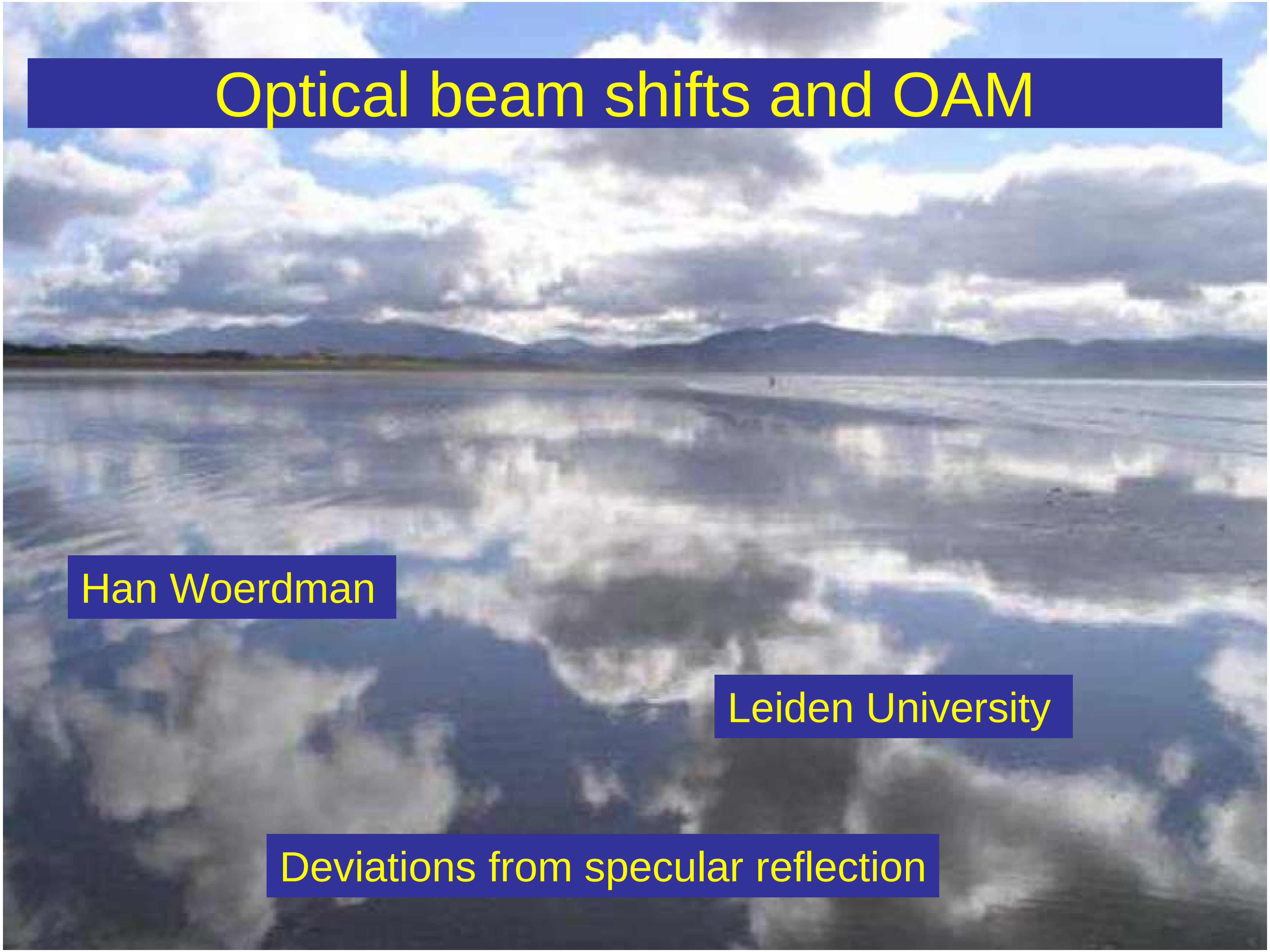
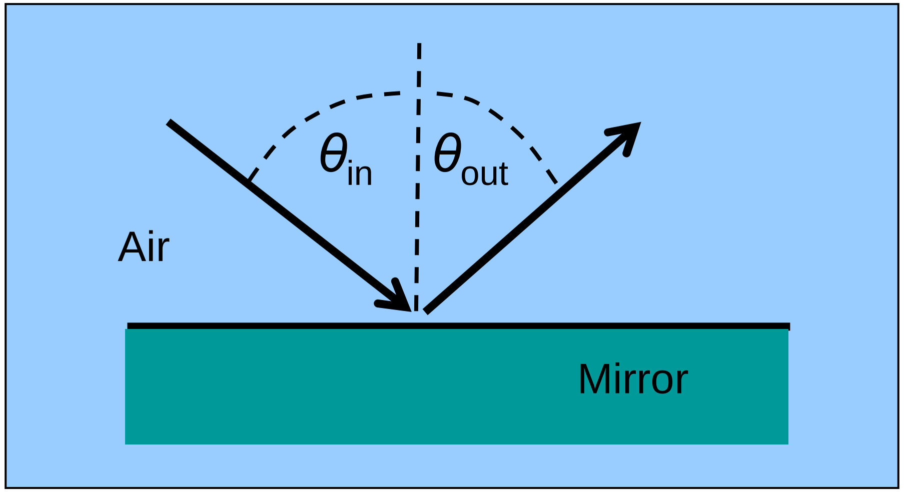

Optical beam shifts and OAM
Han Woerdman
Leiden University
Deviations from specular reflection

Specular reflection of a light ray

Statue of Euclid (Oxford)

Euclid (≈ 300 BC)



$$\theta_{in} = \theta_{out}$$

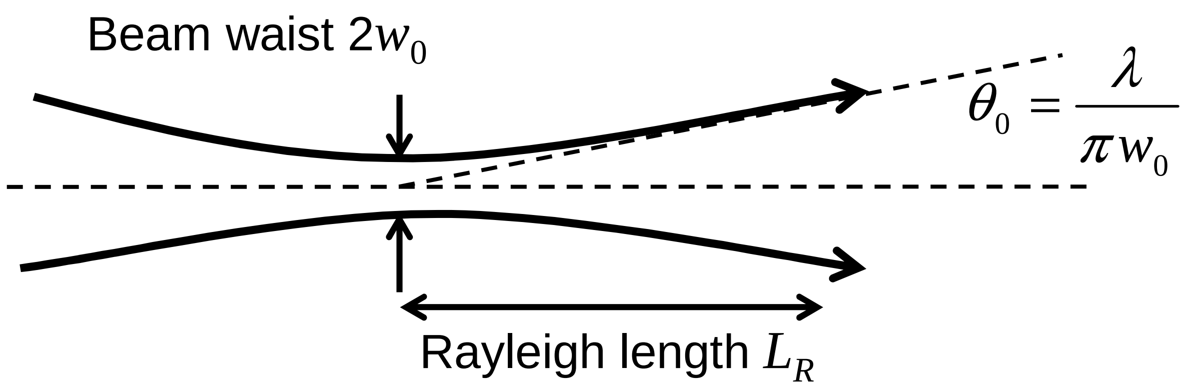
(in his book “*Catoptrics*”)

$\kappa\alpha\tau o\pi\tau\rho o\nu =$ mirror

Wave optics

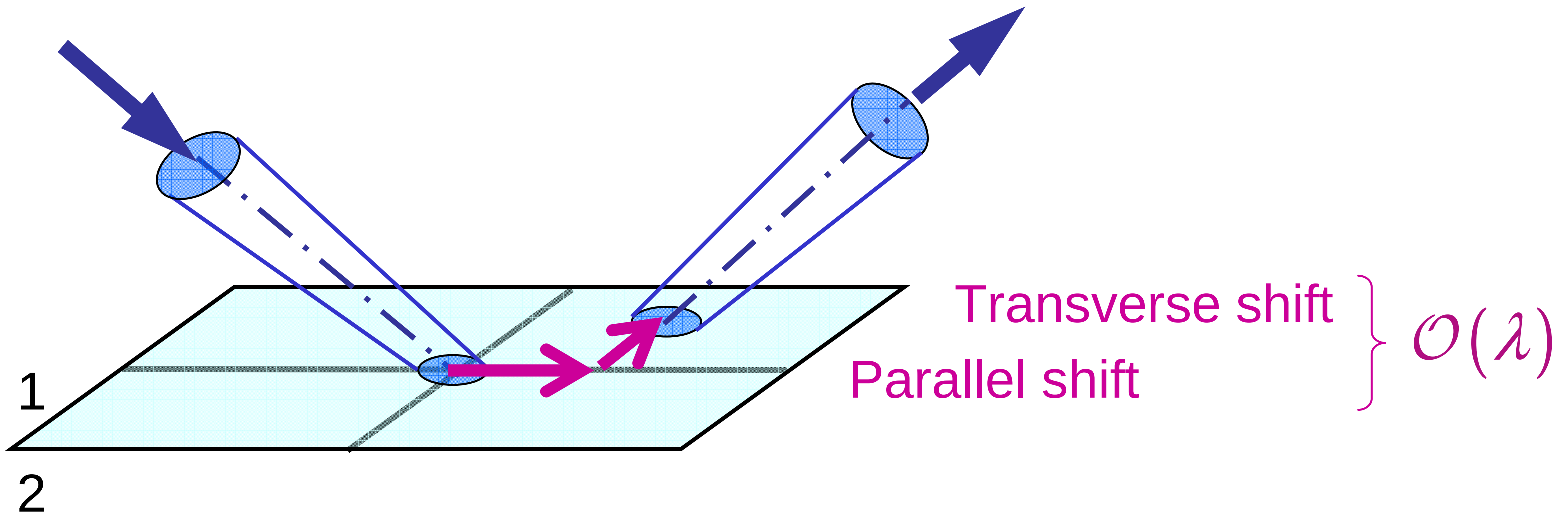
A ray (or a plane wave) is a mathematical construct that does not exist in experimental optics (infinite power !)

Introduce finite-diameter light beam as physical ray:



$\longrightarrow$ Diffractive corrections to ray optics, i.e. “beam shifts”

“Pencil beam” violates specular reflection

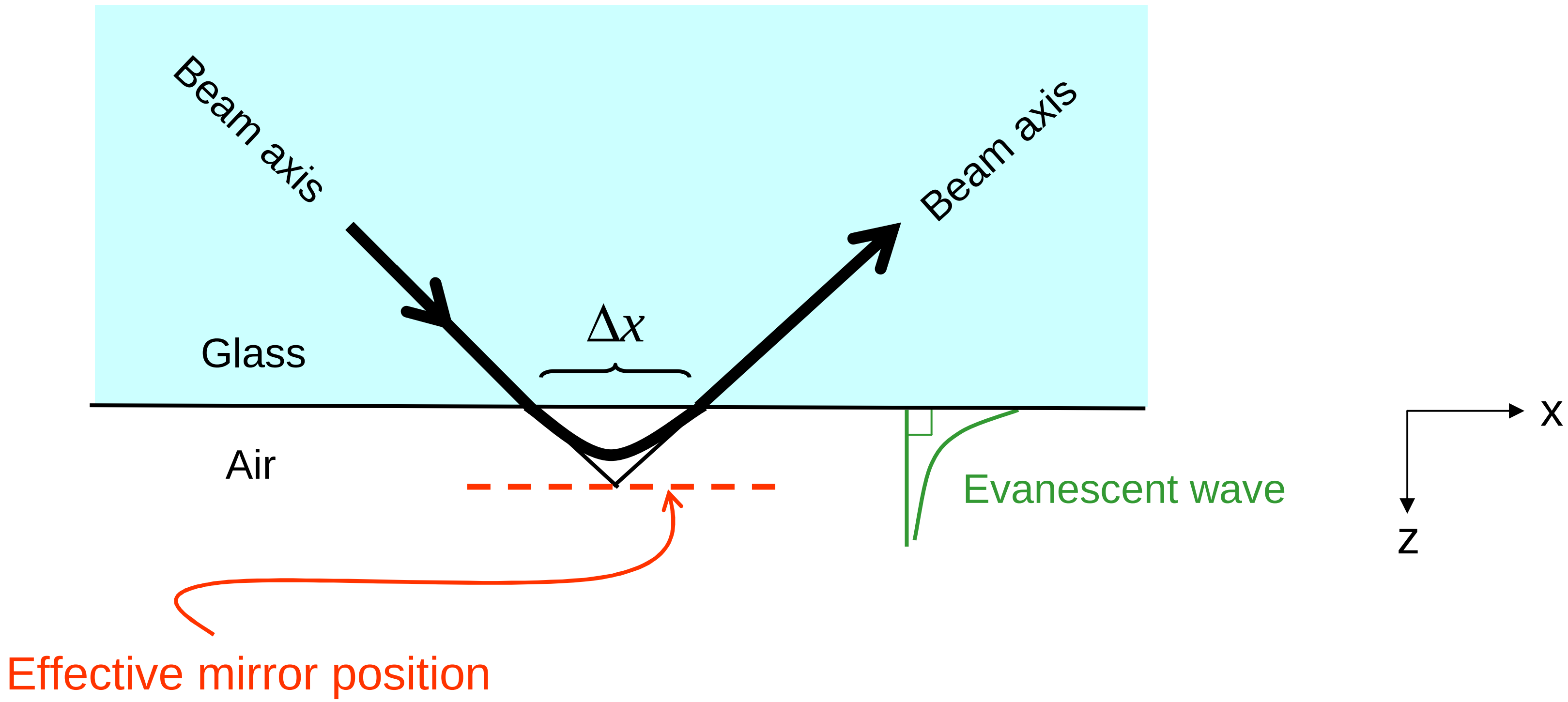


Shifts depend on
- Angle of incidence
- Refractive indices n_1 & n_2
- Input Spin Angular Momentum (polarization)
- Input Orbital Angular Momentum

Shifts do <u>not</u> occur for “ideal” mirror (<u>infinite</u> index contrast)

Goos-Hänchen shift in Total Internal Reflection

Parallel shift of beam



$\Delta x = \mathcal{O}(\lambda) \ll$ beam width

Physical picture:

GH shift is due to propagating surface wave

Annalen der Physik. 6. Folge. Band 1. 1947

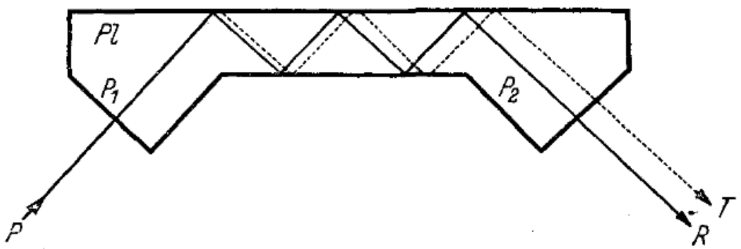
Ein neuer und fundamentaler Versuch zur Totalreflexion

Von F. Goos und H. Hänchen

(Mit 11 Abbildungen)

Inhaltsübersicht

Die Maxwellsche Theorie lehrt, daß bei Totalreflexion Lichtenergie in das dünnere Medium eindringt. Experimentell wurde bisher diese Energie immer nur im dünneren Medium selbst nachgewiesen, dadurch Licht abgezapft und somit die totale Reflexion zunichte gemacht. Es soll nun hier ein neues Experiment beschrieben werden, bei dem die Lichtbewegung im dünneren Medium nachgewiesen wird durch ein Phänomen, welches sich im dichteren Medium abspielt, nachdem das Licht bereits das dünnere Medium durchlaufen und dieses wieder verlassen hat. Dabei wird die totale Reflexion in keiner Weise gestört. Das Phänomen wird quantitativ in Beziehung gesetzt zur Maxwellschen Theorie.



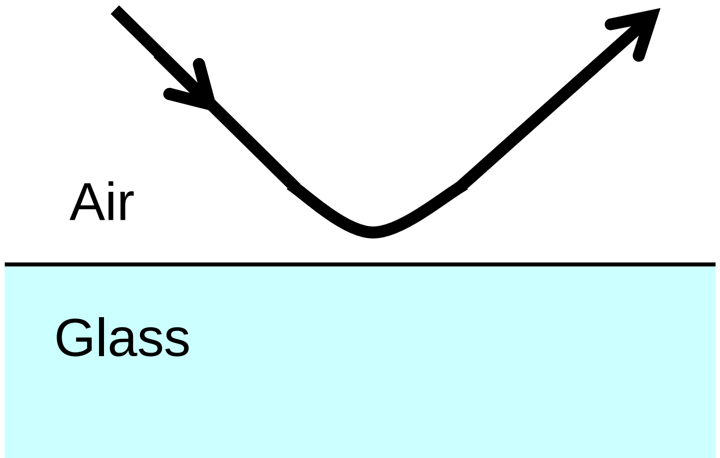
Hamburg, Physikalisches Staatsinstitut, den 25. Oktober 1943.

(Bei der Redaktion eingegangen am 27. 10. 1943.)

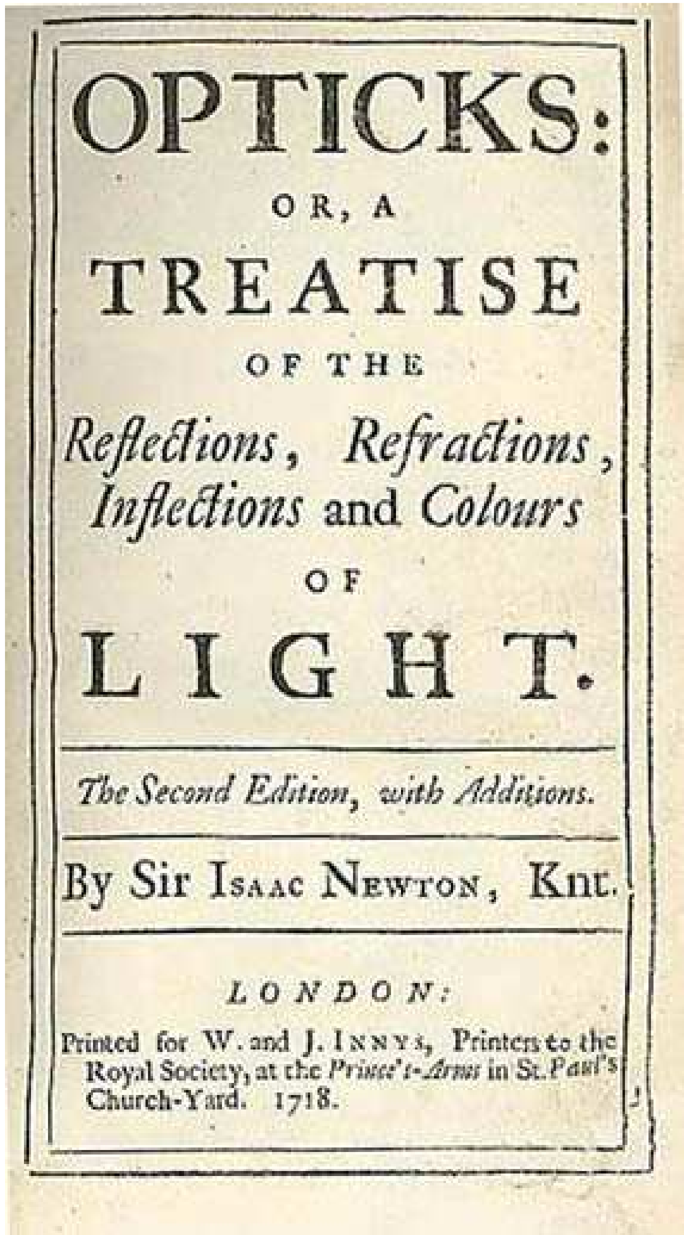
Newton anticipated GH-type shift

QUERY 4.

Do not the Rays of Light which fall upon Bodies begin to bend before they arrive at the Bodies….?



"Negative" GH shift…………


OPTICKS:
OR, A
TREATISE
OF THE
Reflections, Refractions, Inflections and Colours
OF
LIGHT.

The Second Edition, with Additions.

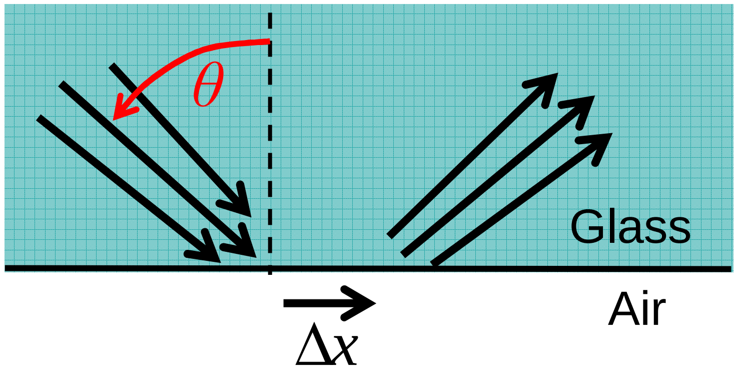
By Sir ISAAC NEWTON, Knt.

LONDON:

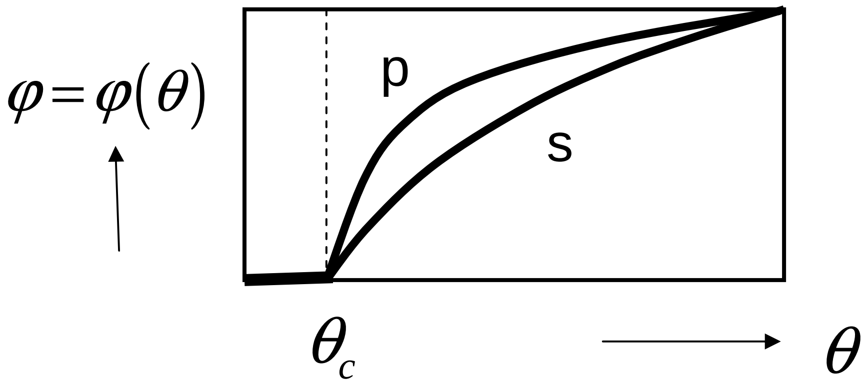
Printed for W. and J. INNYS, Printers to the Royal Society, at the *Prince's-Arms* in St. *Paul's* Church-Yard. 1718.

Theory for GH shift

Finite-diameter beam
(angular wave packet):



Plane-wave reflection phase:



critical angle

$$\Delta x \propto \frac{\lambda}{2\pi}\frac{d\varphi}{d\theta} \propto \frac{\lambda}{2\pi}\frac{1}{\sqrt{\theta-\theta_c}}$$

<u>Different</u> GH shift for *p* and *s* polarization

GH shift appears already in 2D ("sheet beam")

Transverse beam shift: Imbert-Fedorov

PHYSICAL REVIEW D

THIRD SERIES, VOL. 5, 15 February 1972

Calculation and Experimental Proof of the Transverse Shift Induced by Total Internal Reflection of a Circularly Polarized Light Beam

Christian Imbert
Institut d'Optique, Faculté des Sciences, 91-Orsay, France
(Received 20 July 1970)

Wiegrefe, Fedorov, Costa de Beauregard, and Schilling have discussed the transverse energy flux existing in total reflection of an elliptically polarized light beam, the latter two proposing formulas for the transverse shift of the reflected beam. We have calculated the transverse shift by an energy-flux-conservation argument similar to Kristoffel's and to Renard's in their deduction of the longitudinal Goos-Hänchen shift, thus obtaining a formula different from those of the previous authors. We have also tested experimentally the existence of the transverse shift, in the optimal case of circular polarization and quasilimit total reflection, by using two slightly different multiplying procedures. Our measurements definitely vindicate our own formula for the transverse shift against both Costa de Beauregard's and Schilling's. The relevance of our results in connection with noncollinearity of velocity and momentum of the spinning photon inside the evanescent wave is very briefly discussed.

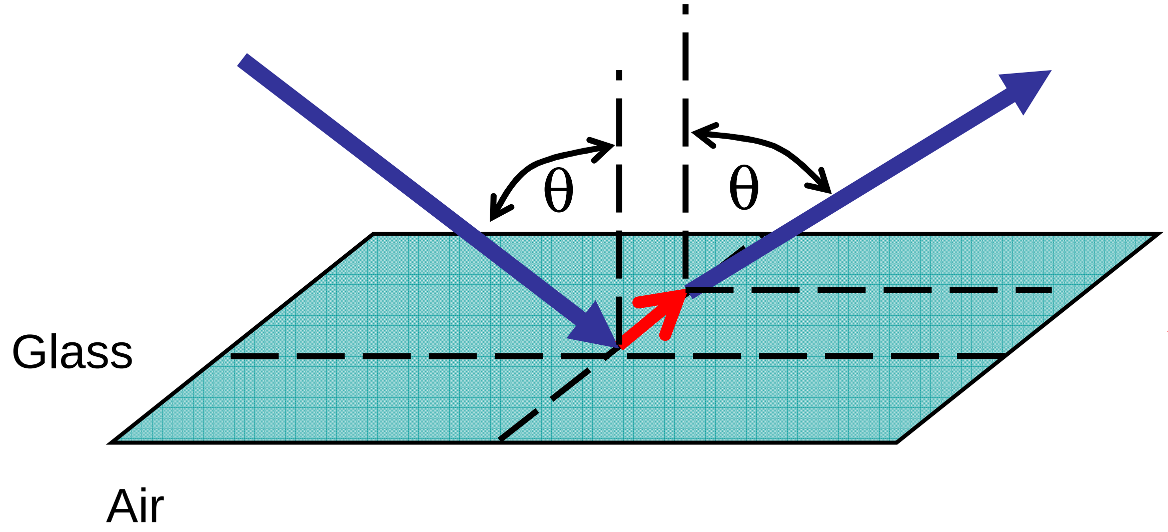
F.I. Fedorov, Dokl. Akad. Nauk SSSR 105, 465(1955)

Imbert-Fedorov shift (IF)

3D pencil beam is essential

Transverse plane-waves with different reflection phases

Input polarization σ^+

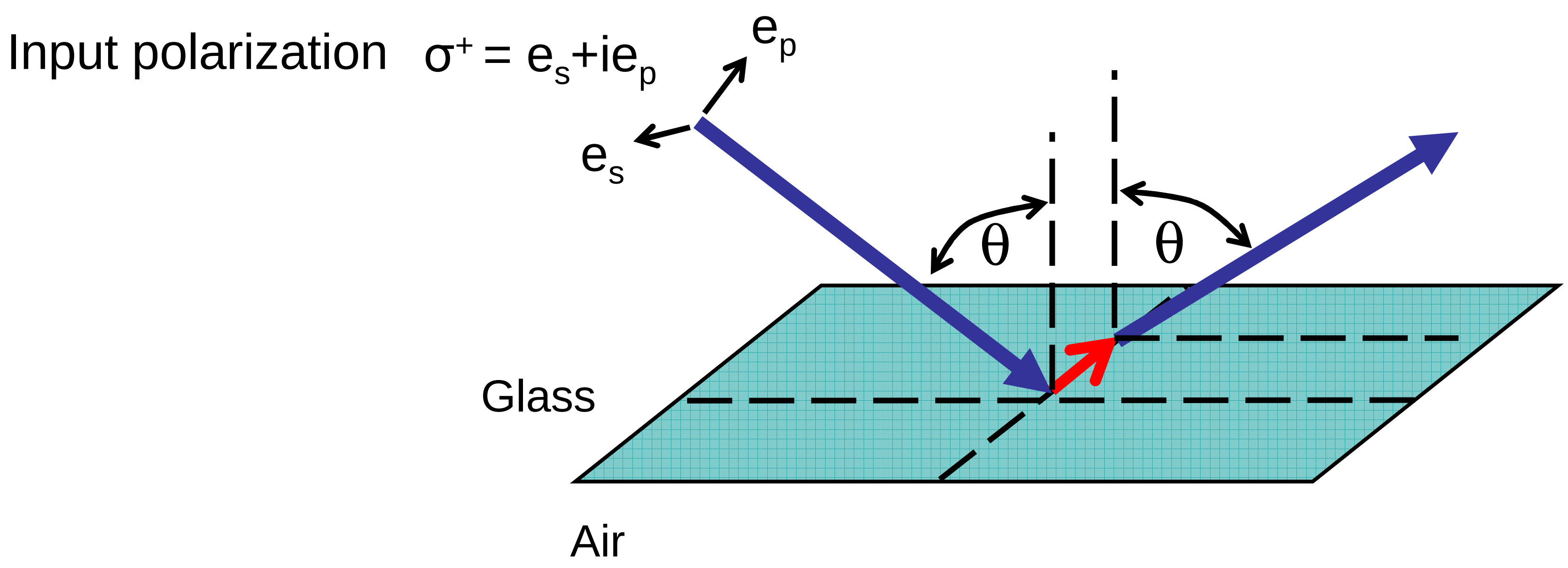


$$\Delta_{IF} \propto \frac{\lambda}{2\pi} f(\theta)$$

$$|f(\theta)| < 1$$

Input polarization σ^- leads to opposite shift

Physical interpretation of IF shift



Spin angular momentum is not conserved

Orbital angular momentum is created by IF shift

$\longrightarrow$ Total angular momentum is conserved

Spin Hall effect of light

Imbert-Fedorov shift has been rediscovered as

Spin Hall Effect of Light (SHEL)

Onoda et al, PRL 93, 083901 (2004)

Bliokh and Bliokh, PRL 96, 073903 (2006)

Hosten and Kwiat, Science 319, 787 (2008)

Qin et al, OL 34, 2551 (2009)

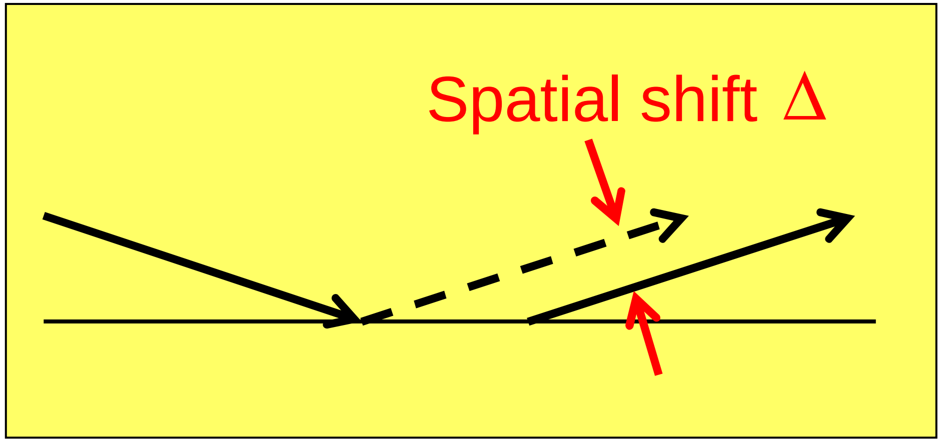
Menard et al, OL 34, 2312 (2009)

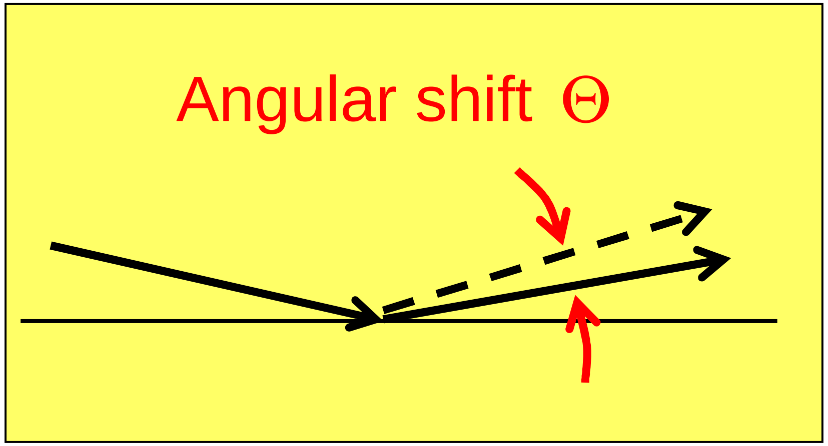
Photonic version of electronic spin Hall effect

Index gradient $n_2 - n_1$ produces a transverse "spin current"

Angular shift

Ra et al, SIAM J. Appl. Math. 24, 396 (1973)
Antar and Boerner, Can. J. Phys. 52, 962 (1974)



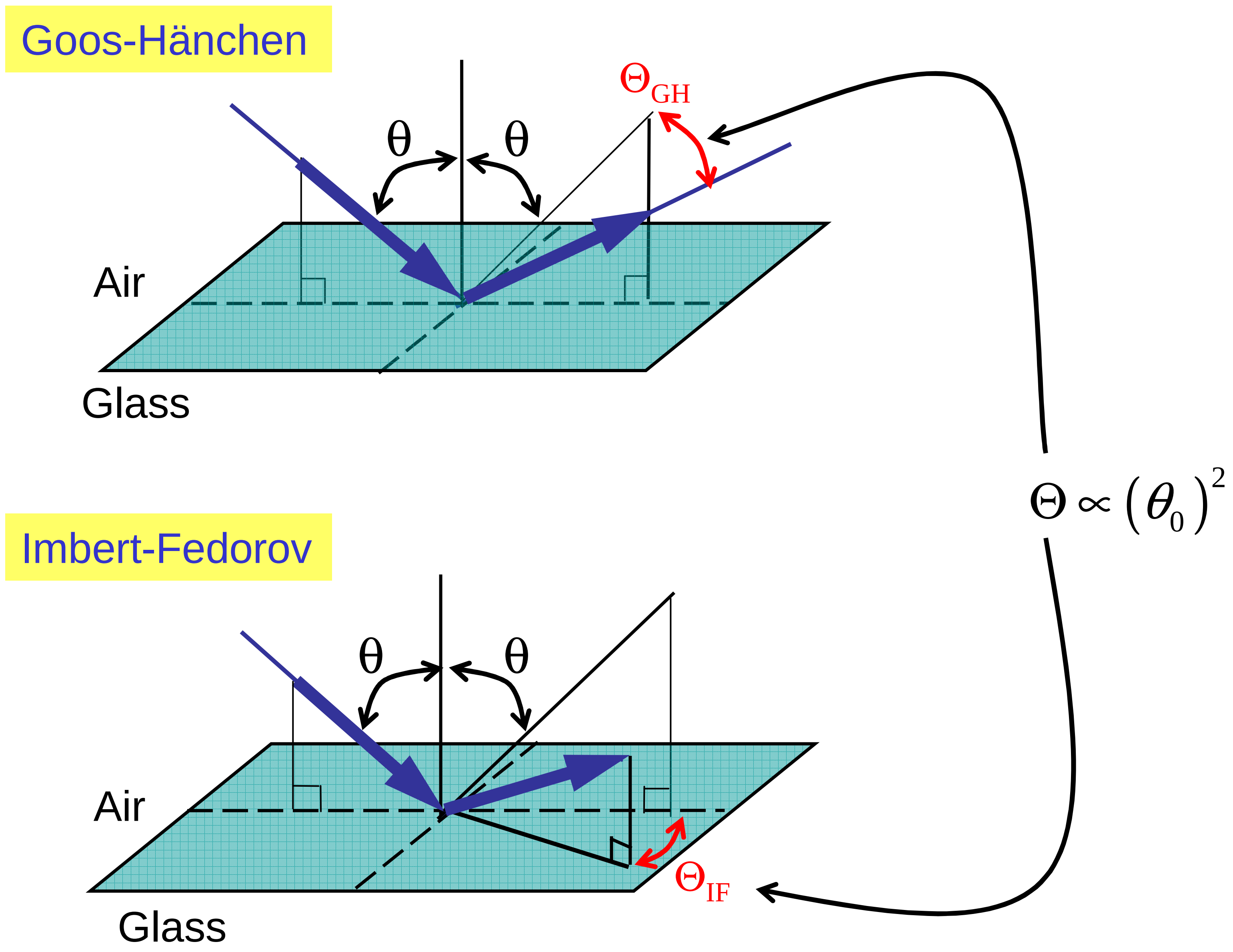


$$r = |r| \exp(i\varphi)$$

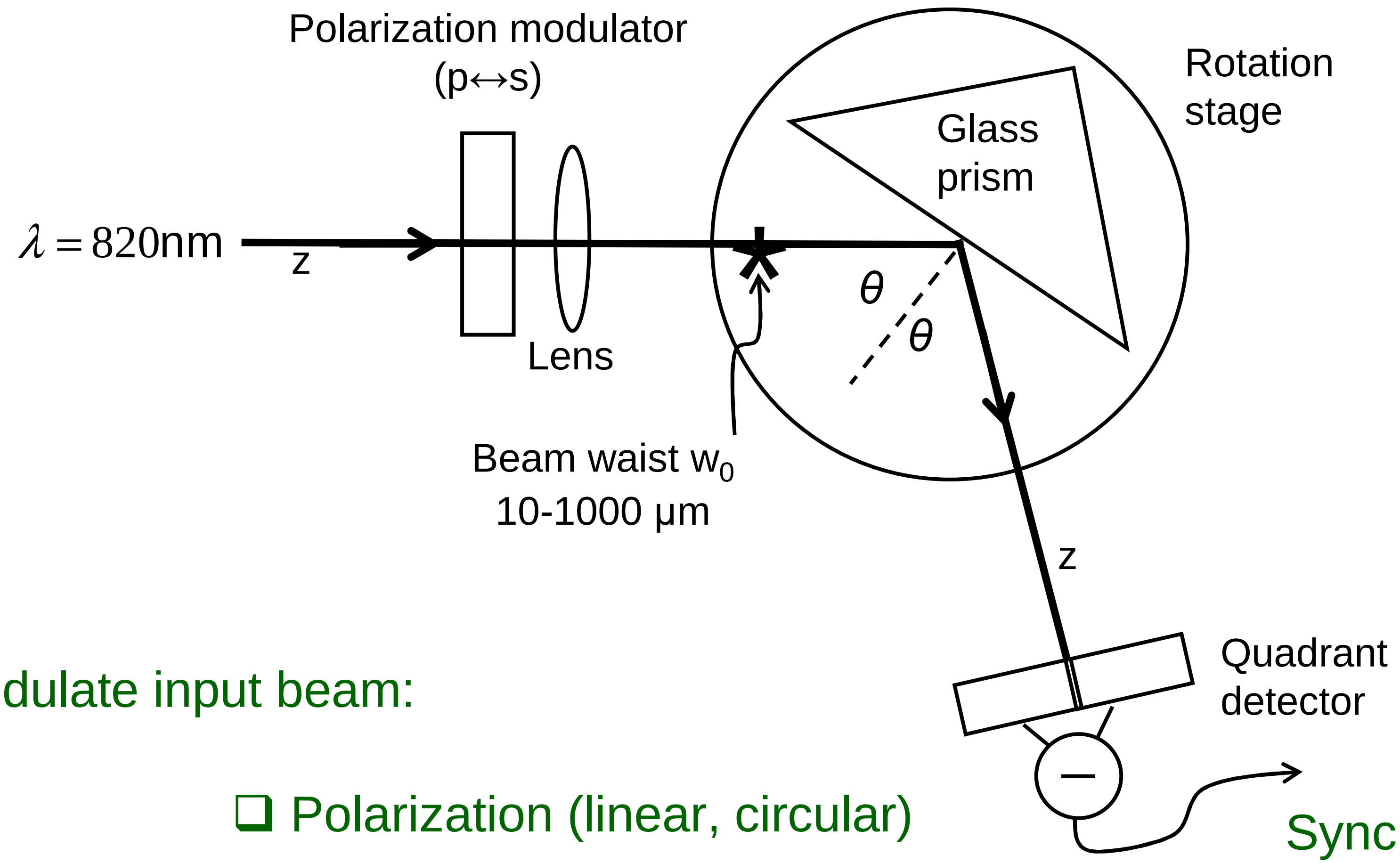
- ❑ Spatial shift due to $\frac{d\varphi}{d\theta} \neq 0$, angular shift due to $\frac{d|r|}{d\theta} \neq 0$

- ❑ Angular shift requires partial reflection, $|r| < 100\%$

Angular GH and IF shifts



Experimental set-up



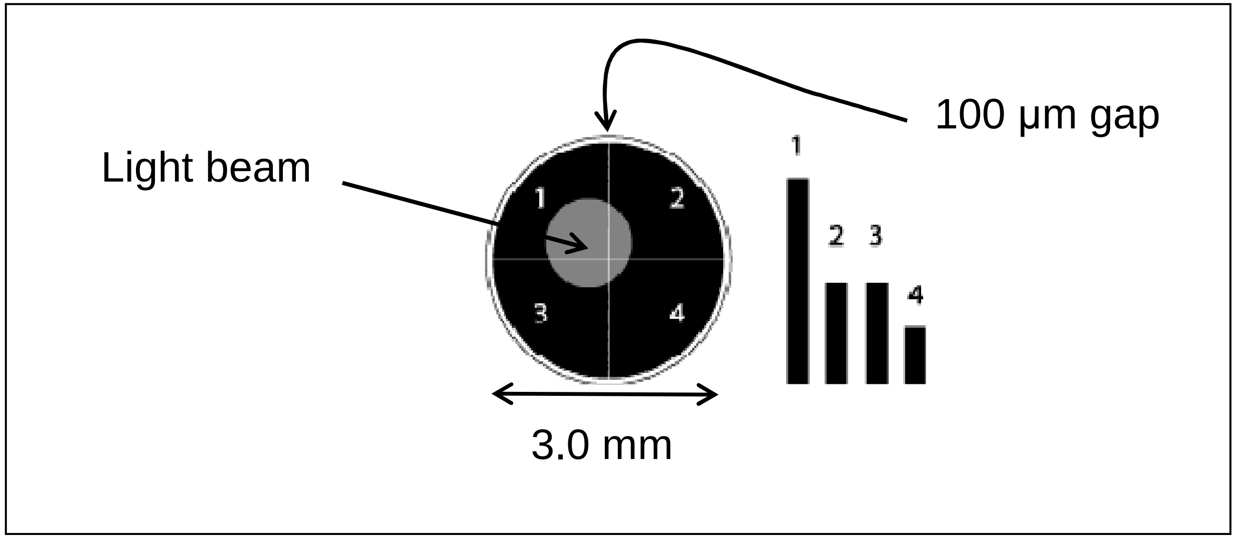
Modulate input beam:

- Polarization (linear, circular)
- Angle of incidence (θ)
- Wavefront curvature (θ_0)

Synchronous detection of beam shift

Quadrant detector

Quadrant detector produces 4 output signals



Resolution limit:
> 20 nm

Measure a <u>spatial</u> shift by pairwise binning:

$X=(1+3)-(2+4)$ parallel shift (GH)

$Y=(1+2)-(3+4)$ transverse shift (IF)

Measure an <u>angular</u> shift by dividing:

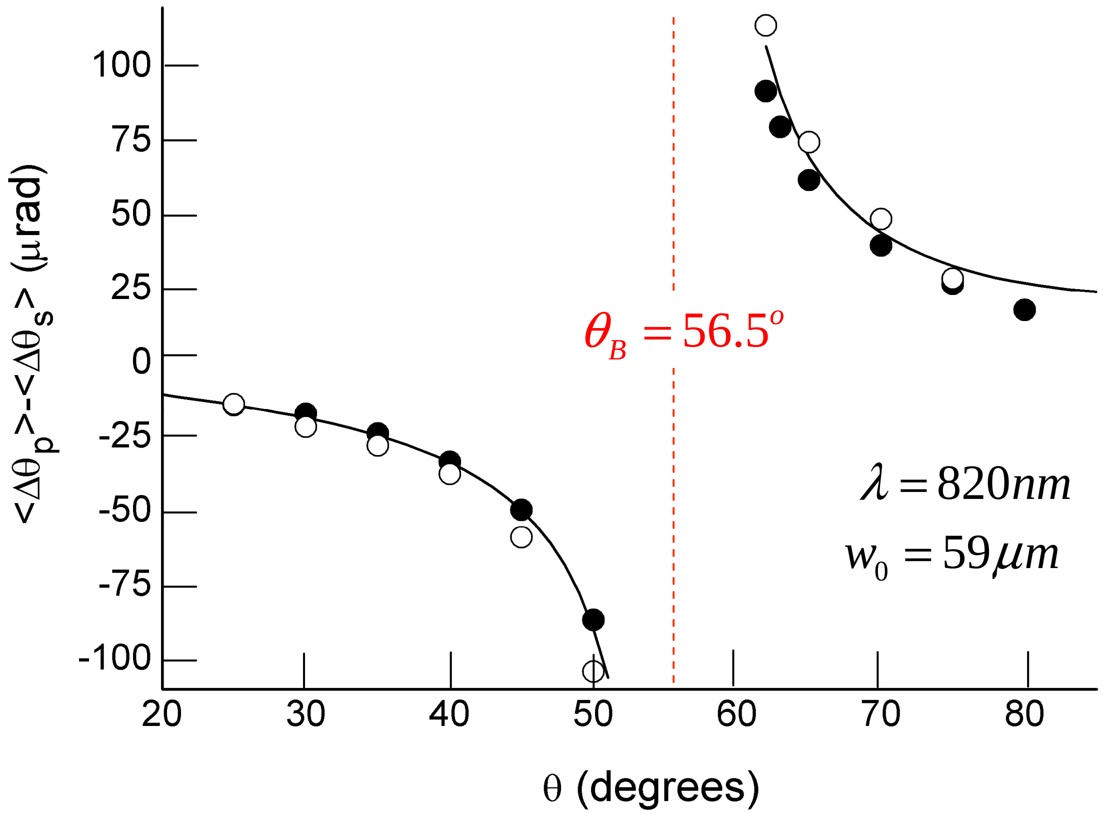
$$\frac{X}{Z} \text{ or } \frac{Y}{Z}$$

Measurement of angular GH shift (*p*-polarization)

$$\Theta_{GH} = \frac{\theta_0^{\,2}}{2}\frac{1}{r}\frac{dr}{d\theta}$$

Singularity at Brewster angle

$$\left(r_p = 0\right)$$

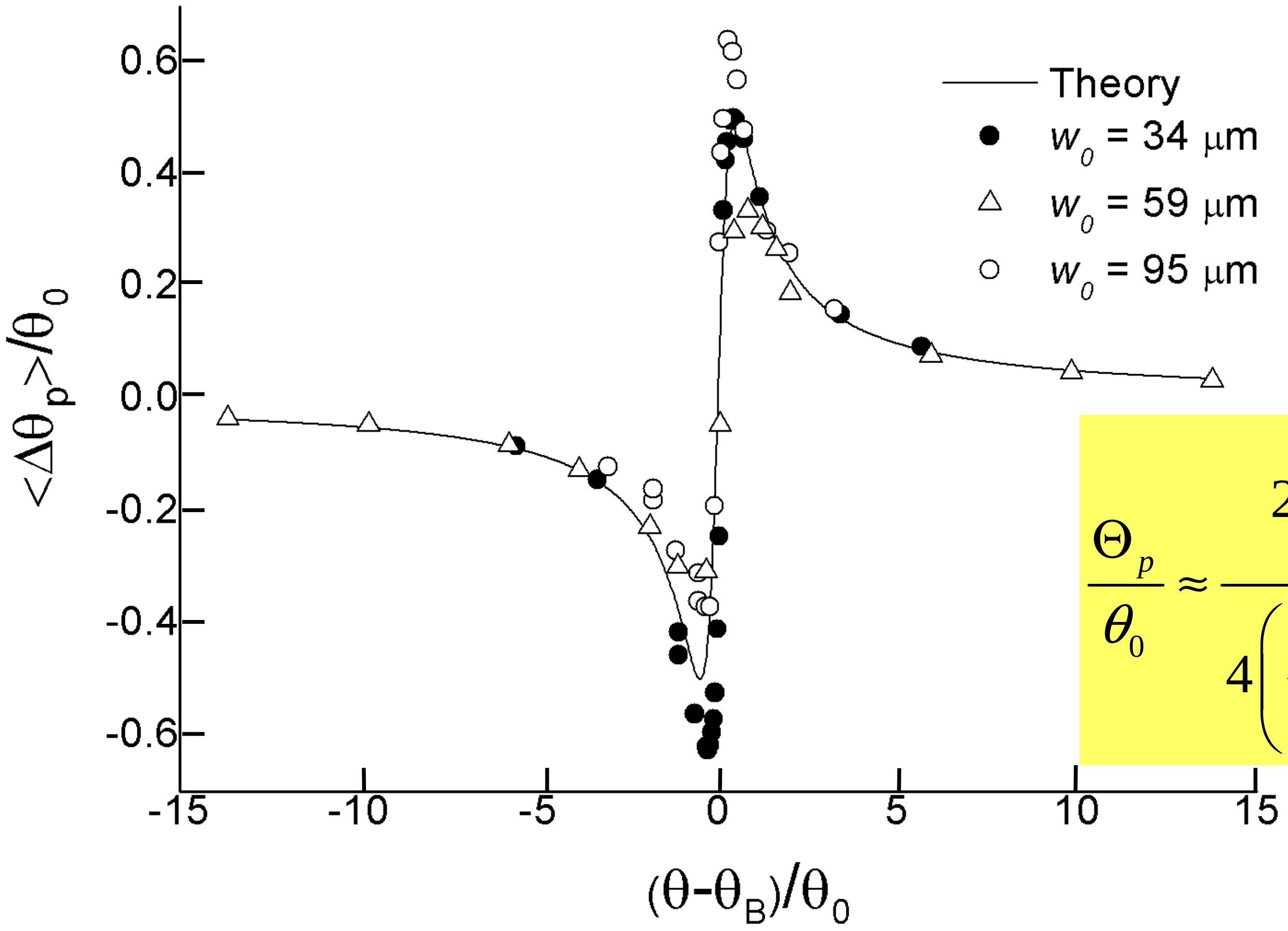


Merano, Aiello, Van Exter & Woerdman, Nat. Phot. (2009)

Angular GH shift near Brewster angle

P-polarization

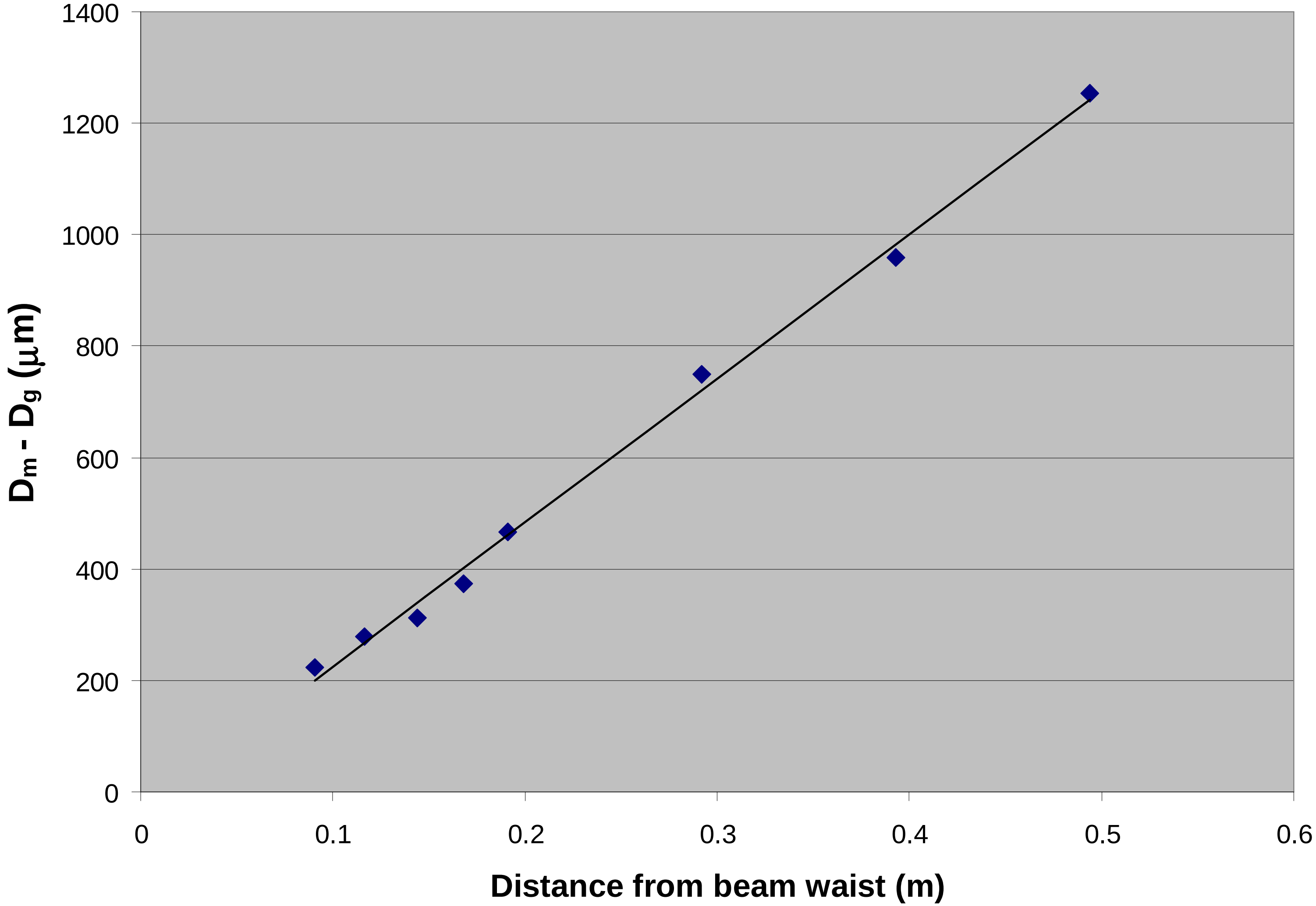
$\lambda = 820\ nm$



$$\frac{\Theta_p}{\theta_0} \approx \frac{2\left(\dfrac{\theta-\theta_B}{\theta_0}\right)}{4\left(\dfrac{\theta-\theta_B}{\theta_0}\right)^2+1}$$

Merano, Aiello, Van Exter & Woerdman, Nat. Phot. (2009)

Angular nature of shift confirmed



Merano, Aiello, Van Exter & Woerdman, Nat. Phot. (2009)

Dielectric *vs* metallic reflection

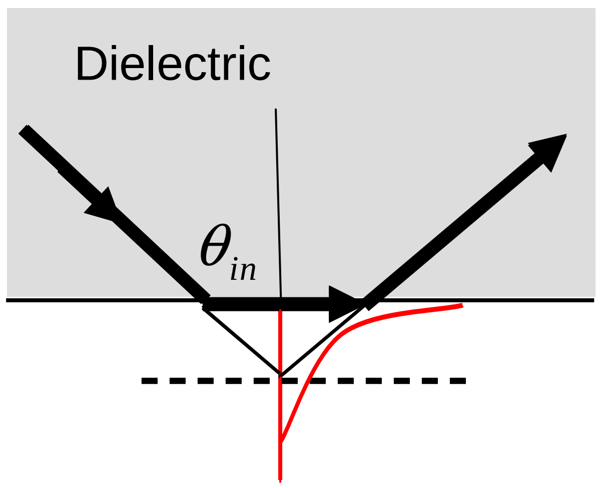
Spatial GH shift

Energy flow in surface wave:

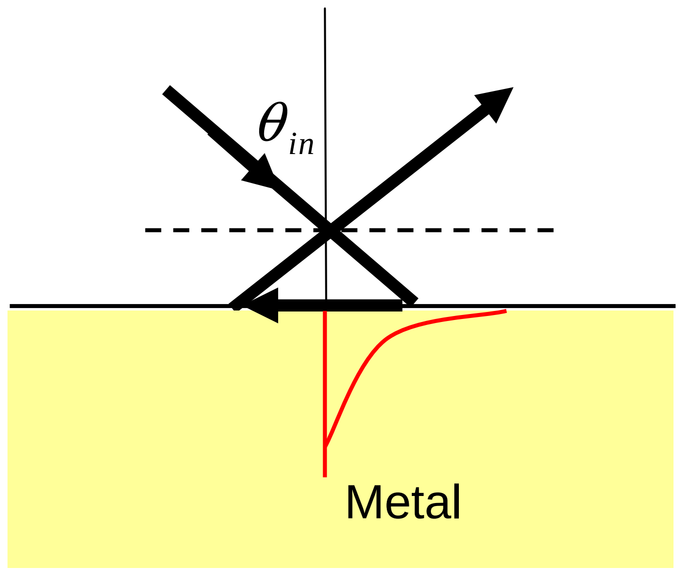
$$S = \frac{c^2 k_x |E|^2}{8\pi\omega\varepsilon}$$

$\varepsilon > 0$ for dielectric

$\varepsilon < 0$ for metal



Positive GH shift



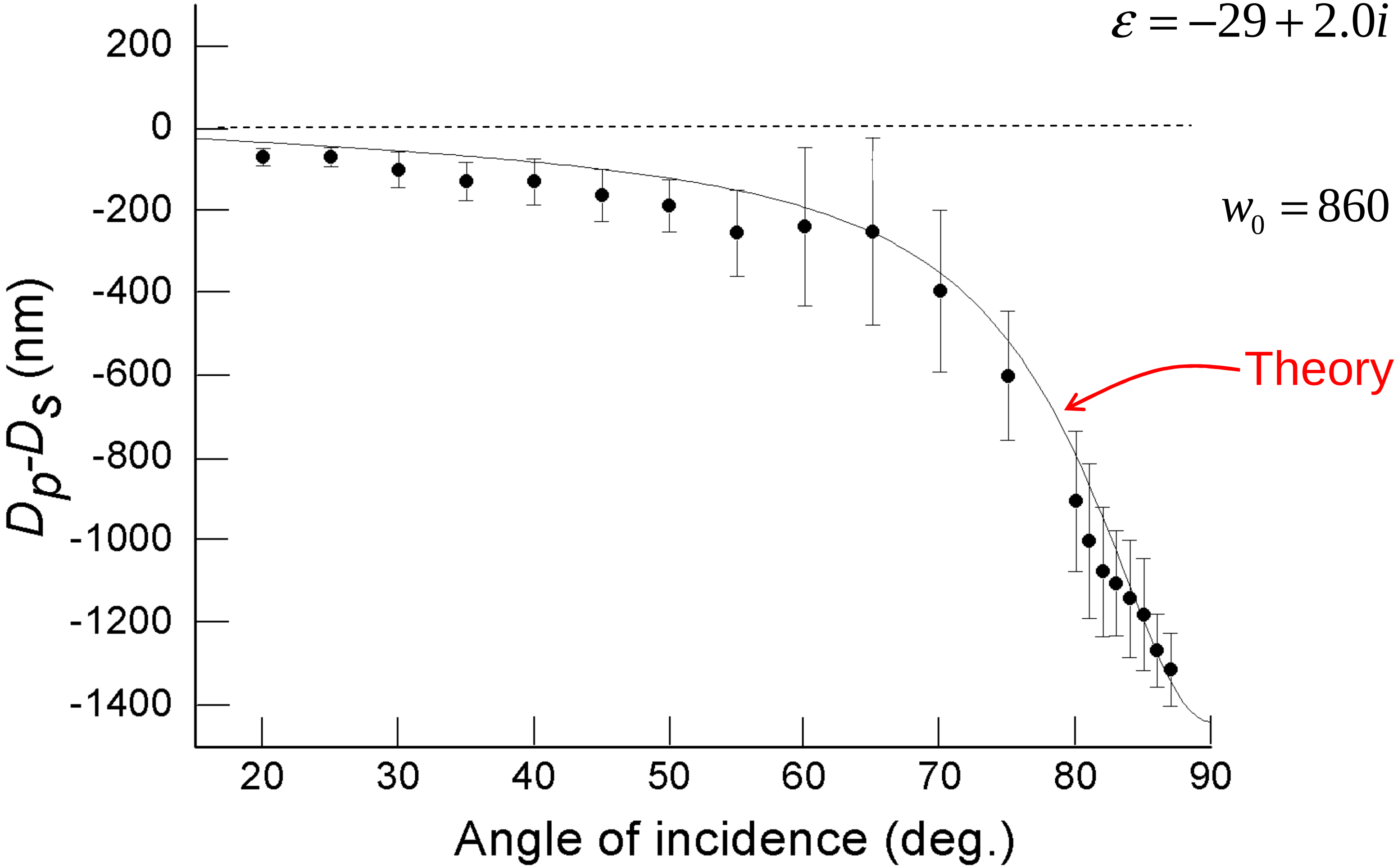
Negative GH shift

Spatial GH shift in reflection by Au mirror

Negative shift !

Au @ 820nm:

$$\varepsilon = -29 + 2.0i = n^2$$

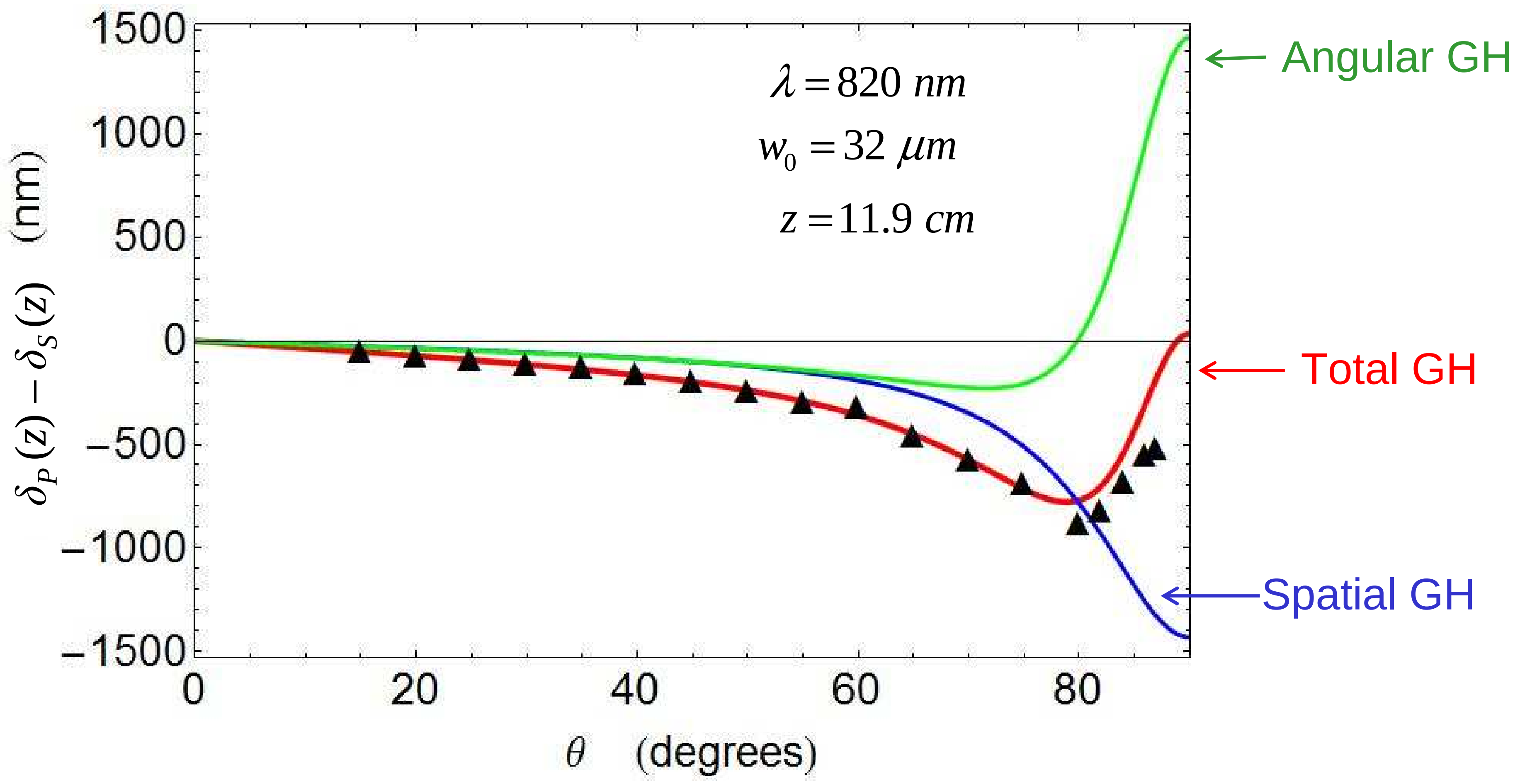


$$w_0 = 860\ \mu m$$

Merano, Aiello, 't Hooft, Van Exter, Eliel & Woerdman, Opt. Express (2007)

Role of metal loss (Au)

Loss couples spatial and angular GH: $\frac{d\varphi}{d\theta} \,\&\, \frac{dR}{d\theta}$



Aiello, Merano & Woerdman, Phys. Rev. A (2009)

Beam shift of OAM carrying beam

Angular momentum of paraxial photon:

$$j = l + s$$

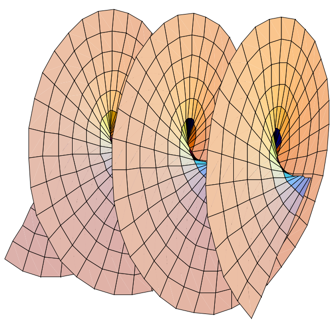
Spin Angular Momentum (circular polarization):

$\pm 1\hbar/$photon

Orbital Angular Momentum:

$0, \pm 1, \pm 2, \ldots \hbar/$photon

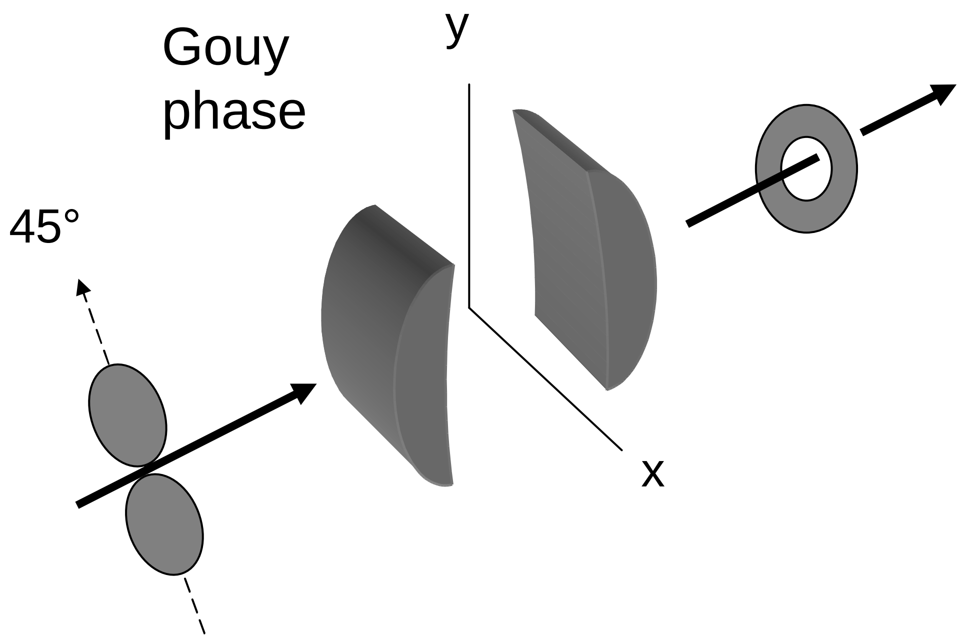
Laguerre-Gauss beam
("donut" beam)

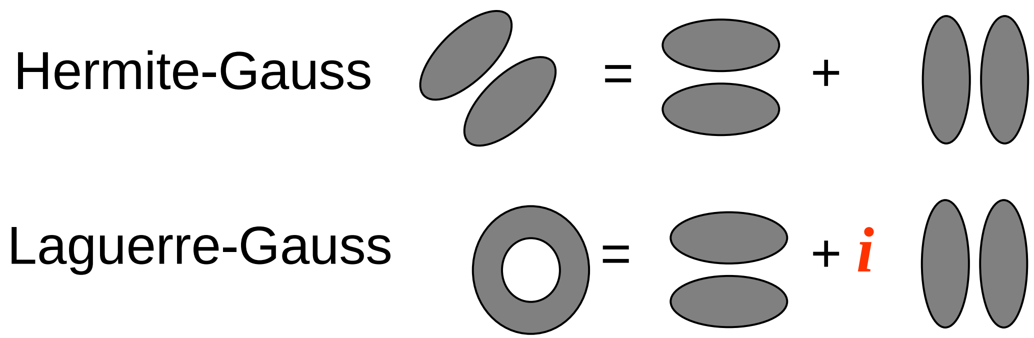


Helical wavefront

Allen, Beijersbergen, Spreeuw & Woerdman, Phys. Rev. A (1992)

How to produce a helical beam

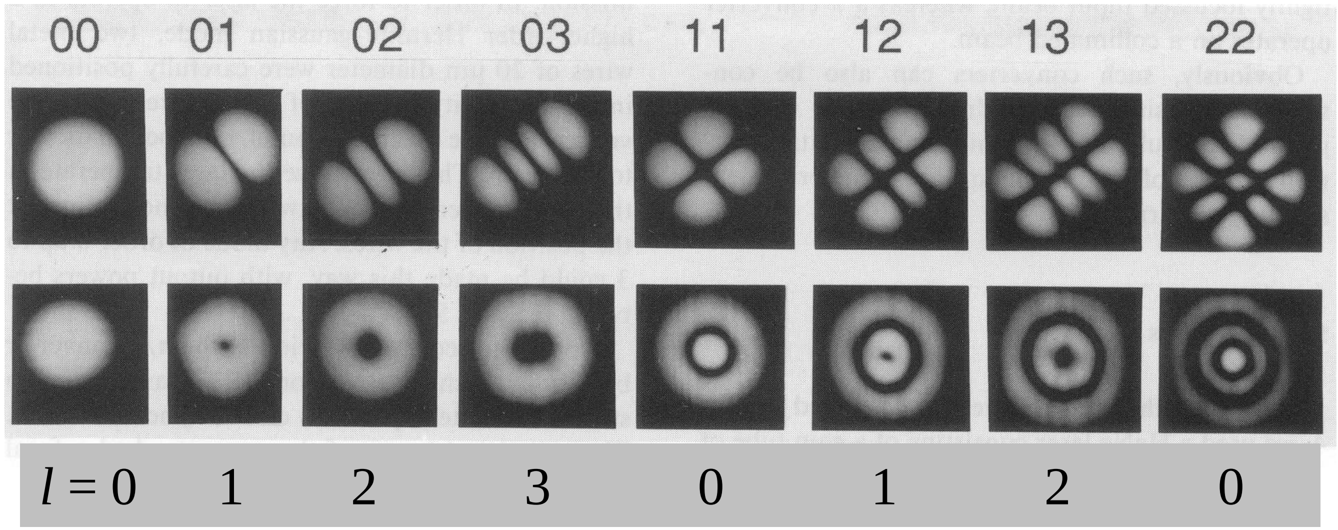




HG ↔LG mode converter

HG_{nm} input

LG_{lp} output



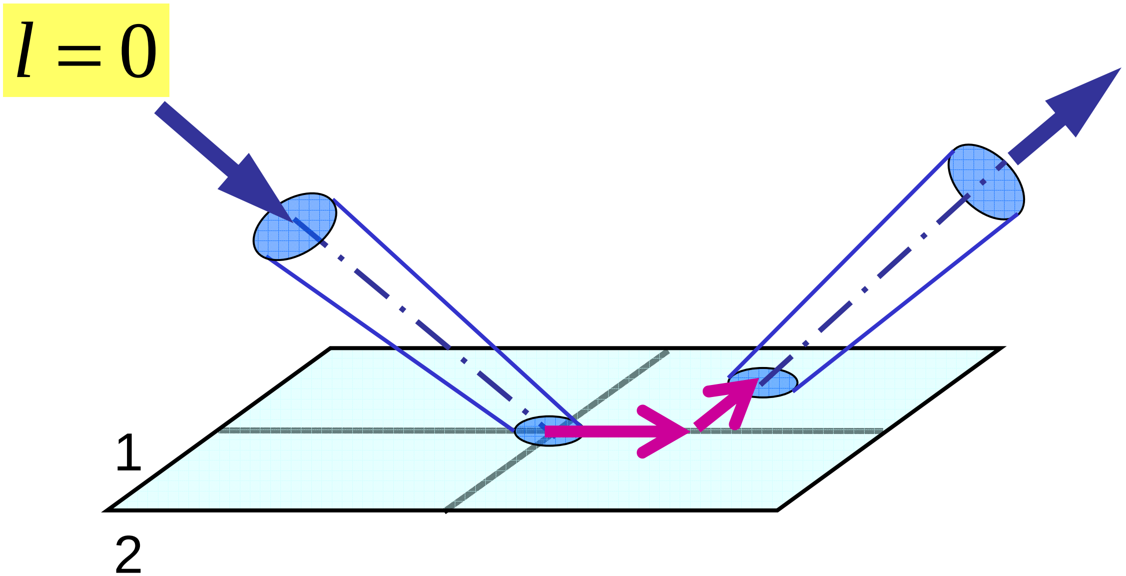
Azimuthal mode number $l = n - m$

("OAM quantum number")

Radial mode index $p = \min(n, m)$

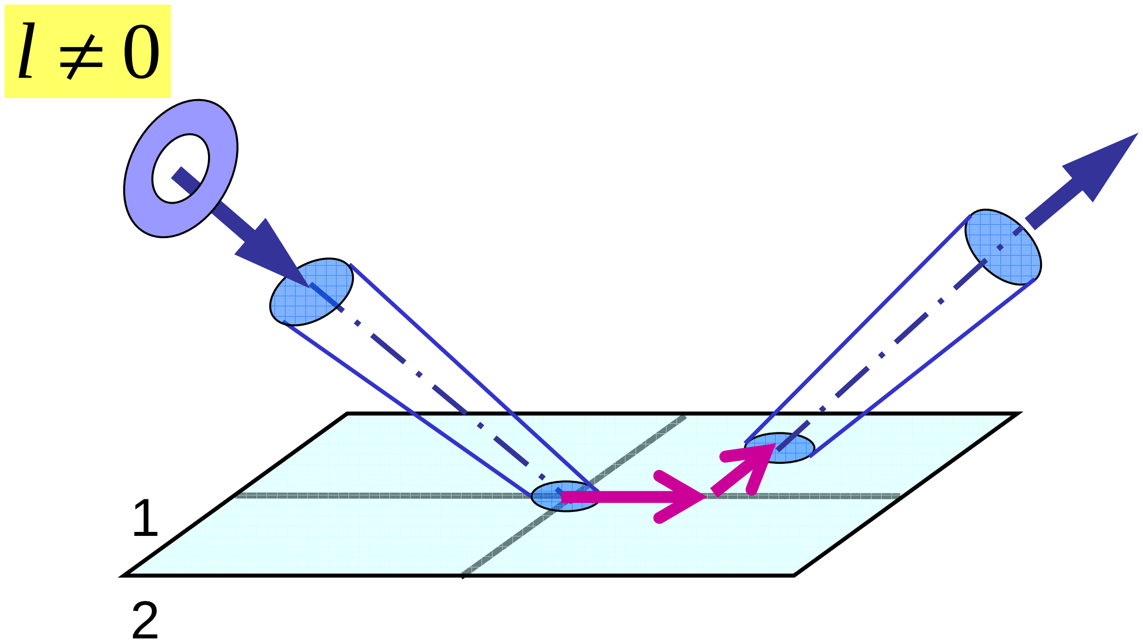
Beijersbergen, Allen, Van der Veen & Woerdman, Opt. Commun. (1993)

Effect of OAM on the 4 beam shifts



$$\begin{bmatrix} \Delta_{GH} \\ \Theta_{IF} \\ \Delta_{IF} \\ \Theta_{GH} \end{bmatrix}$$

Connection ?



$$\begin{bmatrix} \Delta^{l}_{GH} \\ \Theta^{l}_{IF} \\ \Delta^{l}_{IF} \\ \Theta^{l}_{GH} \end{bmatrix}$$

OAM 4x4 mixing matrix

$$\begin{bmatrix} \Delta_{GH}^{l} \\ \Theta_{IF}^{l} \\ \Delta_{IF}^{l} \\ \Theta_{GH}^{l} \end{bmatrix} = \begin{bmatrix} 1 & -2l & 0 & 0 \\ 0 & 1+|2l| & 0 & 0 \\ 0 & 0 & 1 & 2l \\ 0 & 0 & 0 & 1+|2l| \end{bmatrix} \begin{bmatrix} \Delta_{GH} \\ \Theta_{IF} \\ \Delta_{IF} \\ \Theta_{GH} \end{bmatrix}$$

l-dependent beam shifts

Effect of OAM

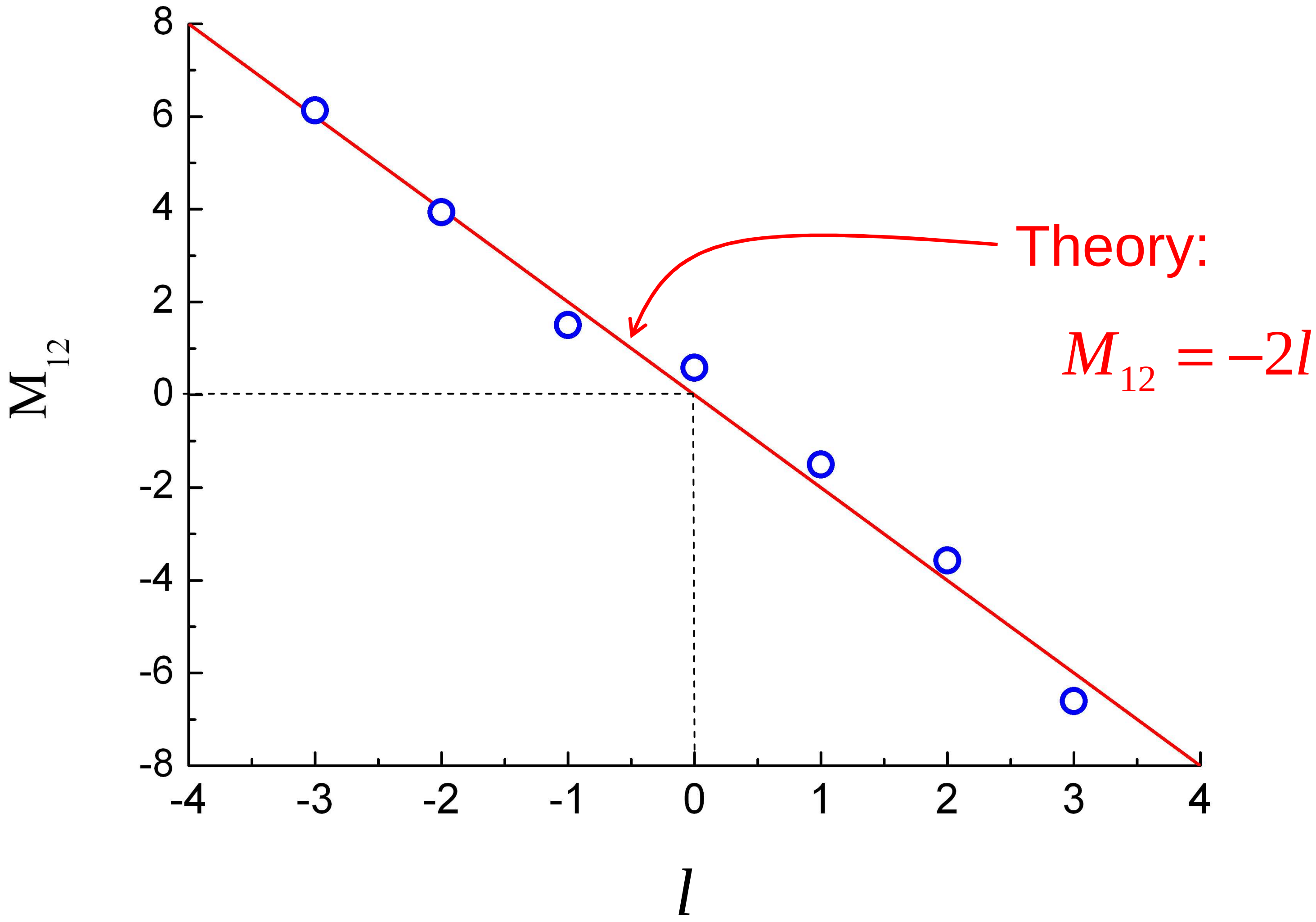
Beam shifts for *l* =0 (depend on polarization, i.e. on SAM)

Off-diagonal elements: OAM mixes GH and IF

On-diagonal elements: OAM enhances angular shifts

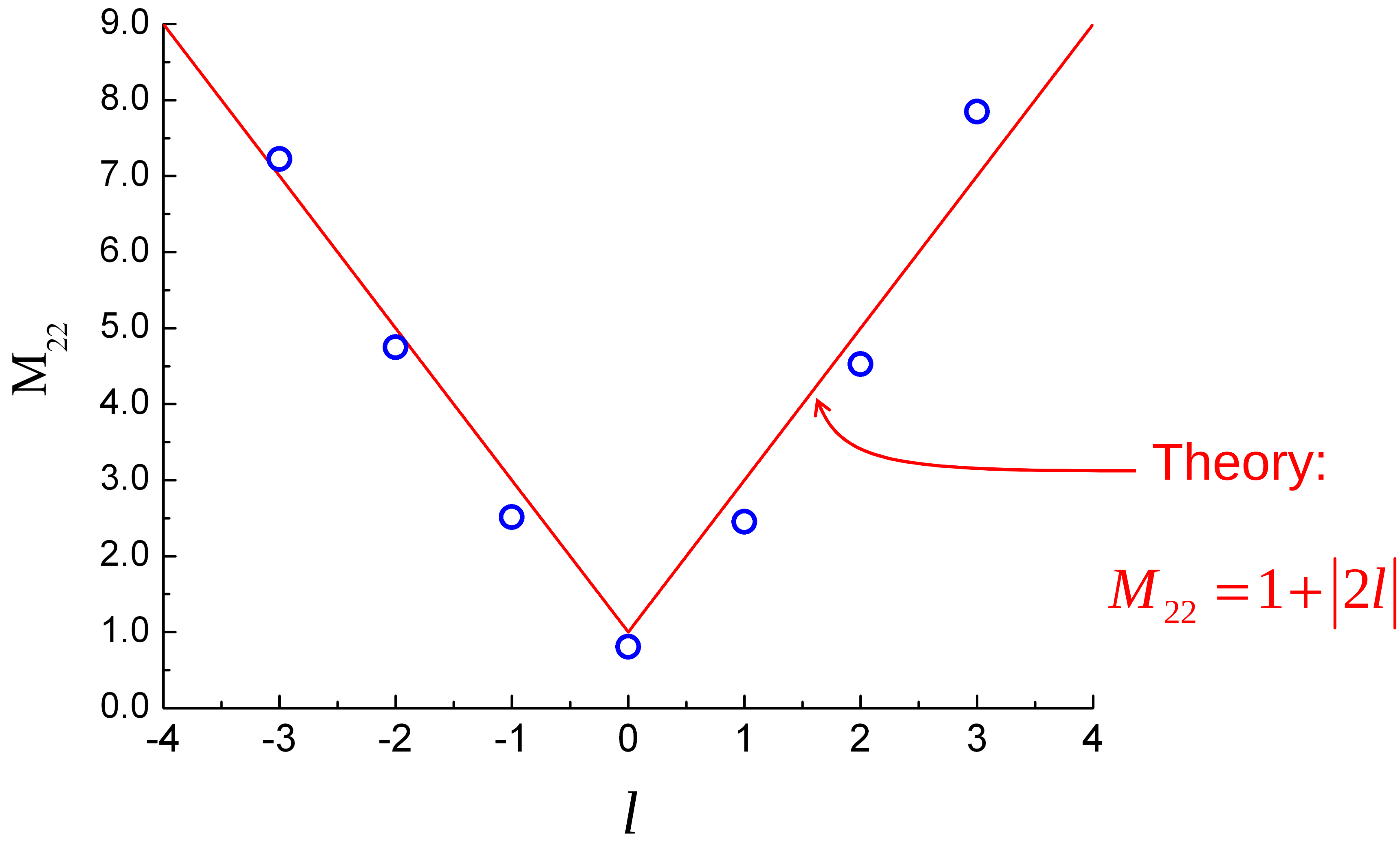
Merano, Hermosa, Woerdman & Aiello, Phys. Rev. A (2010)

Element M_{12} of mixing matrix



Merano, Hermosa, Woerdman & Aiello, Phys. Rev. A (2010)

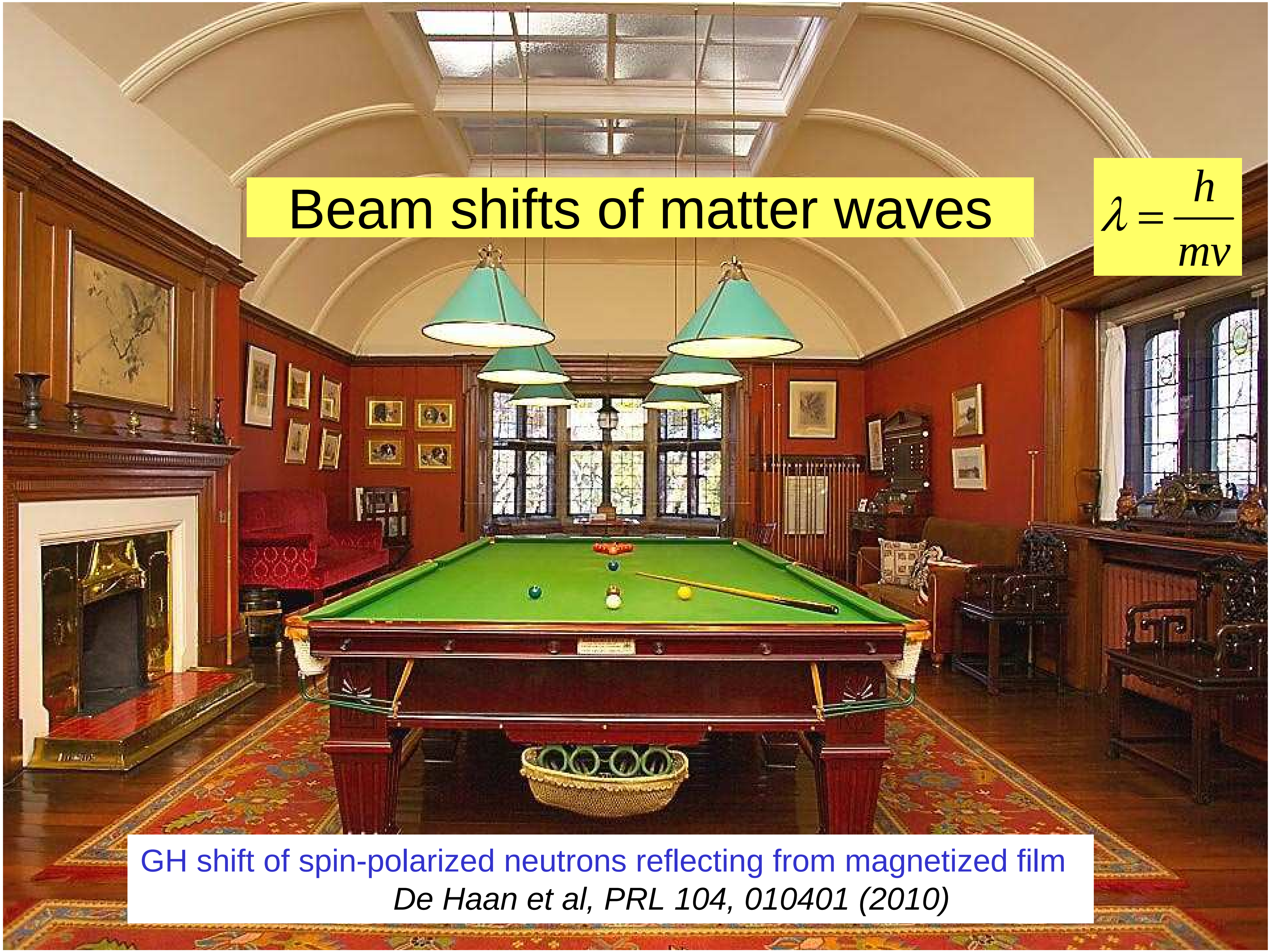
Element M_{22} of mixing matrix



Merano, Hermosa, Woerdman & Aiello, Phys. Rev. A (2010)

Beam shifts of matter waves

$$\lambda = \frac{h}{mv}$$



GH shift of spin-polarized neutrons reflecting from magnetized film

De Haan et al, PRL 104, 010401 (2010)

Acknowledgements

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