fundamental quantum tests II

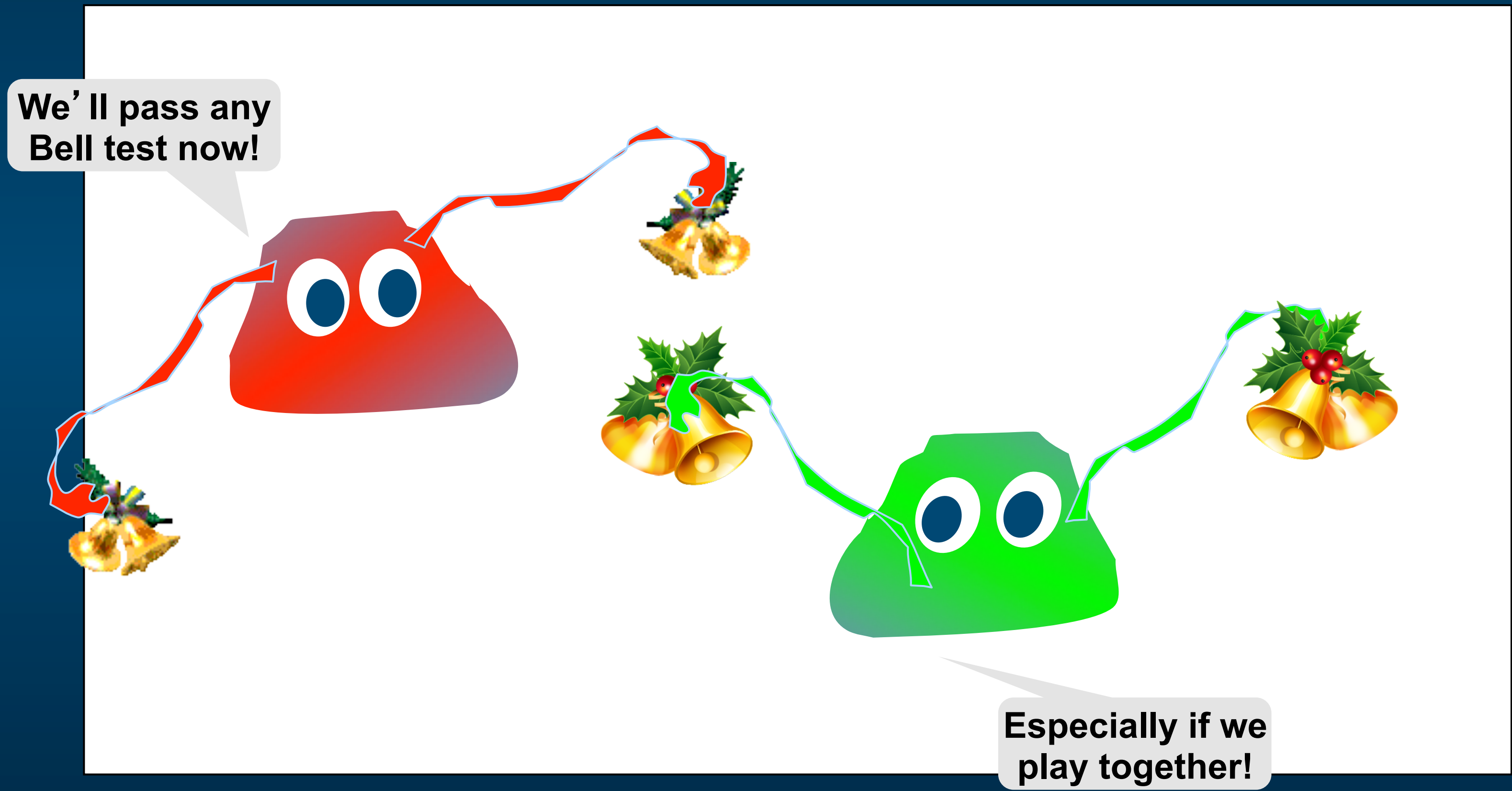
Sonja Franke-Arnold

University of Glasgow, Physics and Astronomy, Optics Group

intro

1. high dimensional entanglement
   - entanglement
   - OAM state space – to infinity and beyond?
   - when OAM entered the quantum world
2. describing quantum states
   - Poincaré sphere
   - density matrices
3. quantum tests in a 2 dimensional OAM subspace
   - nonlocal hidden variable theories and Bell's inequality
   - local hidden variable theories and Leggett's inequality
4. more than 2 dimensions
   - Bell in 3D
   - EPR and entropic uncertainty relation

3. Quantum tests in a two-dimensional OAM subspace



testing quantum mechanics

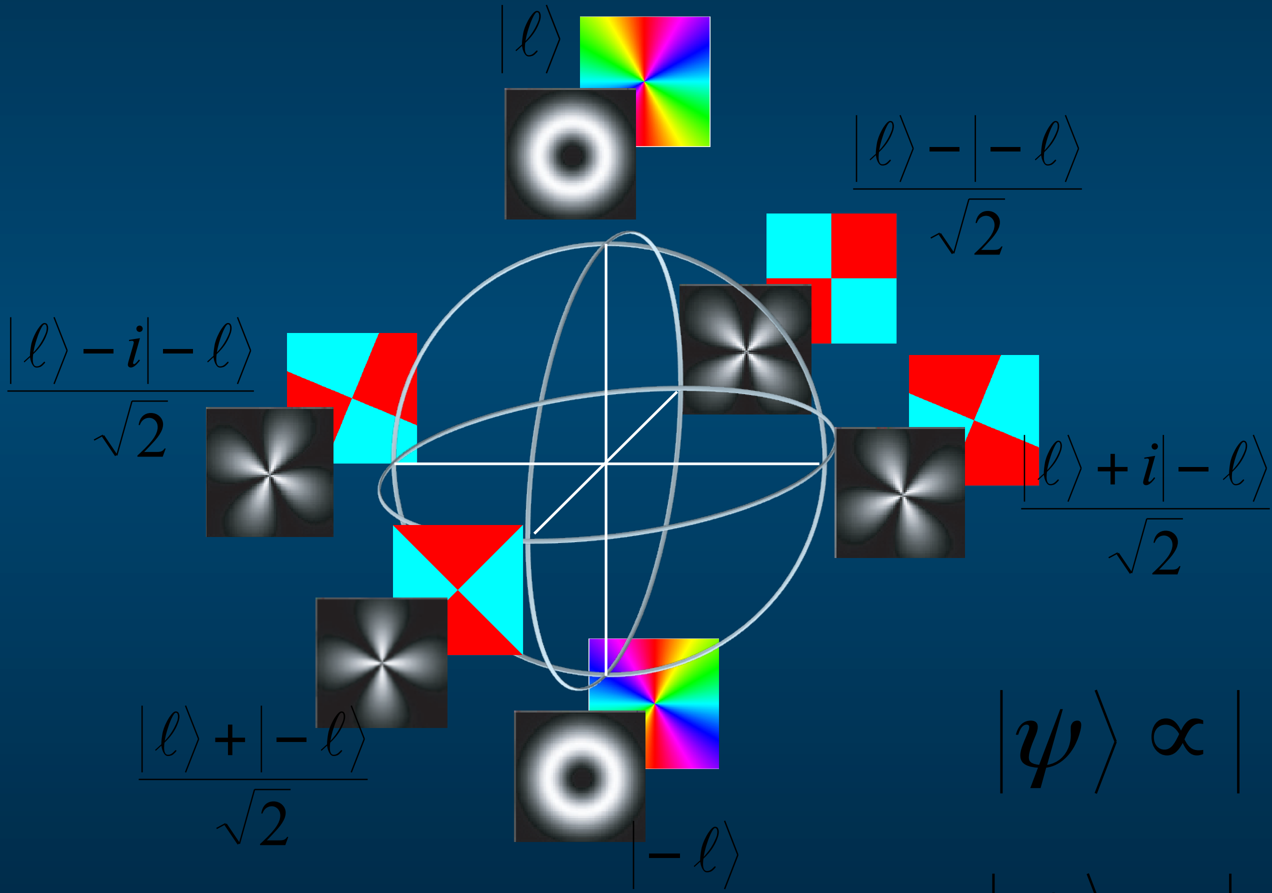
- Quantum theory is already a century old, but not any less puzzling.
- We are now in a position to turn thought experiments into real ones.

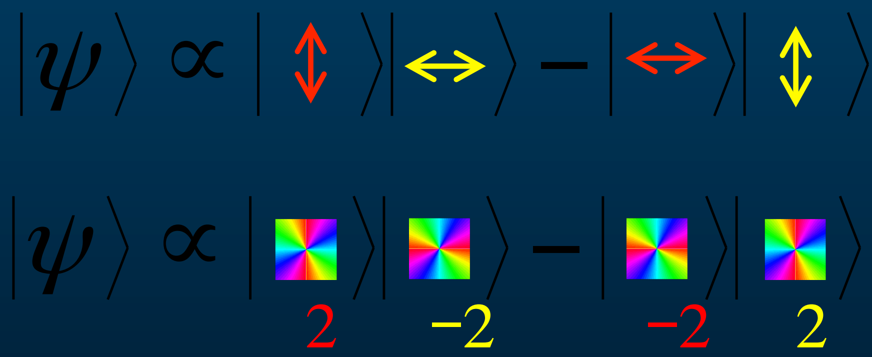
Werner Heisenberg and Niels Bohr

Niels Bohr and Albert Einstein

Two powerful ingredients

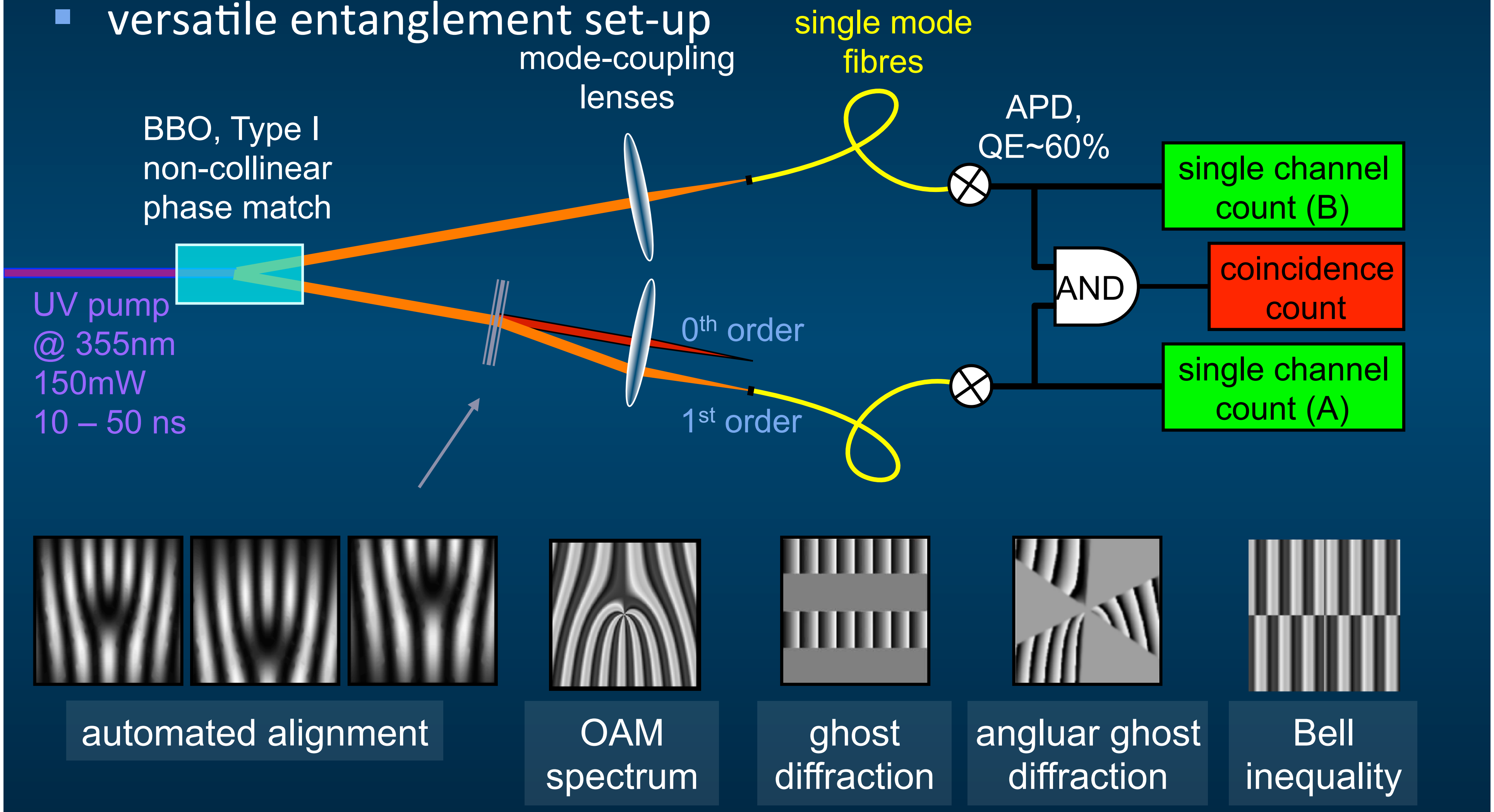
- equivalence between 2D OAM subspace and polarisation





Two powerful ingredients

- versatile entanglement set-up



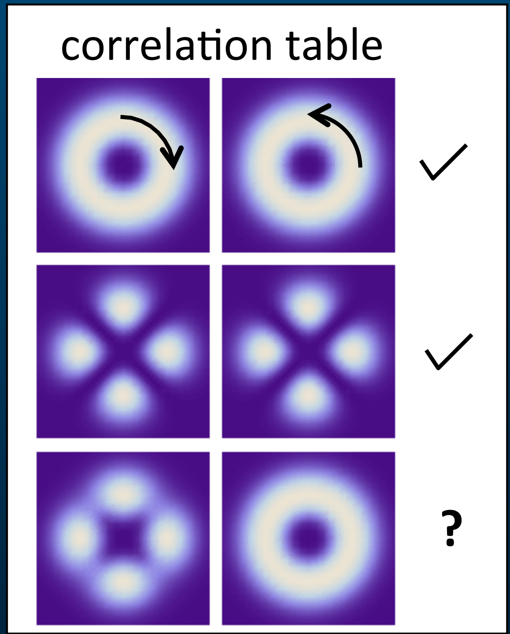
Bell measurements

demonstrate entanglement

=

demonstrate that correlations between photons persist for superposition states

- Quantum correlations are stronger than classically allowed.
- Bell's inequality sets a limit for correlations that are allowed by (classical) local hidden variable theories. Violating a Bell-type inequality is a stringent test for a special class of entangled states.



Saving local realism

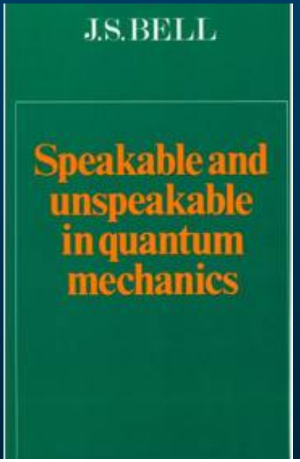
- Correlations can be established in the classical world, by
  - sending messages (signalling)
  - a priori agreement (hidden variables).

- Bell's hidden variable model:
  - Each pair of photons is characterised by a unique value of some "hidden" variable λ.
  - The ensemble of photons is characterised by a statistical distribution of λ values, $\rho(\lambda)$

$$P(\alpha,\beta\,|\,\vec{a},\vec{b}) = \int d\lambda \rho(\lambda) P_{\lambda}(\alpha,\beta\,|\,\vec{a},\vec{b})$$

  - The measurement in system A should only depend on the local measurement setting and the hidden variable,

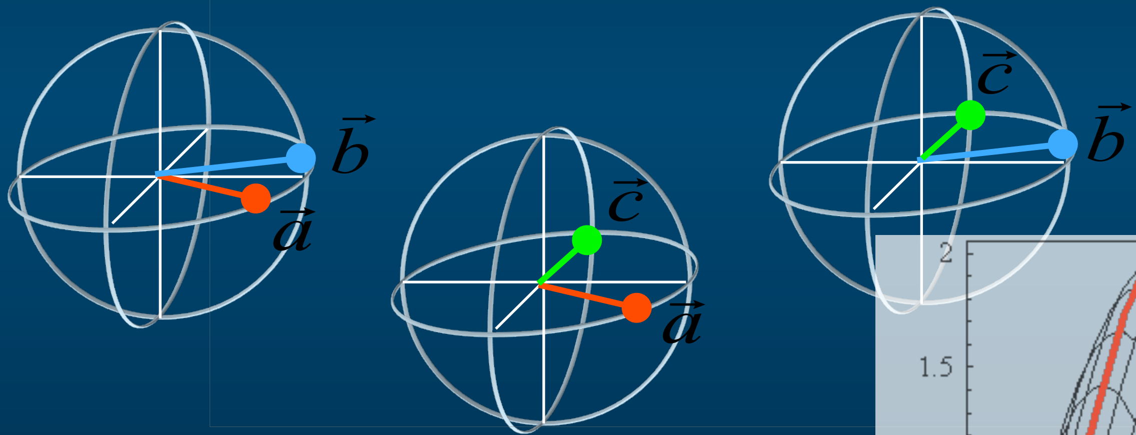
$$P_{\lambda}(\alpha,\beta\,|\,\vec{a},\vec{b}) = A(\vec{a},\lambda) B(\vec{b},\lambda)$$

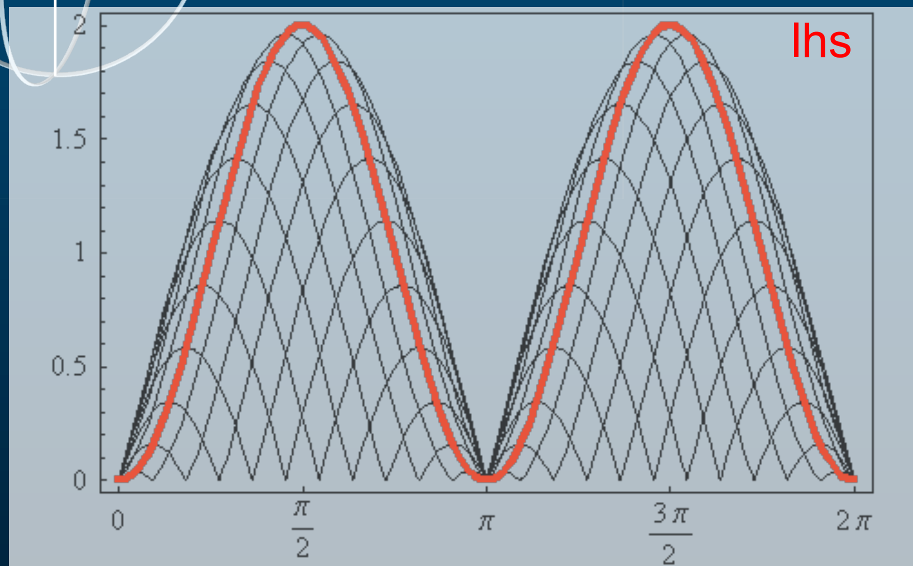


Bell's inequality

Bell devised an inequality that needs to be fulfilled by any local hidden variable theory

$$\left| C(\vec{a},\vec{b}) - C(\vec{a},\vec{c}) \right| \leq 1 - C(\vec{b},\vec{c})$$





… in a convenient notation

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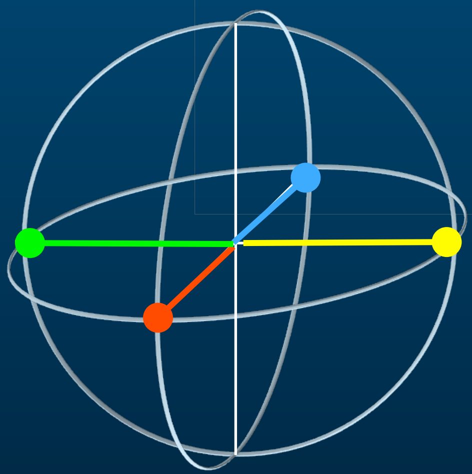
- More convenient: Clauser Horne Shimony Holt inequality.

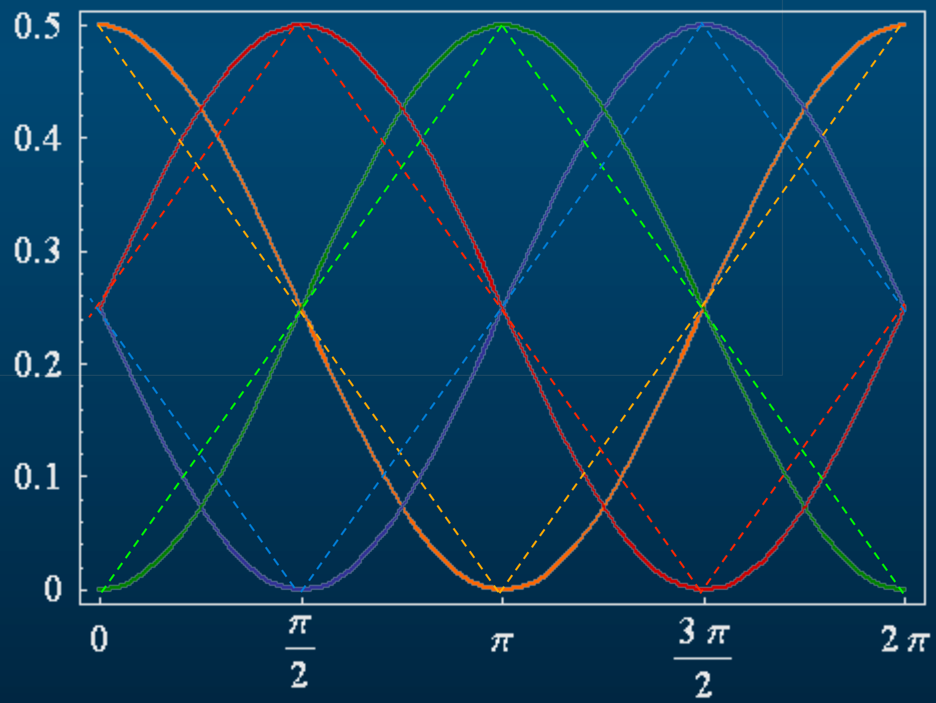
$$S = E(a,b) - E(a,b') + E(a',b) + E(a',b')$$

$$E(a,b) = \frac{C(a,b) + C(-a,-b) - C(a,-b) - C(-a,b)}{C(a,b) + C(-a,-b) + C(a,-b) + C(-a,b)}$$

$$|S_{HV}| \leq 2$$

$$|S_{QM}| = 2\sqrt{2}$$

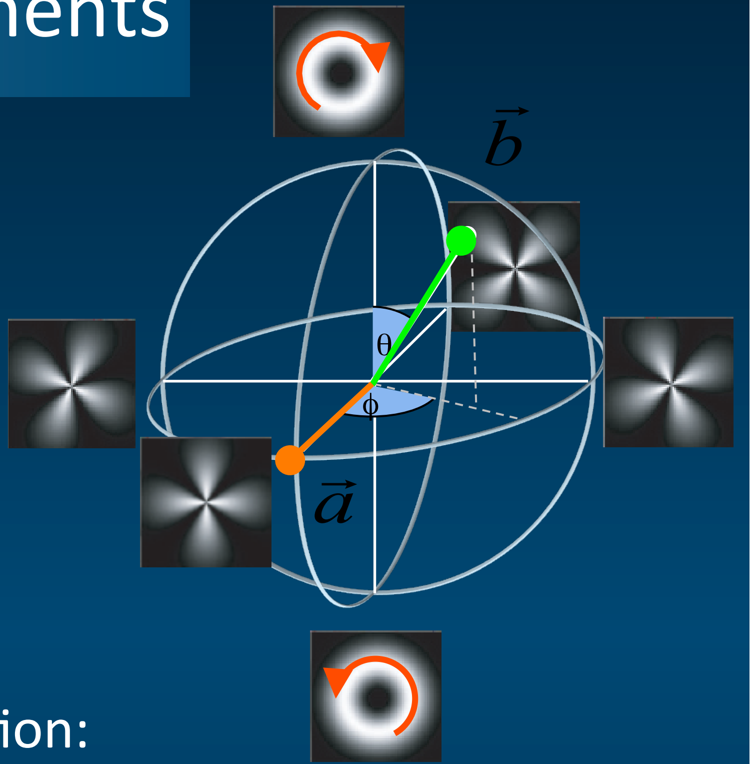




probability measurements

$$|\vec{a}\rangle = \cos\frac{\theta_a}{2}|\ell\rangle + e^{i\phi_a}\sin\frac{\theta_a}{2}|-\ell\rangle$$

$$|\vec{b}\rangle = \cos\frac{\theta_b}{2}|\ell\rangle + e^{i\phi_b}\sin\frac{\theta_b}{2}|-\ell\rangle$$



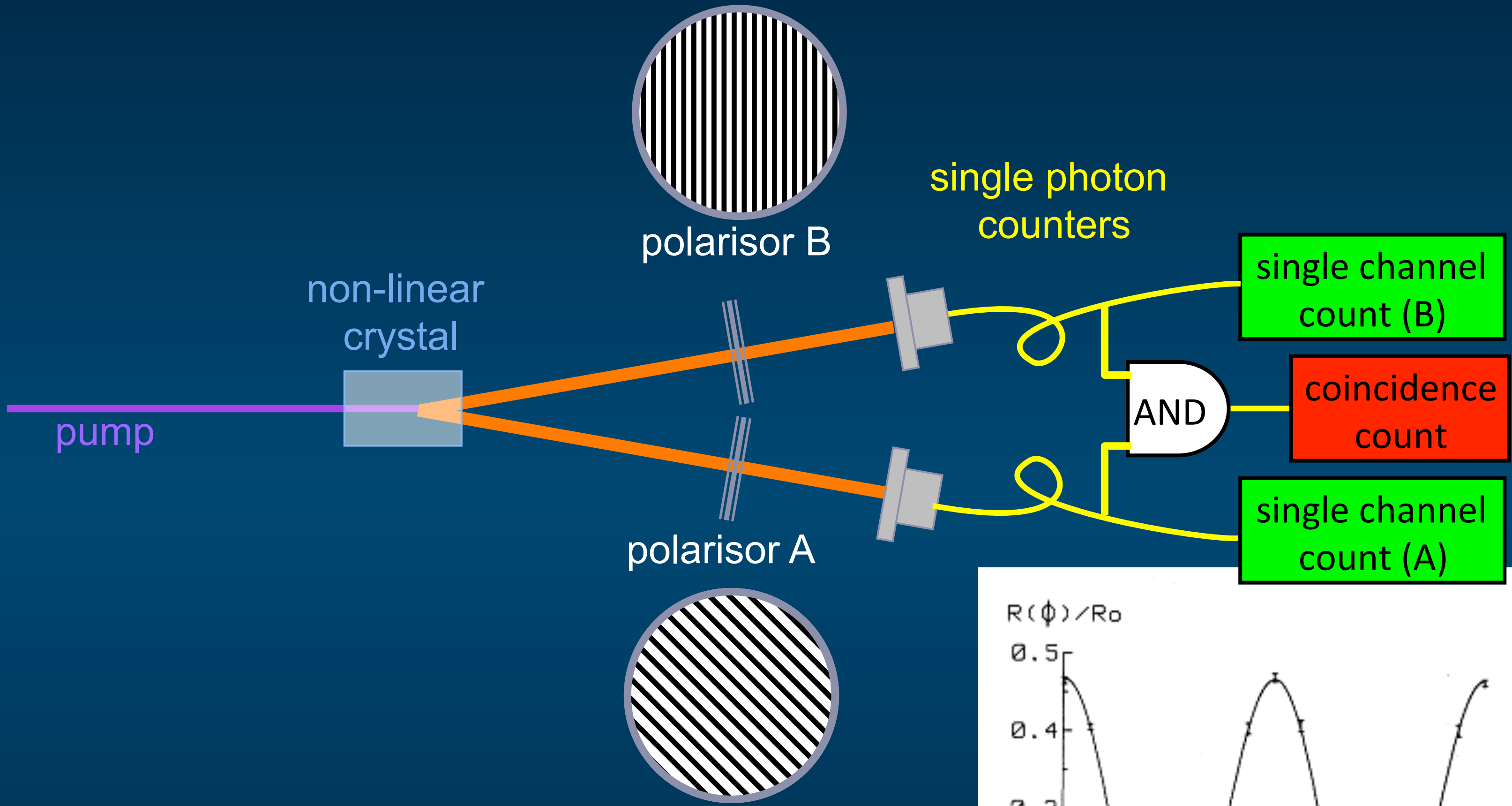
- OAM equivalent to Malus' law for polarisation:
- Each individual photon is incoherent. If measured under an angle ϕ_A to its initial state it will be detected with a probability

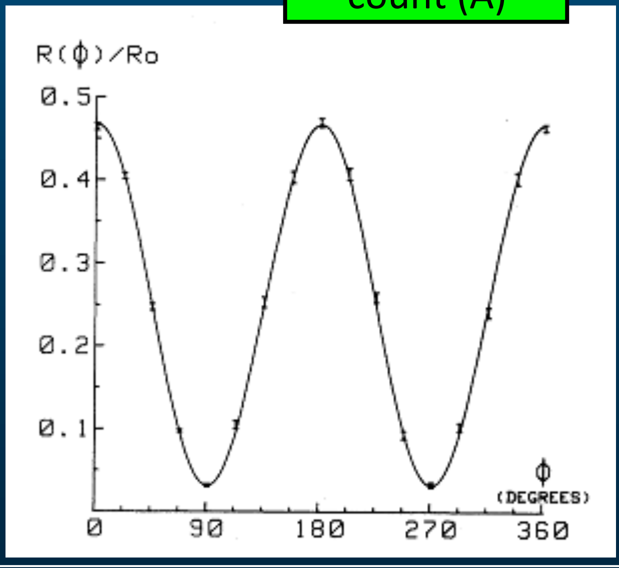
$$A(\vec{a}) \propto 2|\langle\vec{a}|\psi_A\rangle|^2 - 1 = \cos\phi_a$$

- Coincidence measurements

$$C(\vec{a},\vec{b}) = 4|\langle\vec{a}|\langle\vec{b}|\psi\rangle|^2 - 1 = -\cos\theta_a\cos\theta_b + \sin\theta_a\sin\theta_b\cos(\phi_b - \phi_a)$$

Bell for polarisation states

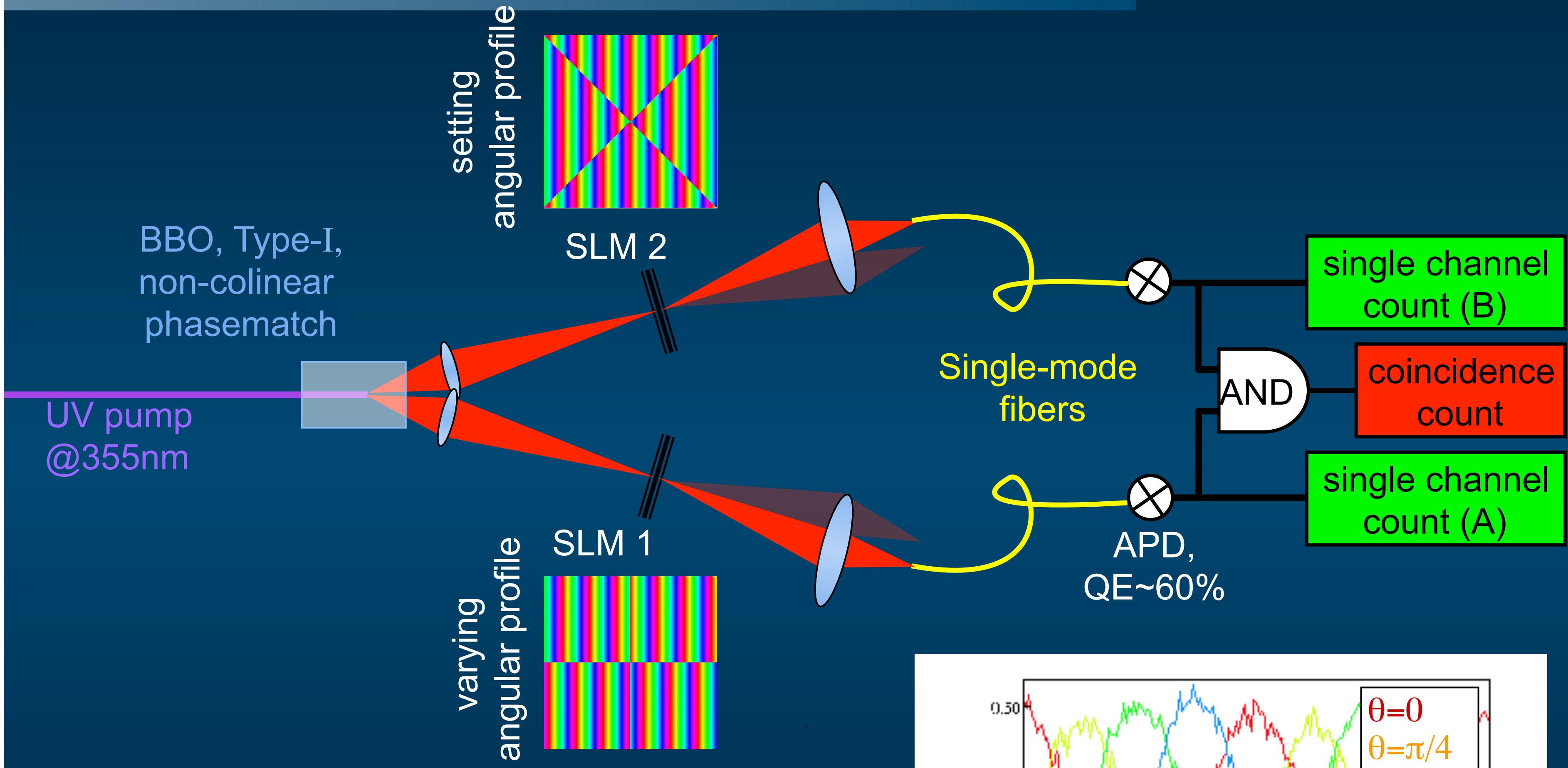


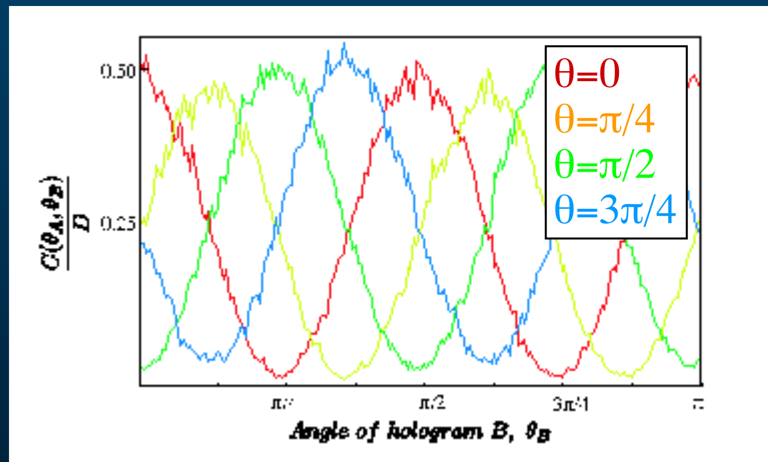


Bell parameter (Freedman) measured in Aspects experiment was 0.0572 (>0 to show a violation), in agreement with quantum mechanics.

Initial experiments (on atomic cascade) by Aspect, Grangier and Roger, PRL **47** 460 (1981)

Bell measurements



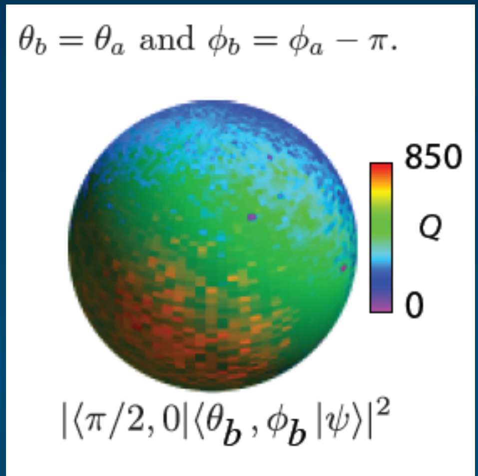


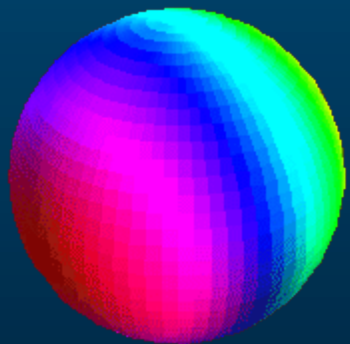
violation of Bell's inequality by σ=35,

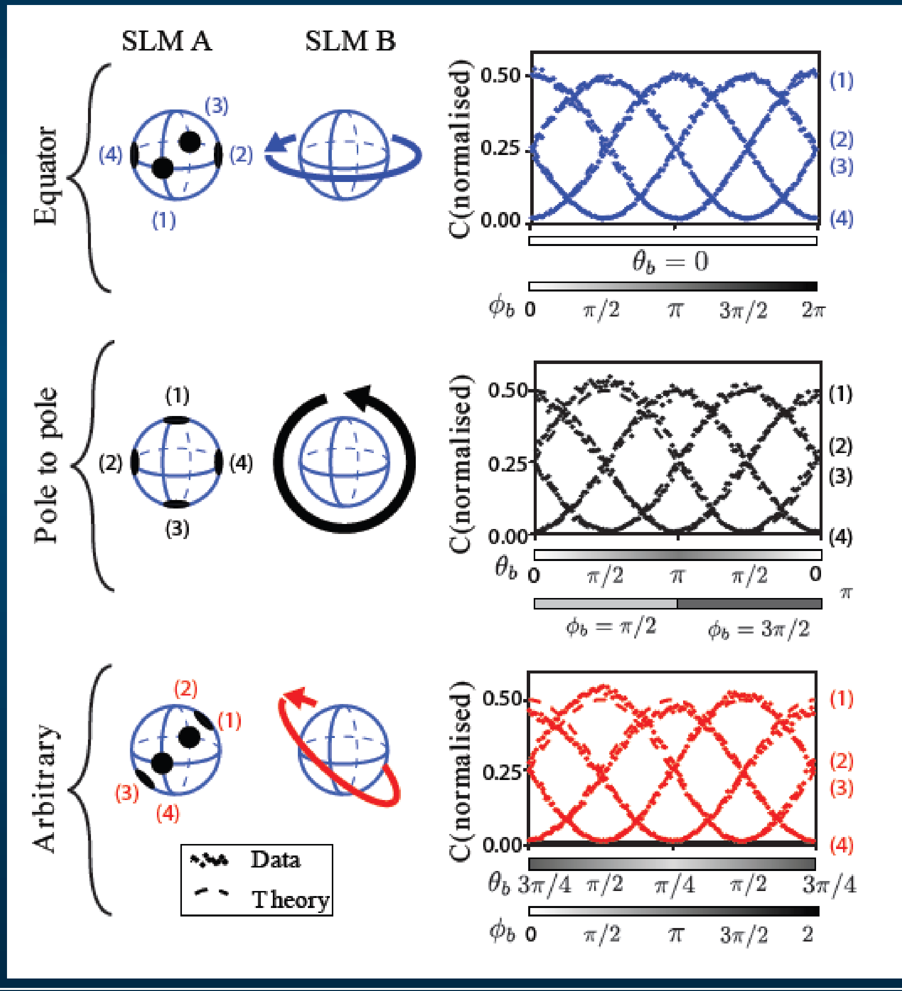
S=2.69±0.02 <2

Bell violation in 2D OAM subspace

a) for different great circles:







S=2.62±0.06

S=2.65±0.05

S=2.64±0.05

B. Jack *et al.* PRA **81**, 043844 (2010)

Bell violation in 2D OAM subspace

b) in a number of two-dimensional subspaces of the higher dimensional OAM Hilbert space

for $\ell = \pm 2$ $S_2 = 2.69$

for $\ell = \pm 3$ $S_3 = 2.55$

for $\ell = \pm 4$ $S_4 = 2.33$

J. Leach *et al.* Opt. Express **17**, 8287 (2009)

beyond local realism

- ”To maintain a local hidden-variable theory in the face of the existing experiments would appear to require believe in a very peculiar conspiracy of nature.” (Leggett, Foundations of Physics, 33, 1469, 2003)

- To do Leggett’s inequality, measurements are required that encompass all 3 dimensions of the Poincaré sphere.

A. Leggett, Found. Phys. **33** 1469 (2003),
S. Gröblacher *et al,* Nature **446** 871 (2007), C. Branciard *et al* Nat. Phys.**4** 681 (2008)

Leggett's hidden variable theory

- Leggett's axioms for a hidden variable theory:
  1. Each pair of photons has a characteristic set of hidden variables .
  2. The ensemble of photon pairs is determined by a statistical distribution of values of λ, ρ(λ), which depends only on the source. $P(\alpha,\beta\,|\,\vec{a},\vec{b}) = \int d\lambda \rho(\lambda) P_\lambda(\alpha,\beta\,|\,\vec{a},\vec{b})$
  3. The outcome of a measurement on each photon may depend on **both** detector settings and the hidden variables, doing away with locality. ~~$P_\lambda(\alpha,\beta\,|\,\vec{a},\vec{b}) = A(\vec{a},\lambda)B(\vec{b},\lambda)$~~
  4. Each photon of the pair individually behaves as if it has well-defined properties, and a (coincidence) measurement on it will show sinusoidal intensity variations (following Malus's law).

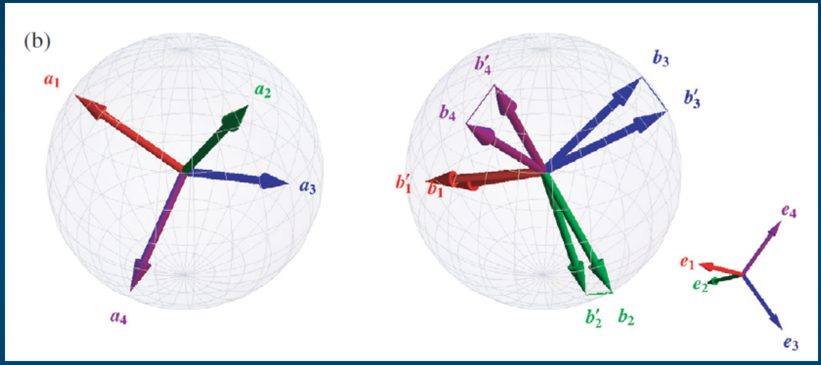
Leggett's inequality

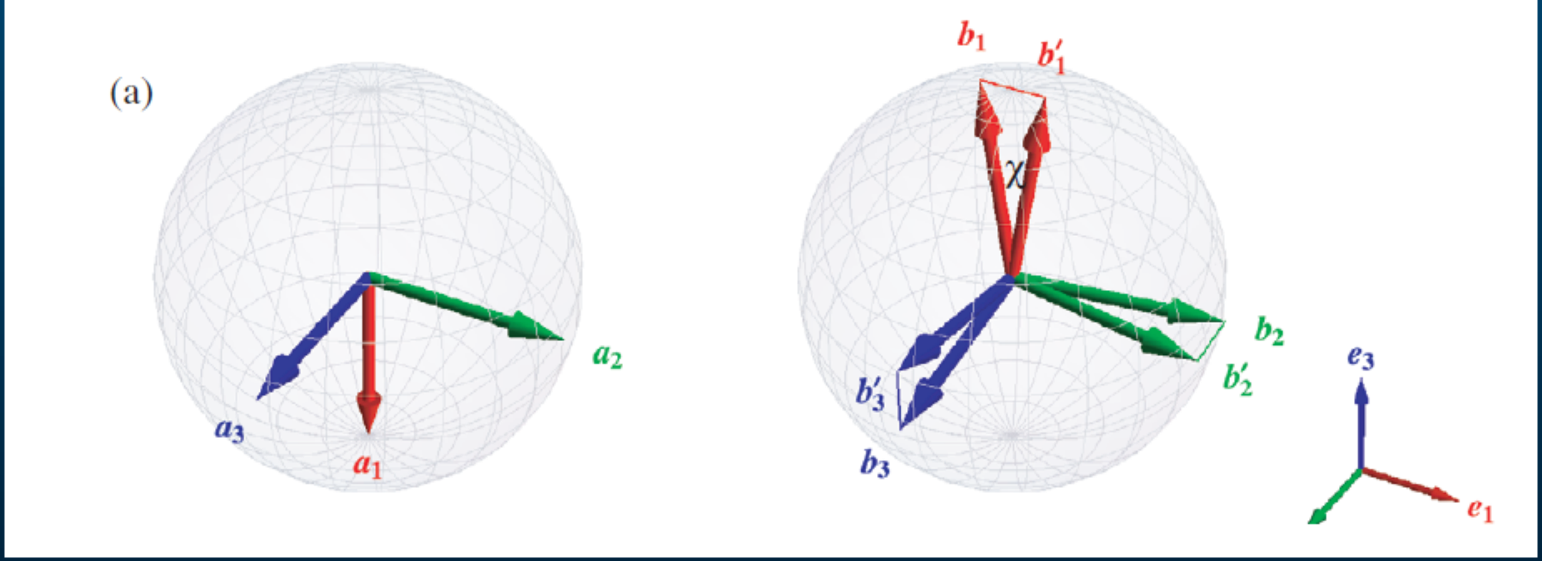
- Leggett's inequality

$$\frac{1}{N}\sum_{i=1}^{N}\left|C(\mathbf{a}_i,\mathbf{b}_i)+C(\mathbf{a}_i,\mathbf{b}_i{}')\right| \equiv L_N(\chi) \le 2-2\eta_N\left|\sin\frac{\chi}{2}\right|$$

- quantum mechanics predicts

$$L_N(\chi)=2\cos\left|\frac{\chi}{2}\right|$$





Leggett violation

- For N=3, we observe maximal violation of L_3 =1.8787±0.0241 at

$$\chi = -42^\circ$$

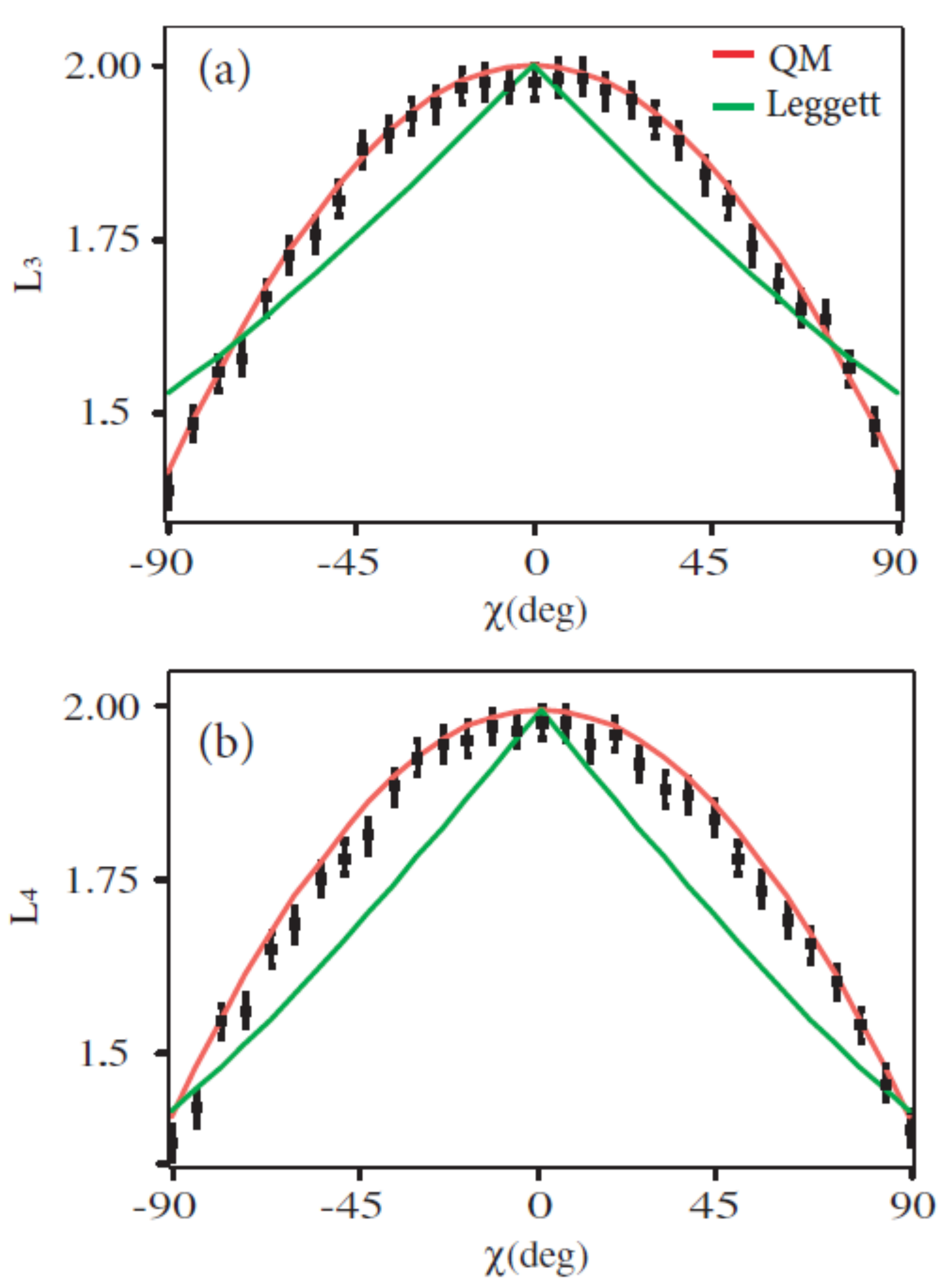
violating the inequality by 5σ.

- For N=4, we observe maximal violation of L_4 =1.9323±0.0239 at

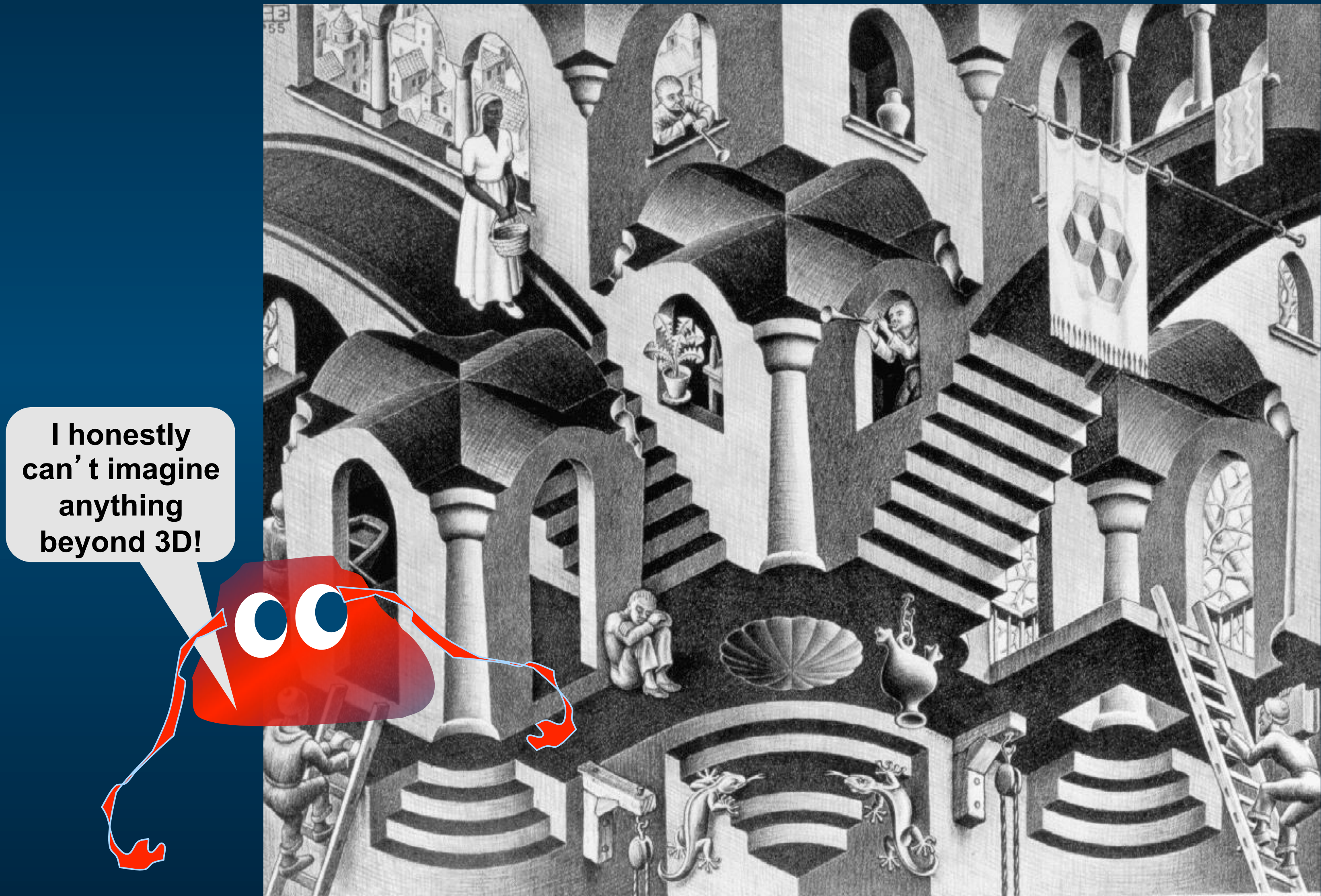
$$\chi = -30^\circ$$

violating the inequality by 6σ.

J. Romero *et al*, New J. Phys. **12**, 123007 (2010)



4. Higher dimensions



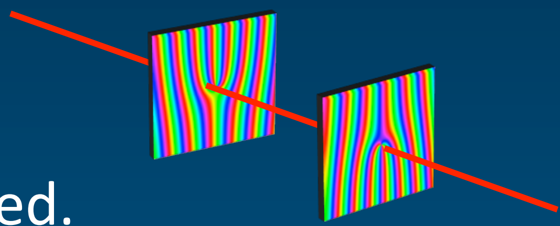
More than 2 dimensions

- There are (at least) 2 different approaches to involve more than 2 OAM dimensions:

1. deliberately address 3 or more OAM modes and test their entanglement
   - e.g. test for qutrit entanglement (Zeilinger)
   - generalised Bell states (Dada)
2. Alternatively, one can take measurements in the Fourier space of OAM, which is represented by the continuous space of angle states. Simultaneous strong correlations between both angle and OAM allow
   - Demonstrating the EPR paradox (Glasgow), and thereby confirming entanglement.
   - Operating in the Fourier space also allows to identify the Shannon dimensionality of OAM systems, (Leiden).

qutrit entanglement

- In 2002, Collins and coworkers formulated a Bell type inequality for 3 and more dimensions, which show a larger violation and are more robust against noise.
- In the same year, Zeilinger *et al* measured these settings by using two subsequent holograms in each arm which could be displaced.

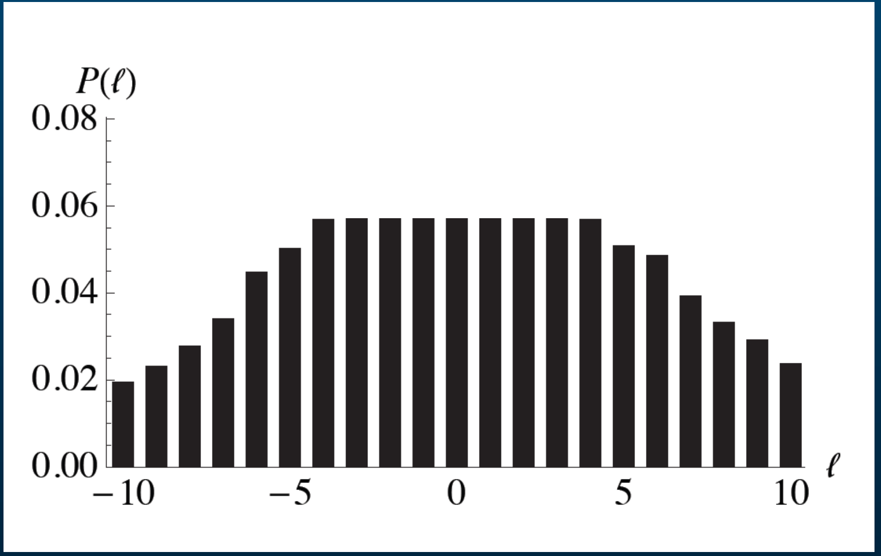


  - These actually produce superpositions including higher modes as well, but only the contributions $|-1\rangle, |0\rangle$, and $|+1\rangle$ were analysed.

- Zeilinger's team analysed over 20 million of combinations of analyser positions and found a maximum violation of the Bell type inequality by 18 standard deviations!

qu11its

- Very recently, demonstration of 11 dimensional entanglement.
- Rather than displacing holograms, the required superposition modes were precisely generated by programmable SLMs.
- Main experimental difficulty: getting a large number of OAM modes at sufficient intensity.
- Allowed solution: entanglement concentration by postselection (Procrustean method) – at the cost of reduced countrates.



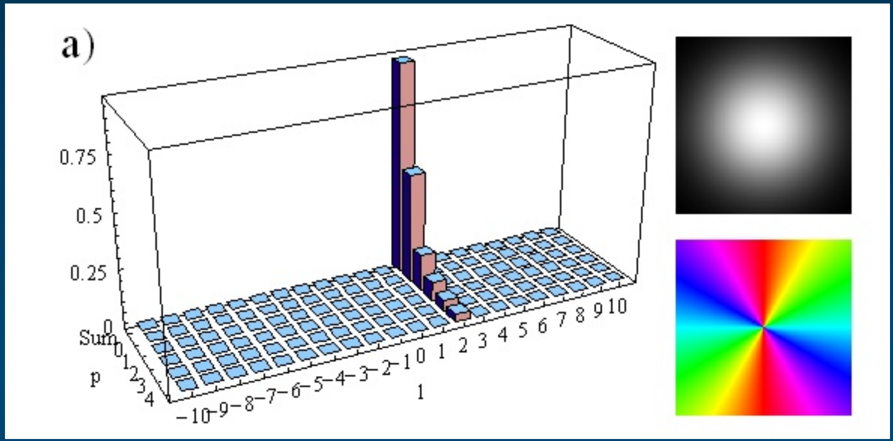
entanglement concentration:
Lee and Jaksch, PRA **80**, 010103R (2009)
11 dimensional entanglement:
A. Dada *et al*, Nat. Physics (2011)

Accessing infinitely many dimensions

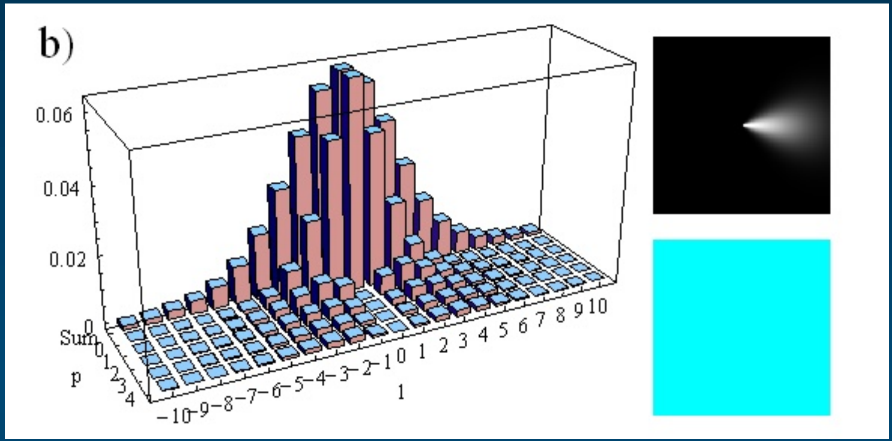
- Restricting the angular range of an OAM beam generates OAM sidebands. Tests that operate in the angular space, i.e. the Fourier space of OAM in a way rely on entanglement in infinitely many dimensions.

step back: generating OAM spectra

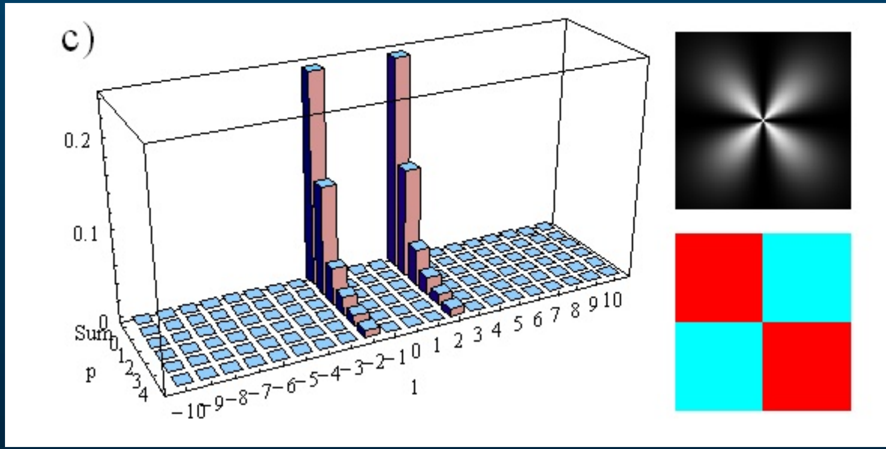
pure OAM mask: $|\ell = 2\rangle$



Gaussian angle and OMA distribution



sinusoidal angle variation: $(|2\rangle + |-2\rangle)/\sqrt{2}$



Modifying the angular profile of a light mode influences its OAM spectrum.

Caused by Fourier optics (classical) but in line with an angular Heisenberg uncertainty relation

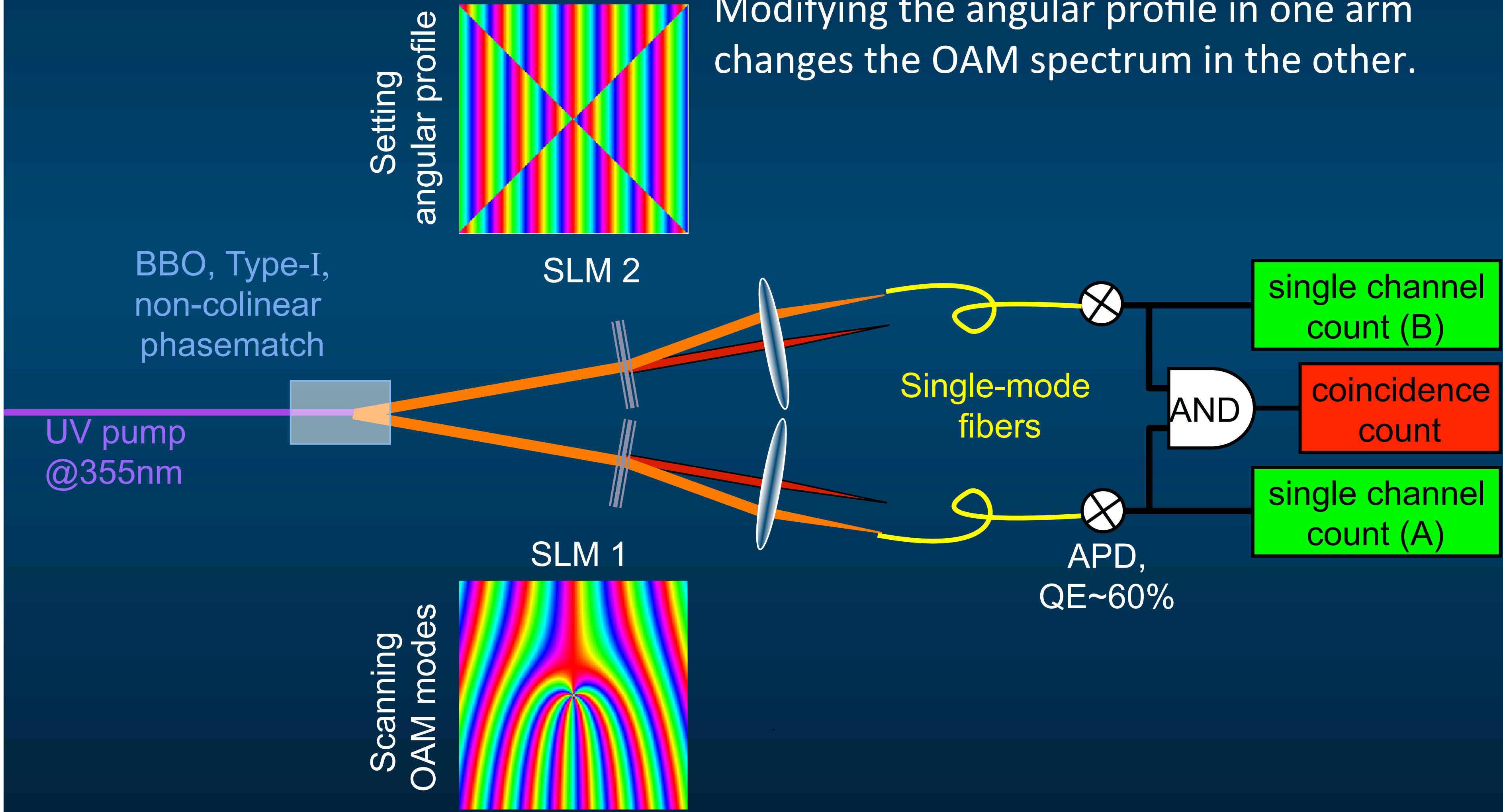
$\Delta\phi_\theta \Delta L_z \geq \hbar/2$ for small angles

S. Franke-Arnold *et al*., NJP **6** 103 (2006).

angular ghost diffraction

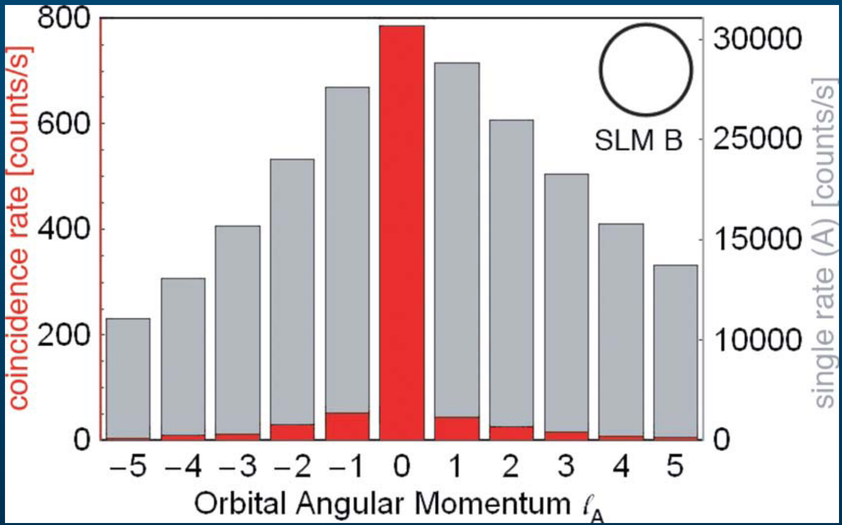
- The Fourier relation still holds for entangled photons:

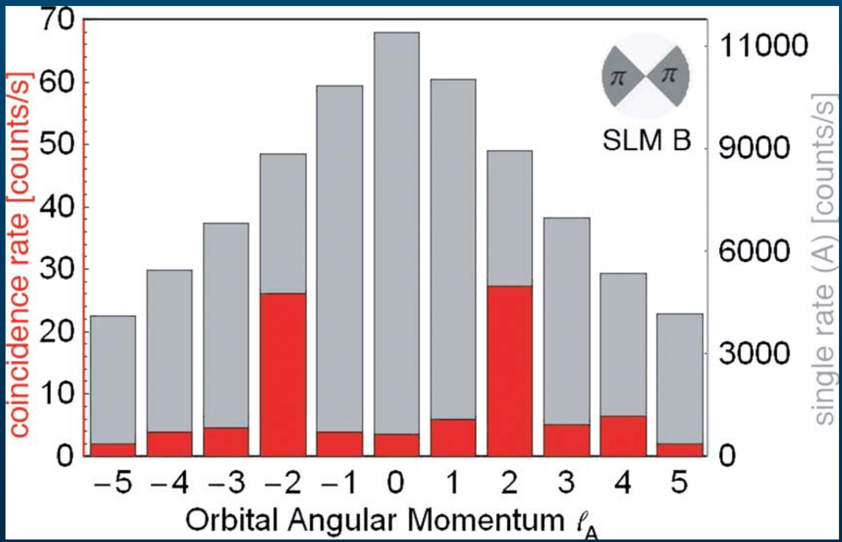
Modifying the angular profile in one arm changes the OAM spectrum in the other.



angular ghost diffraction

- an angular diffraction grating (sector mask) in arm B produces an OAM diffraction pattern in arm A (if measured in coincidence)

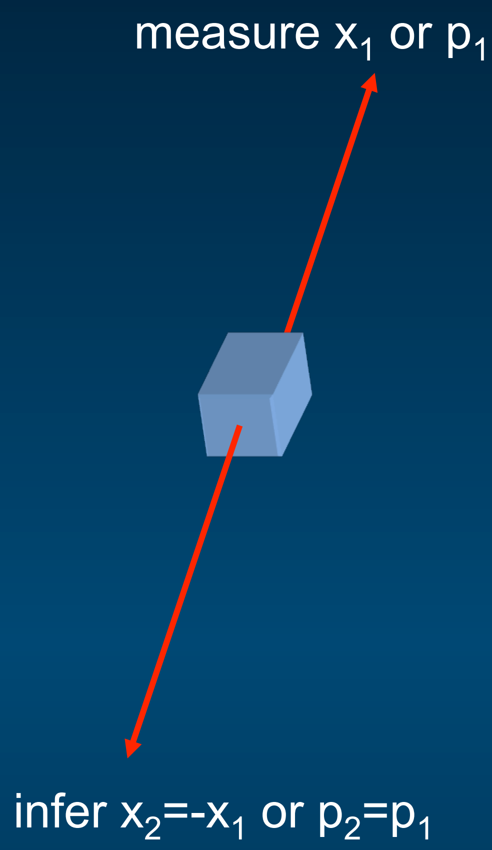




A. Jha *et al.*, Phys. Rev. A **78**, 043810 (2008)

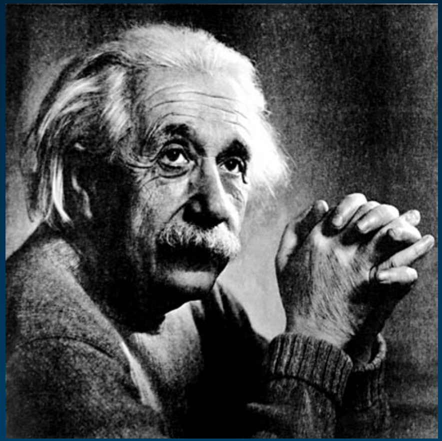
accessing more dimensions via EPR

- In its original form, EPR's paradox highlights strange features of entangled states on the example of position and momentum.
  - Both position and momentum of a photon can be inferred from a measurement on the correlated remote partner photon – i.e. without "disturbing" the photon in question.
  - It seems that both are simultaneously a property of the photon – which is forbidden by the uncertainty relation.
- An angular version: Also OAM and angle are linked by an uncertainty relation.
  - Can we infer OAM and in particular angle with sufficient accuracy from measurements on one photon of an entangled pair to violate the uncertainty relation?



$$\Delta x_2 \Delta p_2 = 0\,?$$

EPR paradox

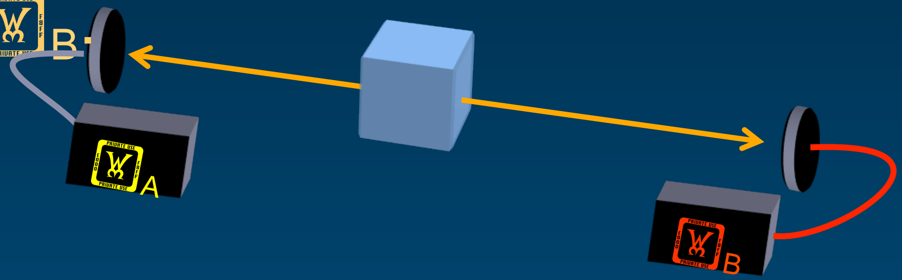


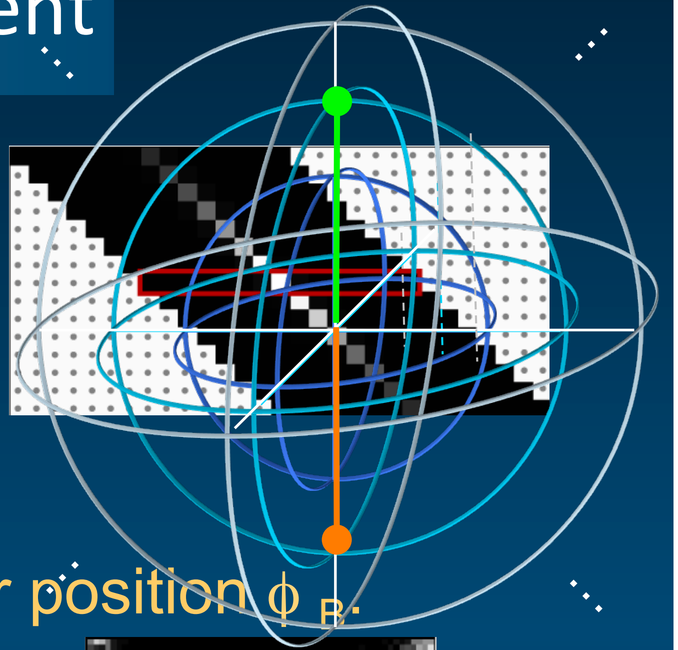
- Can quantum theory be considered complete?
  - A. Einstein, B. Podolsky and N. Rosen, Phys. Rev.47, 777
- Einstein's reality:
  - "If without in any way disturbing a system, we can predict with certainty the value of a physical observable corresponding to this quantity."
- Einstein's paradox:
  - Entangled particles share their properties. We can infer the (angular) position or (angular) momentum of one particle from the (angular) position or (angular) momentum of its remote partner particle. Both *x* and *p* (f and ℓ) of the first particle then have "physical reality" and should be described by its wavefunction.

A paradox arises, if the combined error in inferring these quantities is smaller than allowed by Heisenberg's uncertainty principle.

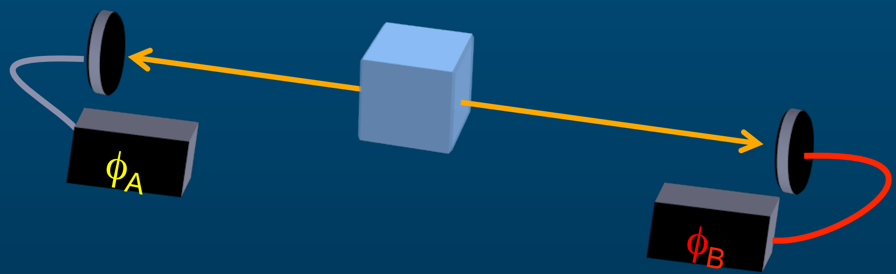
OAM EPR Gedanken experiment

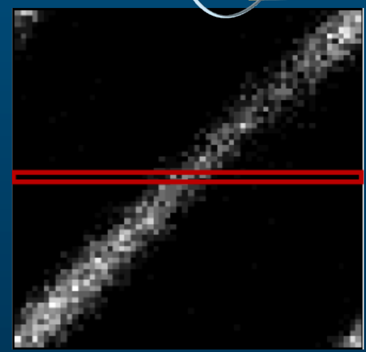
Measure OAM ℓ_A, infer OAM ℓ_B:





Measure angular position ϕ_A, infer angular position ϕ_B.





- However, quantum theory does not allow the simultaneous exact knowledge of ℓ and ϕ.

$$\Delta \ell \hbar \Delta \phi \geq \frac{\hbar}{2} \qquad \text{for small angles}$$

EPR for OAM

- How well do we need to be able to infer angular momentum and position in order to violate Heisenberg?
- In separable systems

$$[\Delta(\ell_B \mid \ell_A)\hbar]^2[\Delta(\phi_B \mid \phi_A)]^2 \geq \frac{\hbar^2}{4}$$

0.171 $\qquad$ 0.140 for $\frac{\pi}{15}$ aperture width

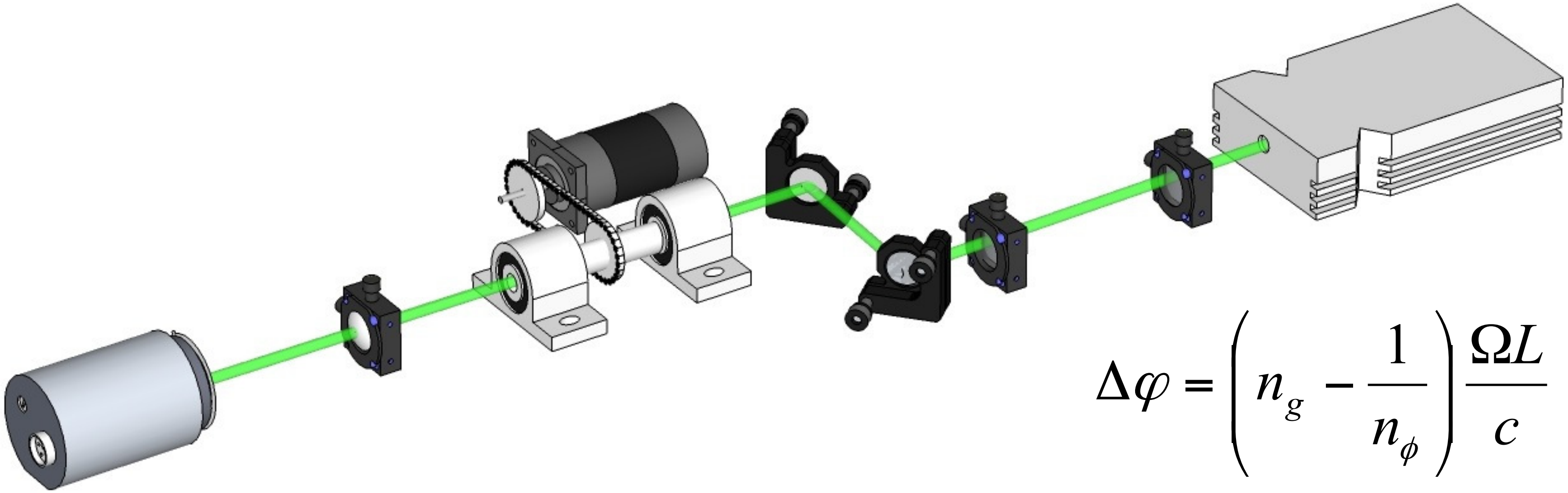
$$[\Delta(\ell_B \mid \ell_A)]^2[\Delta(\phi_B \mid \phi_A)]^2 = 0.024$$

- About a tenth of what's permitted by Heisenberg!

J. Leach et al. Science **329**, 662 (2010)

something completely different

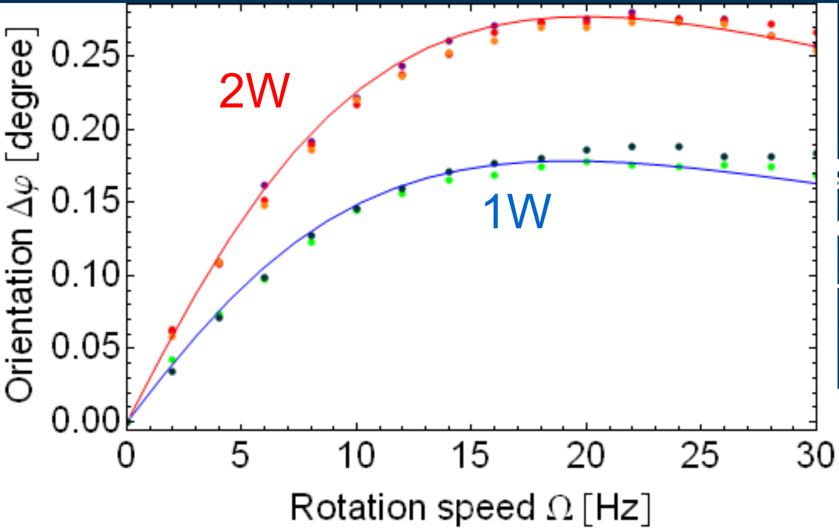
- How does the world look like through a rotating window?

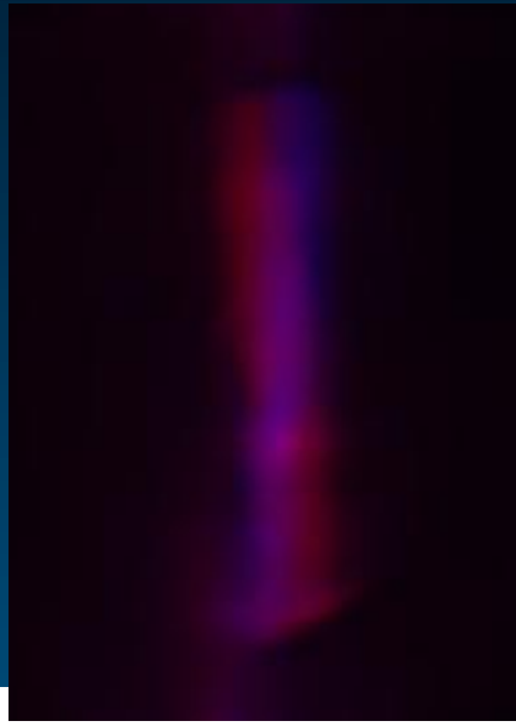


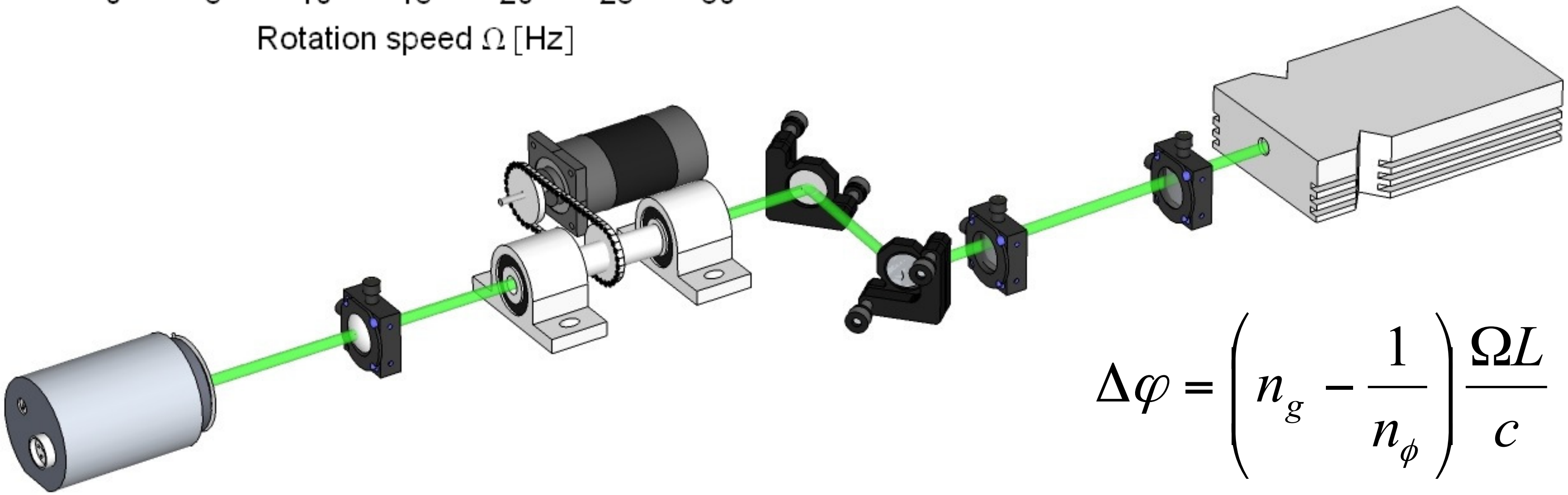
$$\Delta\varphi = \left(n_g - \frac{1}{n_\phi}\right)\frac{\Omega L}{c}$$

Rotary Photon Drag Enhanced by a Slow Light Medium

- Slow light by coherent population oscillations in Ruby.
- illion.
- ge (elliptical beam)
  on of the major axis.
- itially 10cm) length







$$\Delta\varphi = \left(n_g - \frac{1}{n_\phi}\right)\frac{\Omega L}{c}$$

Summary


- Various fundamental quantum tests have been performed, mainly within 2D subspaces, some in higher dimensions.


- OAM and azimuthal angle are conjugate variables, offering an unusual combination of geometries (discrete, continuous and periodic).


- OAM/angle is a rich system, open to be explored for fundamental tests as well as for applications, quantum gates, communication, etc.