QUANTUM OPTICS GROUP

Dipartimento di Fisica, Sapienza Università di Roma

From qubit to qudit with hybrid OAM-polarization quantum state

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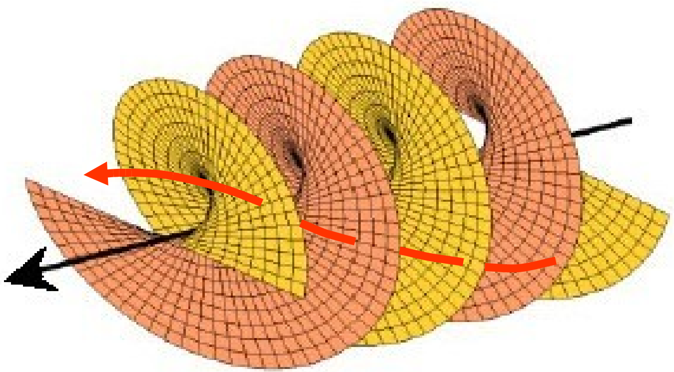
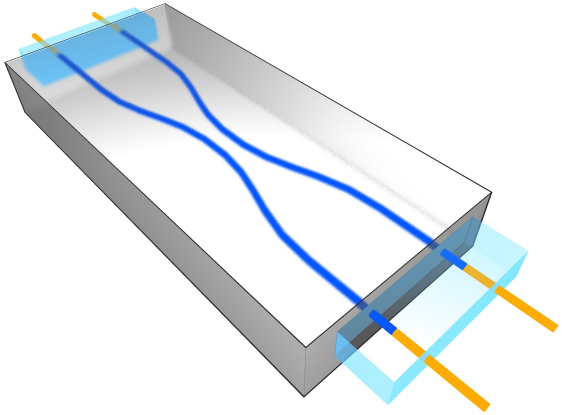
Istituto Nazionale di Ottica, CNR

http:\\quantumoptics.phys.uniroma1.it

High dimensional systems: Orbital angular momentum of light

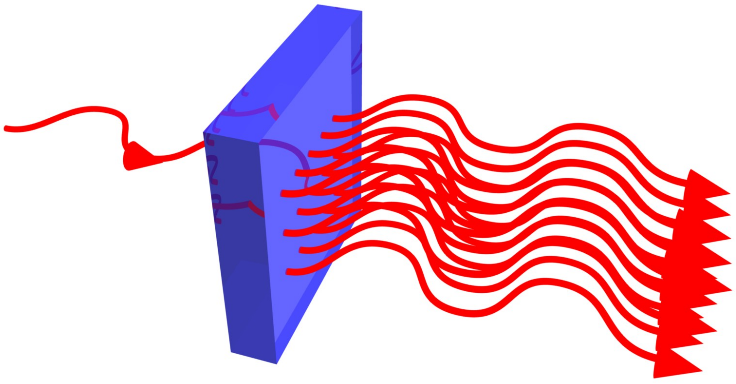


Integrated photonic quantum circuits



Hyper-entanglement generation and manipulation

Amplification of optical quantum states



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Quantum information

Theory of Information

+

Quantum Mechanics → **Quantum Information**

Quantum bit (qubit): quantum state in $\boldsymbol{H_2}$

Challenges: from basic sciences to emerging quantum technologies

(1) Fundamental physics:

Shed light on the boundary between classical and quantum world

Exploiting quantum parallelism to simulate quantum random many-body systems

(1) New cryptographic protocols, quantum imaging, quantum metrology

(2) Quantum computing, quantum simulation

superpositi

many sy

Entanglement:

istic trait

echanics

E. Schrödinger

Entanglement: new resource to elaborate information

Optical implementation

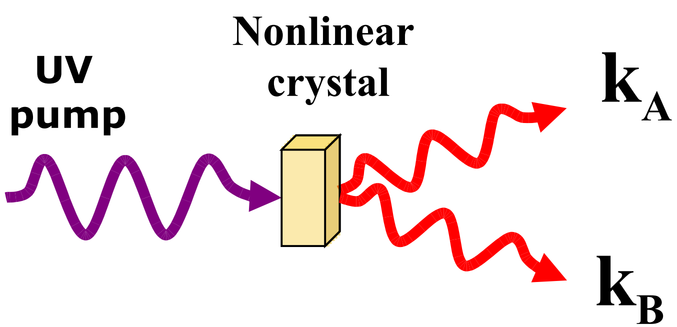
Quantum optics: **excellent experimental test bench for various novel concepts introduced within the framework of the QI theory**

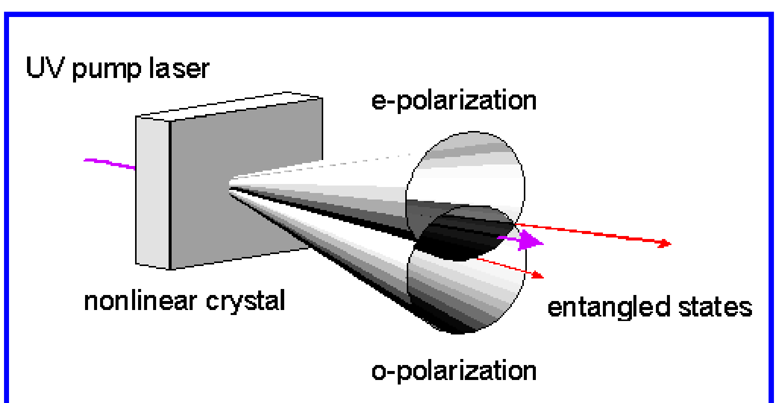
$$\alpha|0\rangle + \beta|1\rangle \longleftrightarrow \alpha|H\rangle + \beta|V\rangle$$

Polarization state of a single photon
H = horizontal; V=vertical

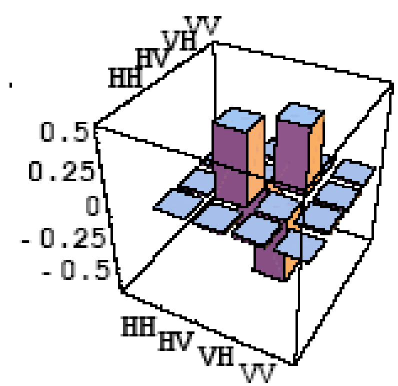
Entangled states:

Generation





Characterization



Applications

- Non-locality tests
- Quantum cryptography
- Quantum teleportation
- Quantum metrology
- Quantum computation
- Simulate quantum random many-body systems

Outline

- Introduction to quantum information

I- OAM qubit

- Qubit implementation via 2-dimensional subspace of OAM
- Quantum transferrer between polarization and OAM
- Generation of hybrid OAM-polarization entangled states
- Resilience of OAM qubit

II – Higher dimensional quantum systems

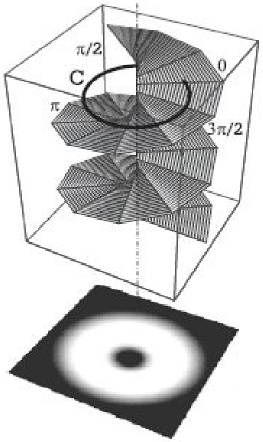
- Realization of π – OAM ququart
- Quantum cryptography based on contextuality

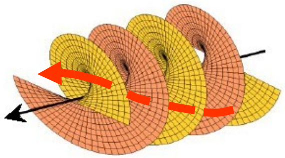
The orbital angular momentum of light for quantum information processing

Quantum information → qubit

Different degrees of freedom:

- Polarization
- Linear Momentum
- Orbital Angular Momentum
- ....

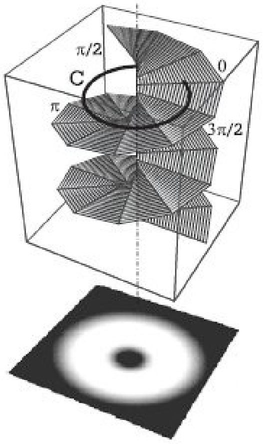




The orbital angular momentum of light for quantum information processing

Quantum information ⇒ **qubit**

Different degrees of freedom:
- Polarization
- Linear Momentum
- Orbital Angular Momentum
- ....

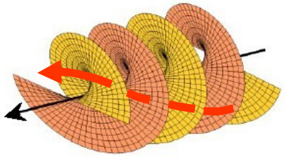


Orbital Angular Momentum (OAM)

⇓

Degree of freedom of light associated with rotationally structured transverse spatial modes

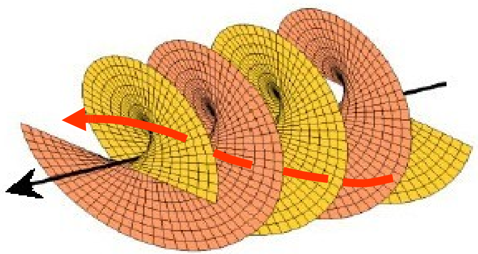
OAM ⇒ Infinite-dimensional degree of freedom ⇒ **Qudit (d > 2)** Multi-level quantum system



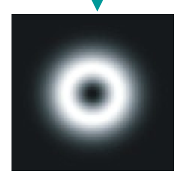
The orbital angular momentum of light

Laguerre-Gauss : $u_{p,l}(r,\varphi,z) \propto u(r,z)e^{-il\varphi}$

Helicoidal phase front
$l = 0\ ,\ \pm 1\ ,\ \pm 2\ ,\ldots$



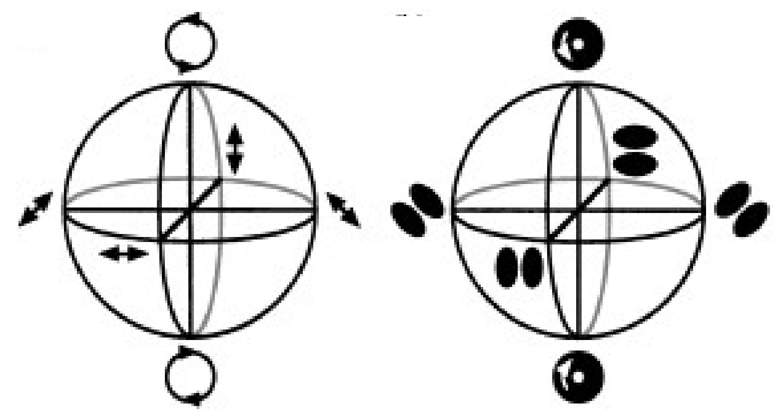
Phase Singularity



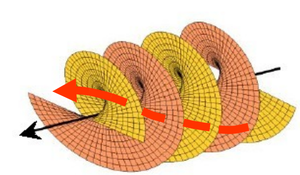
<u>Observation:</u> For a chosen OAM subspace $o_m = \{+m,-m\}$, it is possible to construct a sphere analogous to the Poincaré one, for superpositions of left- and right- handed LG modes.

<u>ex.</u> Subspace $o_2 = \{+2,-2\}$

$$|d_{R,L}\rangle = \frac{1}{\sqrt{2}}(|+2\rangle \pm i|-2\rangle) \qquad |d_{\pm}\rangle = \frac{1}{\sqrt{2}}(|+2\rangle \pm |-2\rangle)$$



Polarization OAM



M. Padgett, and J. Courtial, Optics *Letters* **24**, 430 (1999)

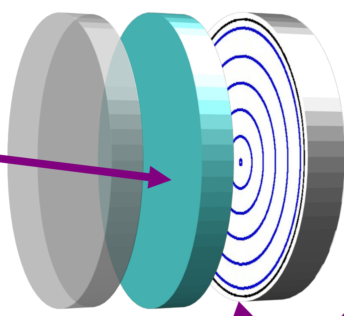
The q-plate device (1/2)

Angular Momentum
- **Spinorial ( )** → Anisotropic medium (birefringent)
- **Orbital (OAM)** → Inhomogenous medium

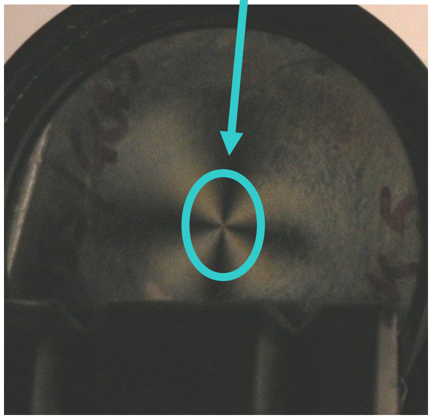
To couple and OAM: Q-plate
anisotropic and inhomogenous

Topological charge q
$\Delta m = 2q\hbar$

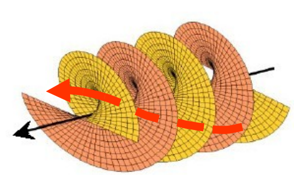
Nematic Liquid Crystal



Circular rubbing of one substrate for the NLC orientation



The q-plate thickness is chosen in order to have half-wave retardation depending on the working wavelength.



L. Marrucci, et al., *Physical Review Letters* **96**, 163905 (2006)

The q-plate device (2/2)

Input State | **Output State**

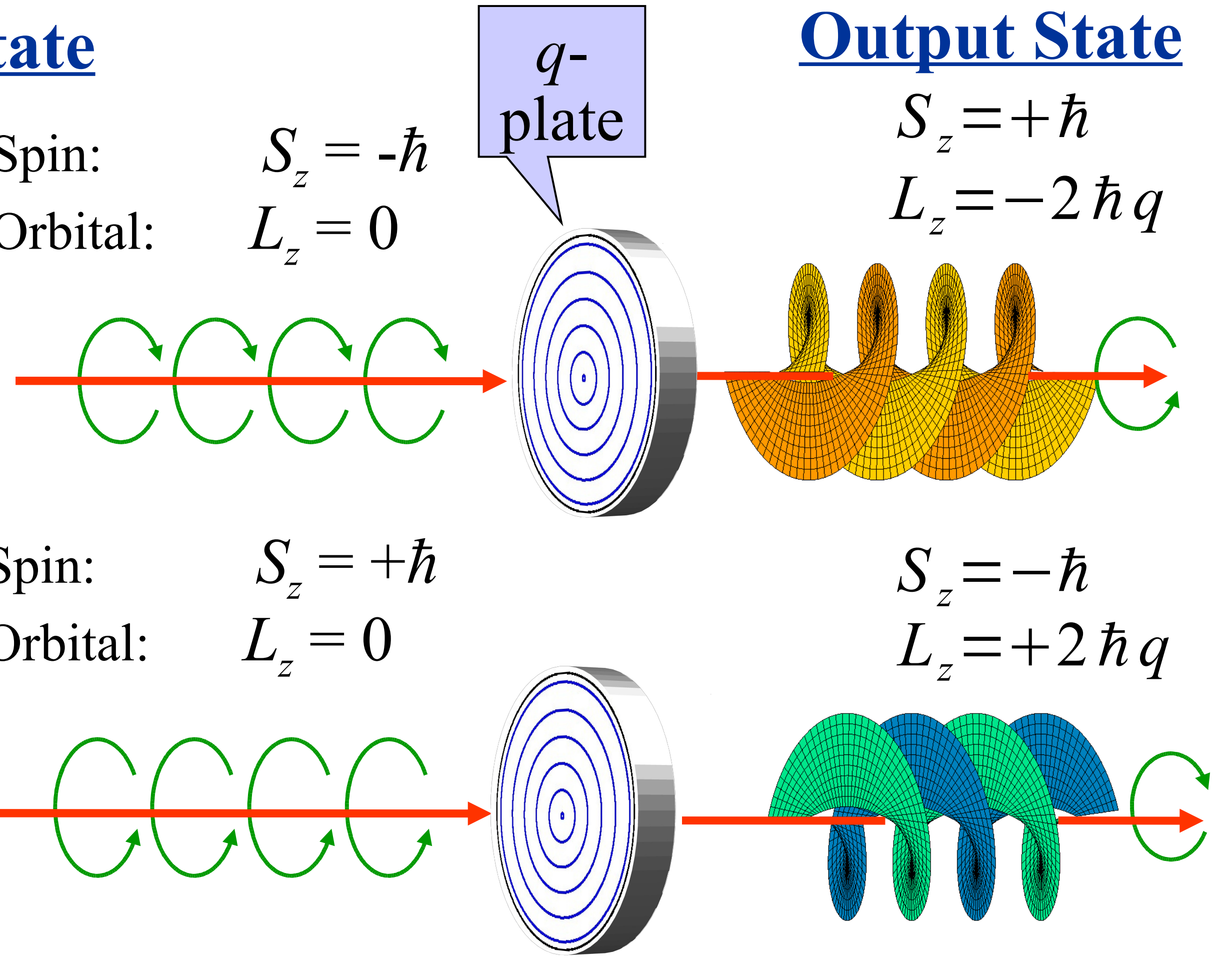
TEM$_{00}$ with **right** circular polarization

Spin: $S_z = -\hbar$

Orbital: $L_z = 0$

$S_z = +\hbar$

$L_z = -2\hbar q$



TEM$_{00}$ with **left** circular polarization

Spin: $S_z = +\hbar$

Orbital: $L_z = 0$

$S_z = -\hbar$

$L_z = +2\hbar q$

Qplate in the quantum regime

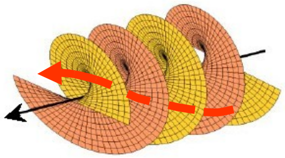
$$|L\rangle_\pi |m\rangle_o \xrightarrow{QP} |R\rangle_\pi |m+2\rangle_o$$

$$|R\rangle_\pi |m\rangle_o \xrightarrow{QP} |L\rangle_\pi |m-2\rangle_o$$

Unitary evolution on a generic input state

$$\alpha|L\rangle_\pi |m\rangle_o + \beta|R\rangle_\pi |m\rangle_o \xrightarrow{QP} \alpha|R\rangle_\pi |m+2\rangle_o + \beta|L\rangle_\pi |m-2\rangle_o$$

- The qplate: a quantum interface between polarization and OAM
    - Single photon entanglement between polarization and OAM
    - Quantum transferrer: polarization ⟶ OAM
    - Quantum transferrer: OAM ⟶ polarization



Single photon entanglement

The q-plate introduces a quantum correlation between the OAM and the polarization π degree of freedom

Single-photon entanglement

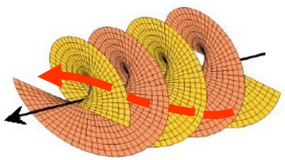
$$\begin{matrix}|H\rangle_\pi|0\rangle_o \\ |V\rangle_\pi|0\rangle_o\end{matrix} \xrightarrow{\textbf{QP}} \frac{1}{\sqrt{2}}\left[|L\rangle_\pi|-2\rangle_{o_2} \pm |R\rangle_\pi|+2\rangle_{o_2}\right]$$

$\Re[\rho]$

$\Im[\rho]$ **Input State** $|H\rangle_\pi|0\rangle_o$

$$\rho_{\pi,o_2} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$C = 0.95 \pm 0.02$$



E. Nagali, *et al., Physical Review Letters* **103,** 013601 (2009)

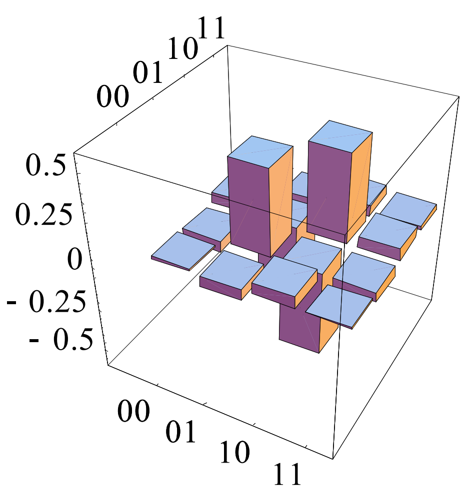
Single photon entanglement

The q-plate introduces a quantum correlation between the OAM and the polarization π degree of freedom

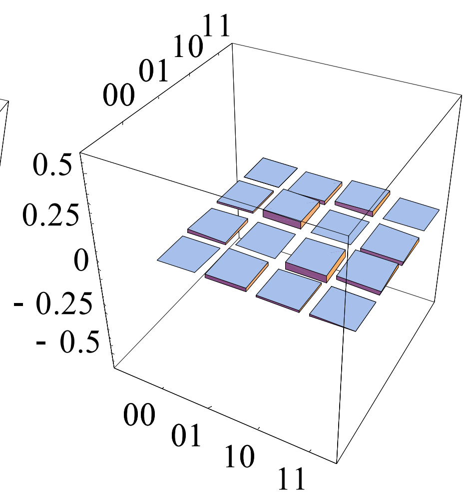
Single-photon entanglement

$$\begin{matrix} |H\rangle_\pi |0\rangle_o \\ |V\rangle_\pi |0\rangle_o \end{matrix} \xrightarrow{\textbf{QP}} \frac{1}{\sqrt{2}}\left[|L\rangle_\pi |-2\rangle_{o_2} \pm |R\rangle_\pi |+2\rangle_{o_2} \right]$$

$\Re[\rho]$



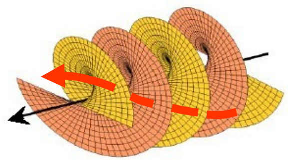
$\Im[\rho]$



Input State $|V\rangle_\pi |0\rangle_o$

$$\rho_{\pi,o_2} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

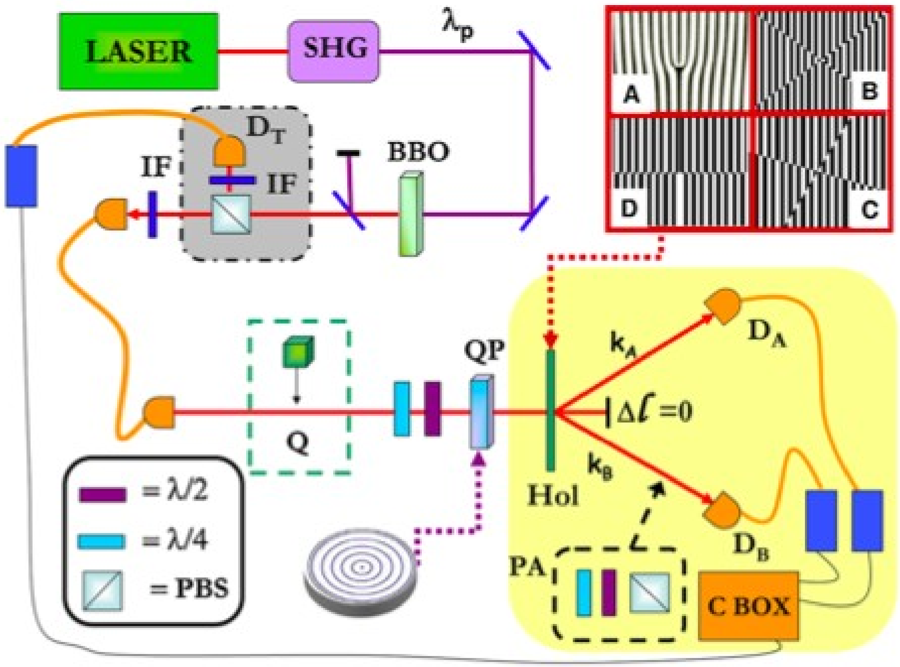
$$C = 0.97 \pm 0.02$$



E. Nagali, *et al.*, *Physical Review Letters* **103,** 013601 (2009)

Single photon entanglement: Quantum state characterization

One photon - 2 qubits
- 1 qubit encoded in polarization
- 1 qubit encoded in OAM



Measurement of 2 qubits
- Holograms:
  Measurement of OAM qubit
- Waveplates + PBS:
  Measurement of polarization qubit

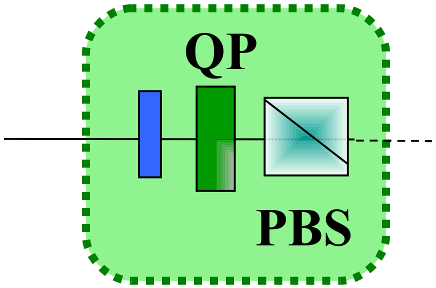
Characterization of 2 qubits statets:
- Quantum state tomography
(analogue for a 2 x 2 space of the measurement of Stokes parameters)
- Reconstruction of the density matrix

E. Nagali, *et al., Physical Review Letters* **103,** 013601 (2009)

Quantum transferrers $\pi \rightarrow$ OAM

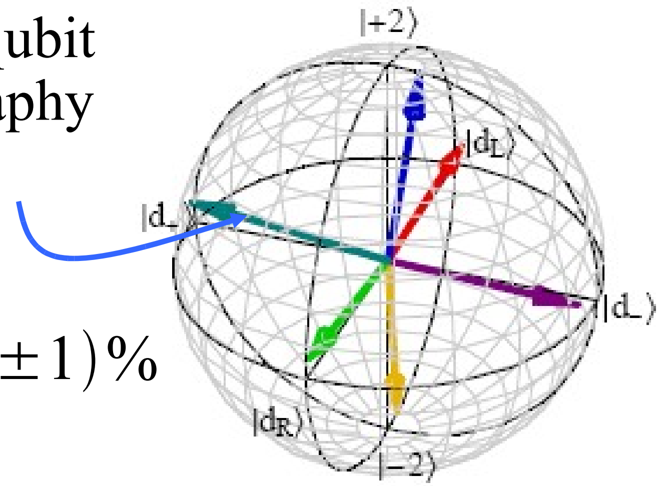
Transferrer $\pi \rightarrow 0_2$

$$|\varphi\rangle_\pi |0\rangle_o$$



$$|H\rangle_\pi |\varphi\rangle_{o_2}$$

Single qubit tomography



$$F = (98 \pm 1)\%$$

$$(\alpha |R\rangle_\pi + \beta |L\rangle_\pi)|0\rangle_{o_2}$$

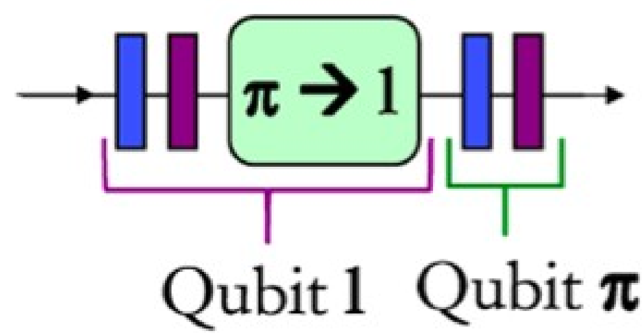
qplate

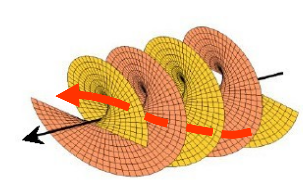
$$\alpha |L\rangle_\pi |-2\rangle_{o_2} + \beta |R\rangle_\pi |+2\rangle_{o_2}$$

Polarising beam splitters

$$|H\rangle_\pi (\alpha |-2\rangle_{o_2} + \beta |+2\rangle_{o_2})$$

Probability of success: 50%



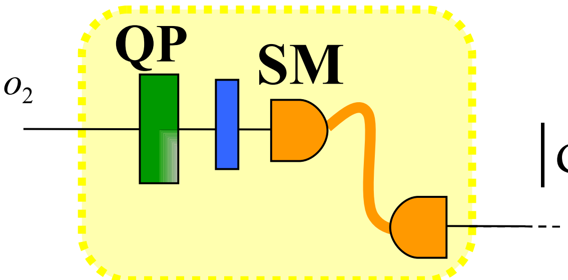


E. Nagali, *et al., Physical Review Letters* **103,** 013601 (2009); *Optics Express* **17,** 18745 (2009)

Quantum transferrers OAM → π

Transferrer $0_2 \rightarrow \pi$

$$|H\rangle_\pi |\varphi\rangle_{o_2}$$



$$|\varphi\rangle_\pi |0\rangle_o$$

$$|H\rangle_\pi (\alpha|-2\rangle_{o_2} + \beta|+2\rangle_{o_2})$$

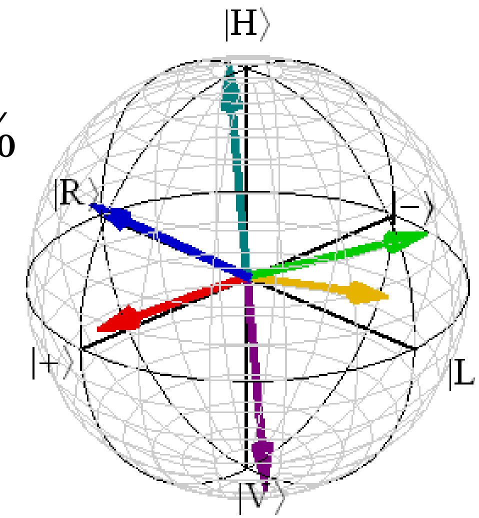
qplate

$$\alpha|R\rangle_\pi|0\rangle_o + \beta|L\rangle_\pi|0\rangle_o$$
$$\alpha|R\rangle_\pi|-4\rangle_{o_4} + \beta|L\rangle_\pi|+4\rangle_{o_4}$$

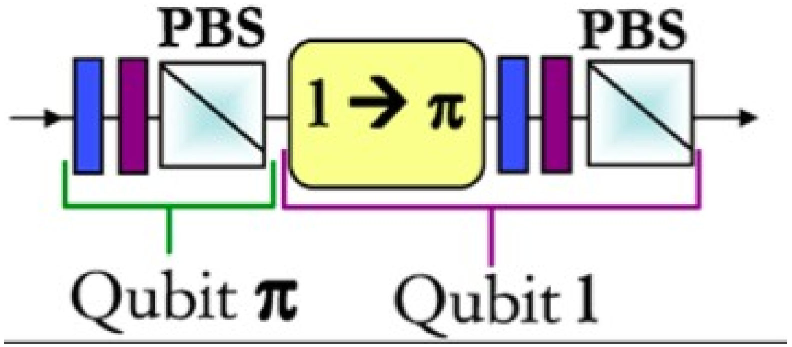
Single mode fiber

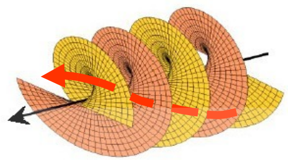
$$(\alpha|R\rangle_\pi + \beta|L\rangle_\pi)|0\rangle_{o_2}$$

$$F = (97 \pm 1)\%$$



Probability of success: 50%



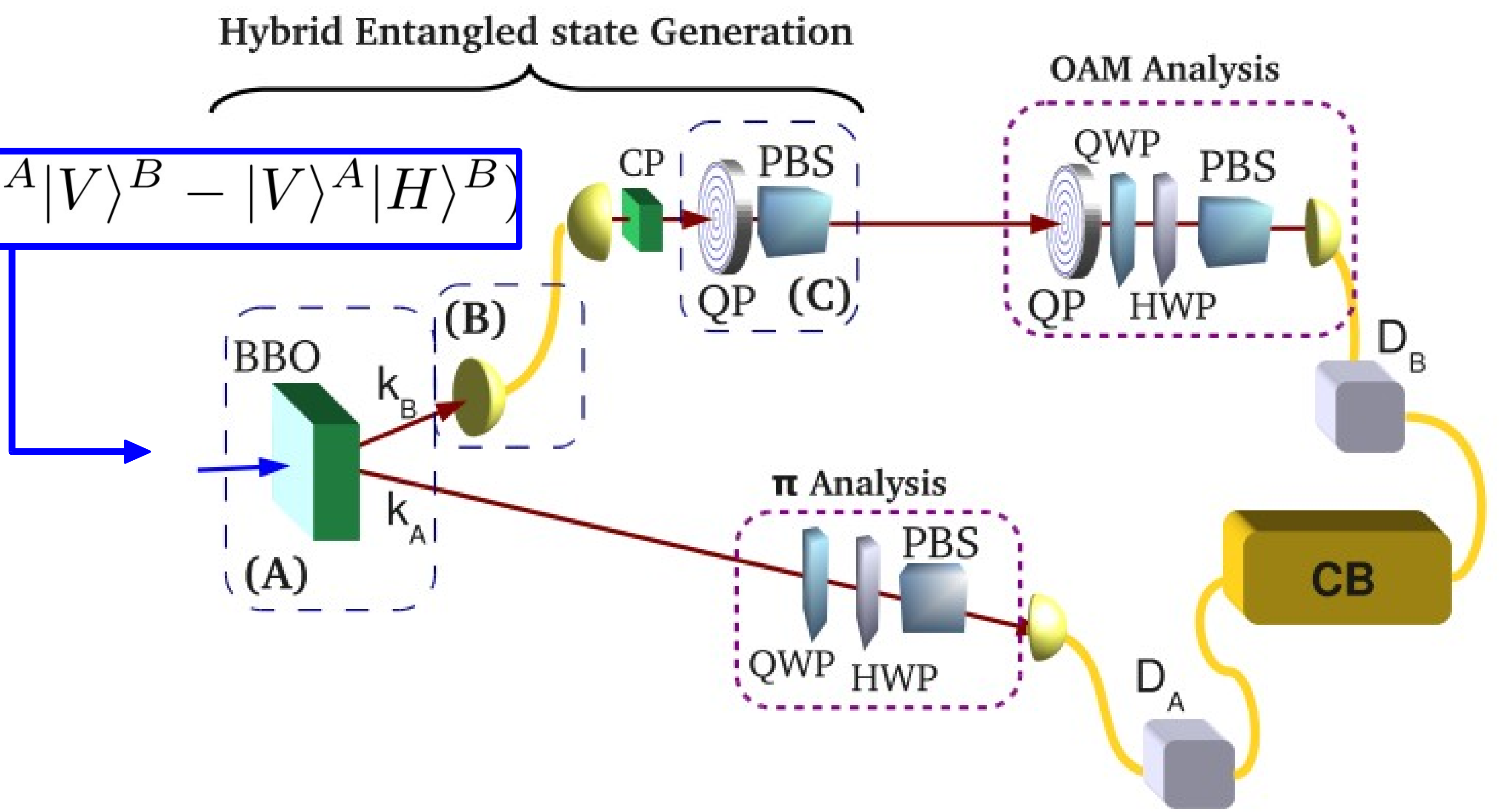


E. Nagali, *et al., Physical Review Letters* **103,** 013601 (2009); *Optics Express* **17,** 18745 (2009)

Hybrid entanglement between π and OAM

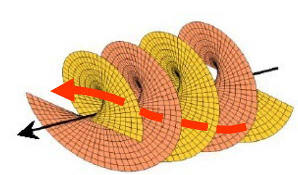
Hybrid entangled states: entanglement between different degrees of freedom of a particle pair

$$\frac{1}{\sqrt{2}}(|H\rangle^A|V\rangle^B - |V\rangle^A|H\rangle^B)$$

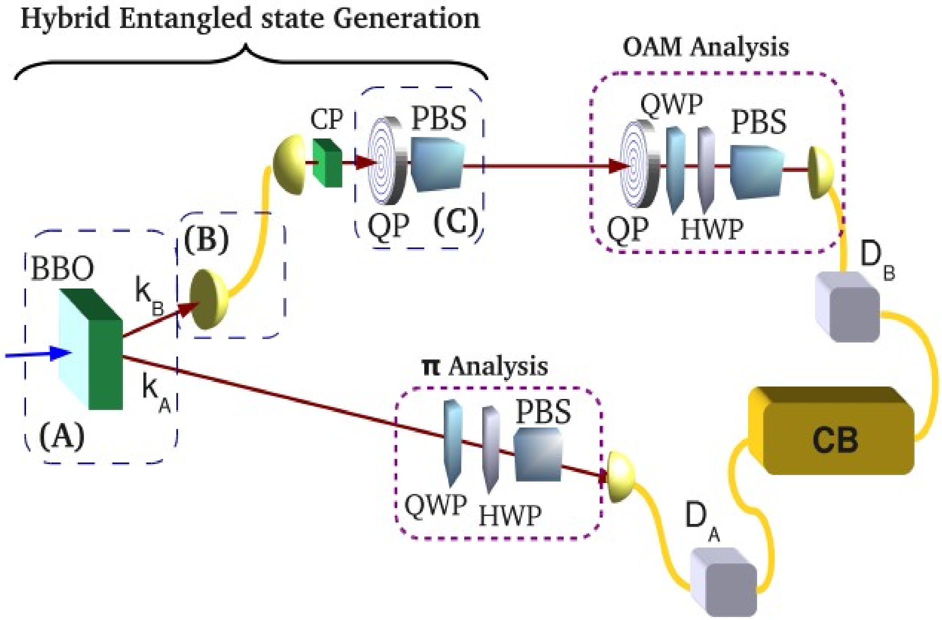


SPDC source + **Quantum transferrers** ➡

$$\frac{1}{\sqrt{2}}(|H\rangle^A_\pi|+2\rangle^B_{o_2} - |V\rangle^A_\pi|-2\rangle^B_{o_2})|0\rangle^A_o|H\rangle^B_\pi$$

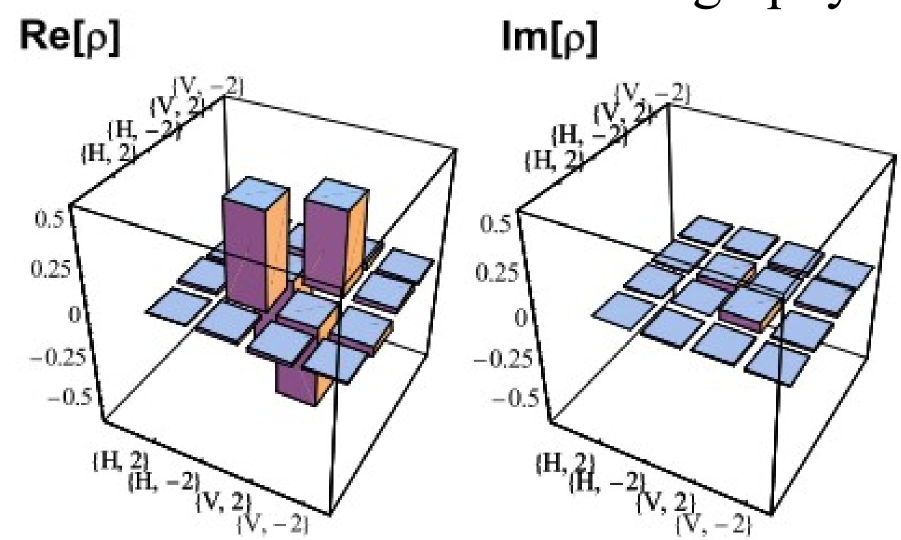


Hybrid entanglement between π and OAM

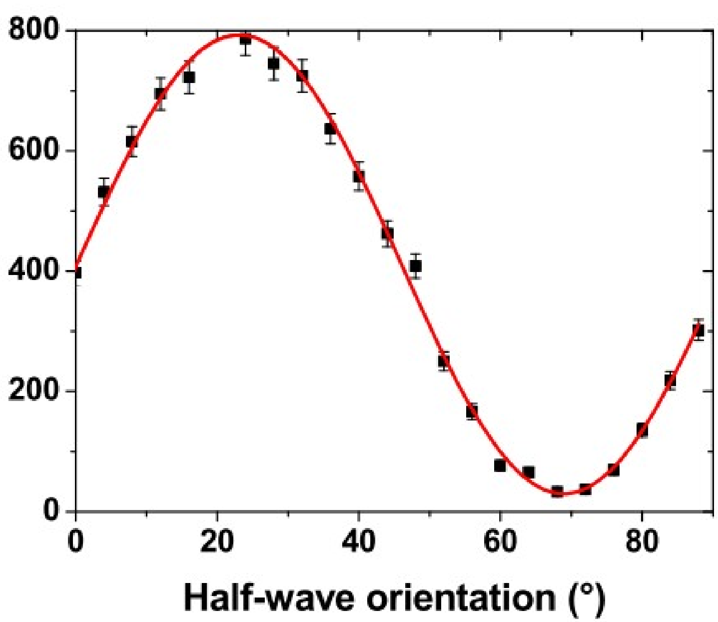


$$\frac{1}{\sqrt{2}}(|H\rangle_\pi^A|+2\rangle_{o_2}^B-|V\rangle_\pi^A|-2\rangle_{o_2}^B)|0\rangle_o^A|H\rangle_\pi^B$$

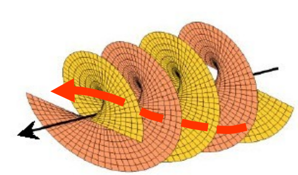
Quantum state tomography



$F=(0.957\pm0.009)$



$S_{CHSH}=(2.51\pm0.02)$



E. Nagali, and F. Sciarrino, *Optics Express* **17,** 18243 (2010)

Decoherence of OAM qubit for partial transmission

PRL 94, 153901 (2005) PHYSICAL REVIEW LETTERS week ending 22 APRIL 2005

Atmospheric Turbulence and Orbital Angular Momentum of Single Photons for Optical Communication

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(Received 8 November 2004; published 18 April 2005)

Free-space information transfer using light beams carrying orbital angular momentum

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Influence of atmospheric turbulence on the propagation of quantum states of light carrying orbital angular momentum

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*Corresponding author: boyd@optics.rochester.edu

PHYSICAL REVIEW A **83**, 042338 (2011)

Resilience of orbital-angular-momentum photonic qubits and effects on hybrid entanglement

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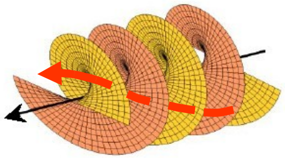
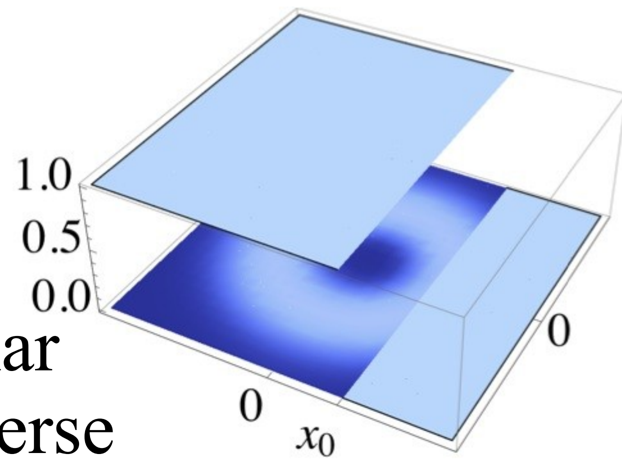
(Received 17 November 2010; published 29 April 2011)



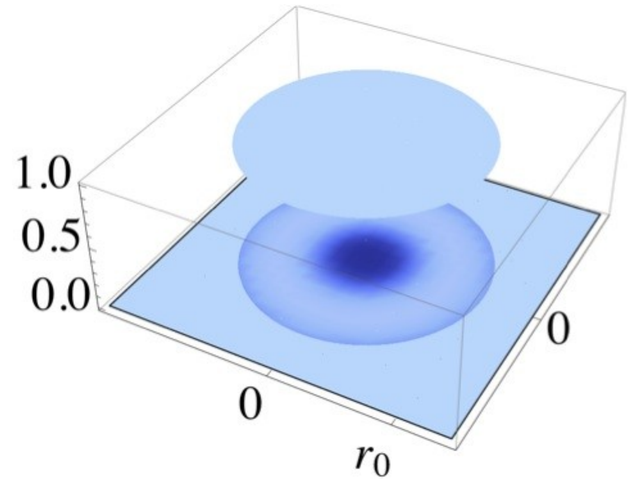
Resilience of OAM qubit (1/4)

How a partial transmission does affect the transmission of information ?

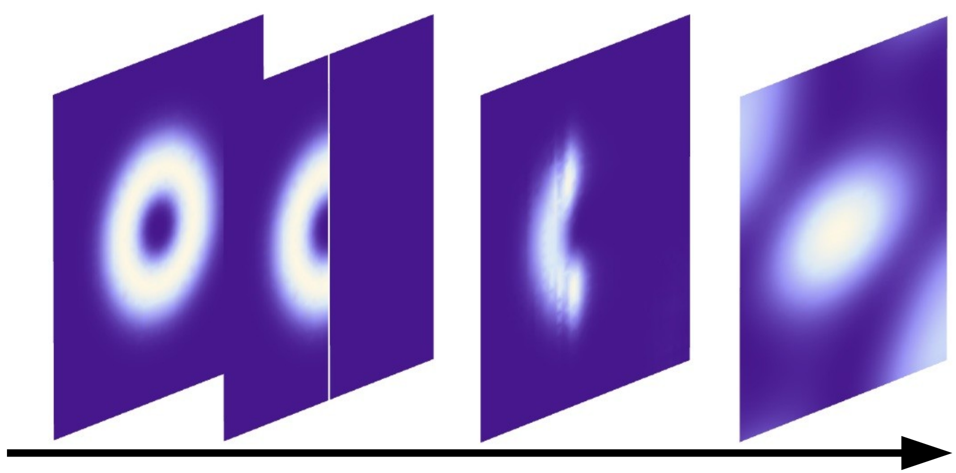
Obstructions:



Planar transverse obstruction

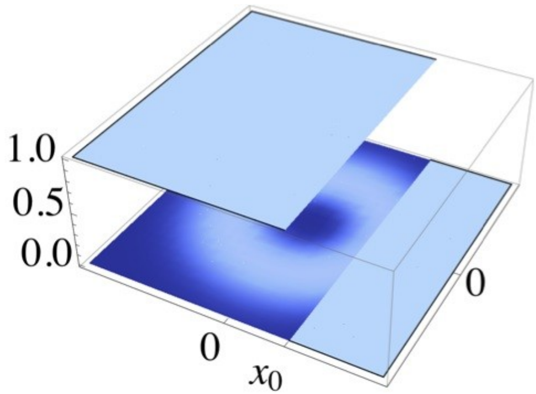


Radial obstruction



Direction of propagation

Resilience of OAM qubit (2/4)



Planar transverse obstruction:

Spread of the OAM values

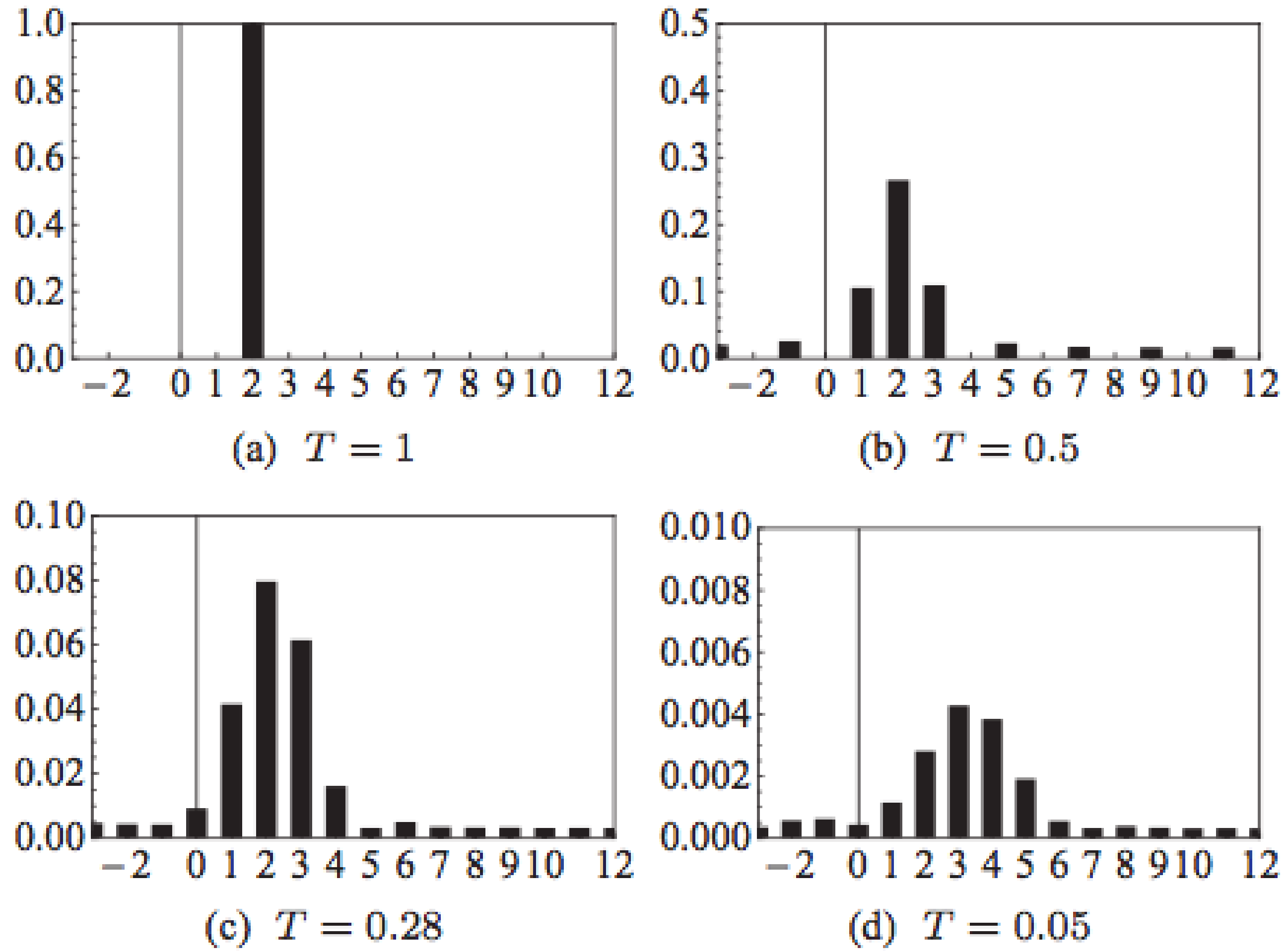
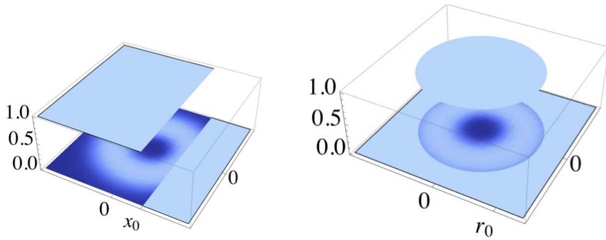


FIG. 3. (Color online) Spread in the measurement probabilities of OAM modes with $\ell' = -2, \ldots, 12$ for various positions x_0 of a $B(x_0)$ aperture inserted into the path of an $\ell = 2$ beam (i.e., for decreasing values of transmittance T).

Resilience of OAM qubit (3/4)

Initial qubit state $|\psi\rangle = |H\rangle_{\pi}(\alpha|-2\rangle_{o_2} + \beta|+2\rangle_{o_2})$



obstruction

Unchanged state component

$$\kappa_{\psi} = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy\ A^*(x,y)\ A'(x,y)$$

Orthogonal state component in the o_2 subspace

$$\kappa_{\psi^{\perp}} = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy [A^{\perp}(x,y)]^* A'(x,y),$$

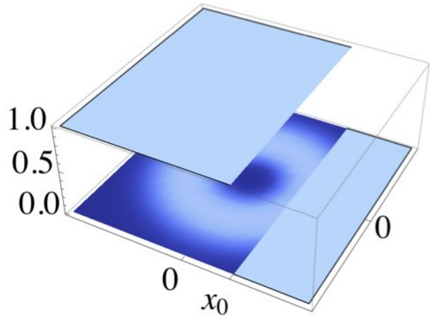
Probability to remain in the encoding subspace

$$P_{o_2} = |\kappa_{\psi}|^2 + |\kappa_{\psi^{\perp}}|^2$$

Fidelity of state transmission

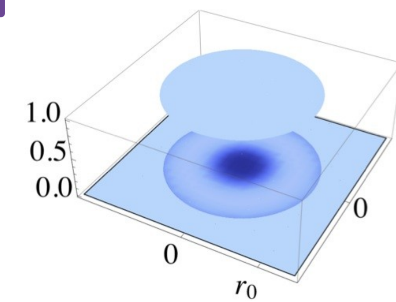
$$F = P(\psi)/[P(\psi) + P(\psi^{\perp})]$$

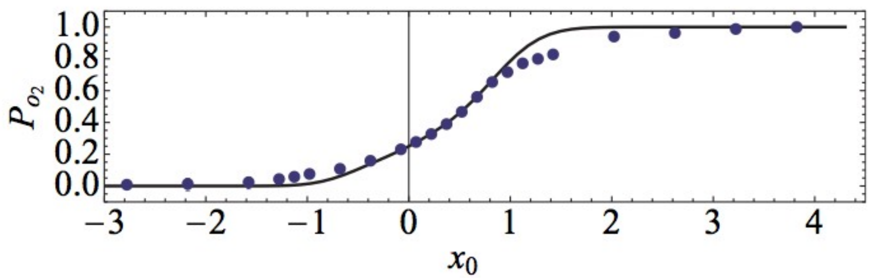
Resilience of OAM qubit (4/4)



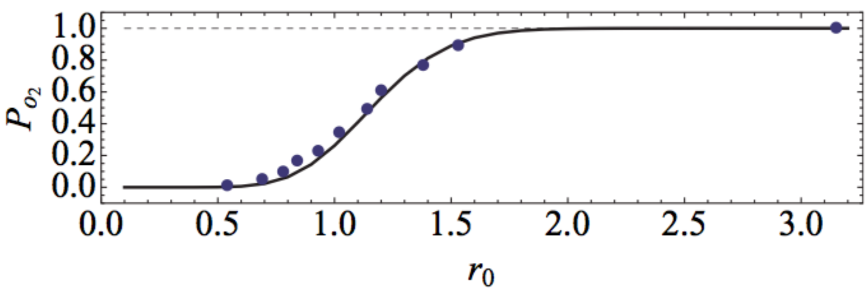
Planar transverse obstruction

Radial obstruction

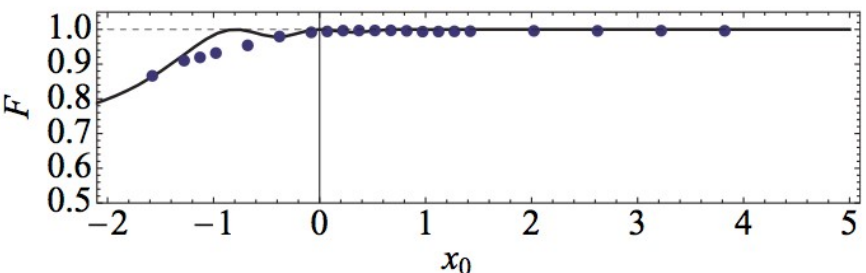




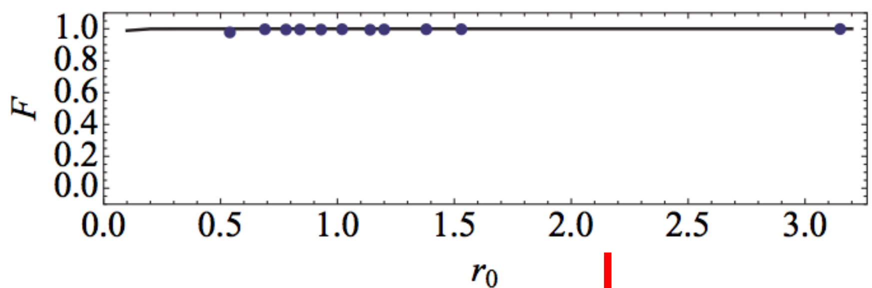
(a)

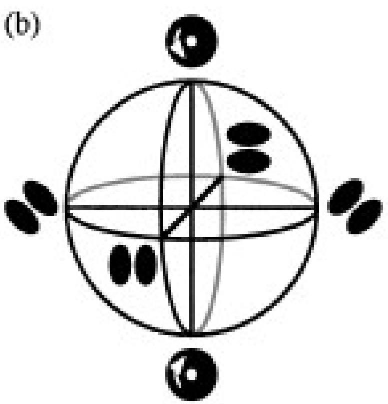


(a)

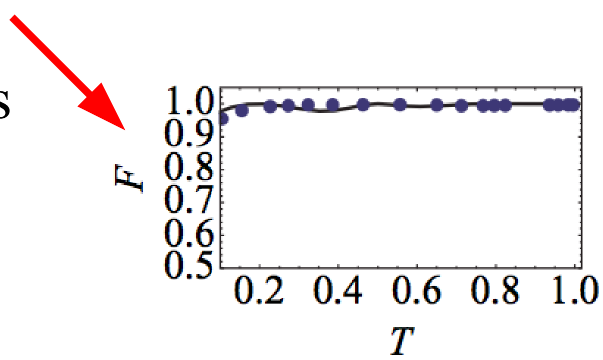


(b)

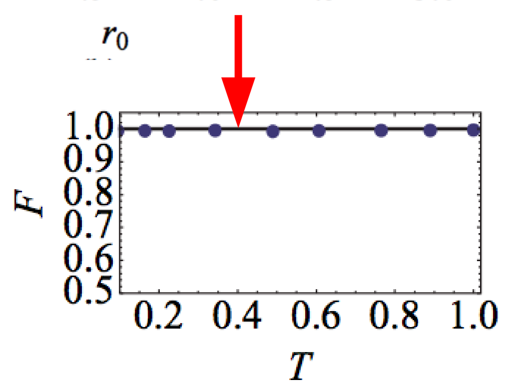




Average values over six states belonging to 3 mutually unbiased basis



(a) $B(x_0)$



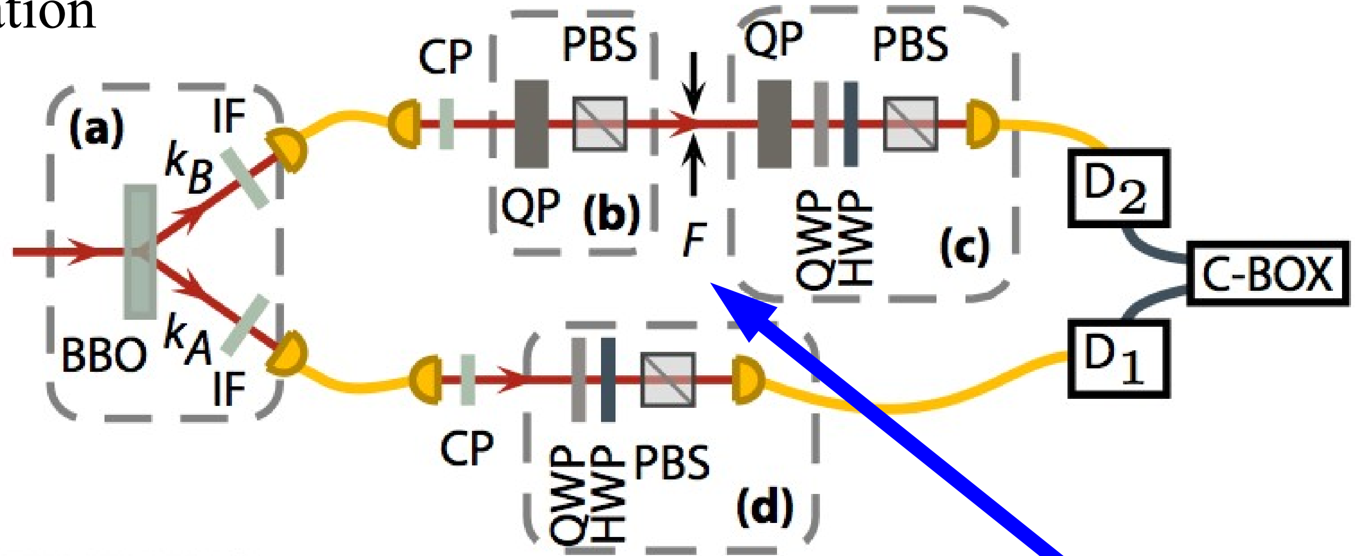
(b) $\Pi(r_0)$

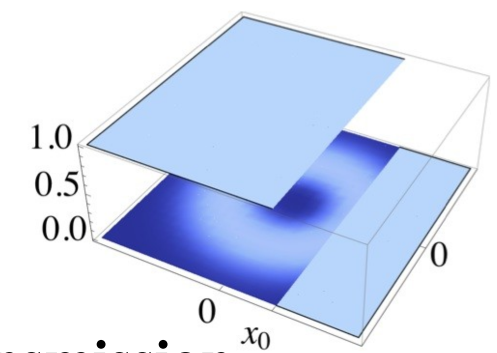
Giovannini, Nagali, Marrucci, and Sciarrino, *Phys. Rev. A* **83**, 042338 (2011)

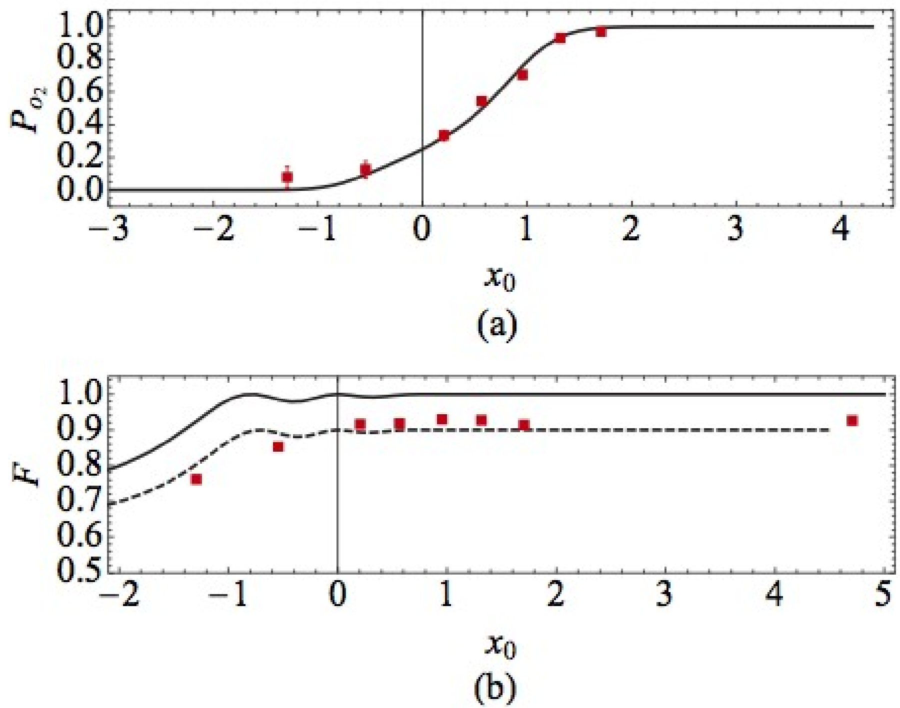
Resilience of hybrid polarization-OAM entanglement

- Source of hybrid polarization OAM entangled states

- Planar obstruction







- High fidelity of transmission at the cost of lower communication rate

- Noisy contributions emerge outside the encoding bidimensional subspace

Giovannini, Nagali, Marrucci, and Sciarrino, *Phys. Rev. A* **83**, 042338 (2011)

The quest for higher quantum dimensionality

- qubit: Hilbert space of dimension 2

- qudit: Hilbert space of dimension d

Quantum systems with $d > 2$ have been proposed as carriers of information in various contexts like quantum cryptography

VOLUME 88, NUMBER 12 PHYSICAL REVIEW LETTERS 25 MARCH 2002

Optimal Eavesdropping in Cryptography with Three-Dimensional Quantum States

D. Bruß[1] and C. Macchiavello[2]

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[2]*Dipartimento di Fisica "A. Volta" and INFM-Unità di Pavia, Via Bassi 6, 27100 Pavia, Italy*

(Received 27 June 2001; published 8 March 2002)

We study optimal eavesdropping in quantum cryptography with three-dimensional systems, and show that this scheme is more secure against symmetric attacks than protocols using two-dimensional states. We generalize the according eavesdropping transformation to arbitrary dimensions, and discuss the connection with optimal quantum cloning.

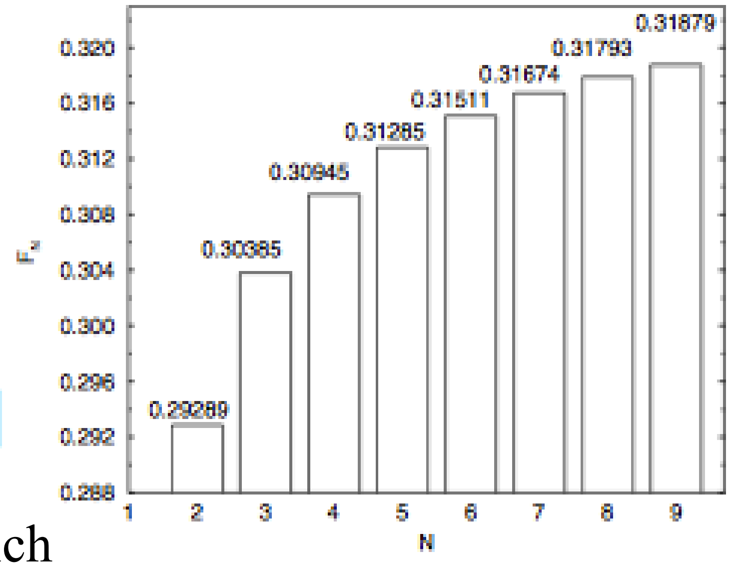
- More robust against isotropic noise
- Higher transmission rates through communication channels
- Increase the noise threshold that quantum key distribution protocols can tolerate

Violation of local realism for qudits expected to grow with d

VOLUME 85, NUMBER 21 PHYSICAL REVIEW LETTERS 20 NOVEMBER

Violations of Local Realism by Two Entangled *N*-Dimensional Systems Are Stronger than for Two Qubits

Dagomir Kaszlikowski,[1] Piotr Gnaciński,[1] Marek Żukowski,[1,2] Wieslaw Miklaszewski,[1] and Anton Zeilinger[2]
[1]*Instytut Fizyki Teoretycznej i Astrofizyki Uniwersytet Gdański, PL-80-952 Gdańsk, Poland*
[2]*Institut für Experimentalphysik, Univesität Wien, Boltzmanngasse 5, A-1090 Wien, Austria*
(Received 15 May 2000)



$$\rho_N(F_N) = F_N \rho_{\text{noise}} + (1 - F_N)|\Psi^N_{\max}\rangle\langle\Psi^N_{\max}|$$

$F_N^{\max}$ is the threshold maximal “noise fraction” for which the state still does not allow a local realistic model

VOLUME 88, NUMBER 4 PHYSICAL REVIEW LETTERS 28 JANUARY 2002

Bell Inequalities for Arbitrarily High-Dimensional Systems

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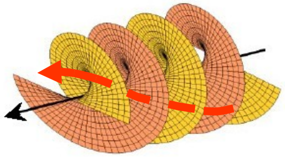
[2]*BRIMS, Hewlett-Packard Laboratories, Stoke Gifford, Bristol BS12 6QZ, United Kingdom*
[3]*Group of Applied Physics, University of Geneva, 20, rue de l'Ecole-de-Médecine, CH-1211 Geneva 4, Switzerland*
[4]*Department of Mathematics, Bristol University, University Walk, Bristol BS8 1TW, United Kingdom*
[5]*Service de Physique Théorique, Université Libre de Bruxelles, CP 225, Boulevard du Triomphe, B1050 Bruxelles, Belgium*
(Received 23 July 2001; published 10 January 2002)



Closing the detection loophole in Bell's test

Higher violation of Bell's inequalities

Detection efficiencies required for closing the detection loophole in Bell tests can be significantly lowered using quantum system of dimension larger than two.

For four dimensional systems the detection efficiency can be lowered to 61.8%.

PRL 104, 060401 (2010) PHYSICAL REVIEW LETTERS week ending 12 FEBRUARY 2010

Closing the Detection Loophole in Bell Experiments Using Qudits

Tamás Vértesi,[1] Stefano Pironio,[2] and Nicolas Brunner[3]
[1]*Institute of Nuclear Research of the Hungarian Academy of Sciences, H-4001 Debrecen, P.O. Box 51, Hungary*
[2]*Group of Applied Physics, University of Geneva, CH-1211 Geneva 4, Switzerland*
[3]*H.H. Wills Physics Laboratory, University of Bristol, Bristol, BS8 1TL, United Kingdom*
(Received 8 October 2009; published 11 February 2010)

We show that the detection efficiencies required for closing the detection loophole in Bell tests can be significantly lowered using quantum systems of dimension larger than two. We introduce a series of asymmetric Bell tests for which an efficiency arbitrarily close to $1/N$ can be tolerated using N-dimensional systems, and a symmetric Bell test for which the efficiency can be lowered down to 61.8% using four-dimensional systems. Experimental perspectives for our schemes look promising considering recent progress in atom-photon entanglement and in photon hyperentanglement.

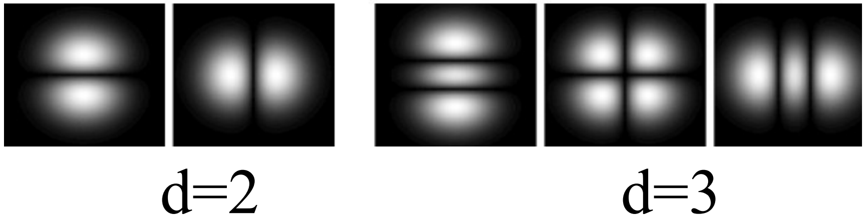
DOI: 10.1103/PhysRevLett.104.060401 PACS numbers: 03.65.Ud, 42.50.Ex

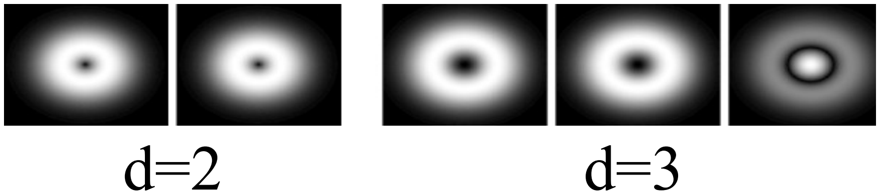
Higher dimensionality based on OAM

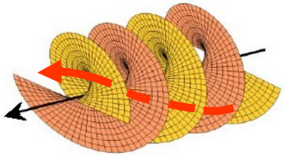
The OAM is a natural candidate for the experimental implementation of single-photon *d*-dimensional states.

Two possible strategies for qudit implementation

→ d-dimensional subspace of OAM







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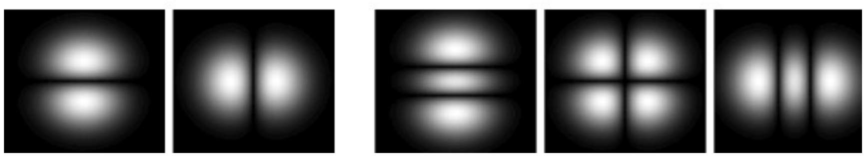
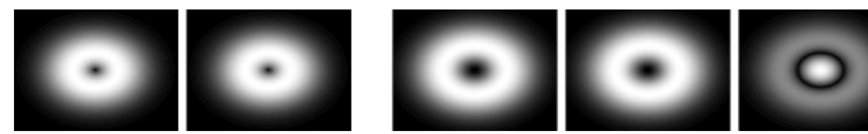


d=2 d=3

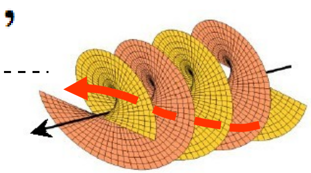


d=2 d=3

→ hybrid implementation based on OAM and other degree of freedom

Ququart (d=4) implemented by exploiting polarization and bidimensional subspace of OAM

$$\{|1\rangle,|2\rangle,|3\rangle,|4\rangle\} \Longleftrightarrow \{|H,+2\rangle,|H,-2\rangle,|V,+2\rangle,|V,-2\rangle\},$$



Mutually unbiased bases (MUBs) in dimension d

d-dimensional space:

k orthonormal basis are said to be **mutually unbiased**

if the basis states $\left| e_j^{\beta} \right\rangle$ satisfy the relation:

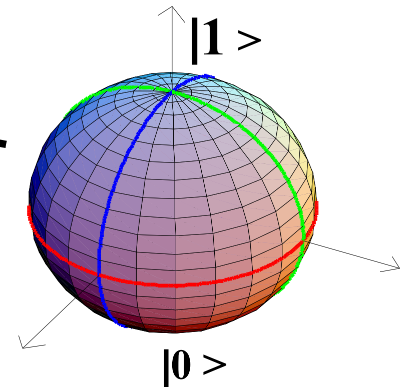
$$\left| \left\langle e_i^{\alpha} \middle| e_j^{\beta} \right\rangle \right| = \begin{cases} \delta_{ij} & \text{if } \alpha = \beta, \\ \frac{1}{\sqrt{d}} & \text{if } \alpha \neq \beta, \end{cases} \quad \text{where} \quad i, j = 1, \ldots, k$$

How many MUBs can one introduce in a given Hilbert space (HS)?

- In a d-dimensional HS the number of MUBs can not exceed d+1
(W. K. Wootters, 1989)
- If d is a prime number or a prime power, the number of MUBs is d+1
(W. K. Wootters, 1989)
- Given a composite dimension $d=d_1 \ldots d_N$ there will be a number of separable MUBs corresponding to the ones of an Hilbert space with dimension $min\{d_j\}$
(Wiesnak 2011)

Mutually unbiased bases (MUBs) in dimension d

d	MUBs
2	3
3	4
4	5
6	?? (at least 3)
7	8



Why working with MUBs?

Experimental quantum tomography of photonic qudits via mutually unbiased basis

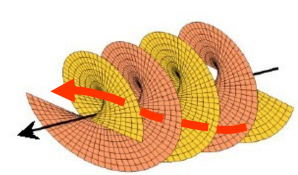
G. Lima,[1,2,*] L. Neves,[1,2], R. Guzmán,[1,3], E. S. Gómez,[1,2]
W. A. T. Nogueira,[1,2] A. Delgado,[1,2] A. Vargas,[1,3] and C. Saavedra[1,2]

[1]Center for Optics and Photonics, Universidad de Concepción, Casilla 4016, Concepción, Chile
[2]Departamento de Física, Universidad de Concepción, Casilla 160-C, Concepción, Chile
[3]Departamento de Ciencias Físicas, Universidad de La Frontera, Temuco, Casilla 54-D, Chile
[*]Corresponding author: glima@udec.cl

To recontruct the density matrix of a quantum state in a d-dimensional space, at least (d+1) orthonormal bases are needed to determine the (d-1)(d-1) parameters that describe

MUBs – based quantum state tomography requires projection for a minimal number of bases to be performed.

MUBs- quantum cryptography



Realization of -OAM ququart

Generate ququart states (4-dimensional) encoded in a single photon by manipulating the OAM and polarization degrees of freedom

Logic ququart basis $\{|1\rangle, |2\rangle, |3\rangle, |4\rangle\}$ → $\{|H,+2\rangle, |H,-2\rangle, |V,+2\rangle, |V,-2\rangle\}$,

The complete characterization of a ququart state is achieved by defining and measuring five mutually unbiased bases with four states each.

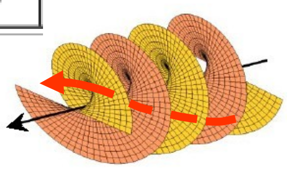
Separable states

Theory		
Ququart States		
	Ququart Logic Bases	*OAM − π*
I	$|1\rangle$	$|H,+2\rangle$
	$|2\rangle$	$|H,-2\rangle$
	$|3\rangle$	$|V,+2\rangle$
	$|4\rangle$	$|V,-2\rangle$
II	$\frac{1}{2}(|1\rangle+|2\rangle+|3\rangle+|4\rangle)$	$|A,h\rangle$
	$\frac{1}{2}(|1\rangle-|2\rangle+|3\rangle-|4\rangle)$	$|A,v\rangle$
	$\frac{1}{2}(|1\rangle+|2\rangle-|3\rangle-|4\rangle)$	$|D,h\rangle$
	$\frac{1}{2}(|1\rangle-|2\rangle-|3\rangle+|4\rangle)$	$|D,v\rangle$
III	$\frac{1}{2}(|1\rangle+i|2\rangle+i|3\rangle-|4\rangle)$	$|R,a\rangle$
	$\frac{1}{2}(|1\rangle-i|2\rangle+i|3\rangle+|4\rangle)$	$|R,d\rangle$
	$\frac{1}{2}(|1\rangle+i|2\rangle-i|3\rangle+|4\rangle)$	$|L,a\rangle$
	$\frac{1}{2}(|1\rangle-i|2\rangle-i|3\rangle-|4\rangle)$	$|L,d\rangle$

Entangled states

Theory		
Ququart States		
	Ququart Logic Bases	*OAM − π*
IV	$\frac{1}{2}(|1\rangle+|2\rangle+i|3\rangle-i|4\rangle)$	$\frac{1}{\sqrt{2}}(|R,+2\rangle+|L,-2\rangle)$
	$\frac{1}{2}(|1\rangle-|2\rangle+i|3\rangle+i|4\rangle)$	$\frac{1}{\sqrt{2}}(|R,+2\rangle-|L,-2\rangle)$
	$\frac{1}{2}(|1\rangle+|2\rangle-i|3\rangle+i|4\rangle)$	$\frac{1}{\sqrt{2}}(|L,+2\rangle+|R,-2\rangle)$
	$\frac{1}{2}(|1\rangle-|2\rangle-i|3\rangle-i|4\rangle)$	$\frac{1}{\sqrt{2}}(|L,+2\rangle-|R,-2\rangle)$
V	$\frac{1}{2}(|1\rangle+i|2\rangle+|3\rangle-i|4\rangle)$	$\frac{1}{\sqrt{2}}(|H,a\rangle+|V,d\rangle)$
	$\frac{1}{2}(|1\rangle+i|2\rangle-|3\rangle+i|4\rangle)$	$\frac{1}{\sqrt{2}}(|H,a\rangle-|V,d\rangle)$
	$\frac{1}{2}(|1\rangle-i|2\rangle+|3\rangle+i|4\rangle)$	$\frac{1}{\sqrt{2}}(|H,d\rangle+|V,a\rangle)$
	$\frac{1}{2}(|1\rangle-i|2\rangle-|3\rangle-i|4\rangle)$	$\frac{1}{\sqrt{2}}(|H,d\rangle-|V,a\rangle)$

E. Nagali, L. Sansoni, L. Marrucci, E. Santamato, and F. Sciarrino, *Phys. Rev. A* **81**, 052317 (2010).



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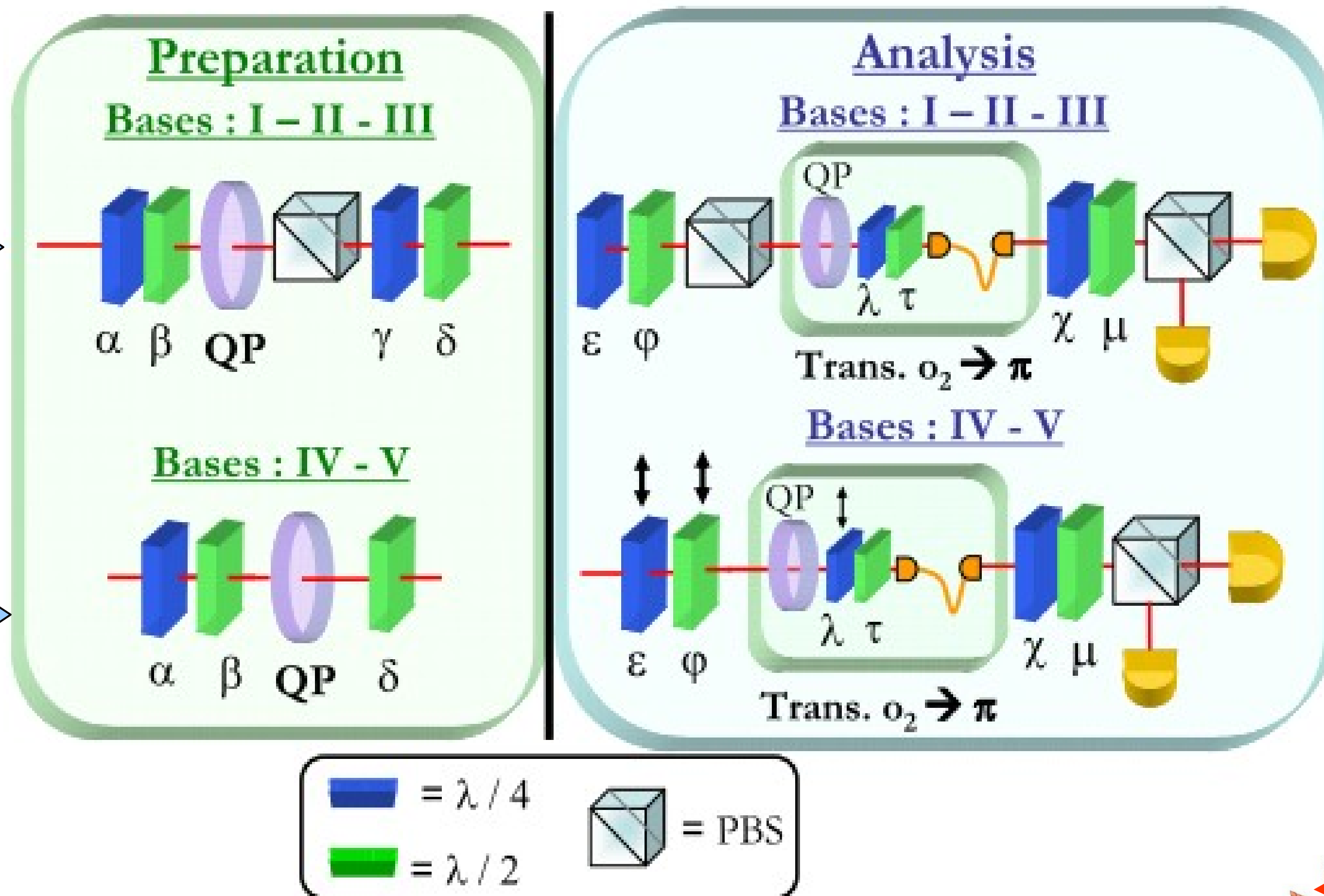
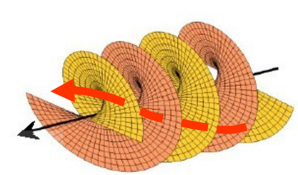
Separable states of polarization and OAM →

Entangled states of polarization and OAM →

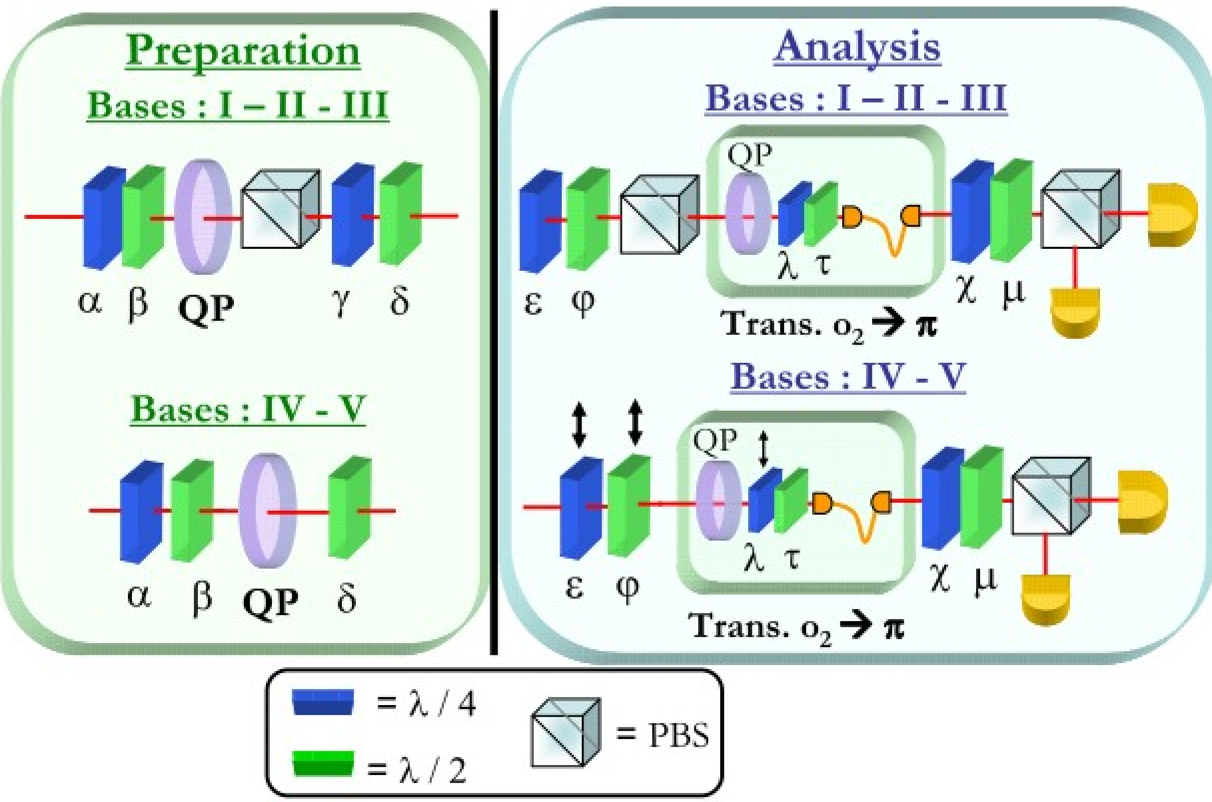




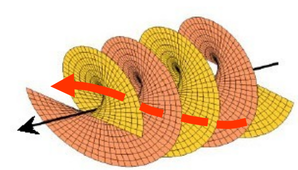
E. Nagali, L. Sansoni, L. Marrucci, E. Santamato, and F. Sciarrino, *Phys. Rev. A* **81**, 052317 (2010).

Realization of π-OAM ququart

- By combining qplates, waveplates, PBS, single mode fibers all the states belonging to the mutually unbiased basis can be generated and characterized



Preparation				F_{exp}
α	β	γ	δ	
−45	0	0	0	$(99.9 \pm 0.4)\%$
+45	0	0	0	$(94.6 \pm 0.4)\%$
−45	0	0	+45	$(99.9 \pm 0.4)\%$
+45	0	0	+45	$(95.8 \pm 0.4)\%$
0	0	0	+22.5	$(95.0 \pm 0.4)\%$
0	+45	0	+22.5	$(89.2 \pm 0.4)\%$
0	0	0	−22.5	$(97.7 \pm 0.4)\%$
+45	0	0	−22.5	$(95.0 \pm 0.4)\%$
0	−22.5	+45	0	$(96.3 \pm 0.4)\%$
0	+22.5	+45	0	$(95.7 \pm 0.4)\%$
0	−22.5	−45	+45	$(94.1 \pm 0.4)\%$
0	+22.5	−45	+45	$(94.5 \pm 0.4)\%$
0	0	−	−	$(84.8 \pm 0.4)\%$
0	+45	−	−	$(91.4 \pm 0.4)\%$
0	0	−	+45	$(89.4 \pm 0.4)\%$
0	+45	−	+45	$(88.4 \pm 0.4)\%$
0	+22.5	−	−	$(89.7 \pm 0.4)\%$
0	−22.5	−	−	$(86.1 \pm 0.4)\%$
0	+22.5	−	+45	$(88.4 \pm 0.4)\%$
0	−22.5	−	+45	$(92.0 \pm 0.4)\%$



E. Nagali, L. Sansoni, L. Marrucci, E. Santamato, and F. Sciarrino, *Phys. Rev. A* **81**, 052317 (2010)

Tests on the Foundations of Quantum Mechanics

- Can Quantum Mechanics theory be completed by a more general theory which provides a complete description of reality (**L**ocal **H**idden **V**ariables) ?

Tests on the Foundations of Quantum Mechanics

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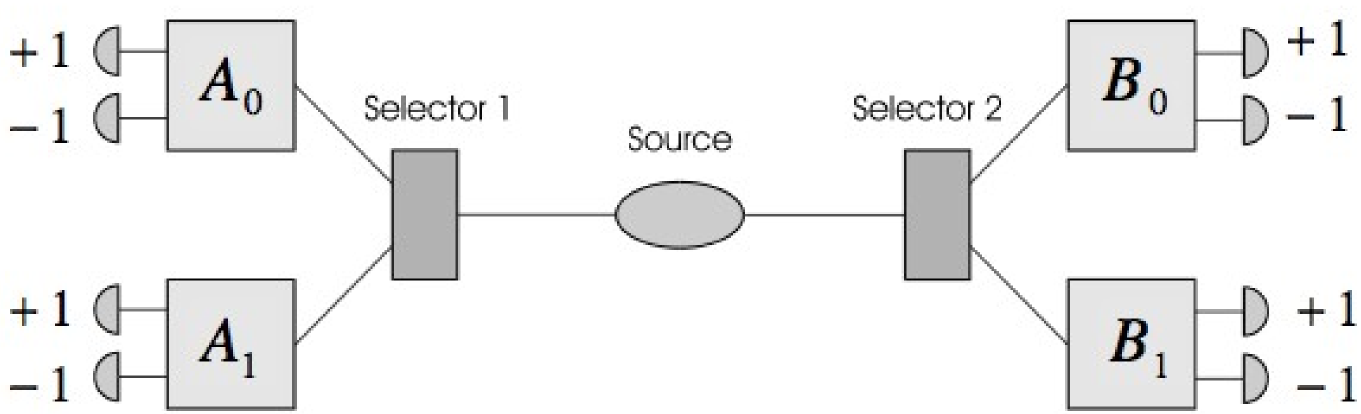
- Entanglement: usefull resource to test Bell's inequalities satisfied by LHV

$$\left|\langle A_0 B_0\rangle + \langle A_0 B_1\rangle + \langle A_1 B_0\rangle - \langle A_1 B_1\rangle\right| \leq 2$$

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right) \quad \Rightarrow \quad \beta_{QM} = 2\sqrt{2} > 2$$



Local Hidden Variables
Non-contextual theory

- Other approach to test local hidden variables...

It exploits the concept of ***contextuality***

A result is *non-contextual* if is independent of the context of observation, that is, of which other *compatible* observables are jointly measured.

Compatibility: compatible observables are those which can be measured
"without disturbing each other"
(in QM ↔ commutating observables).

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Compatibility: compatible observables are those which can be measured "without disturbing each other" (in QM $\leftrightarrow$ commutating observables).

LOCAL HIDDEN VARIABLE (LHV) THEORIES:
pre-assigned value for the observables

↓

Result is independent of context of observation
LHV non-contextual theory

Quantum contextuality: Kochen-Specker theorem

For *any physical system*, *in any state*, there exist a *finite* set of observables such that it is impossible to pre-assign them noncontextual results respecting the predictions of QM.

(any physical system in which observables can belong to more than one context, i.e., those represented in QM by a Hilbert space of dimension $d > 2$)

A result is *noncontextual* if is independent of which other *compatible* observables are jointly measured.

E. P. Specker, A. Specker, and S. Kochen, Zürich, early 1963.

S. Kochen and E.P. Specker, J. Math. Mech. **17**, 59 (1967).

Higher quantum dimensionality to test contextuality

4 dimensional system:

Logic ququart basis

$\{|1\rangle, |2\rangle, |3\rangle, |4\rangle\}$

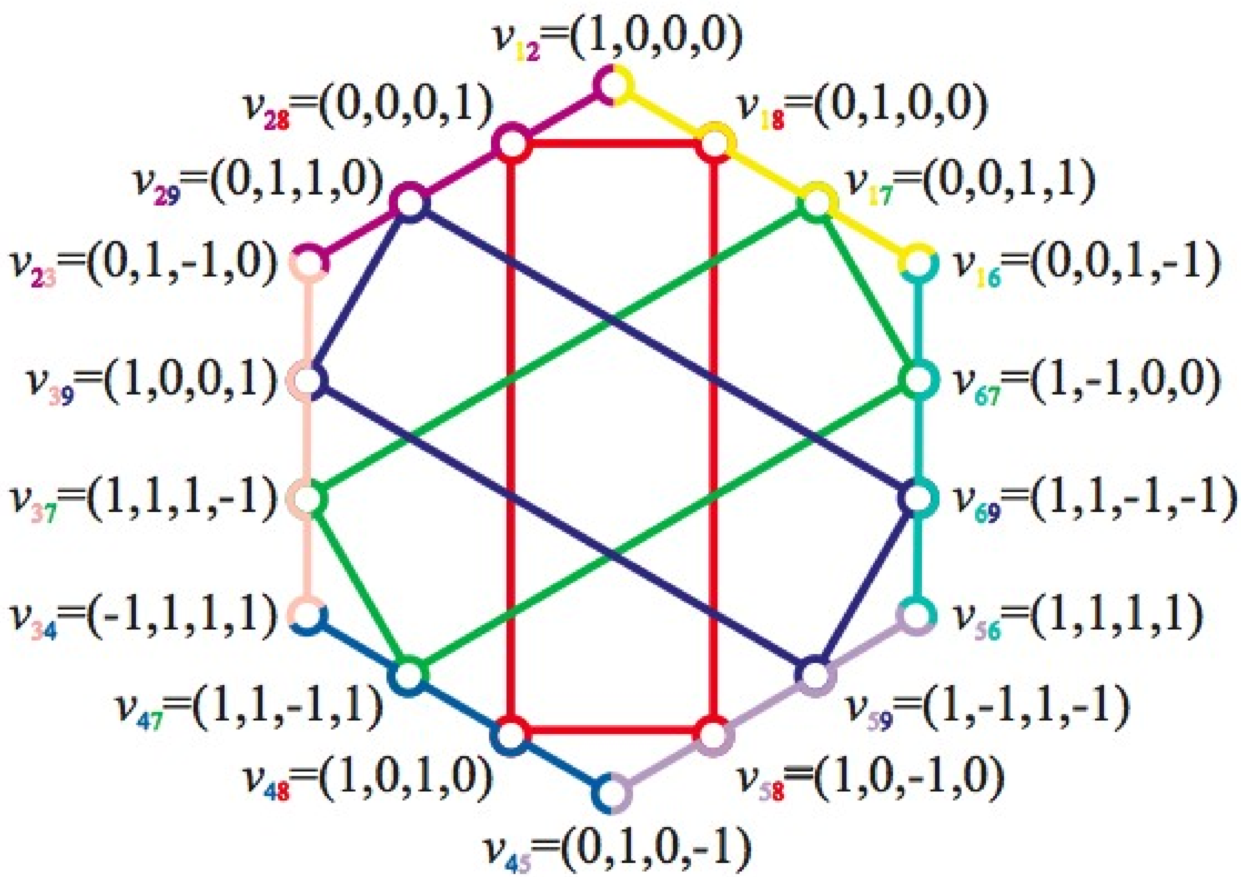
$\alpha|1\rangle + \beta|2\rangle + \gamma|3\rangle + \delta|4\rangle$

$(\alpha, \beta, \gamma, \delta)$

18 states

Grouped in 9 basis

Each state belongs to two different basis (not possible for dimension 2)

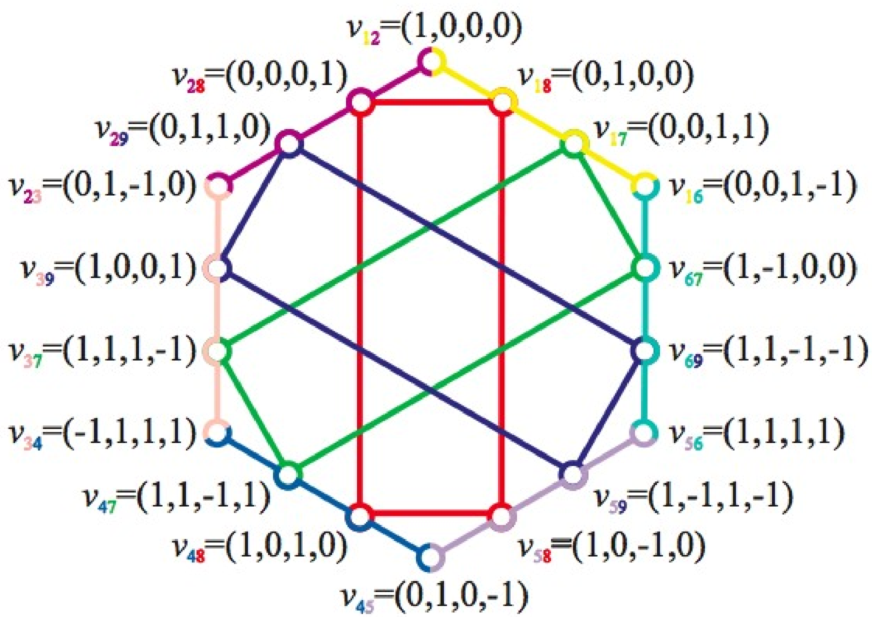


A. Cabello, J. M. Estebaranz, and G. Garćıa-Alcaine, *Phys. Lett. A* **212**, 183 (1996).

Higher quantum dimensionality to test contextuality

1000	1111	1111	1000	1001	1001	111-1	111-1	100-1
0100	11-1-1	1-11-1	0010	0100	1-11-1	1-100	0101	0110
0011	1-100	10-10	0101	0010	11-1-1	0011	10-10	11-11
001-1	001-1	010-1	010-1	100-1	0110	11-11	1-111	1-111

- Each vector represents an elementary yes-no test (described in QM by the projection operator onto the corresponding normalized vector; for instance, 111-1 represents the projector onto the vector (1,1,1,-1)/2).



Higher quantum dimensionality to test contextuality

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0100	11-1-1	1-11-1	0010	0100	1-11-1	1-100	0101	0110
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001-1	001-1	010-1	010-1	100-1	0110	11-11	1-111	1-111

- Each vector represents an elementary yes-no test (described in QM by the projection operator onto the corresponding normalized vector; for instance, 111-1 represents the projector onto the vector (1,1,1,-1)/2).

- Each column contains four orthogonal four-dimensional vectors, so the corresponding projectors commute (i.e., represent compatible tests) and sum the identity. Therefore, in any assignment of "yes" (1) or "no" (0) answers that satisfies the predictions of QM, each column must have assigned the answer "yes" to one and only one vector.

Higher quantum dimensionality to test contextuality

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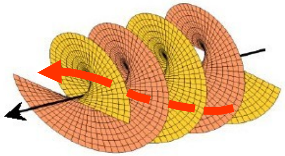
Higher quantum dimensionality to test contextuality

1000 0100 0011 001-1	1111 11-1-1 1-100 001-1	1111 1-11-1 10-10 010-1	1000 0010 0101 010-1	1001 0100 0010 100-1	1001 1-11-1 11-1-1 0110	111-1 1-100 0011 11-11	111-1 0101 10-10 1-111	100-1 0110 11-11 1-111

- A noncontextual assignment is impossible: Each vector appears in two columns, so the total number of "yes" answers must be an even number, but the total number of "yes" answers must also be equal to the number of columns, which is an odd number.

Assign to each observable a defined value (non-contextual theory)

CONTRADICTION:
QUANTUM CONTEXTUALITY !



A. Cabello, J. M. Estebaranz, and G. Garćıa-Alcaine, *Phys. Lett. A* **212**, 183 (1996).

Quantum cryptography protected by Kochen-Specker contextuality

- **Quantum cryptography:**

  Quantum key distribution (QKD)

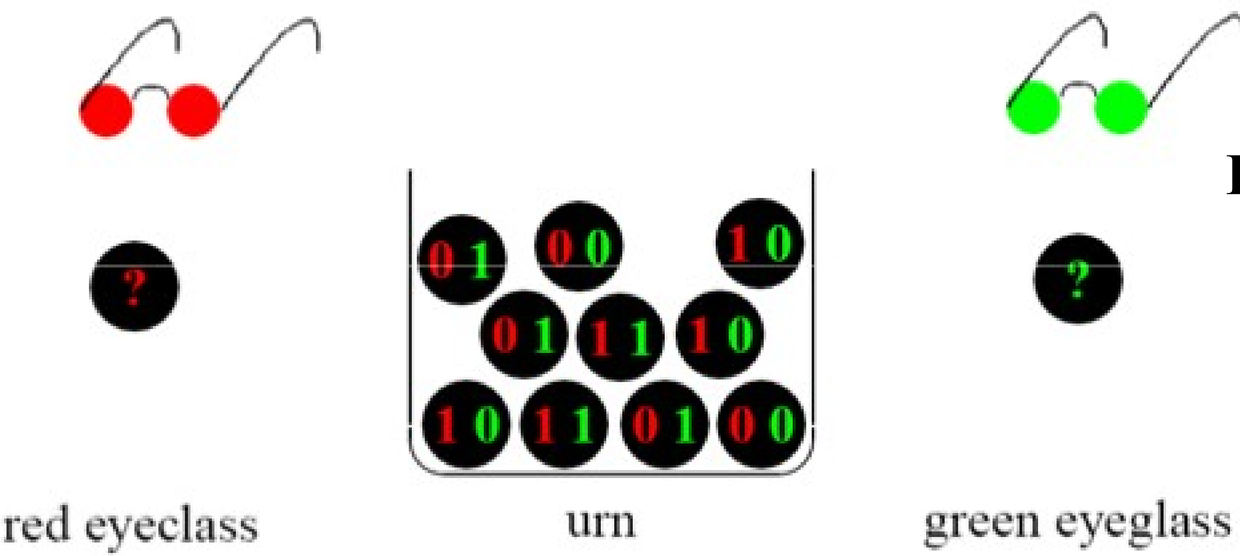
  Exploits transmission of qubit on two mutually unbiased basis

- **QKD + Quantum contextuality**

  Extrasecurity to check that the experimental apparatus behaves in a purely quantum mechanical way

Classically "mimicking" BB84 QKD protocol

Balltyp	red	green
Typ 1	0	0
Typ 2	0	1
Typ 3	1	0
Typ 4	1	1

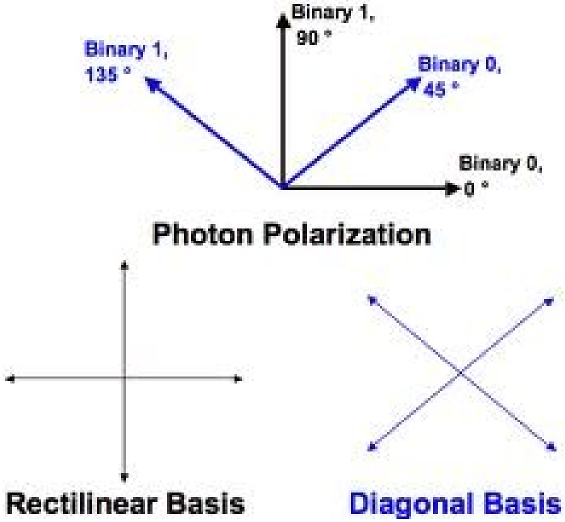


Part of the data discarded (50%)

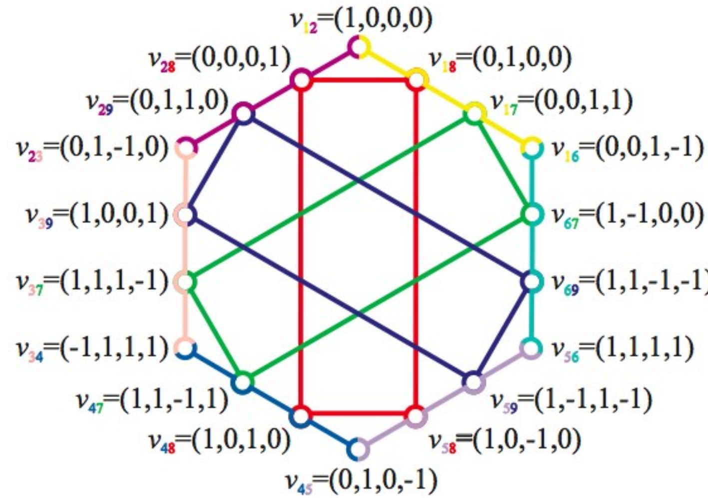
HOW CAN WE TEST THE QKD APPARATUS TO CHECK THAT IT IS *TRULY QUANTUM* ?

K. Svozil, *Am. J. Phys.* **74**, 800 (2006).

Quantym cryptography
certified by quantum contextuality



BB84 - QKD



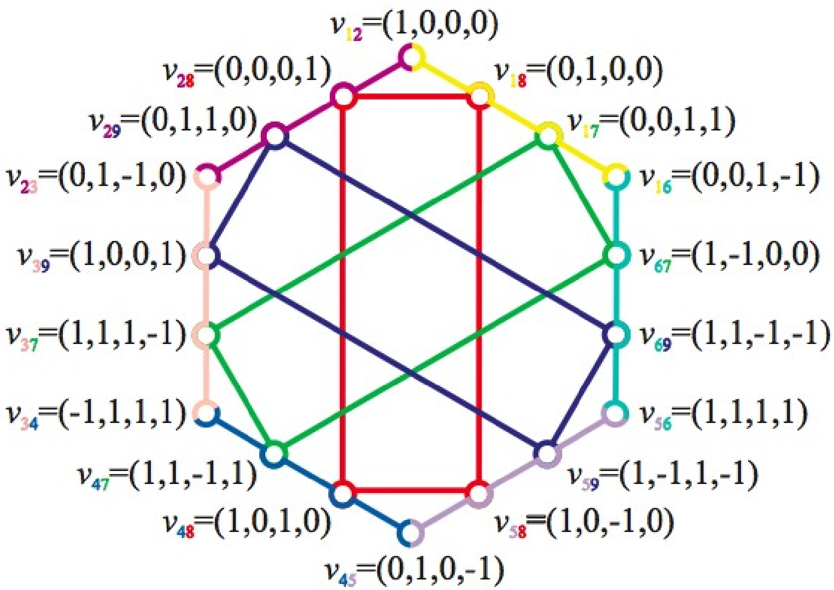
KS - QKD

Svozil, arXiv:0903.0231

- 9 basis with four states each
- probability that Alice and Bob use the same basis p=1/9
- 2 bit are exchanged

- Quantumness of apparata can be directly checked
- Tolerate a communication noise of about 6.5%

Higher quantum dimensionality via hybrid polarization-OAM ququart

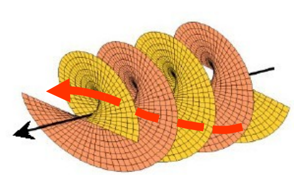


$$\alpha|1\rangle+\beta|2\rangle+\gamma|3\rangle+\delta|4\rangle$$

optical implementation

$$\alpha|H,+1\rangle+\beta|H,1-1\rangle+\gamma|V,+1\rangle+\delta|V,-1\rangle$$

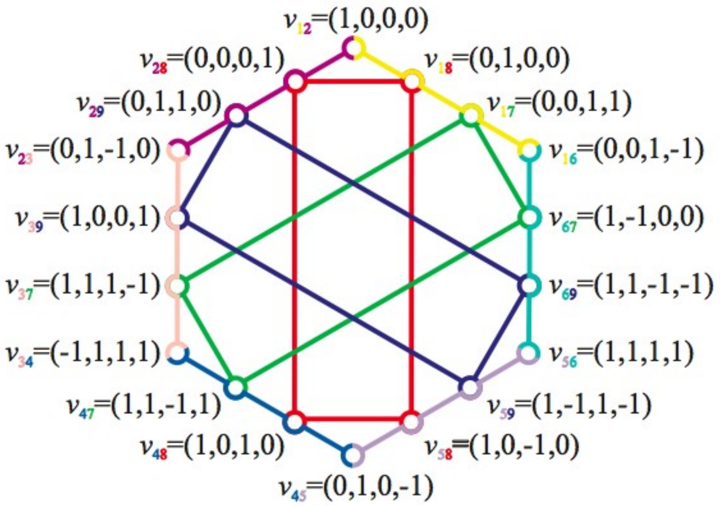
Basis	Class	Example		
		Set	Logic	π-OAM
$I-II$ $III-VIII$	P_2	I	1000	$\|H,+1\rangle$
			0100	$\|H,-1\rangle$
			0011	$\|V,h\rangle$
			001-1	$\|V,v\rangle$
V	B_2	V	1-111	$\frac{1}{\sqrt{2}}(\|H,v\rangle+\|V,h\rangle)$
			100-1	$\frac{1}{\sqrt{2}}(\|H,+1\rangle-\|V,-1\rangle)$
			0110	$\frac{1}{\sqrt{2}}(\|H,-1\rangle+\|V,+1\rangle)$
IX	M_1	IX	0001	$\|V,-1\rangle$
			0110	$\frac{1}{\sqrt{2}}(\|H,-1\rangle+\|V,+1\rangle)$
			1000	$\|H,+1\rangle$
			01-10	$\frac{1}{\sqrt{2}}(\|H,-1\rangle-\|V,+1\rangle)$
$IV-VI-VII$	M_2	VII	111-1	$\frac{1}{\sqrt{2}}(\|A,+1\rangle+\|D,-1\rangle)$
			1-100	$\|H,v\rangle$
			0011	$\|V,h\rangle$
			11-11	$\frac{1}{\sqrt{2}}(\|D,+1\rangle+\|A,-1\rangle)$

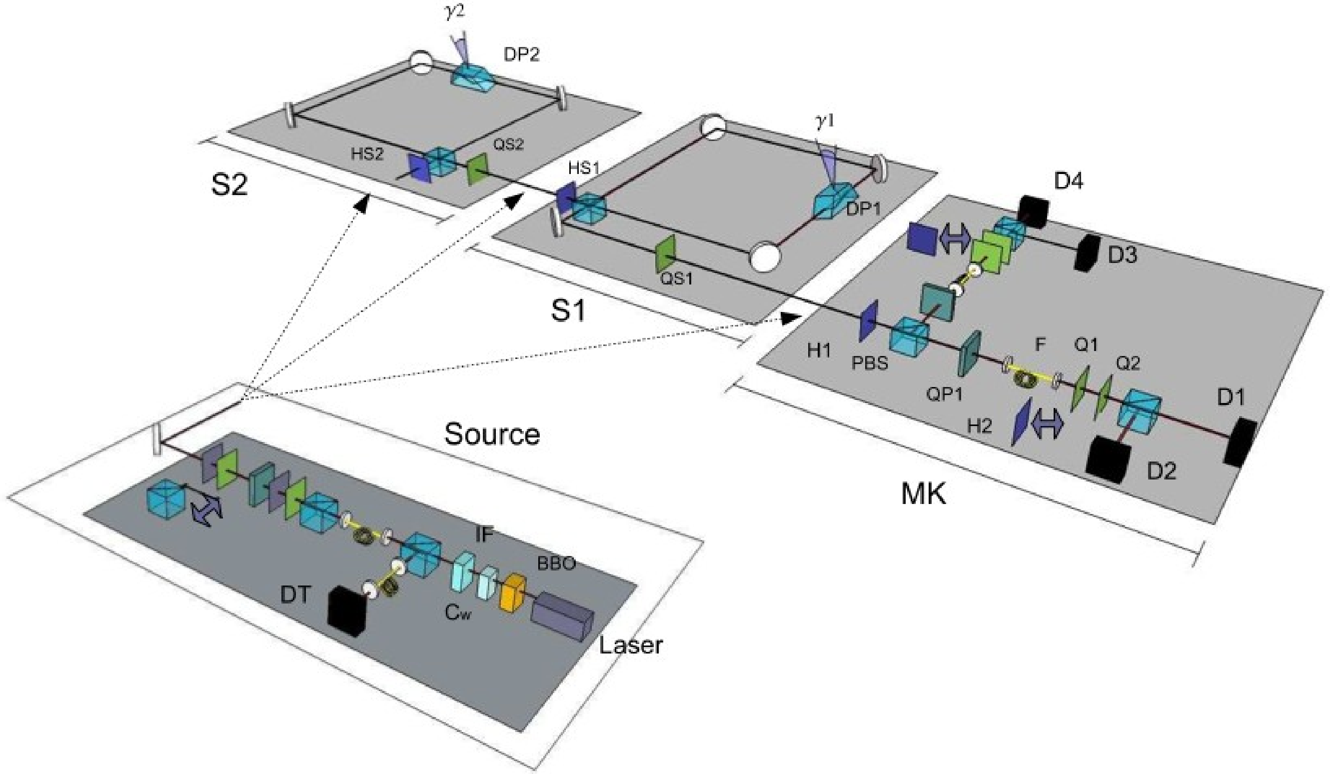


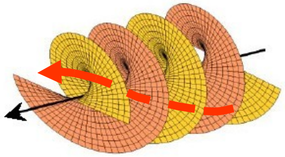
Cabello, D'Ambrosio, Nagali, Sciarrino, *in preparation*

Optical schemes for polarization-OAM implementation

Finally... optical schemes (v 0.1)
not straightforward...

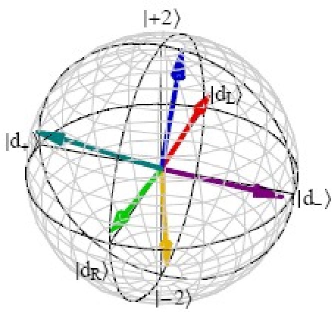


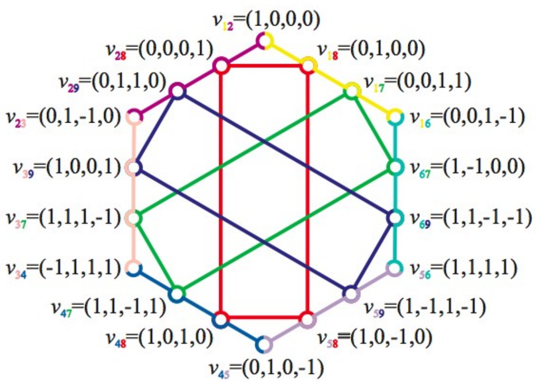


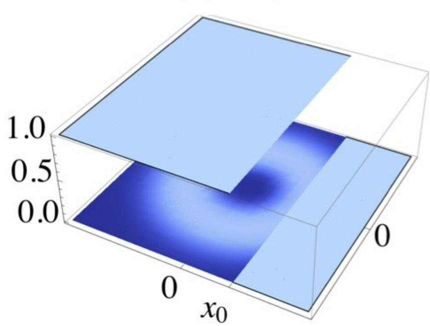


Cabello, D'Ambrosio, Nagali, Sciarrino, *in preparation*

Conclusions and perspectives







- The qplate is a reliable interface between OAM and polarization
  - Qubit transferrer from polarization to OAM and viceversa
  - Resilience to partial transmission
  - Experimental implementation and manipulation of ququart states
  - NEXT STEPS: Higher dimensionality for fundamental test
  - and protocols of quantum information

http:\\quantumoptics.phys.uniroma1.it

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