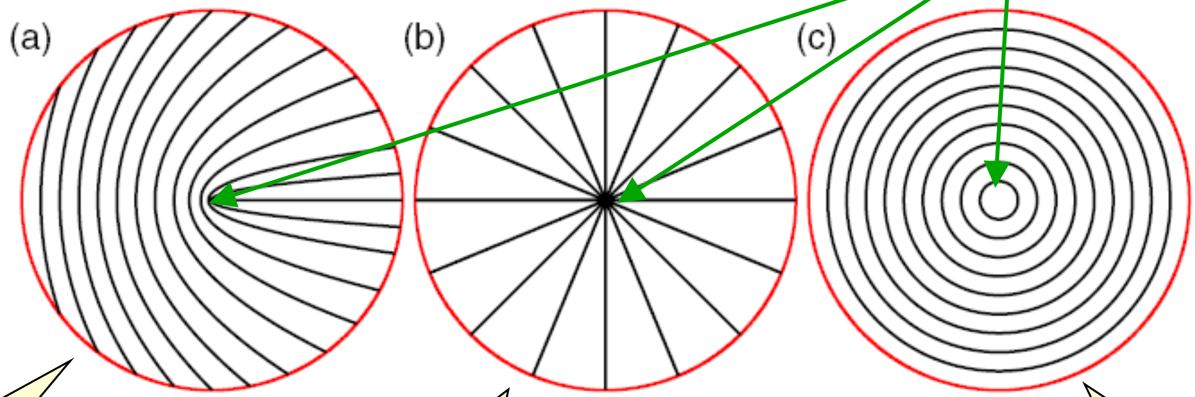
General pattern:

$$\alpha(x,y)=\alpha(r,\varphi)=q\varphi+\alpha_0$$

with q integer or half-integer

Three examples:

Topological defect of charge *q* in the center



$q = ½$

$(\alpha_0 = 0)$

$q = 1$

$(\alpha_0 = 0)$

$q = 1$

$(\alpha_0 = \pi/2)$

q-plate optical effect: Jones calculus

Jones matrix of an *α*-oriented half-wave plate: $\mathbf{M} = \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{bmatrix}$

Let us apply it to an **input left-circular** polarized plane wave:

$$\mathbf{M} \times \begin{bmatrix} 1 \\ i \end{bmatrix} E_0 = \begin{bmatrix} \cos 2\alpha + i \sin 2\alpha \\ -i \cos 2\alpha + \sin 2\alpha \end{bmatrix} E_0 = \begin{bmatrix} 1 \\ -i \end{bmatrix} e^{i2\alpha} E_0$$

The output polarization is **uniform right-handed circular**

The wave has acquired a **phase retardation**

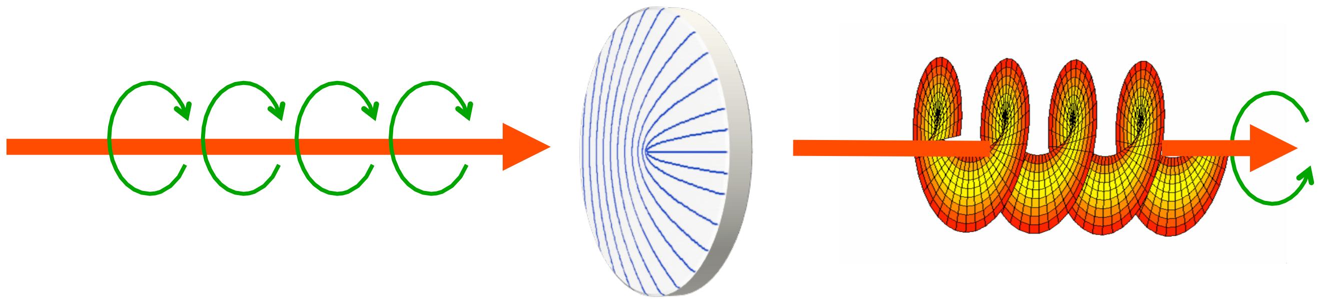
$$\Delta\Phi = 2\alpha$$

Pancharatnam-Berry geometrical phase

(unrelated with optical path length)

q-plate optical effect

For a non-uniform optical axis orientation:



The wavefront gets reshaped!

For the specific q-plate pattern:

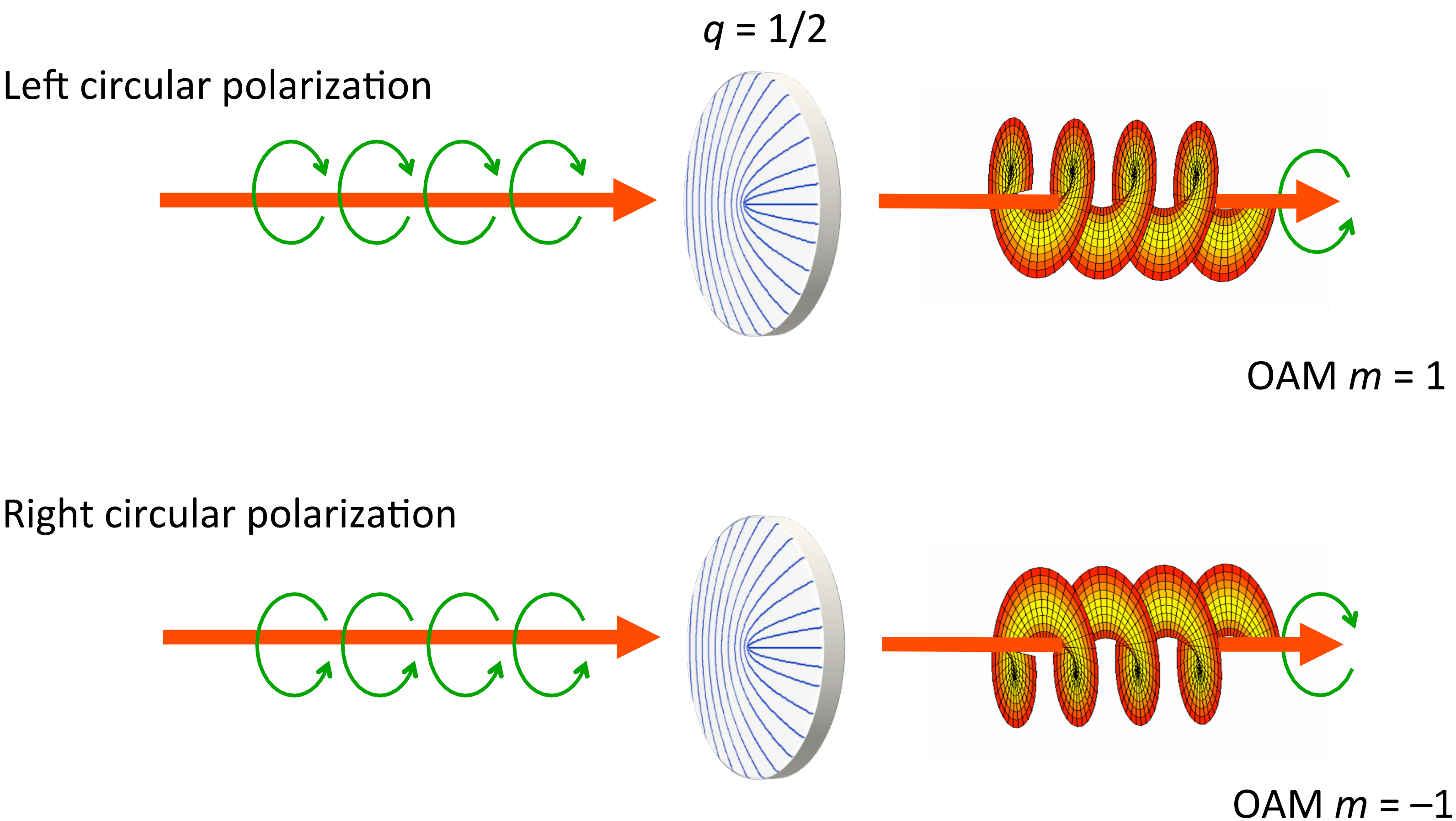
$$\alpha(r,\varphi) = q\varphi + \alpha_0$$

$$\Delta\Phi(x,y) = \pm 2\alpha = \pm 2q\varphi + \left(\pm 2\alpha_0\right) = m\varphi + \text{cost.}$$

Helical phase with $m = \pm 2q$ **!**

q-plate optical effect

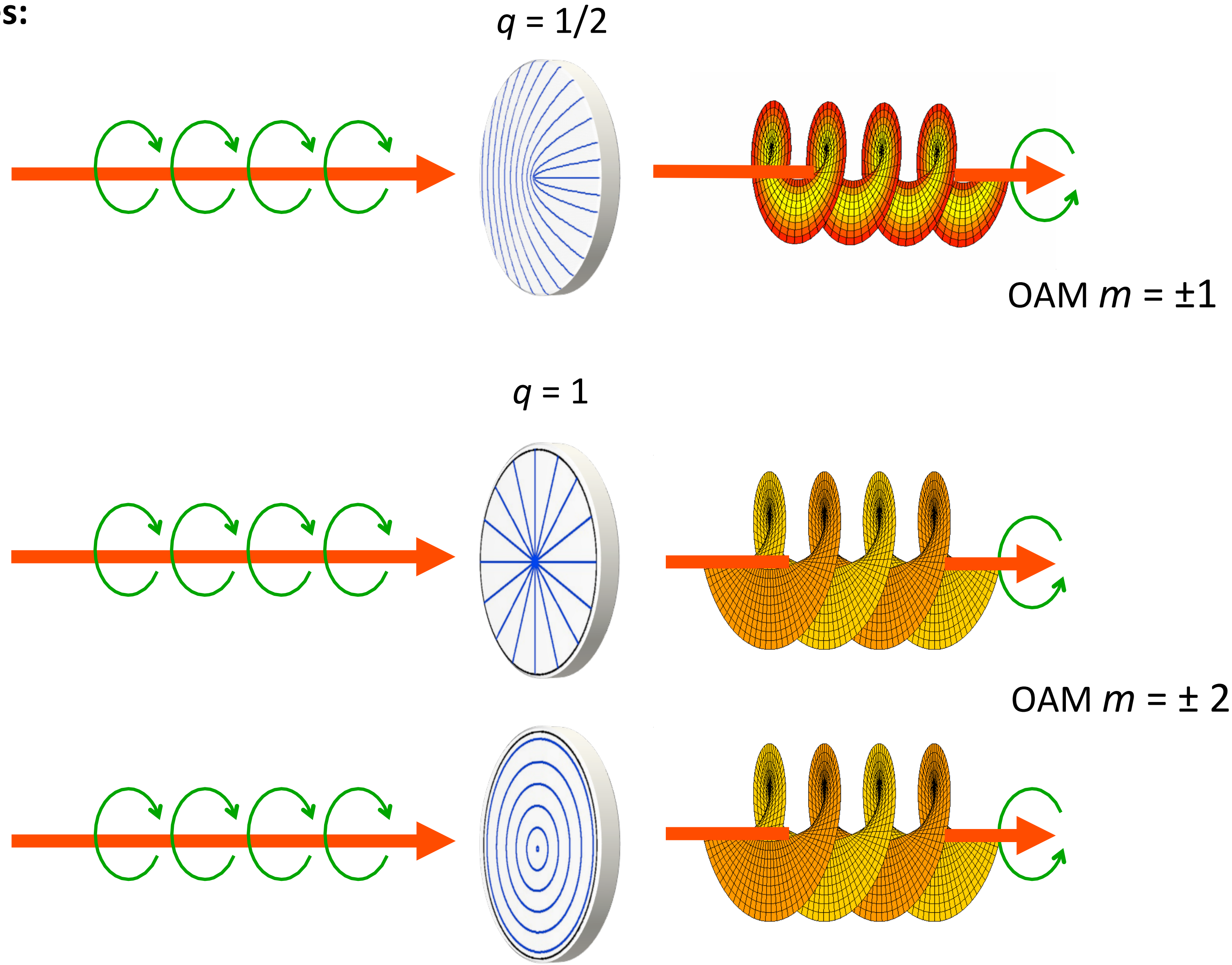
Examples:



Polarization controlled OAM handedness

q-plate optical effect

Examples:



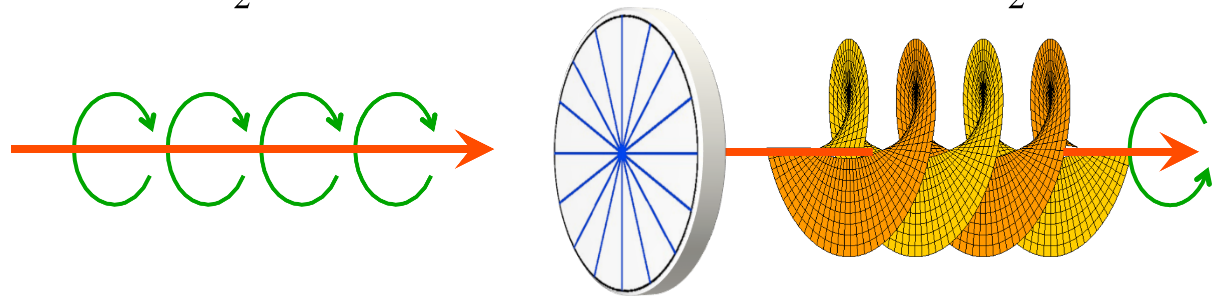
Photon angular momentum balance: case $q = 1$

Left-circular input:

Spin: $S_z = +\hbar$
Orbital: $L_z = 0$
Total: $J_z = +\hbar$

q-plate

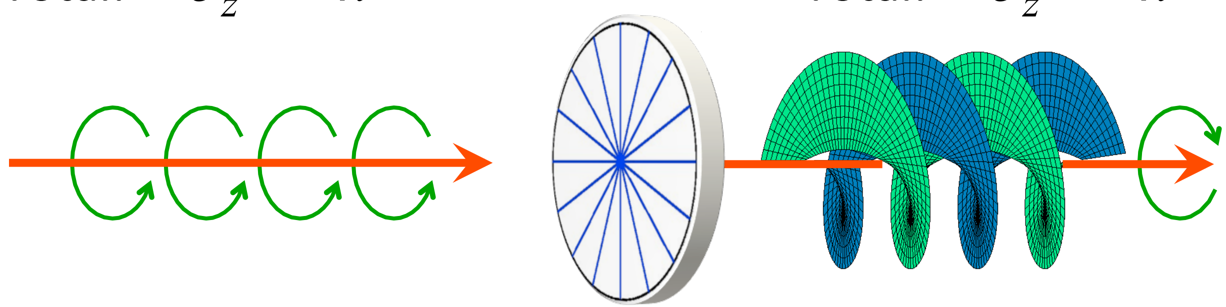
Spin: $S_z = -\hbar$
Orbital: $L_z = 2\hbar$
Total: $J_z = +\hbar$



Right-circular input:

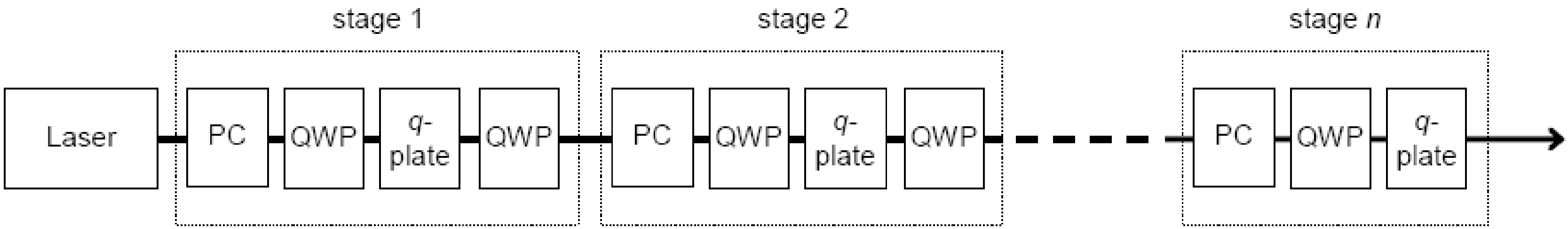
Spin: $S_z = -\hbar$
Orbital: $L_z = 0$
Total: $J_z = -\hbar$

Spin: $S_z = +\hbar$
Orbital: $L_z = -2\hbar$
Total: $J_z = -\hbar$



Spin-to-orbital conversion of optical angular momentum

Cascading q-plates



By multiple polarization control, one can access any value of OAM

In principle, the switching can be as fast as GHz rates (MHz are fairly easy)

[L. Marrucci, C. Manzo, D. Paparo, APL **88**, 221102 (2006)]

Let's "go quantum": single photons with OAM

Notice: we will actually be using the quantum language and notation also for describing optical processes which are fully within the scope of classical electromagnetism

Single photons with OAM

Notation: a photon having a given polarization (SAM) and OAM state

$$|\psi\rangle = |\mathrm{SAM}\rangle|\mathrm{OAM}\rangle = |h\rangle_\pi |m\rangle_o$$

SAM (π): a 2D space

$h = H,\ V$ (linear polarizations)
$h = L,\ R$ (circular polarizations)

OAM (*o*): an ∞D space

$$m = 0, \pm 1, \pm 2, \pm 3, \ldots$$

(Interesting for quantum information: lots of room in just one photon!)

Quantum OAM superpositions

Polarization superpositions: $$\left|\psi\right\rangle = \alpha\left|L\right\rangle_\pi + \beta\left|R\right\rangle_\pi = \alpha'\left|H\right\rangle_\pi + \beta'\left|V\right\rangle_\pi$$

A polarization "qubit"

OAM superpositions: $$\left|\psi\right\rangle = \alpha\left|+2\right\rangle_o + \beta\left|-2\right\rangle_o$$

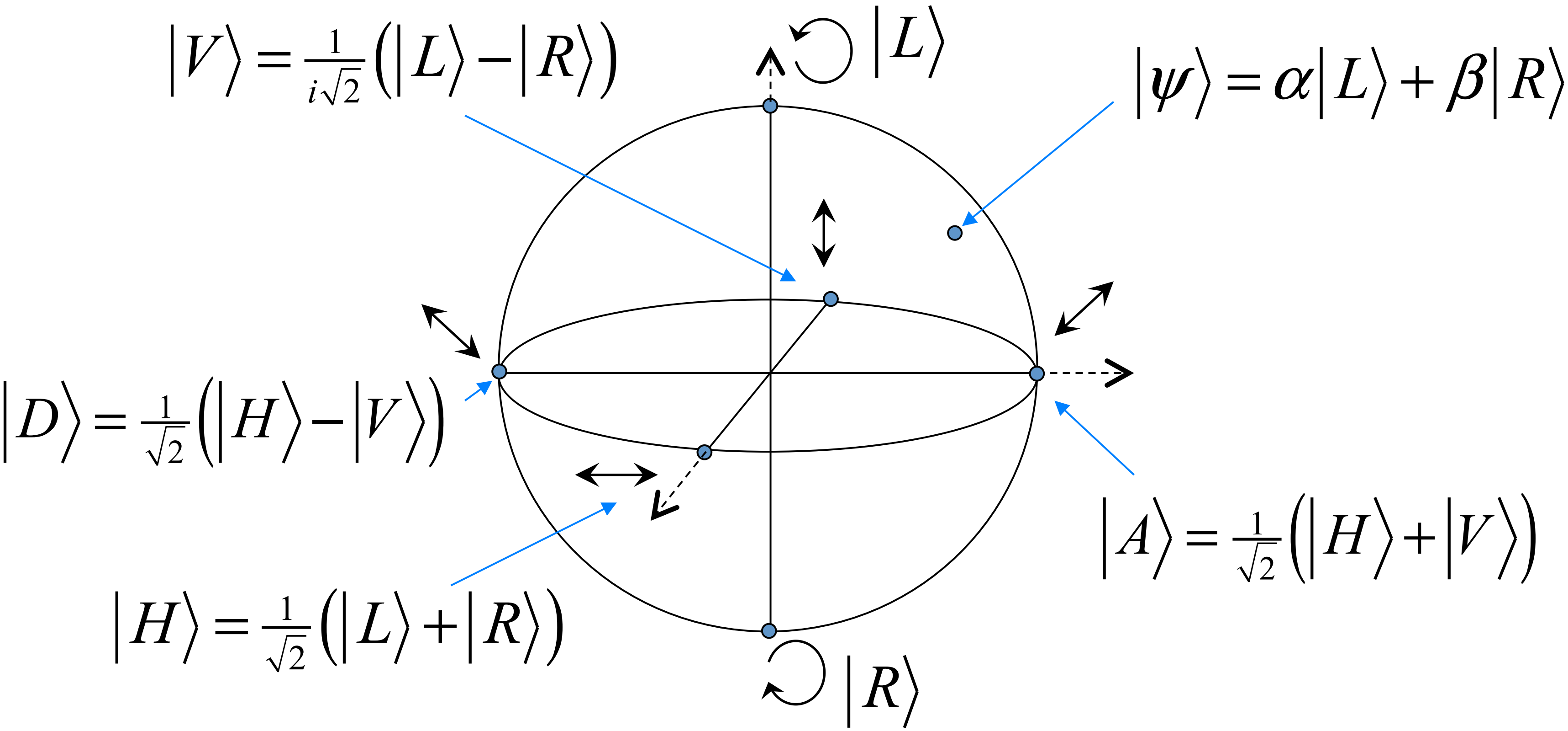
An OAM "qubit"

Higher-dimensional superpositions are also possible with OAM ("**qudits**")

In the following we will consider only OAM qubits with $m = \pm 2$

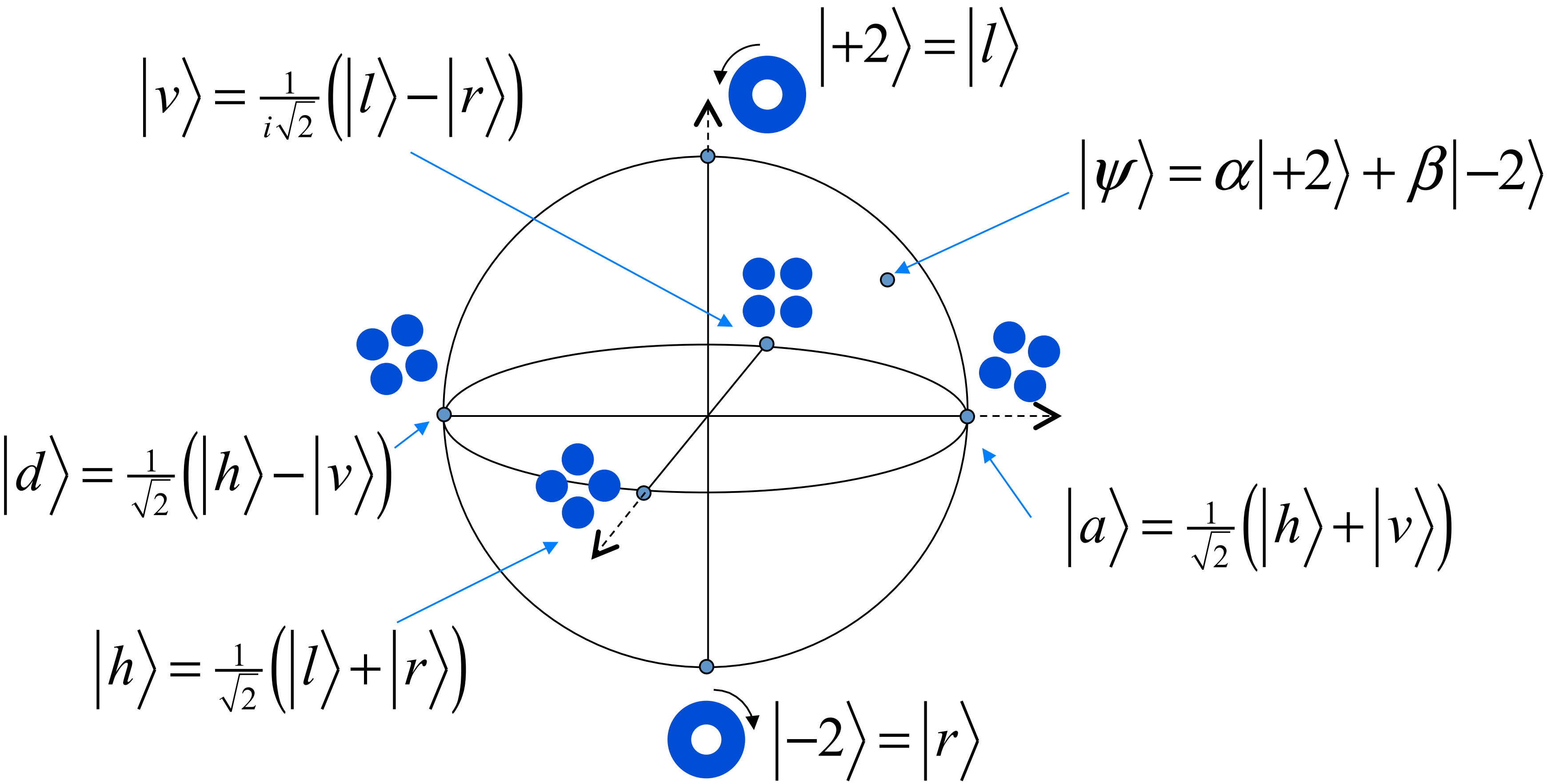
Quantum superpositions: Poincaré (or Bloch) sphere

The (well known) case of polarization:



Quantum superpositions: Poincaré-like sphere

The case of an OAM subspace (|*m*|=2):



[M. J. Padgett & J. Courtial, Opt. Lett. **24**, 430 (1999)]

What is the behavior of a *q*-plate in the quantum domain?

***q*-plate effect on single photons**

[L. Marrucci, *Proc. SPIE* **6587**, 658708 (2007)]

[E. Nagali, F. Sciarrino, F. De Martini, L. Marrucci, B. Piccirillo, E. Karimi, E. Santamato, PRL **103**, 013601 (2009)]

q-plate quantum effect on single photons

For SAM-OAM eigenstates, nothing new:

$$|L\rangle_\pi |0\rangle_o \Rightarrow$$

$$\Rightarrow |R\rangle_\pi |+2\rangle_o$$

$$|R\rangle_\pi |0\rangle_o \Rightarrow$$

$$\Rightarrow |L\rangle_\pi |-2\rangle_o$$

What happens with quantum superpositions?

q-plate quantum effect on single photons

The *q*-plate is also expected to preserve the superpositions (it is coherent):

$$|\psi\rangle = \alpha|L\rangle_\pi|0\rangle_o + \beta|R\rangle_\pi|0\rangle_o \Rightarrow$$

$$\Rightarrow \alpha|R\rangle_\pi|+2\rangle_o + \beta|L\rangle_\pi|-2\rangle_o$$

In particular for a linearly polarized input (H or V):

$$|H\rangle_\pi|0\rangle_o = \tfrac{1}{\sqrt{2}}\left(|L\rangle_\pi + |R\rangle_\pi\right)|0\rangle_o \Rightarrow$$

$$\Rightarrow \tfrac{1}{\sqrt{2}}\left(|R\rangle_\pi|+2\rangle_o + |L\rangle_\pi|-2\rangle_o\right)$$

$$|V\rangle_\pi|0\rangle_o = \tfrac{1}{i\sqrt{2}}\left(|L\rangle_\pi - |R\rangle_\pi\right)|0\rangle_o \Rightarrow$$

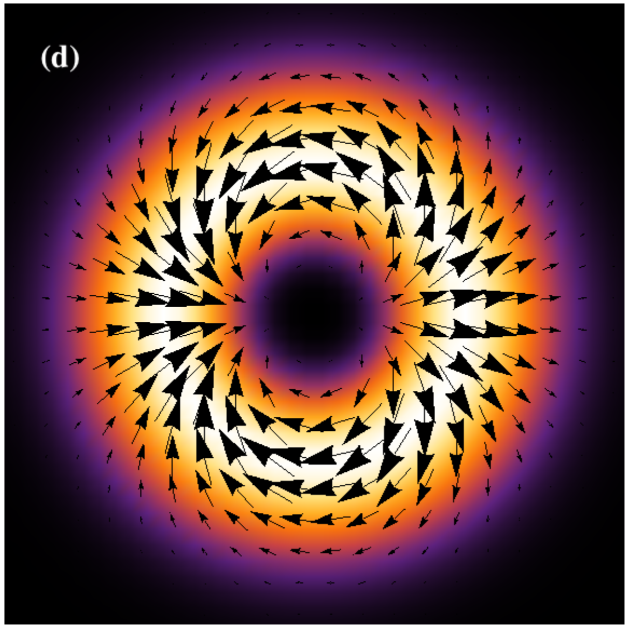
$$\Rightarrow \tfrac{1}{i\sqrt{2}}\left(|R\rangle_\pi|+2\rangle_o - |L\rangle_\pi|-2\rangle_o\right)$$

<u>Entangled state</u> of spin and orbital angular momentum of the same photon!

q-plate quantum effect on single photons

Notice: this single-photon entanglement is not a "non-local" property and can be also understood classically

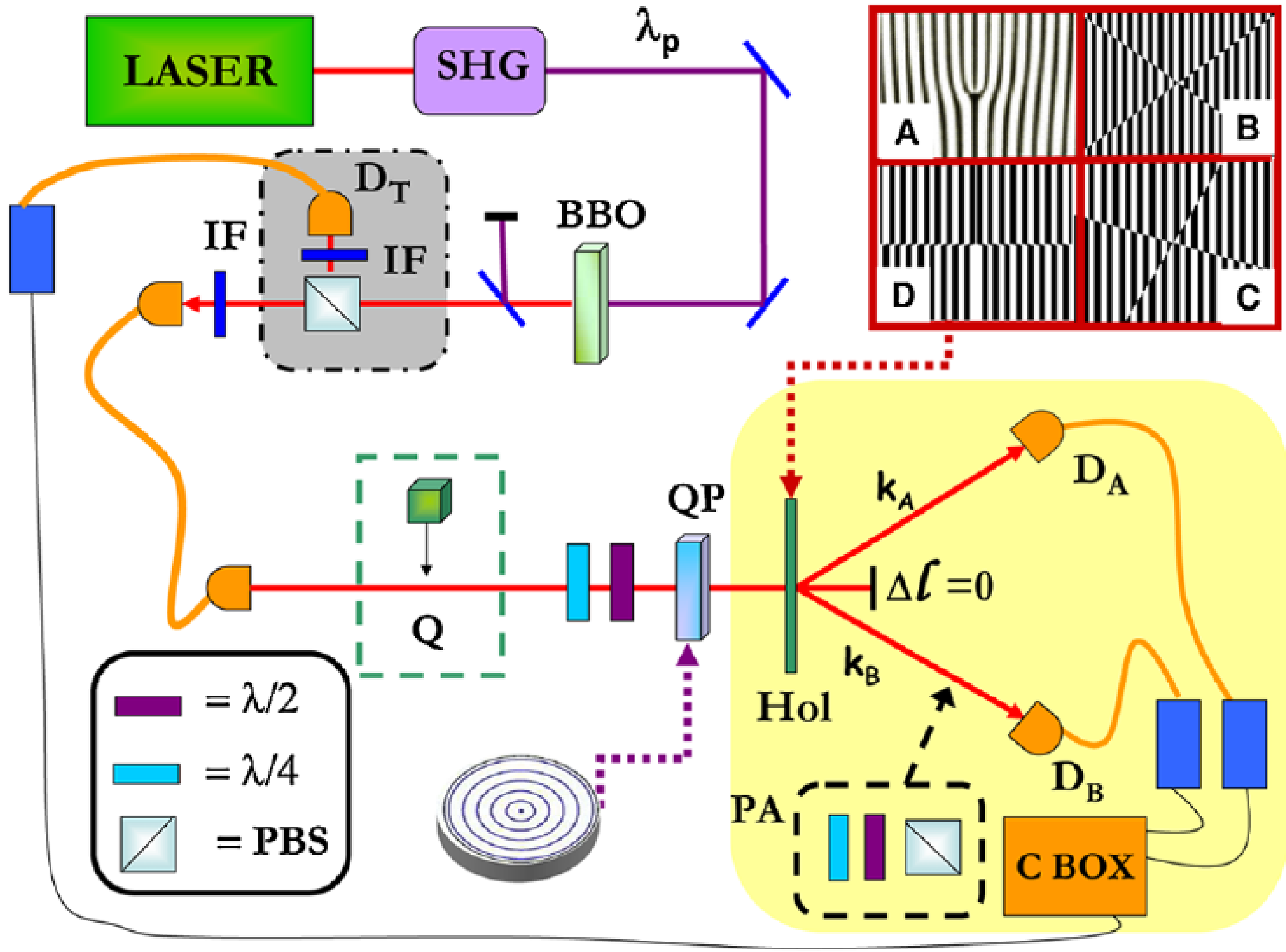
$$\frac{1}{\sqrt{2}}\left(\left|R\right\rangle_{\pi}\left|+2\right\rangle_{o}+\left|L\right\rangle_{\pi}\left|-2\right\rangle_{o}\right) \quad =$$



A non-separable polarization – spatial mode distribution

Still interesting for quantum information protocols and for making some fundamental tests on quantum mechanics

q-plate effect on single photons: experiment

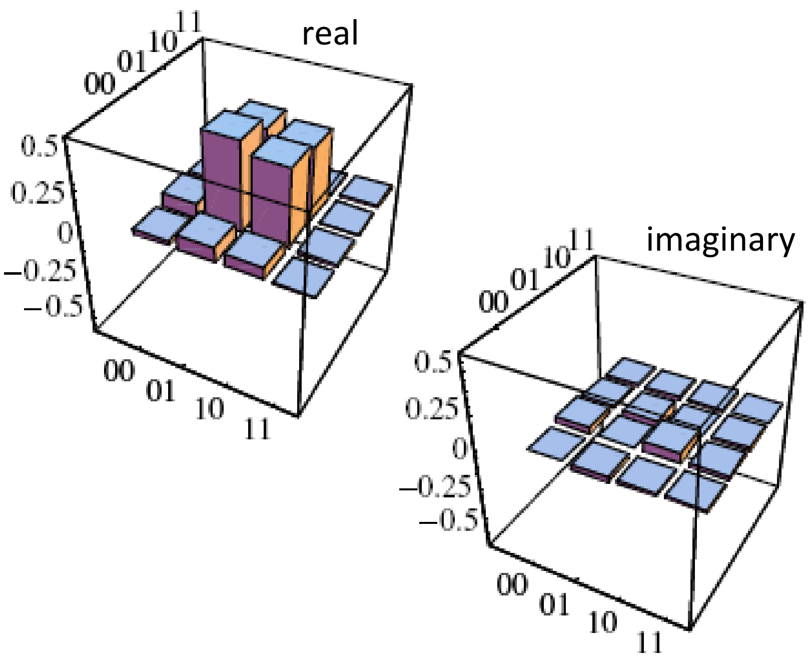


[E. Nagali, F. Sciarrino, F. De Martini, L. Marrucci, B. Piccirillo, E. Karimi, E. Santamato, PRL **103**, 013601 (2009)]

q-plate effect on single photons: experiment

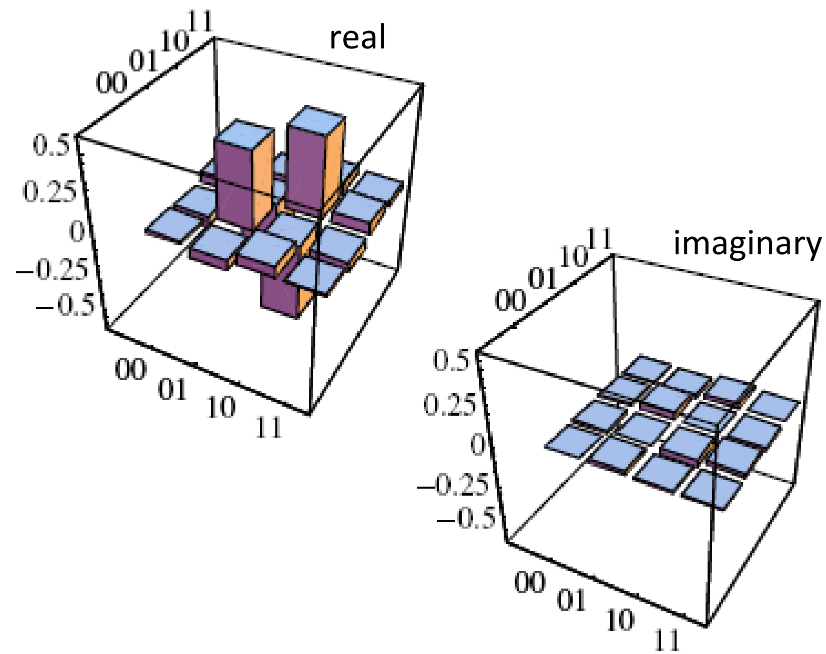
Quantum tomography of polarization-OAM entangled states

Input H photons



$$|\psi\rangle = \tfrac{1}{\sqrt{2}}\left(|R\rangle_\pi |+2\rangle_o + |L\rangle_\pi |-2\rangle_o\right)$$

Input V photons



$$|\psi\rangle = \tfrac{1}{\sqrt{2}}\left(|R\rangle_\pi |+2\rangle_o - |L\rangle_\pi |-2\rangle_o\right)$$

Single-photon entanglement confirmed

q-plate quantum effect: what can we do with it?

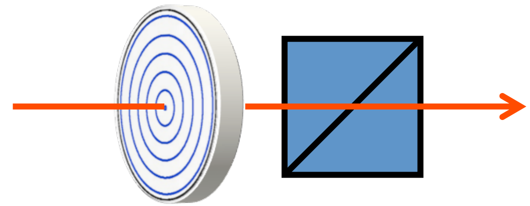
A "quantum interface":

Quantum information transfer SAM $\leftrightarrow$ OAM

[E. Nagali, F. Sciarrino, F. De Martini, L. Marrucci, et al., *Phys. Rev. Lett*. **103**, 013601 (2009)]
[E. Nagali, F. Sciarrino, F. De Martini, L. Marrucci, et al., *Opt*. *Express* **17**, 18745-18759 (2009)]

Quantum information transfer SAM ↔ OAM

1) SAM → OAM

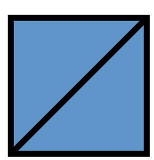


Arbitrary polarization qubit: $$|\psi\rangle_\pi = |\psi\rangle_\pi |0\rangle_o = \left(\alpha|L\rangle_\pi + \beta|R\rangle_\pi\right)|0\rangle_o$$

q-plate effect:

$$\alpha|R\rangle_\pi|+2\rangle_o + \beta|L\rangle_\pi|-2\rangle_o$$

H polarizer:



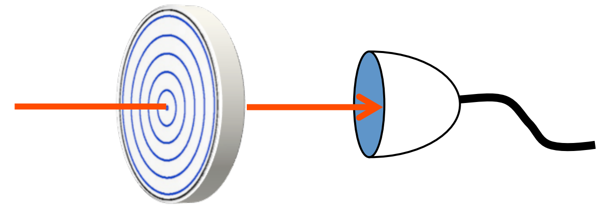
$$\tfrac{1}{\sqrt{2}}|H\rangle_\pi\left(\alpha|+2\rangle_o + \beta|-2\rangle_o\right) = \tfrac{1}{\sqrt{2}}|H\rangle_\pi|\psi\rangle_o$$

Transfer completed!

But successful only 50% of times

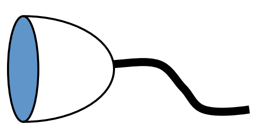
Quantum information transfer SAM ↔ OAM

2) OAM → SAM



Arbitrary OAM qubit: $$|\psi\rangle_o = |H\rangle_\pi |\psi\rangle_o = |H\rangle_\pi \left(\alpha|+2\rangle_o + \beta|-2\rangle_o\right)$$

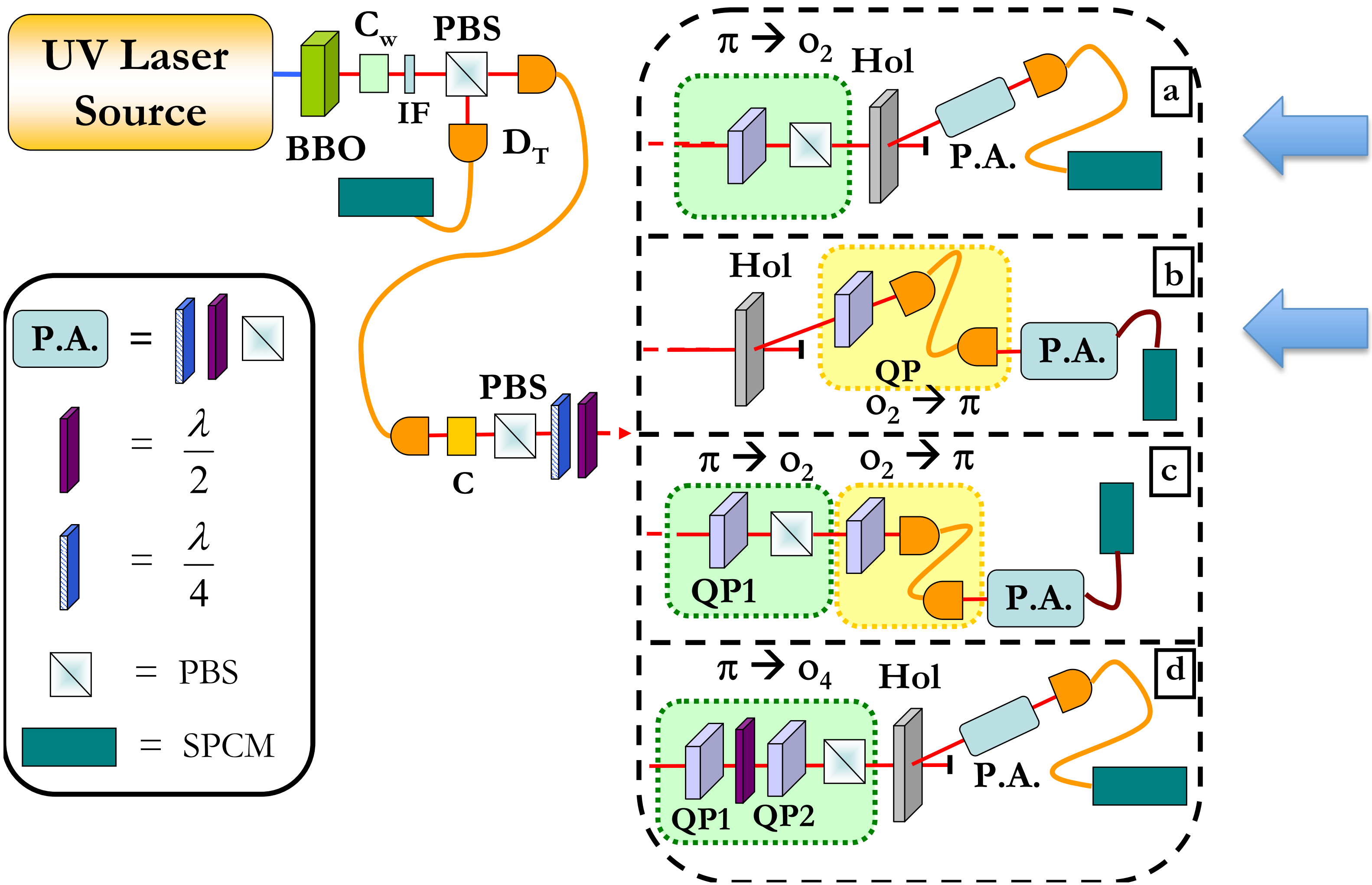
q-plate effect:  ⇒ $$\frac{\alpha}{\sqrt{2}}\left(|R\rangle_\pi|+4\rangle_o + |L\rangle_\pi|0\rangle_o\right) + \frac{\beta}{\sqrt{2}}\left(|R\rangle_\pi|0\rangle_o + |L\rangle_\pi|-4\rangle_o\right)$$

Coupling to single mode fiber:  ⇒ $$\tfrac{1}{\sqrt{2}}\left(\alpha|L\rangle_\pi + \beta|R\rangle_\pi\right)|0\rangle_o = \tfrac{1}{\sqrt{2}}|\psi\rangle_\pi|0\rangle_o$$

Transfer completed

Again successful only 50% of times

Quantum information transfer SAM ↔ OAM: the experiment

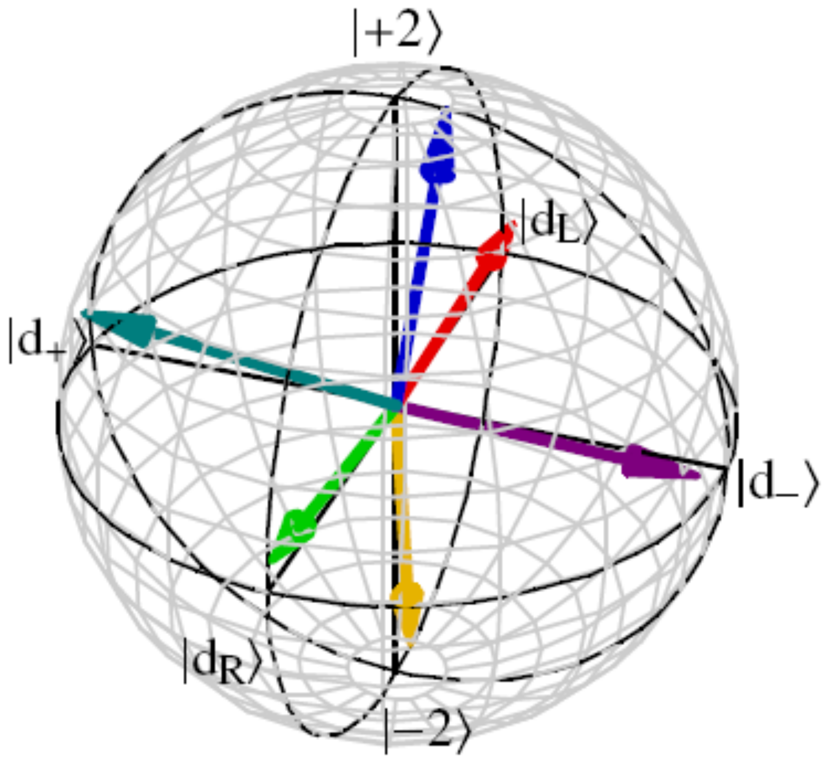


[E. Nagali, F. Sciarrino, F. De Martini, L. Marrucci, et al., *Opt. Express* **17**, 18745-18759 (2009)]

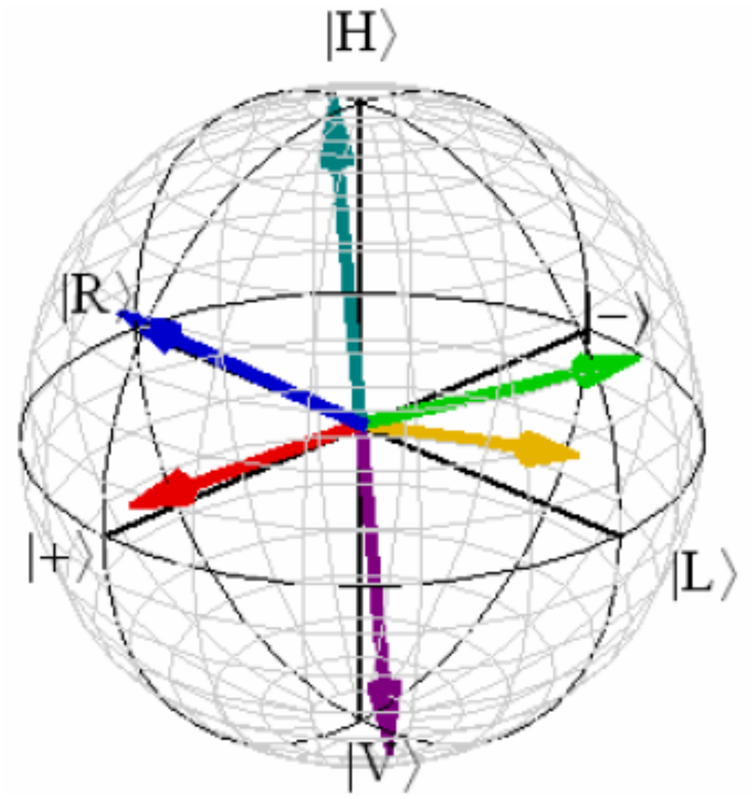
Quantum information transfer SAM ↔ OAM: the experiment

Poincaré sphere state reconstructions

SAM → OAM

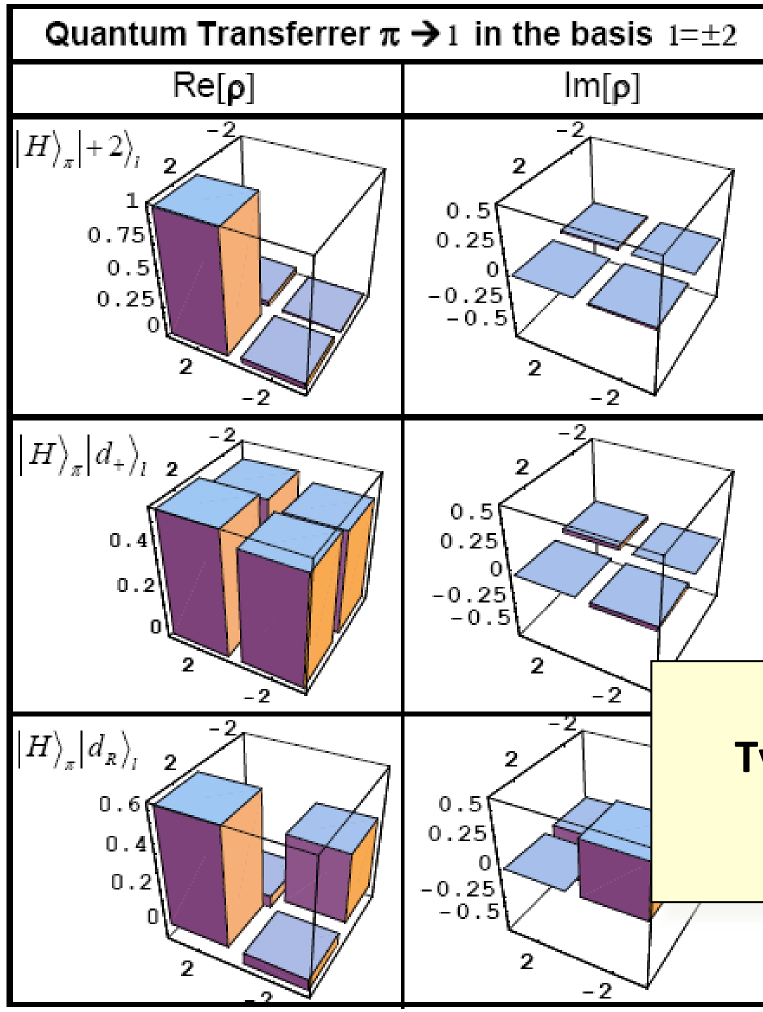


OAM → SAM



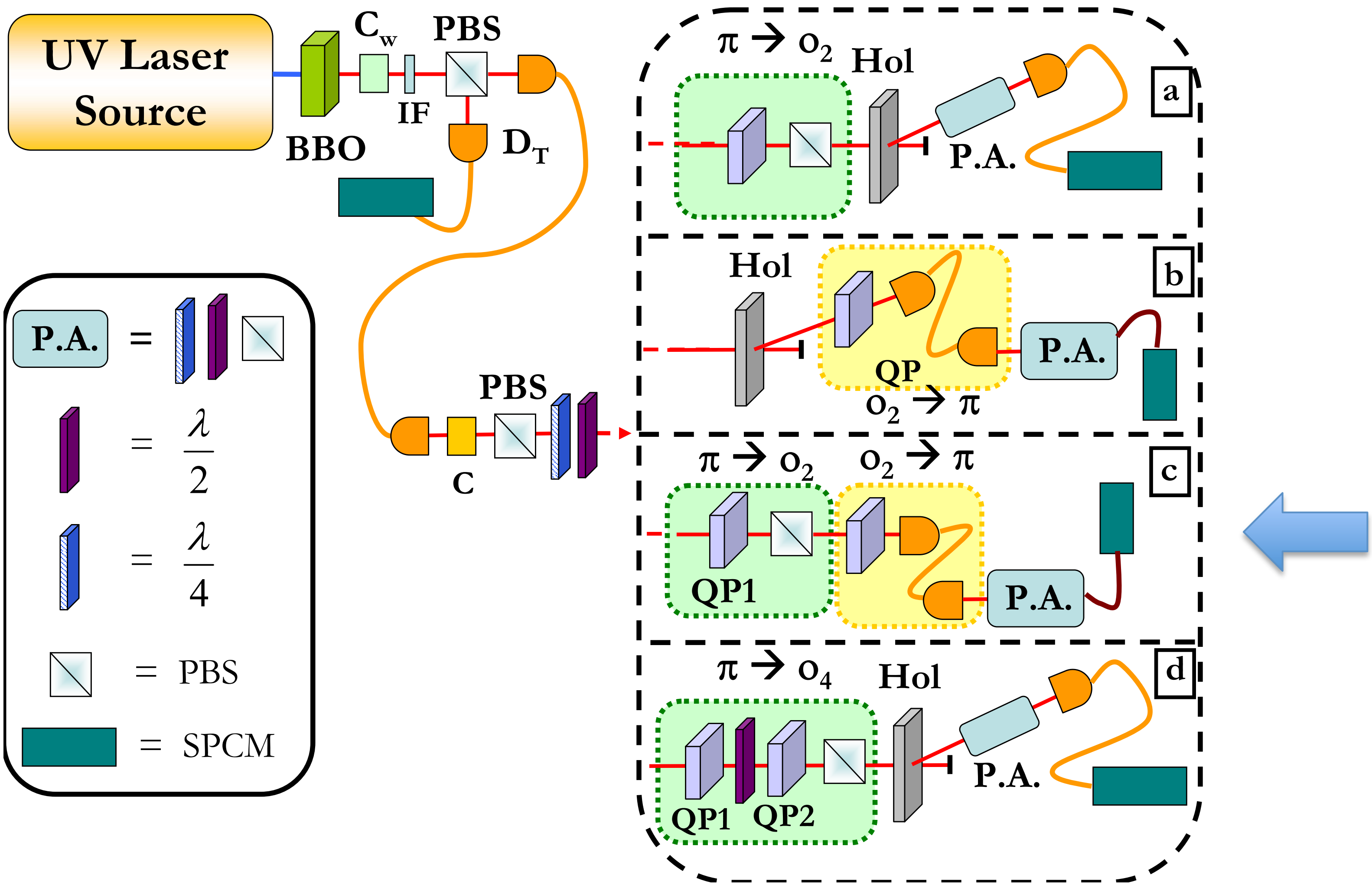
Quantum information transfer SAM ↔ OAM: the experiment

Typical quantum tomography results (SAM→OAM):



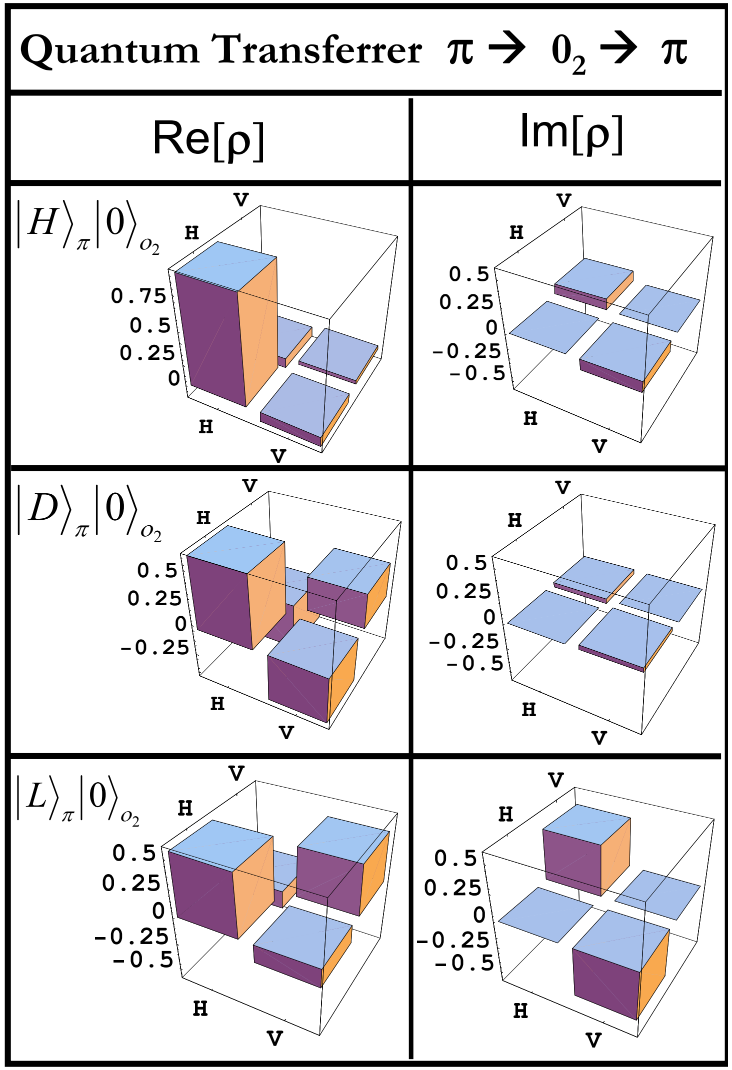
Typical experimental fidelities = 98%!

Quantum information transfer SAM ↔ OAM: back and forth

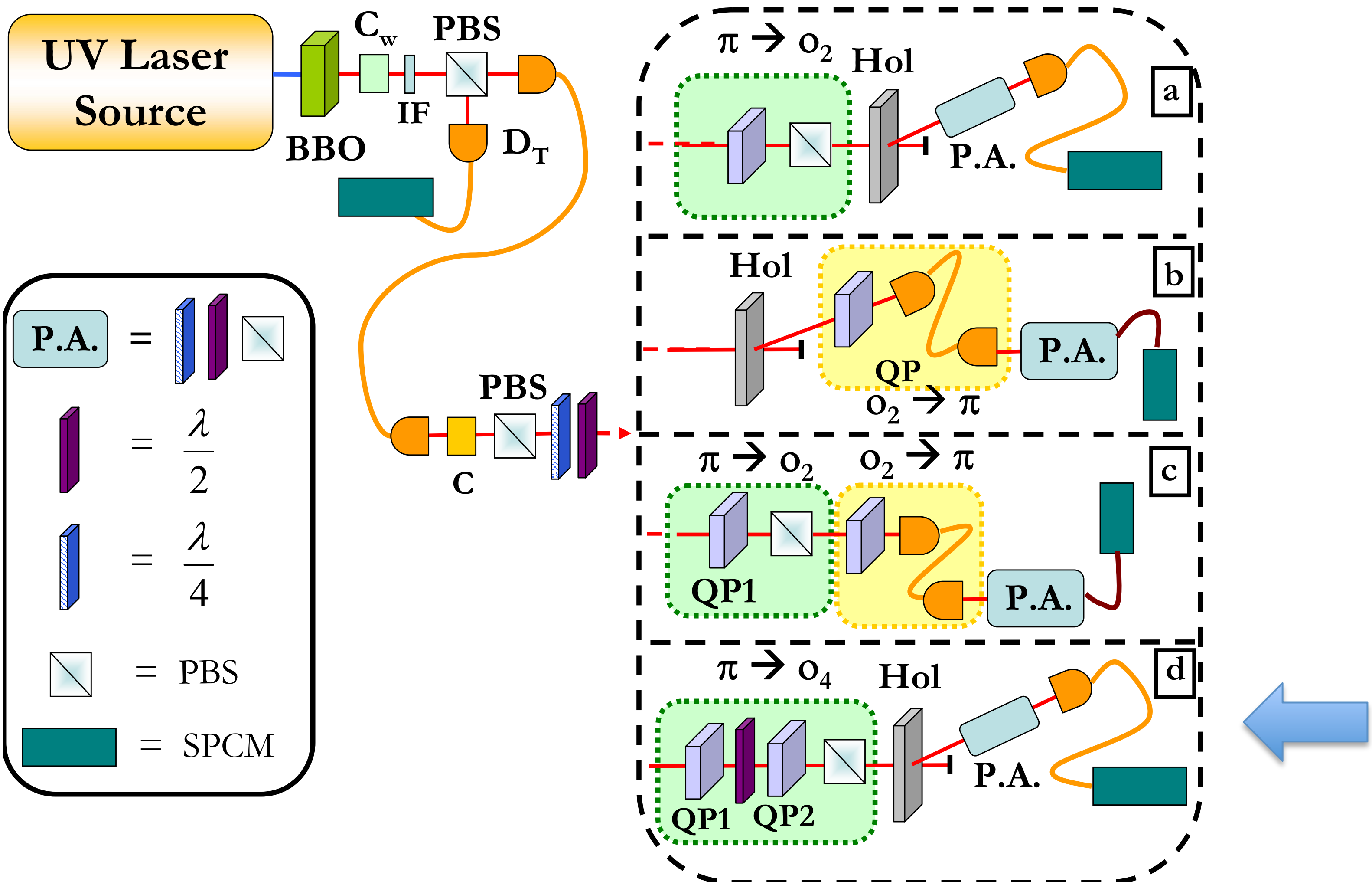


[E. Nagali, F. Sciarrino, F. De Martini, L. Marrucci, et al., *Opt. Express* **17**, 18745-18759 (2009)]

Quantum information transfer SAM ↔ OAM: back and forth



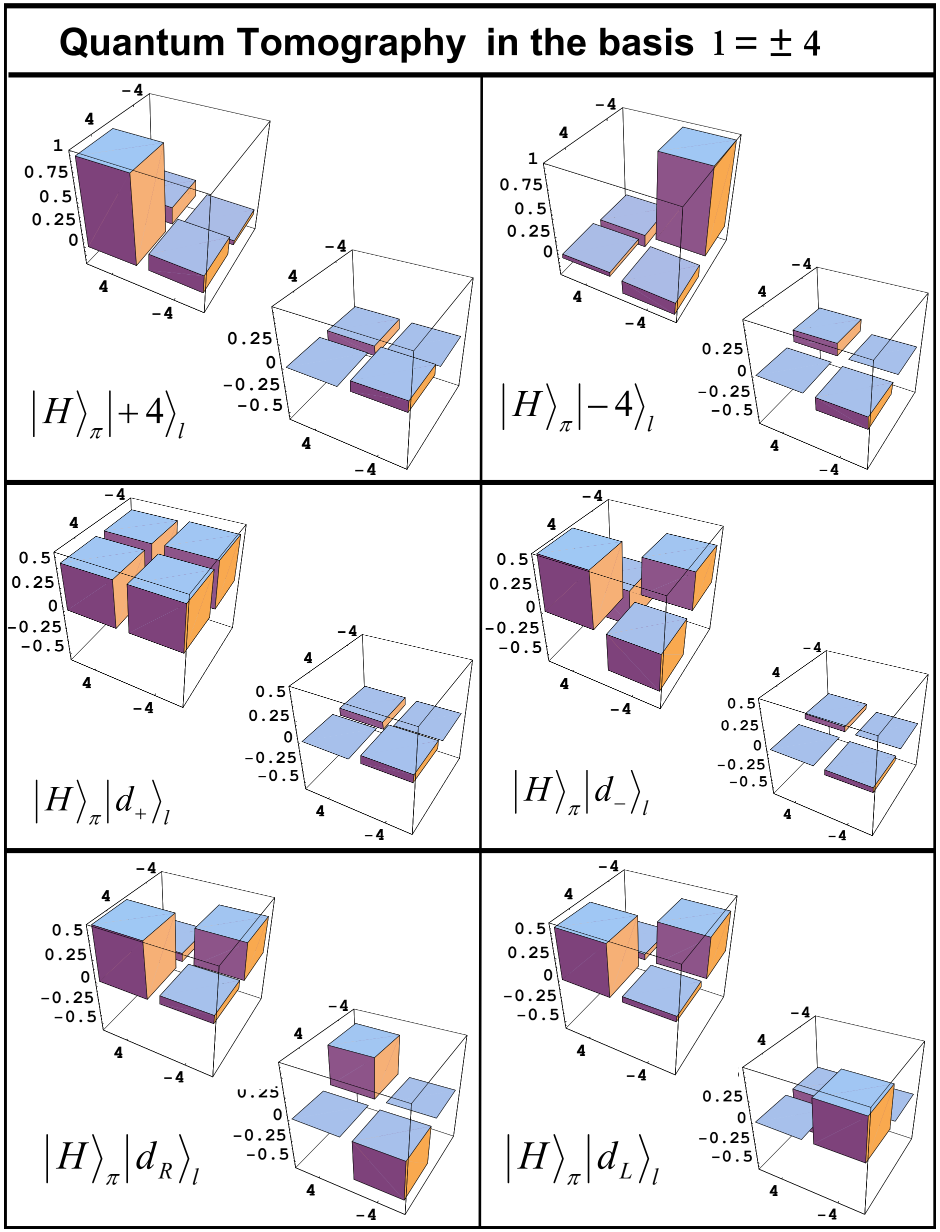
Quantum information transfer SAM ↔ OAM: cascaded transfer



[E. Nagali, F. Sciarrino, F. De Martini, L. Marrucci, et al., *Opt. Express* **17**, 18745-18759 (2009)]

Quantum information transfer SAM ↔ OAM: cascaded transfer

SAM → OAM (subspace ±4)



Thus far only probabilistic (lossy) transfer, with 50% success probability.
Can we do better?

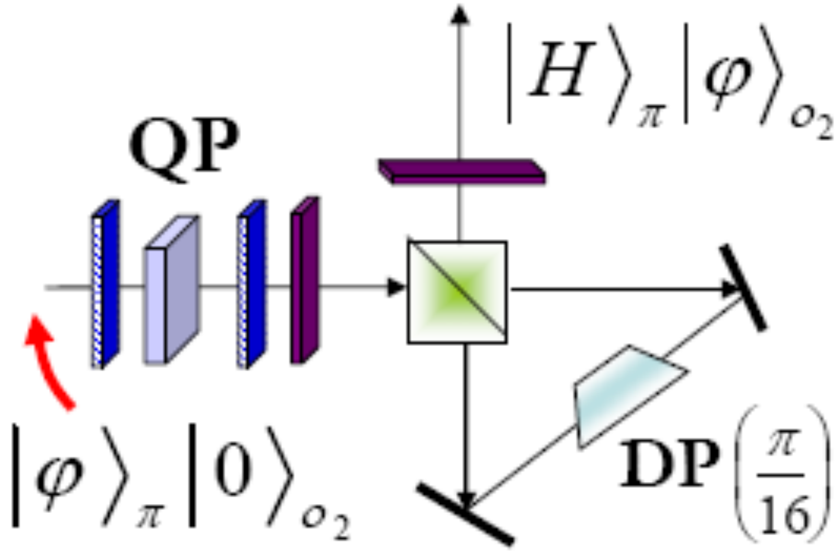
Coherent unitary mapping
SAM ↔ OAM
(or determistic reversible quantum information transfer)

[E. Nagali, F. Sciarrino, F. De Martini, L. Marrucci, et al., *Opt*. *Express* **17**, 18745-18759 (2009)]

[E. Karimi, S. Slussarenko, B. Piccirillo, L. Marrucci, E. Santamato, PRA **81**, 053813 (2010)]

Coherent unitary mapping SAM ↔ OAM

Yes: reversible and deterministic transfer is possible (ideally 100% success probability) :



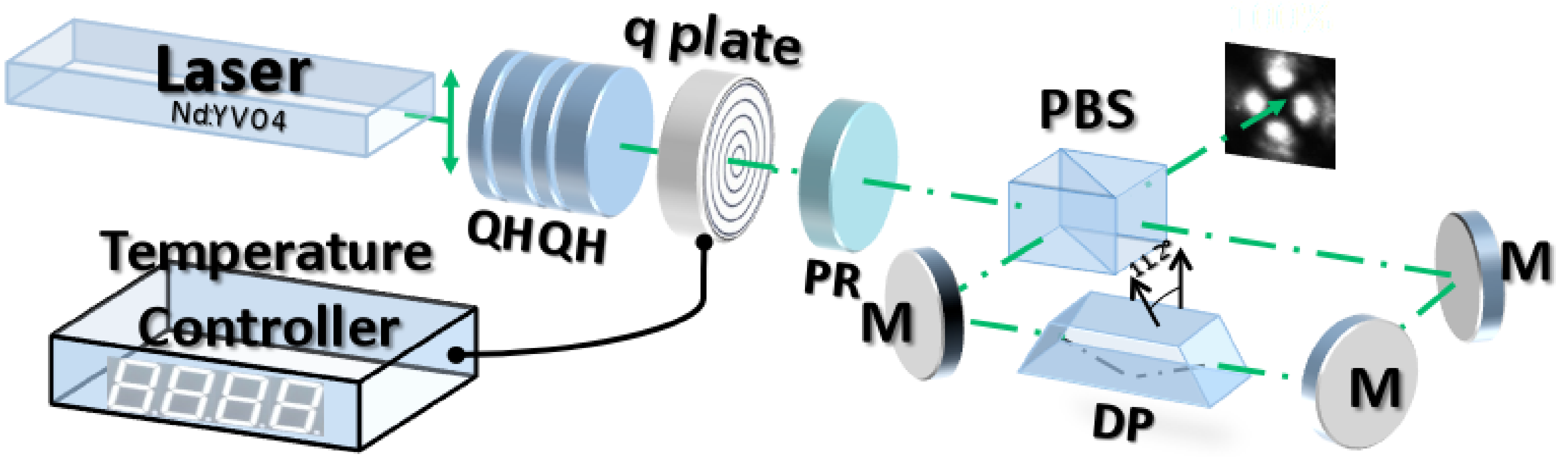
Sagnac interferometer with PBS input/output and Dove prism (DP)

This scheme has not been tested yet in the single photon regime…

… but we did it in the (equivalent) classical regime

Coherent unitary mapping SAM ↔ OAM

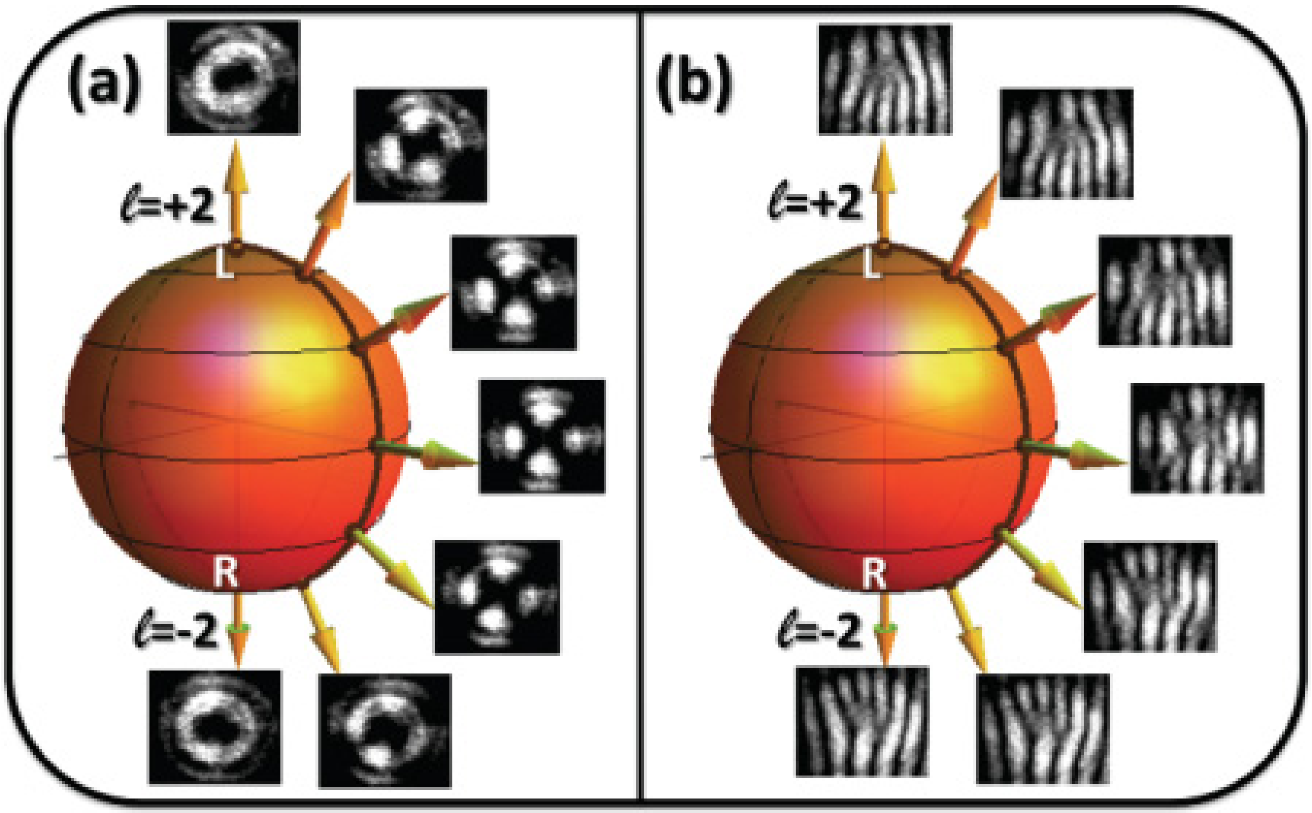
A 3D version of the setup:



[E. Karimi, S. Slussarenko, B. Piccirillo, L. Marrucci, E. Santamato, PRA **81**, 053813 (2010)]

Coherent unitary mapping SAM ↔ OAM

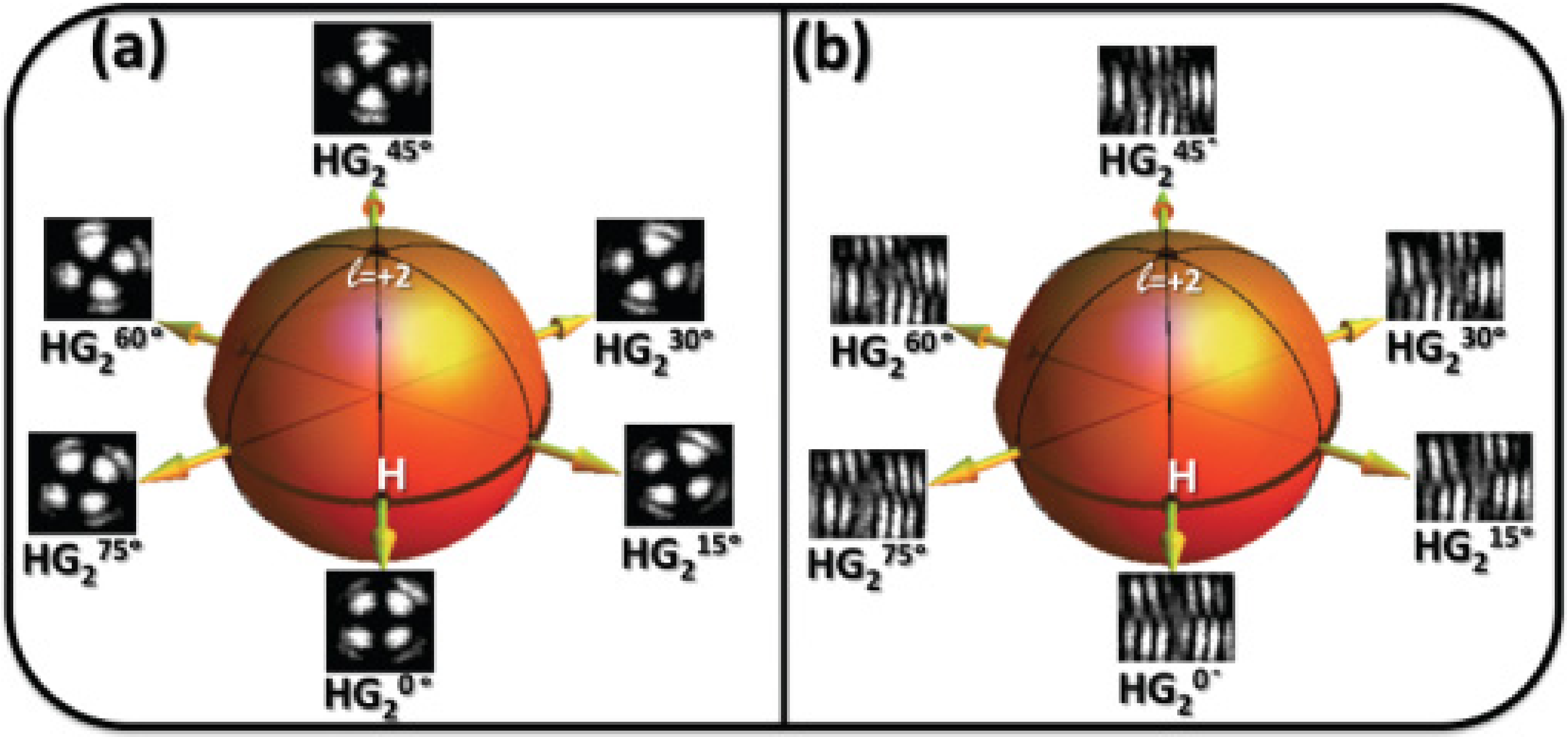
Experimental results (output mode images and interference patterns):



All OAM states on the OAM Poincaré-like sphere can be reproduced using polarization control only.

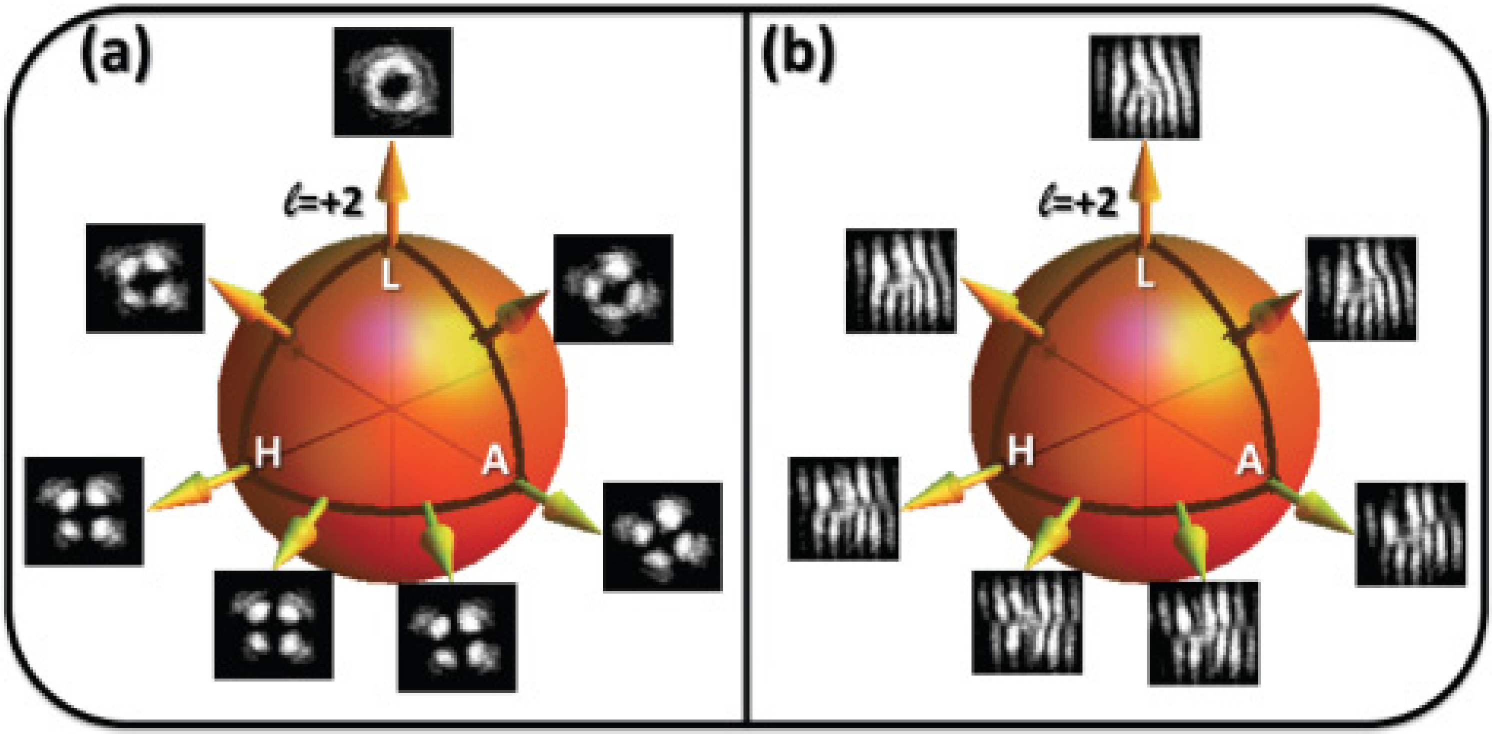
Coherent unitary mapping SAM ↔ OAM

A different (closed) path on the Poincaré sphere:



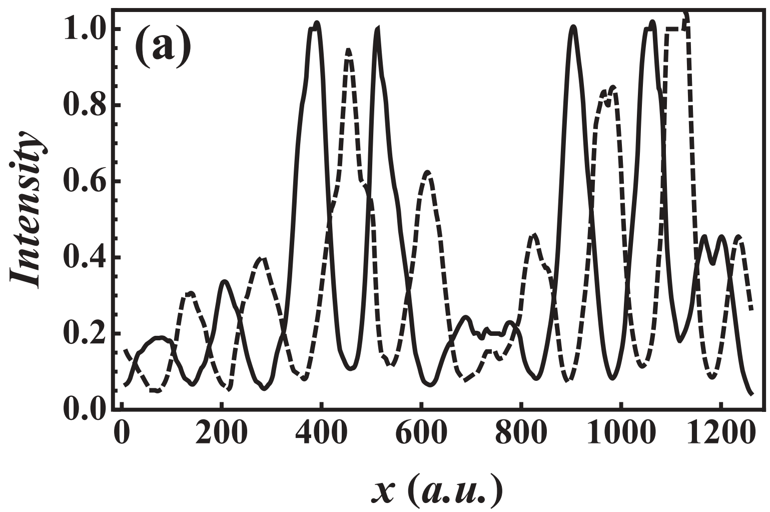
Coherent unitary mapping SAM ↔ OAM

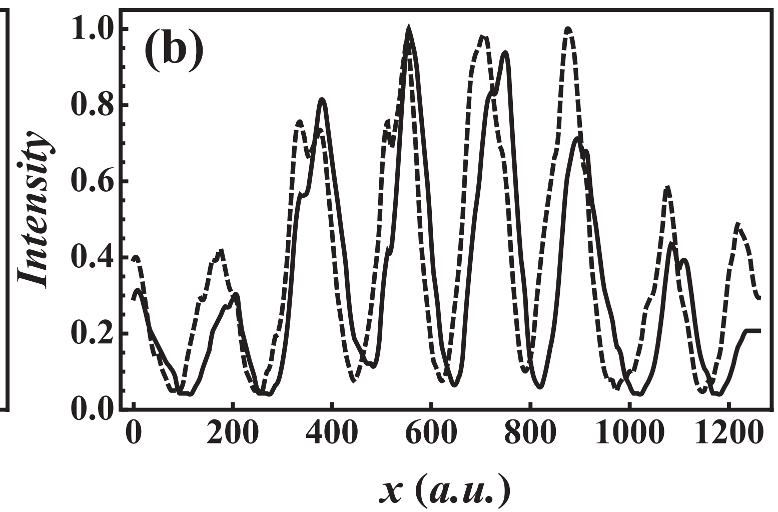
Yet another path:

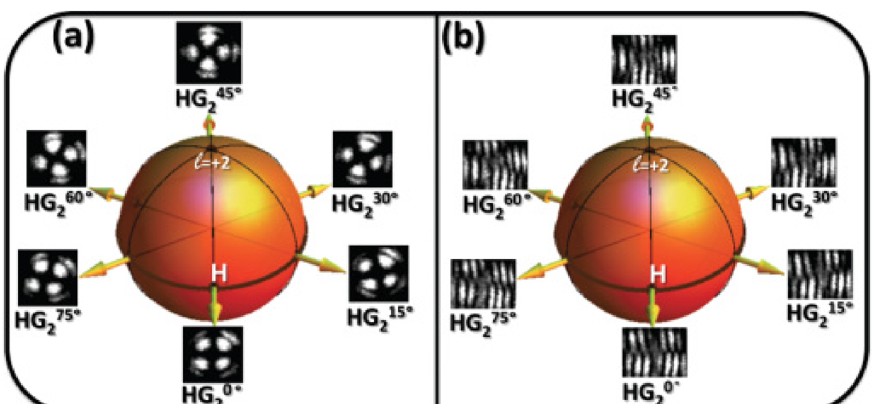


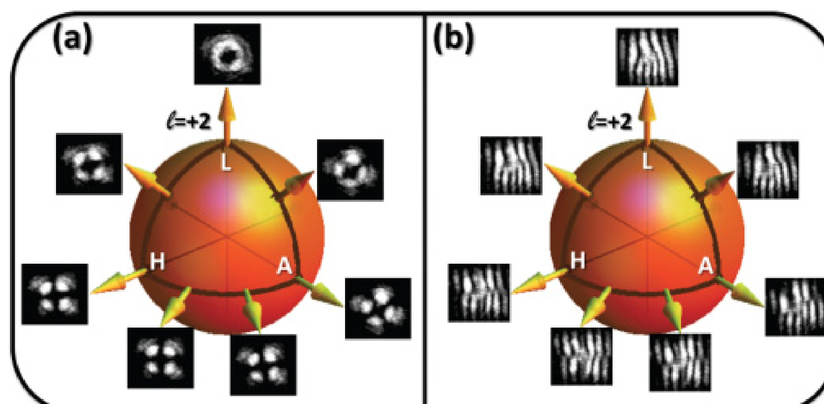
Coherent unitary mapping SAM ↔ OAM

Pancharatnam geometric phase resulting in the closed paths also transferred:









Single photon: not uniquely quantum effects
(just like classical optics, but at lower intensity)

We need to test the case of two (or more) photons for having truly quantum correlation effects

Generating a 2-photon quantum state with OAM correlations

[E. Nagali, F. Sciarrino, F. De Martini, L. Marrucci, B. Piccirillo, E. Karimi, E. Santamato, PRL **103**, 013601 (2009)]

2-photon quantum correlations in OAM

Consider 2 photons with orthogonal linear polarizations H, V:

$$|\psi\rangle = |H\rangle_1 |V\rangle_2$$

Same state in the circular-polarization basis:

$$|\psi\rangle = \tfrac{1}{\sqrt{2}}\left(|L\rangle_1 + |R\rangle_1\right)\tfrac{1}{i\sqrt{2}}\left(|L\rangle_2 - |R\rangle_2\right) = \frac{1}{2i}\left(|L\rangle_1|L\rangle_2 + |R\rangle_1|L\rangle_2 - |L\rangle_1|R\rangle_2 - |R\rangle_1|R\rangle_2\right)$$

Coalescence enhancement

2-photon quantum interference

<u>For identical photons</u>:

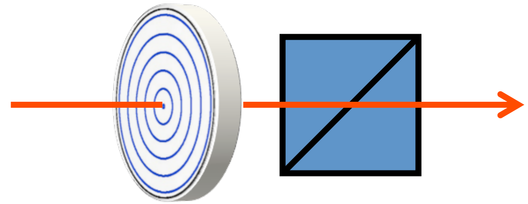
$$|\psi\rangle = \frac{1}{i\sqrt{2}}\left(|L\rangle|L\rangle - |R\rangle|R\rangle\right)$$

When identical, the two photons must always have the same polarization handedness: quantum correlations!

In polarization this has been already demonstrated, but now…

2-photon quantum correlations in OAM

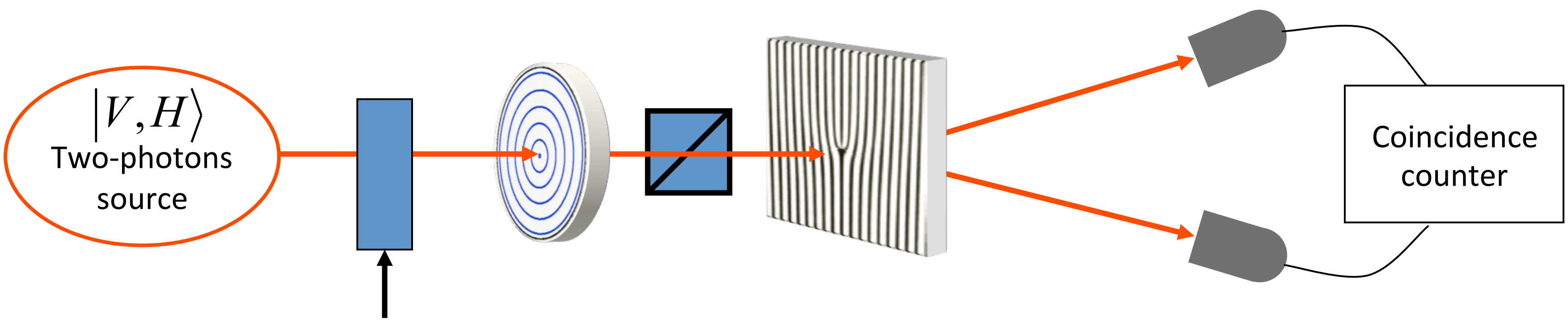
SAM → OAM



Obtained a 2-photon state with OAM quantum correlations!

$$|\psi\rangle = \frac{1}{i\sqrt{2}}\left(|+2\rangle|+2\rangle - |-2\rangle|-2\rangle\right)$$

How to verify?

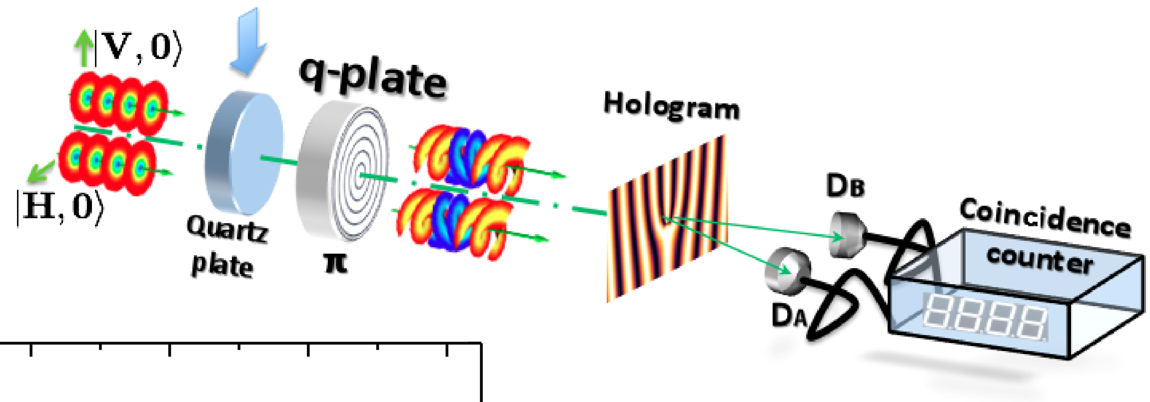


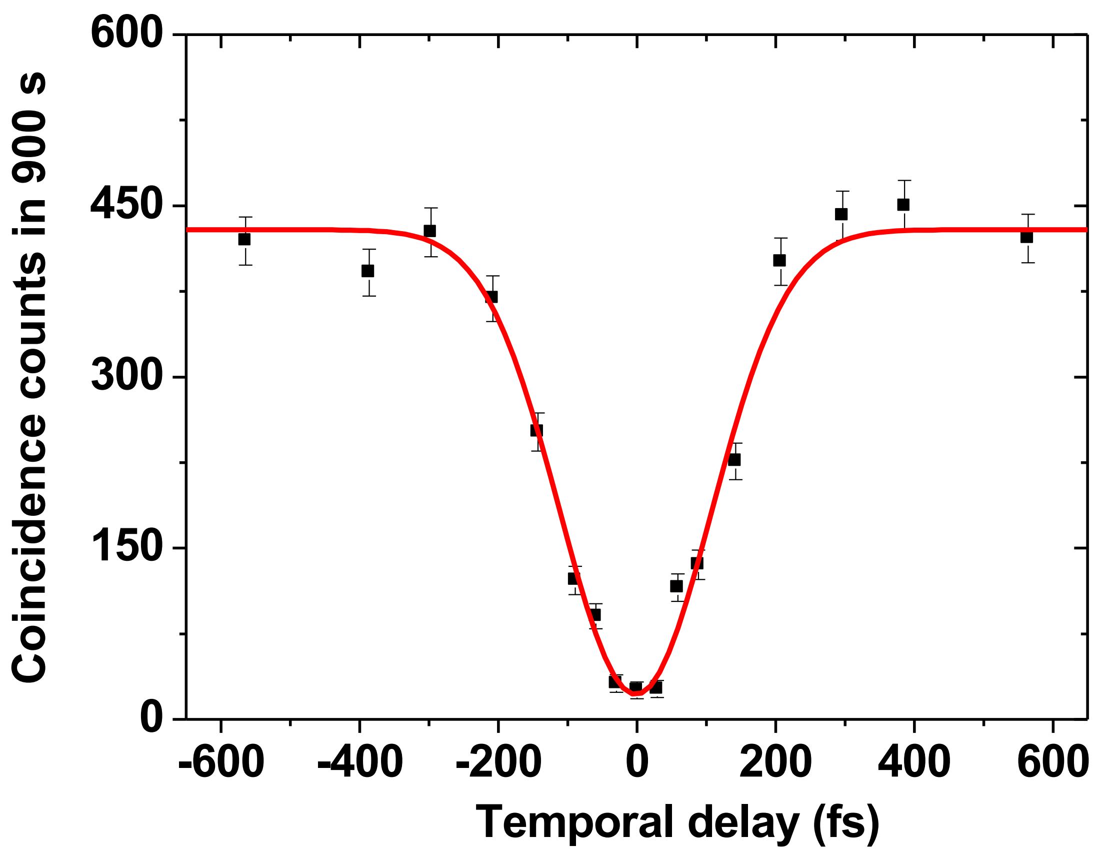
Photons “separator” (introducing a delay)

There should be no coincidences when the photons are identical

2-photon quantum correlations in OAM

Our experimental results:



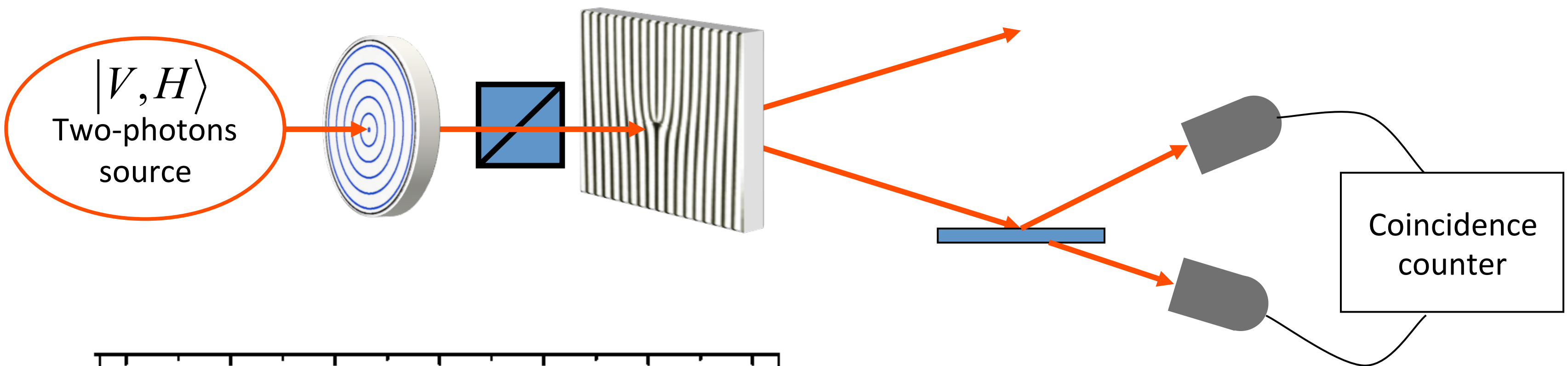


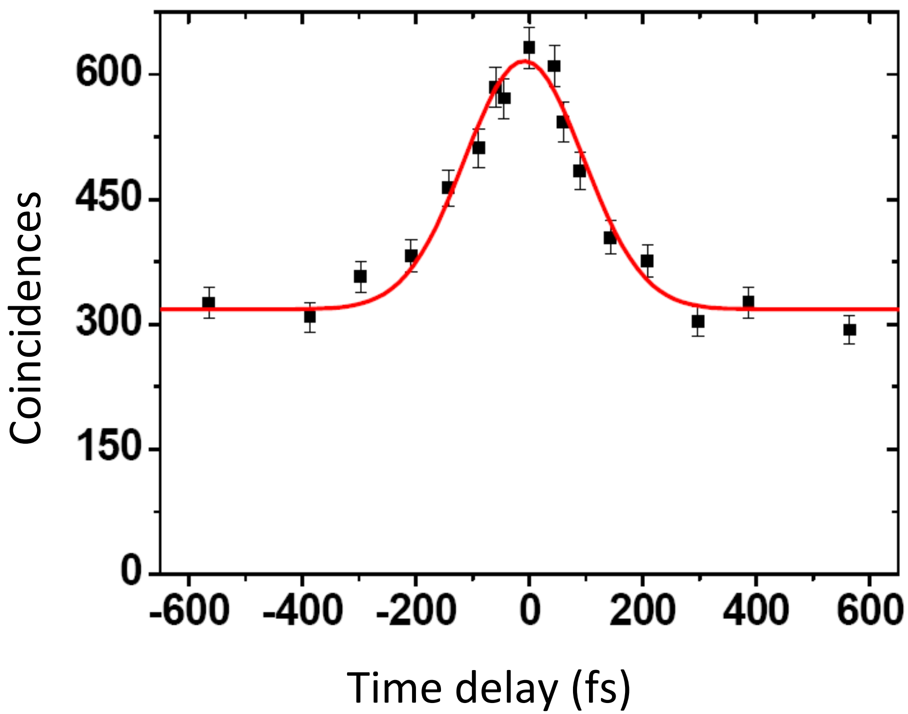
But, where are the photons going?

Coalescence enhancement

2-photon quantum correlations in OAM

Verifying coalescence enhancement:



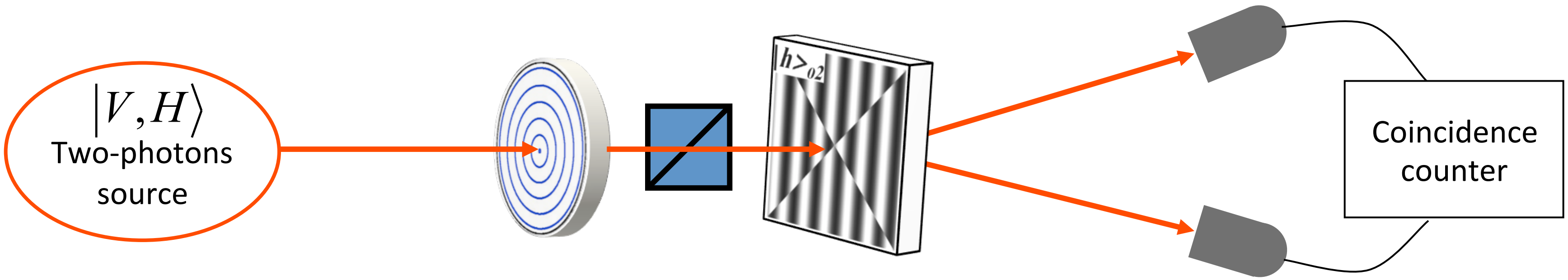


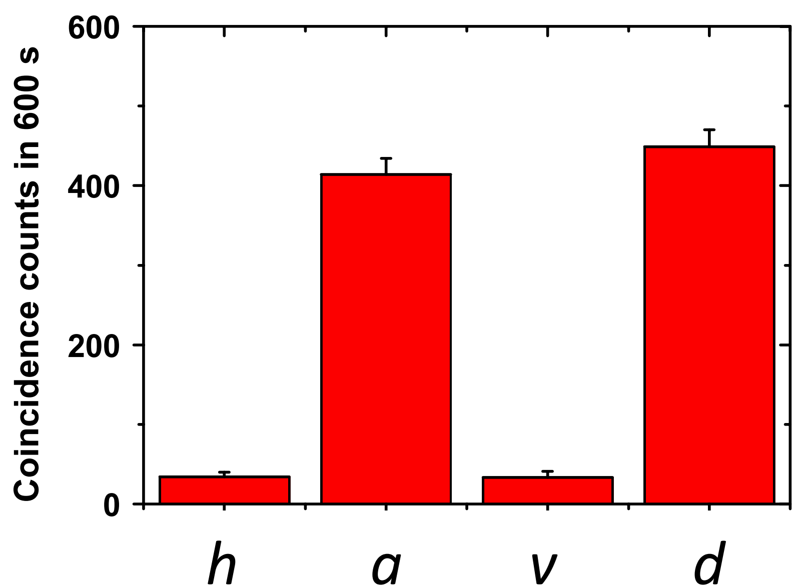
Enhancement factor = 1.94 ≈ 2

2-photon quantum correlations in OAM

Coherence check:

$$|\psi\rangle = \frac{1}{i\sqrt{2}}\left(|+2\rangle|+2\rangle - |-2\rangle|-2\rangle\right) = |h\rangle|v\rangle = \frac{1}{\sqrt{2}}\left(|a\rangle|a\rangle - |d\rangle|d\rangle\right)$$

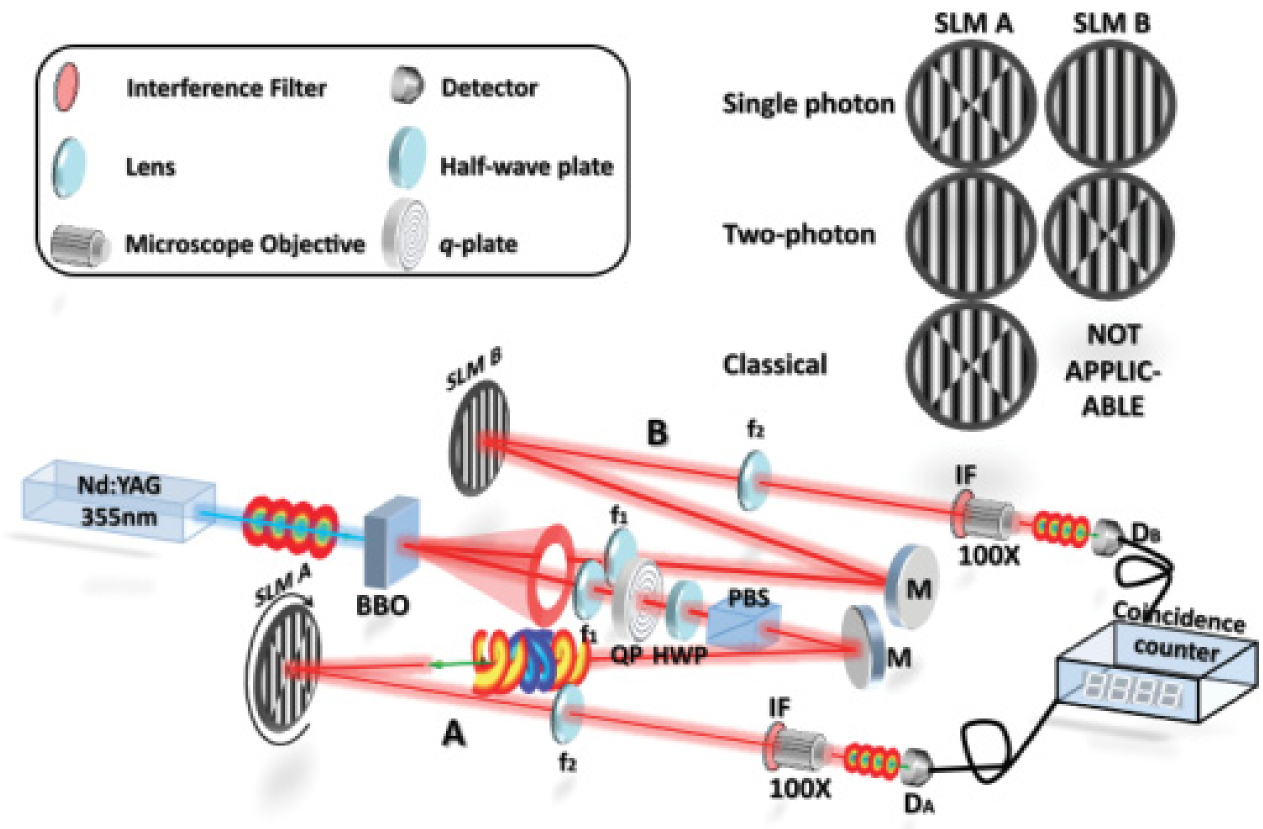




But what can we do with these quantum information transfer devices?

Quantum information transfer: some examples of applications

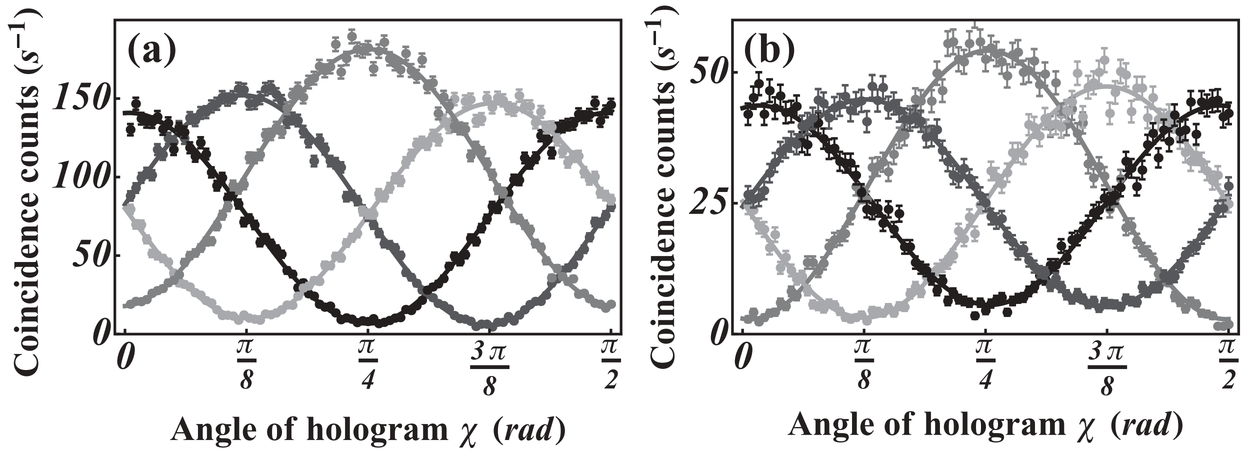
Hybrid OAM – SAM entanglement and quantum contextuality tests

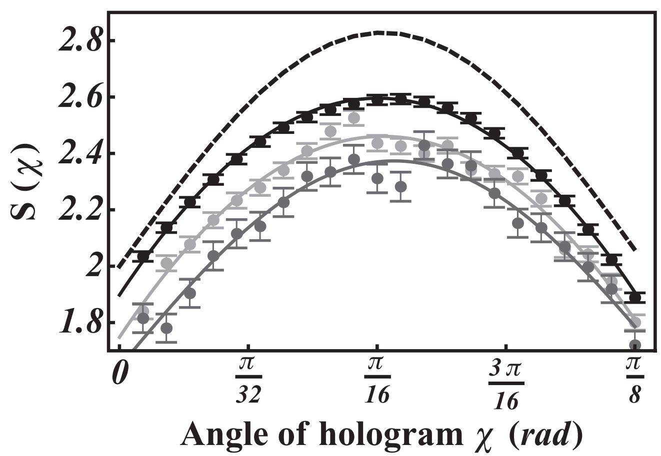


E. Karimi, J. Leach, S. Slussarenko, B. Piccirillo, L. Marrucci, L. Chen, W. She, S. Franke-Arnold, M. J. Padgett, E. Santamato, *PRA* **82**, 022115 (2010)

Hybrid OAM – SAM entanglement and quantum contextuality tests

Experimental results:





Bell-kind (Clauser-Horne-Shimony-Holt) inequality tested for demonstrating quantum contextuality in different regimes

Similar results (with a slightly different technique) also reported in [E. Nagali and F. Sciarrino, *Opt. Express* **18**, 18243 (2010)]
➔ see Fabio's talk

E. Karimi, J. Leach, S. Slussarenko, B. Piccirillo, L. Marrucci, L. Chen, W. She, S. Franke-Arnold, M. J. Padgett, E. Santamato, *PRA* **82**, 022115 (2010)

Quantum cloning of OAM qubits and SAM – OAM qudits

LETTERS
PUBLISHED ONLINE: 22 NOVEMBER 2009 | DOI: 10.1038/NPHOTON.2009.214
nature photonics

Optimal quantum cloning of orbital angular momentum photon qubits through Hong–Ou–Mandel coalescence

Eleonora Nagali[1], Linda Sansoni[1], Fabio Sciarrino[1,2]*, Francesco De Martini[1,3], Lorenzo Marrucci[4,5]*, Bruno Piccirillo[4,6], Ebrahim Karimi[4] and Enrico Santamato[4,6]

PRL 105, 073602 (2010) PHYSICAL REVIEW LETTERS week ending 13 AUGUST 2010

Experimental Optimal Cloning of Four-Dimensional Quantum States of Photons

E. Nagali,[1] D. Giovannini,[1] L. Marrucci,[2,3] S. Slussarenko,[2] E. Santamato,[2] and F. Sciarrino[1,4,*]

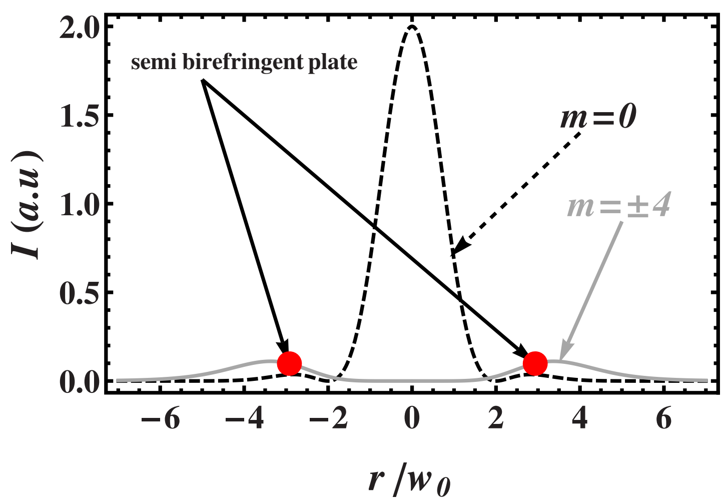
➔ See Fabio's talk

The next step:

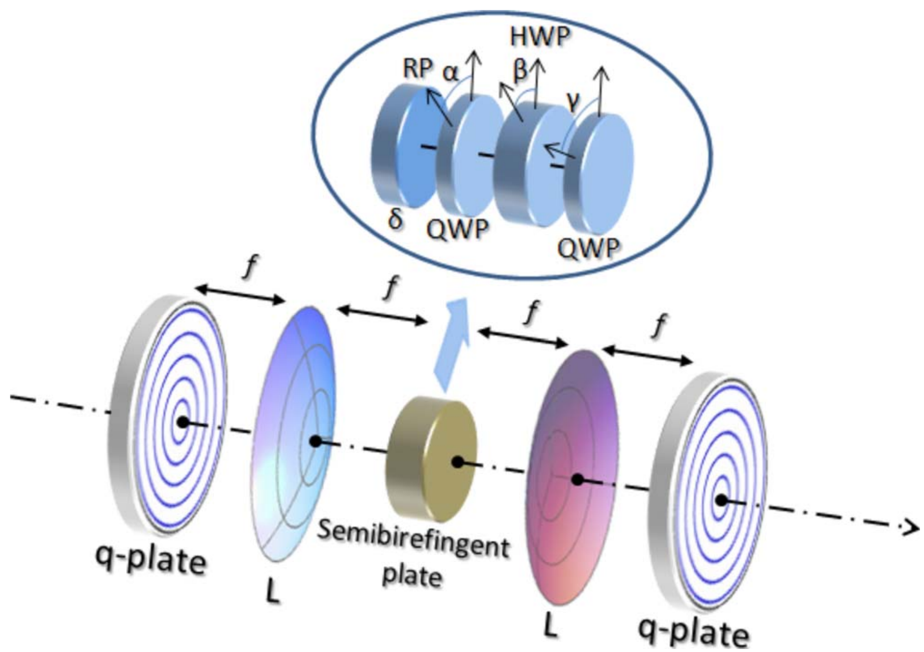
Moving further up in the photon space dimensionality

A single-beam universal quantum gate in SAM – OAM space:

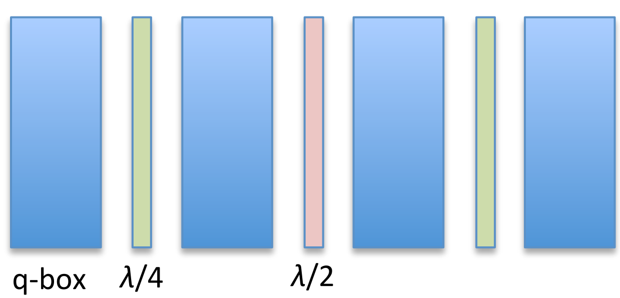
Main idea: to exploit OAM – radial profile correlations arising in free propagation



A "q-box":

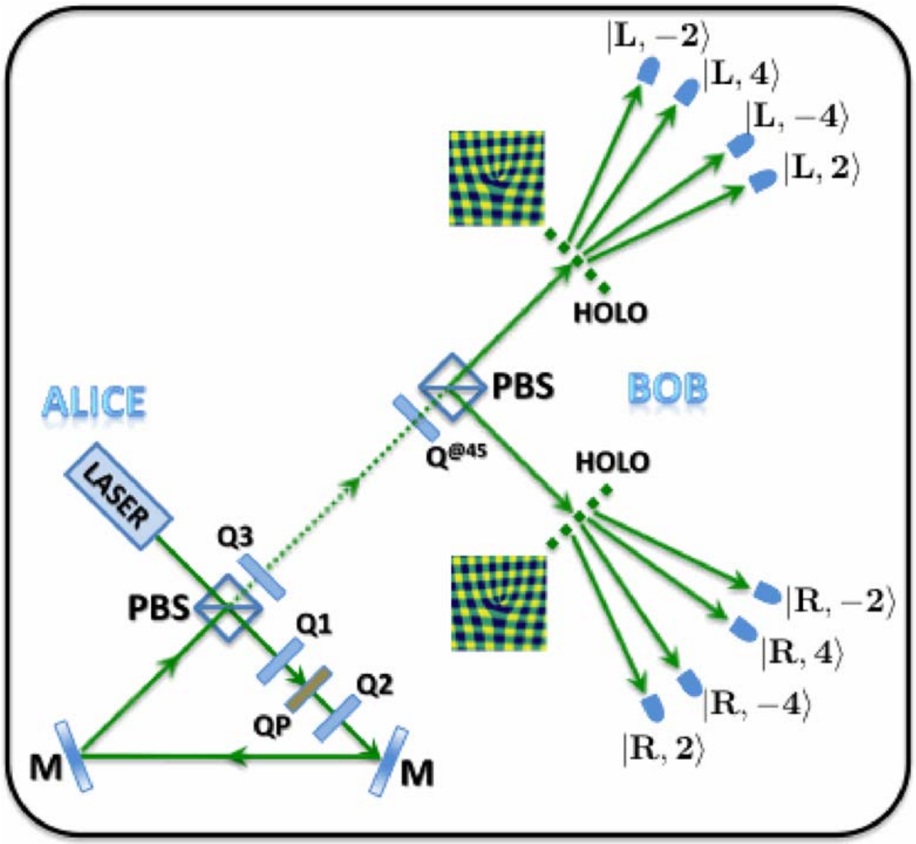


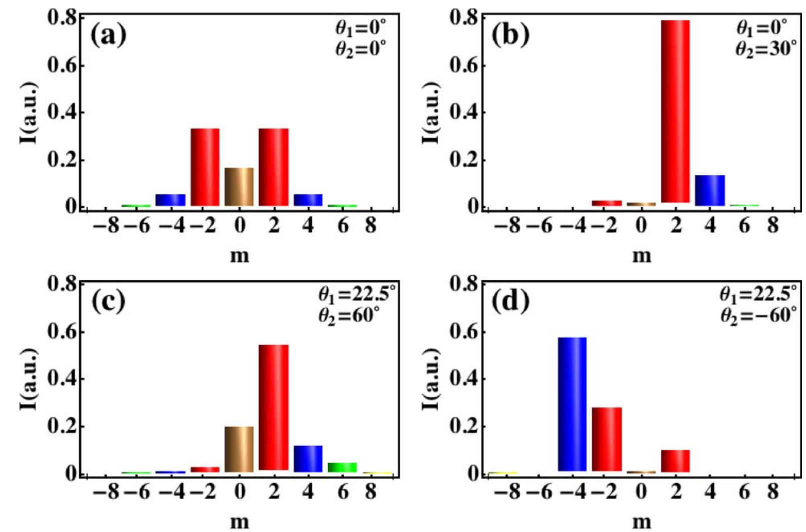
The complete device:

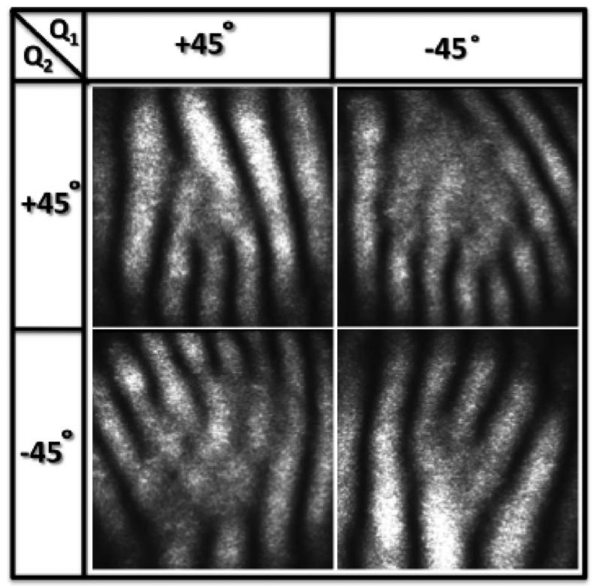


[S. Slussarenko, E. Karimi, B. Piccirillo, L. Marrucci, E. Santamato, PRA **80**, 022326 (2009)]

Controlling a higher-dimensional OAM subspace with a single q-plate



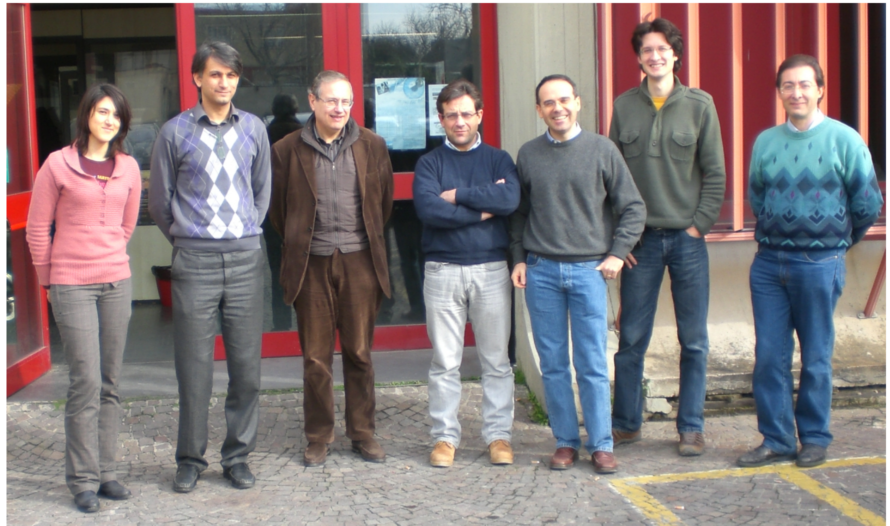




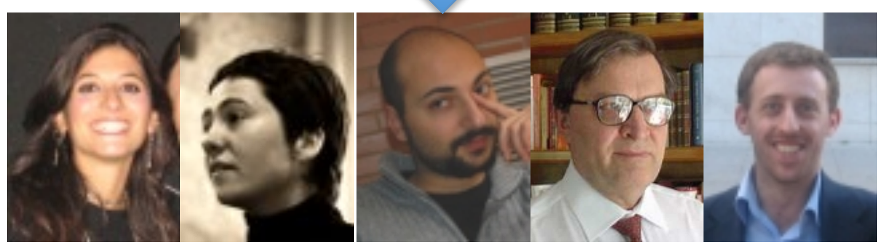
[S. Slussarenko, E. Karimi, B. Piccirillo, L. Marrucci, E. Santamato, JOSA A **28**, 61-65 (2011)]

Acknowledgments

Naples group:



Rome group:
(OAM line)



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SEVENTH FRAMEWORK PROGRAMME

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