Optimal cloning of photonic quantum states

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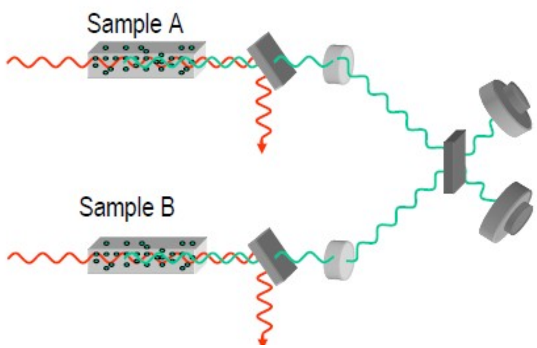
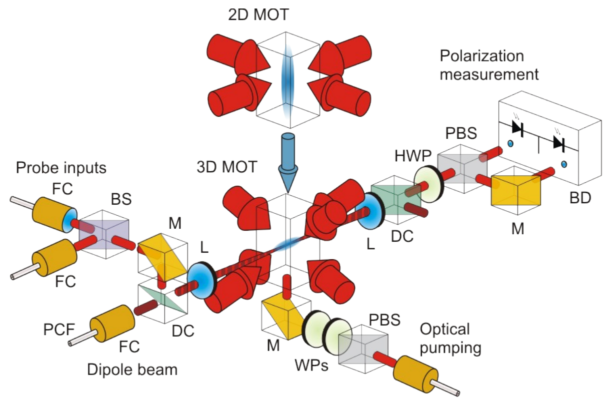
http:\\quantumoptics.phys.uniroma1.it

Motivations...

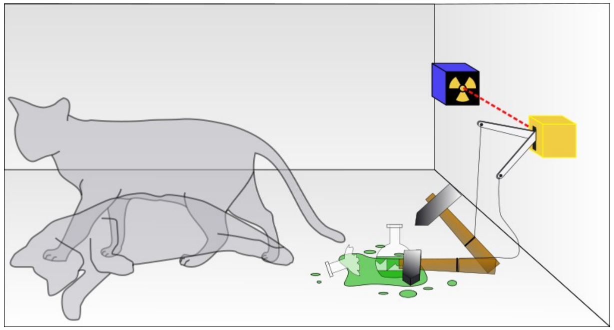
Quantum information processing:
computation and communication



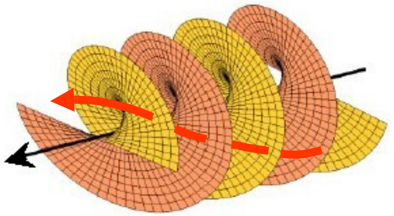
Quantum sensing:
Exploit quantum resources to enhance measurement precision



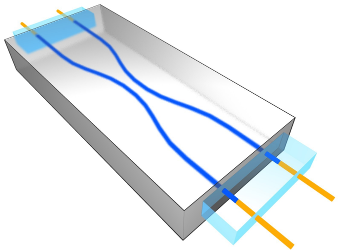
Foundations of Quantum mechanics:
Macroscopic entanglement “Schroedinger's cat”



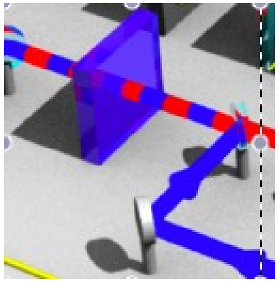
I) High dimensional quantum systems: Orbital angular momentum of light



II) Integrated Photonic quantum circuits



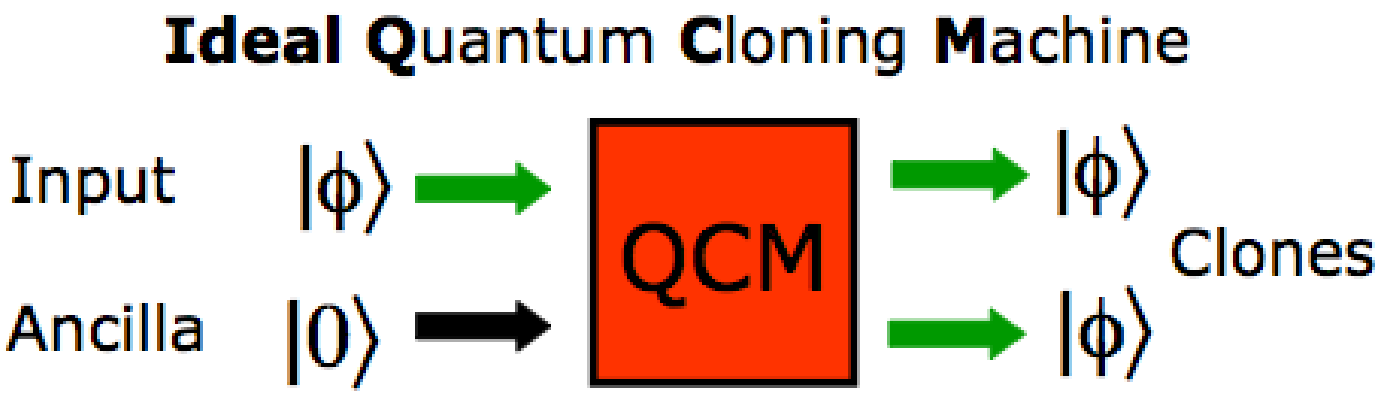
III) Amplification of optical quantum states



Outline

- Introduction to quantum cloning theory
- Optimal cloning via stimulated emission
- Optimal cloning via qubit symmetrization
- Hong-Ou-Mandel coalescence of OAM state
- Optimal cloning of OAM qubit
- Hong-Ou-Mandel coalescence of single photon entangled states
- Optimal cloning of π – OAM ququart

Quantum cloning ?

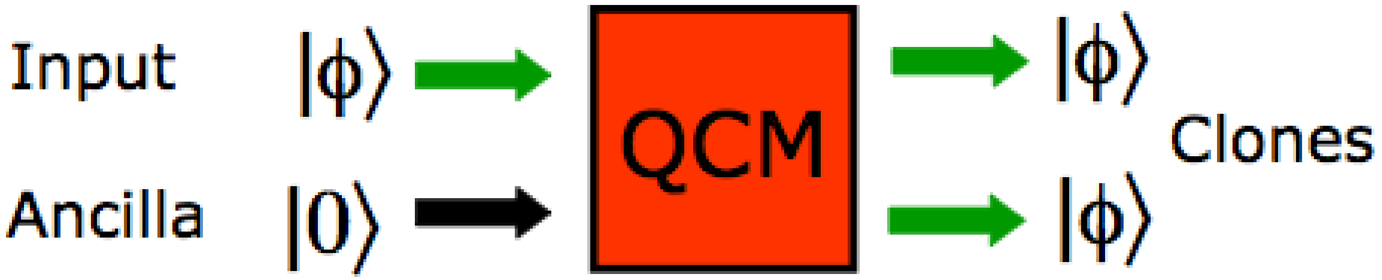


- Perfect copies of the input state
- Universal: all the state can be cloned

Is it allowed by Quantum Mechanics?

Quantum cloning ?

Ideal Quantum Cloning Machine

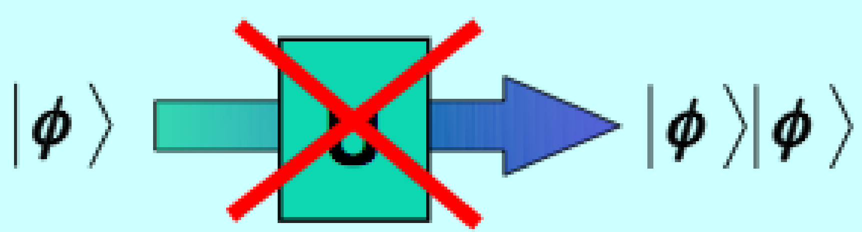


- Perfect copies of the input state
- Universal: all the state can be cloned

Is it allowed by Quantum Mechanics?

No cloning theorem

"Unknown quantum states cannot be cloned"



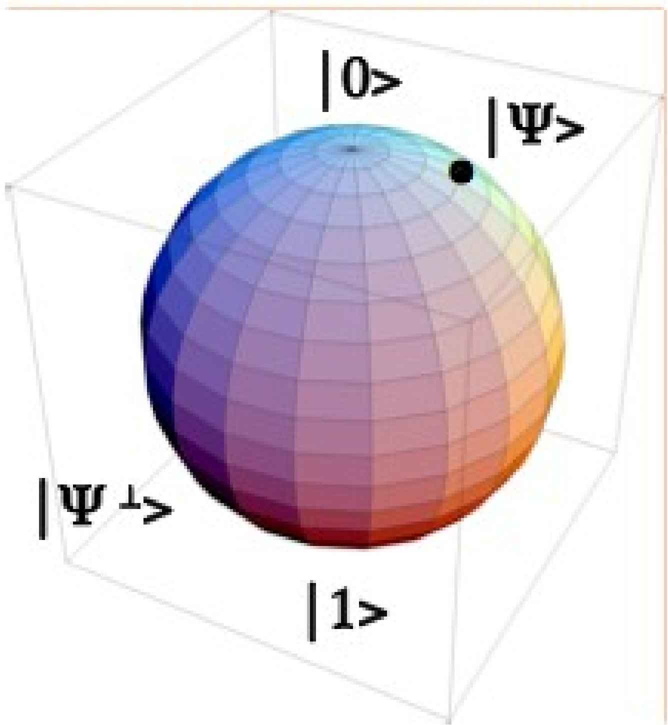
Wootters, Zurek, *Nature* ***299***, 802 (1982)

NOT gate of an unknown qubit

$$|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle \longrightarrow \boxed{\text{NOT GATE}} \longrightarrow |\Psi^{\perp}\rangle = \beta^{*}|0\rangle - \alpha^{*}|1\rangle$$

Inversion of the Bloch sphere: Flipping of a qubit on the symmetric point of the Bloch sphere

NOT GATE ANTI-UNITARY not physically realizable with Fidelity = 1



TRANSPOSE

$$\begin{pmatrix} |\alpha|^2 & \alpha\beta^{*} \\ \alpha^{*}\beta & |\beta|^2 \end{pmatrix} \Rightarrow \begin{pmatrix} |\alpha|^2 & \alpha^{*}\beta \\ \alpha\beta^{*} & |\beta|^2 \end{pmatrix}$$

Important for criteria of separability of bipartite state

$$E_{NOT}(\rho) \leftarrow \xrightarrow{\sigma_Y} E_{PT}(\rho)$$

Optimal Quantum Machines

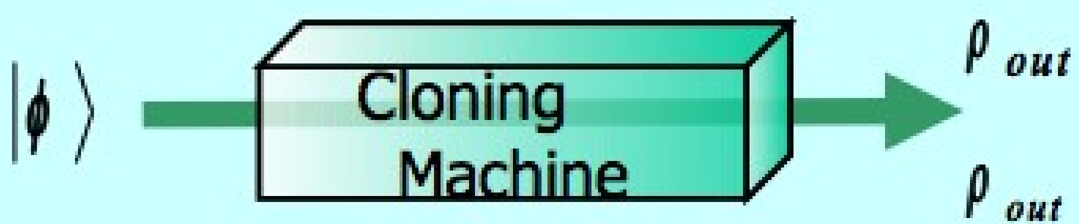
No cloning theorem: "It is not possible to clone an arbitrary unknown quantum state"

No quantum NOT gate: "A universal NOT gate cannot be realized"

What the best physical approximations of the these two machines ?

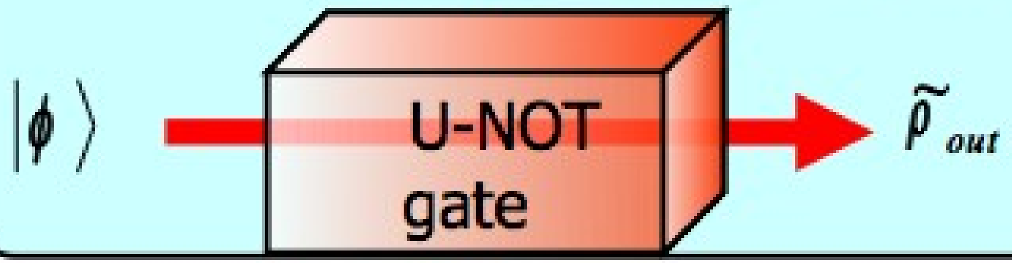
Fidelity F: $0 \leq F \leq 1$, $F = 1$ perfect realization, forbidden by quantum mechanics

Optimal Universal Quantum Cloning 1→ 2



$$F = \int_{|\phi\rangle \in H} \langle \phi | \rho_{out} | \phi \rangle d\phi$$

Optimal Universal NOT gate 1→1



$$F = \int_{|\phi\rangle \in H} \langle \phi^{\perp} | \tilde{\rho}_{out} | \phi^{\perp} \rangle d\phi$$

Optimal transformations

Initial qubit $|\phi\rangle_S$ + 2 ancilla qubits $|0\rangle_A |0\rangle_B$

$\hat{U}_{clon}$

$$|\Sigma(\phi)\rangle_{SAB} = \sqrt{\frac{2}{3}}|\phi\rangle_S|\phi\rangle_A|\phi^{\perp}\rangle_B - \frac{1}{\sqrt{6}}(|\phi\rangle_S|\phi^{\perp}\rangle_A + |\phi^{\perp}\rangle_S|\phi\rangle_A)|\phi\rangle_B$$

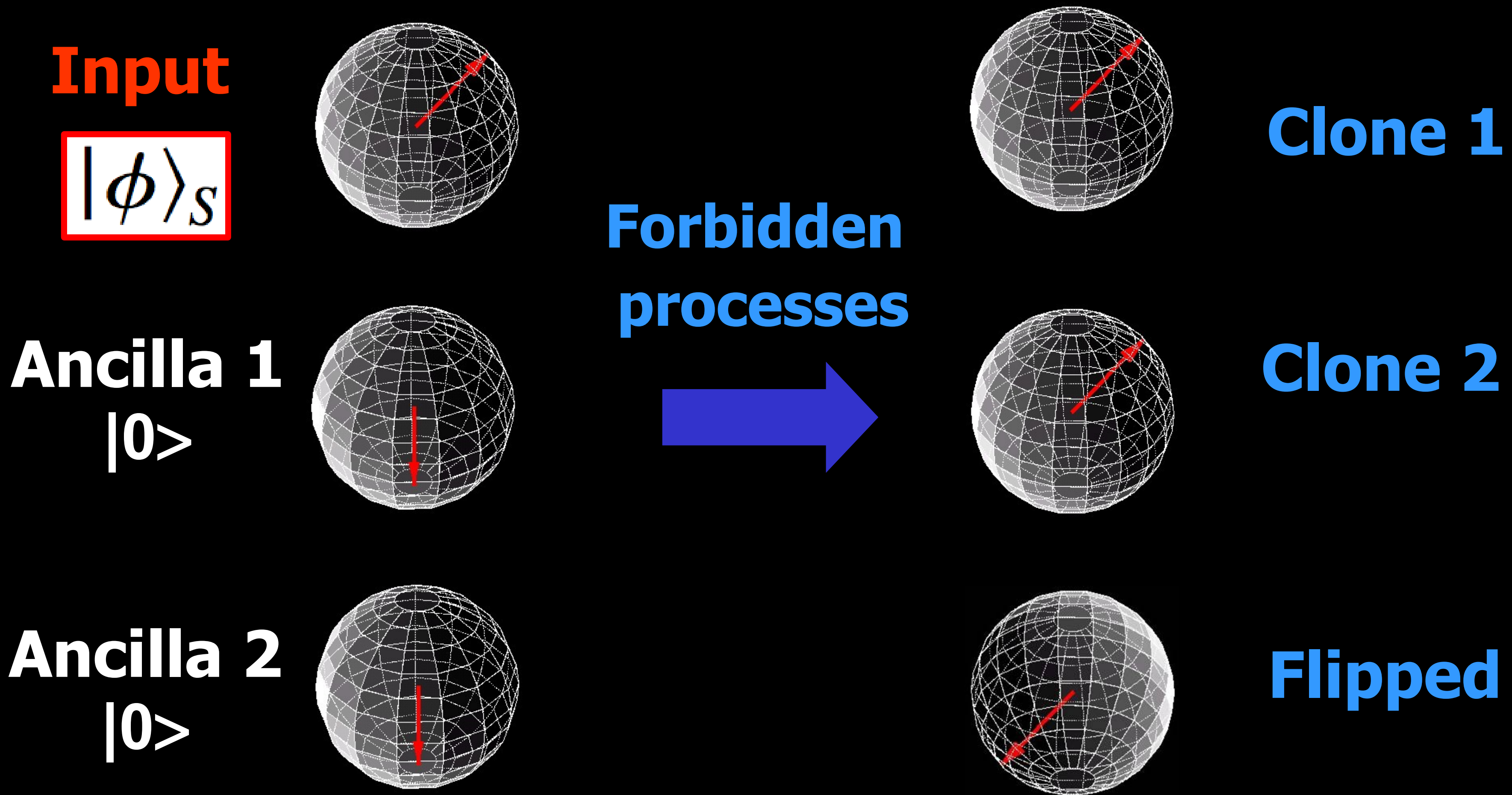
Qubits S and A: optimal quantum cloned qubits (F=5/6)

Qubit B: optimal flipped qubit (F=2/3)

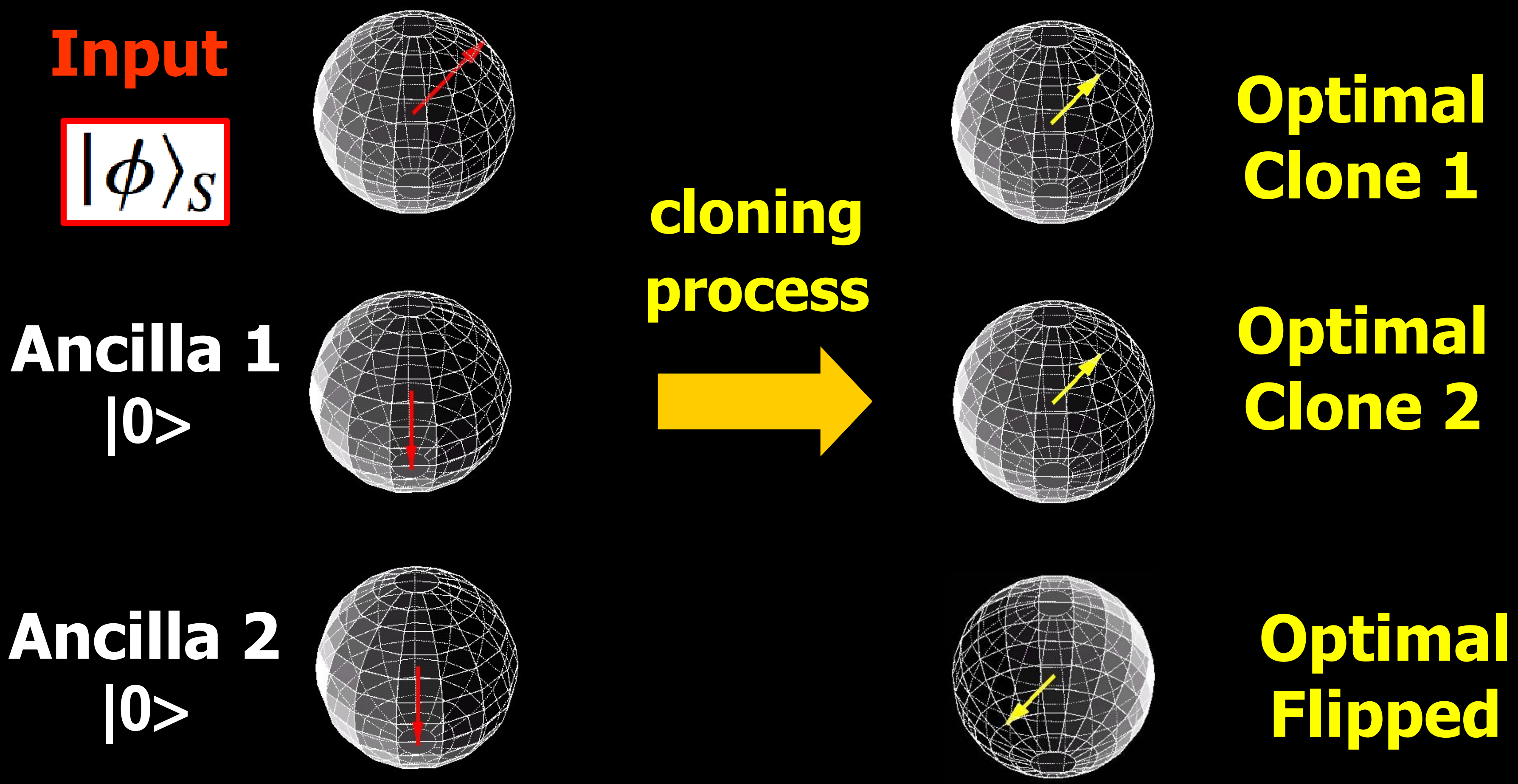
Redistribution of the initial information content into 3 qubits entangled state

- **To understand analogies and differences between classical and quantum information processing**
  - **Quantum state estimation**
- **Useful for some quantum computation tasks**

Ideal machines



Optimal Quantum Cloning and Universal NOT Gate



Quantum cloning by stimulated emission

Input qubit: polarization state of a single photon

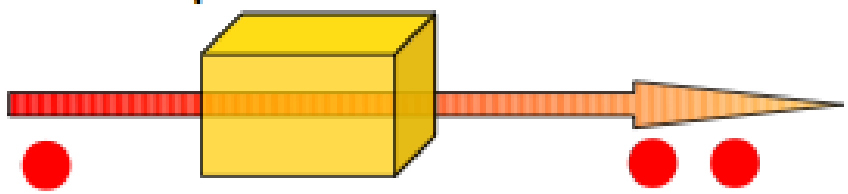
$$|\phi_{IN}\rangle = \alpha|0\rangle + \beta|1\rangle \Leftrightarrow |\phi_{IN}\rangle = \alpha|H\rangle + \beta|V\rangle$$

Implementation based on the stimulated emission process

I) Medium with inverted population: same gain for all the polarizations

II) Optical parametric amplification

Injected photon



Amplifier

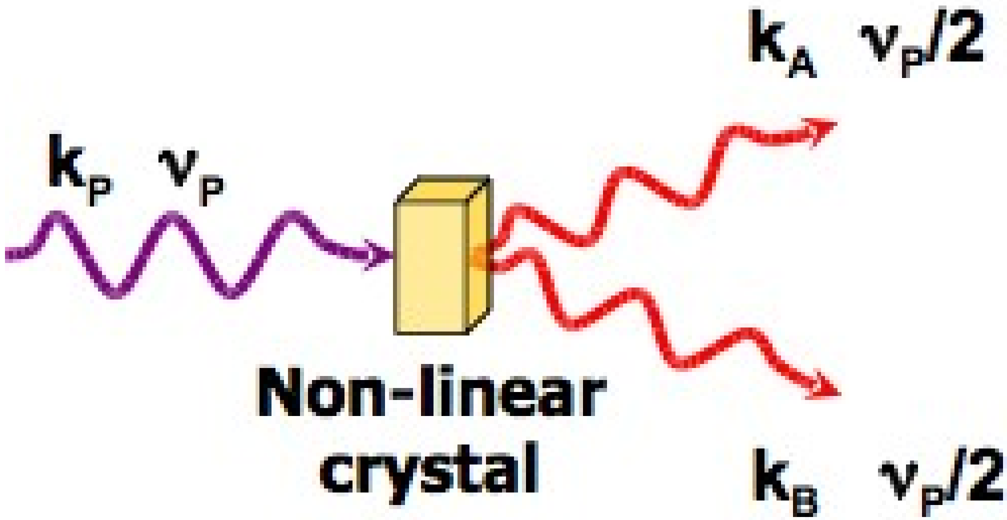
Universality of the cloning
Universality of
the amplification

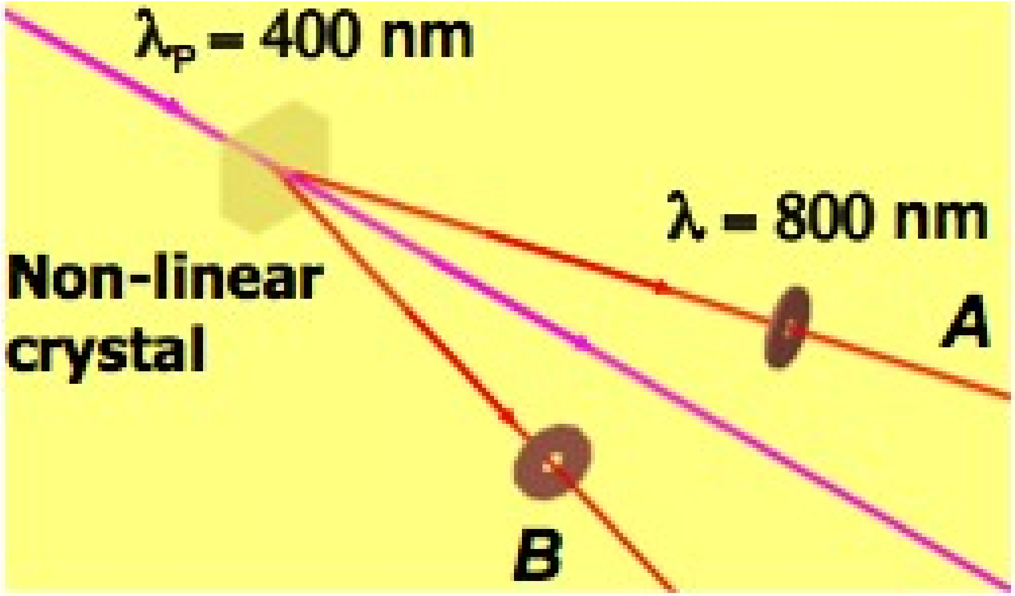
↔

Spontaneous emission for all the states
Noise → F<1

C. Simon, G .Weihs, and A. Zeilinger, Phys. Rev. Lett. **84**, 2993 (2000)

Parametric interaction: generation of entangled states





Spontaneous emission ⟺ $\hat{U}|0\rangle_A|0\rangle_B \propto \left(|H\rangle_A|V\rangle_B - |V\rangle_A|H\rangle_B\right)$

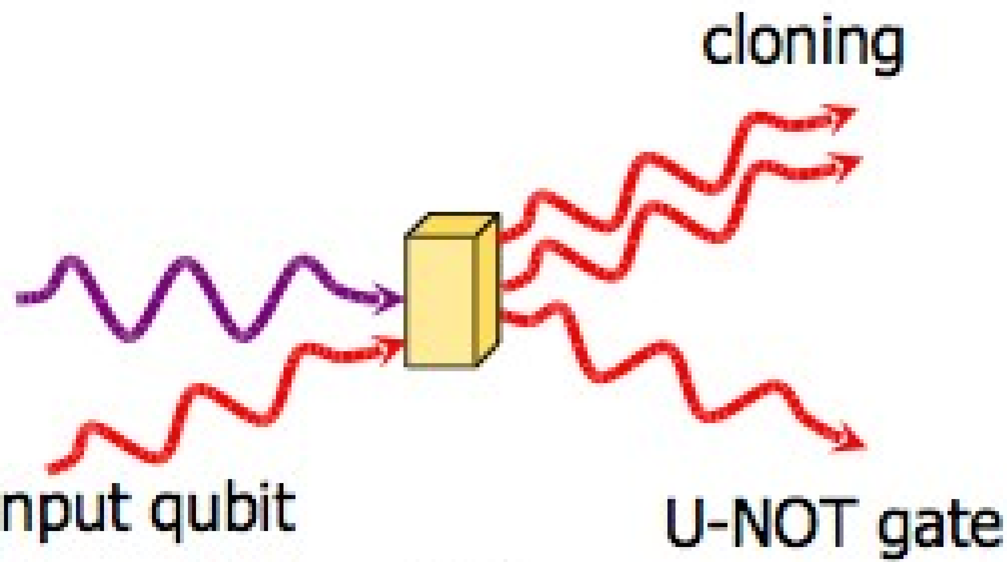
Hamiltonian of interaction

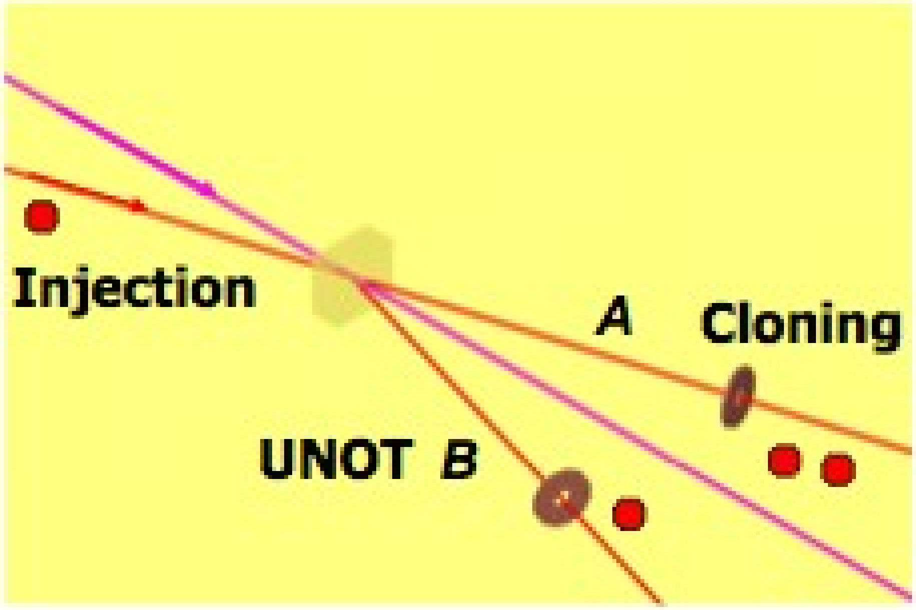
$$H_I = i\hbar\chi\left(\hat{a}_V^+\hat{b}_H^+ - \hat{a}_H^+\hat{b}_V^+\right) + h.c.$$

Unitary evolution

$$\hat{U} = \exp\left(-iH_I t/\hbar\right)$$

Quantum Injected Optical Parametric Amplifier





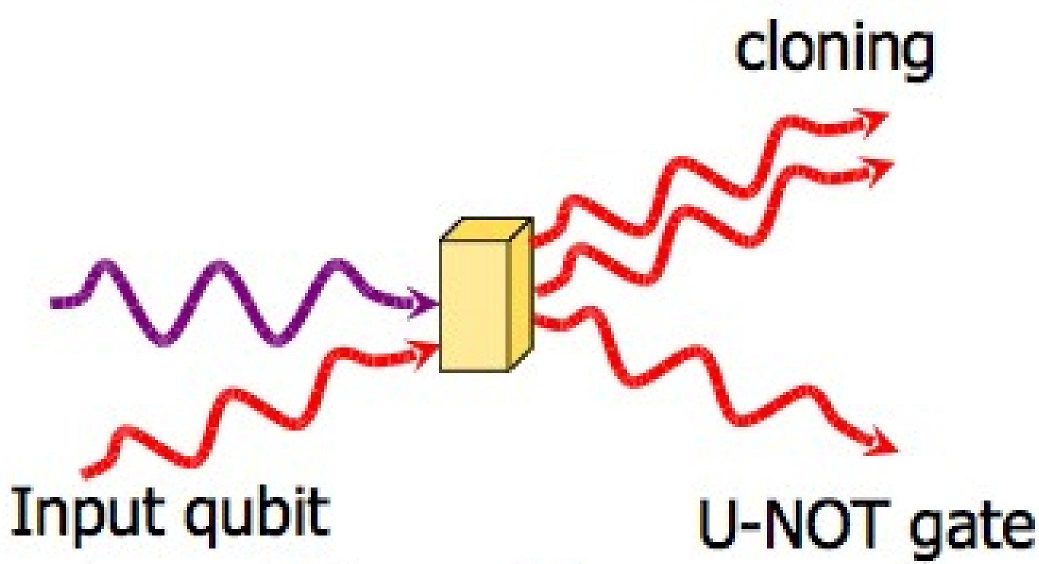
$$|\phi\rangle_A = \alpha|H\rangle_A + \beta|V\rangle_A$$

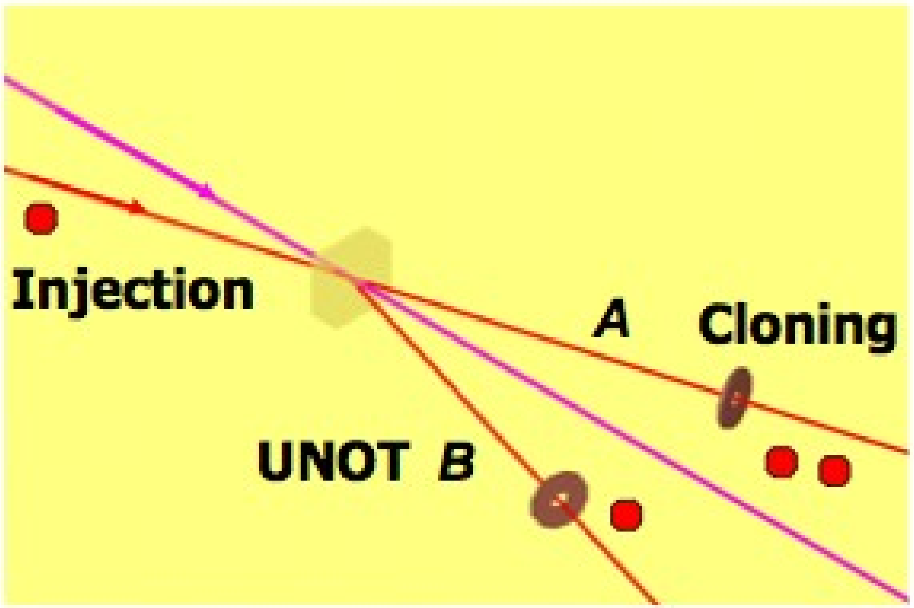
Stimulated emission $\Longleftrightarrow \hat{U}|\phi\rangle_A|0\rangle_B \propto \left(2^{1/2}|\phi\,\phi\rangle_A|\phi^{\perp}\rangle_B - |\phi\,\phi^{\perp}\rangle_A|\phi\rangle_B\right)$

Mode A: Universal cloning process
Mode B: UNOT gate

De Martini, et al., *Nature* **419**, 815 (2002); Lamas-Linares, et al., *Science* **296**, 712 (2002)
De Martini, Pelliccia, Sciarrino, *Phys. Rev. Lett.* **92**, 067901(2004)

Quantum Injected Optical Parametric Amplifier





$|\phi_{IN}\rangle_A = \alpha|H\rangle_A + \beta|V\rangle_A$

Stimulated emission $\Leftrightarrow \hat{U}|\phi\rangle_A|0\rangle_B \propto \left(2^{1/2}|\phi\ \phi\rangle_A|\phi^{\perp}\rangle_B - |\phi\ \phi^{\perp}\rangle_A|\phi\rangle_B\right)$

$H_I = i\hbar\chi\left(a_V^{\dagger}b_H^{\dagger} - a_H^{\dagger}b_V^{\dagger}\right) + h.c.$ classical and undepleted pump $\chi \propto E_P$

Universality of the amplifier : the interaction Hamiltonian can be recast in the following way (SU(2) invariance) $H_I = i\hbar\chi\left(\hat{a}_{\phi}^{\dagger}\hat{b}_{\phi\perp}^{\dagger} - \hat{a}_{\phi\perp}^{\dagger}\hat{b}_{\phi}^{\dagger}\right) + h.c.$

Quantum Injected Optical Parametric Amplifier

$$H_I = i\hbar\chi\left(\hat{a}^{+}_{\phi}\hat{b}^{+}_{\phi\perp} - \hat{a}^{+}_{\phi\perp}\hat{b}^{+}_{\phi}\right) + h.c.$$

$\hat{U} = \exp\left(-iH_I t/\hbar\right)$ gain parameter $g = \chi t < <1$ g^2 terms neglected

Spontaneous parametric down-conversion

$$\hat{U}|0\rangle_A|0\rangle_B \approx |0\rangle_A|0\rangle_B + g\left(|\phi\rangle_A|\phi^{\perp}\rangle_B - |\phi^{\perp}\rangle_A|\phi\rangle_B\right)$$

Stimulated emission by injection of the state $|\Psi_{in}\rangle = |\phi\rangle_A|0\rangle_B$

$$\hat{U}|\phi\rangle_A|0\rangle_B \approx |\phi\rangle_A|0\rangle_B + g\,2^{1/2}\left(|\phi\,\phi\rangle_A|\phi^{\perp}\rangle_B - |\phi\,\phi^{\perp}\rangle_A|\phi\rangle_B\right)$$

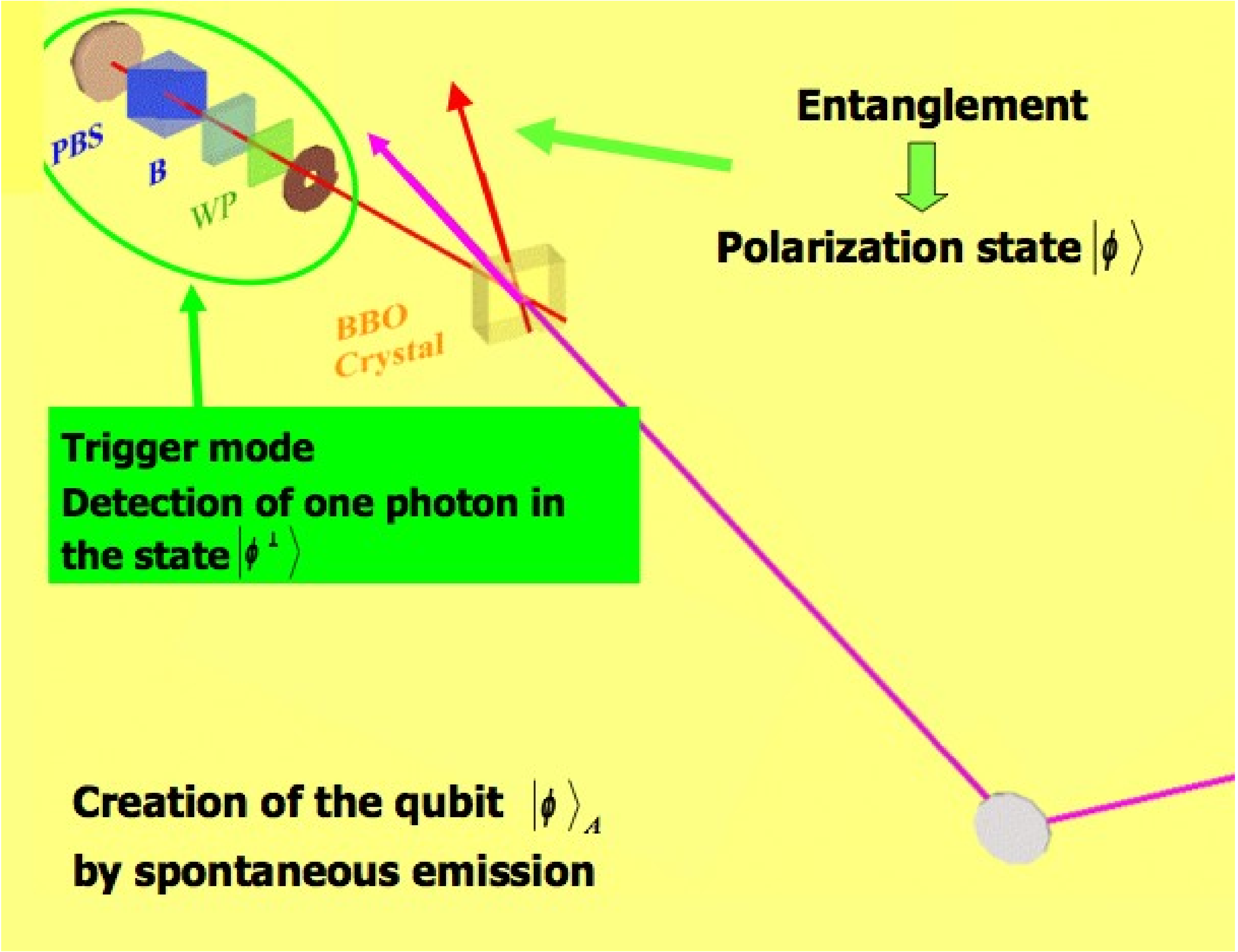
probability of emitting $|\phi\rangle$ **over mode A increased by a factor R=2**

probability of emitting $|\phi^{\perp}\rangle$ **over mode B increased by a factor R*=2**

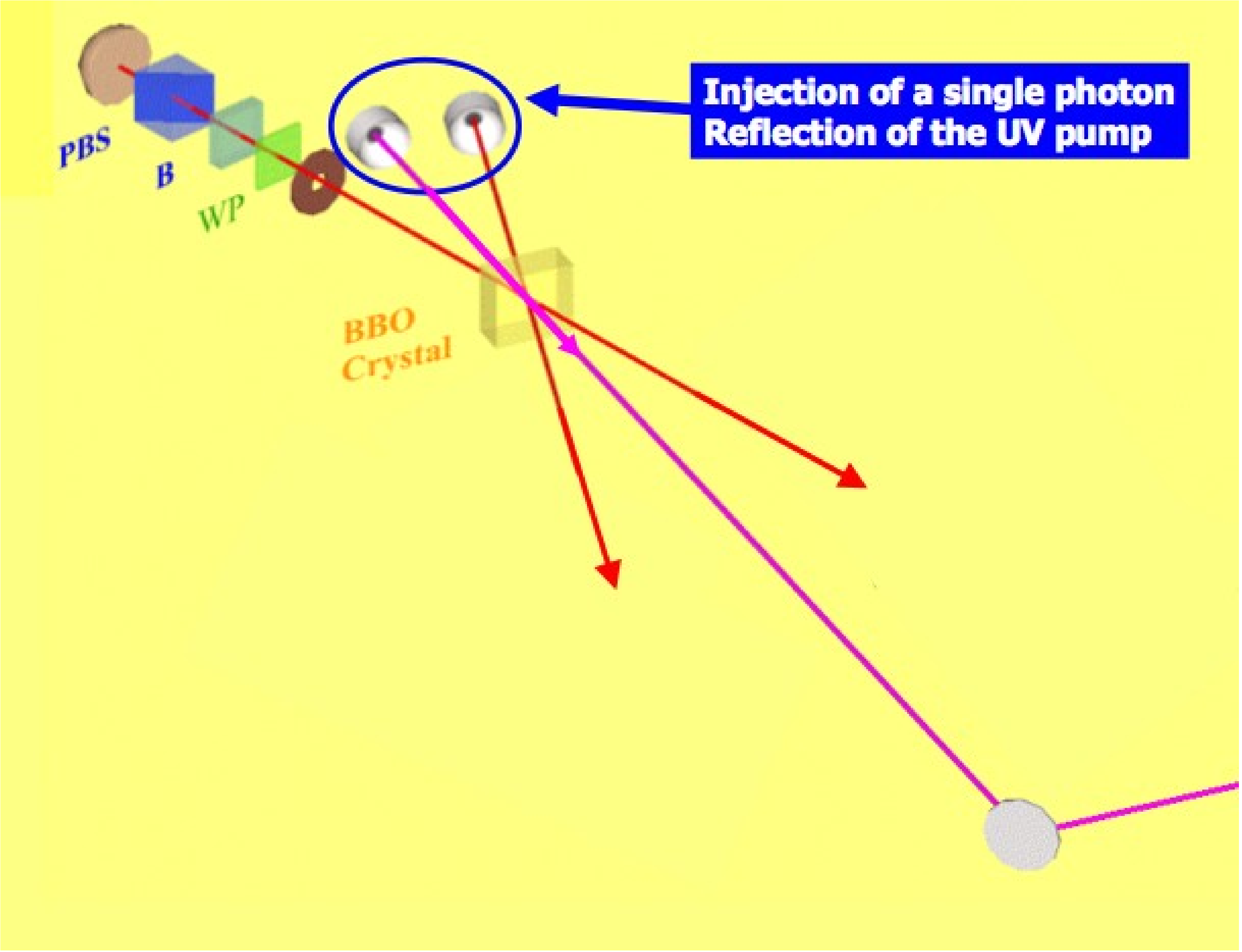
A: Cloning mode: 2 photons in the state $\rho_A = \frac{5}{6}|\phi\rangle\langle\phi| + \frac{1}{6}|\phi^{\perp}\rangle\langle\phi^{\perp}|$

B: UNOT mode: 1 photon in the state $\rho_B = \frac{1}{3}|\phi\rangle\langle\phi| + \frac{2}{3}|\phi^{\perp}\rangle\langle\phi^{\perp}|$

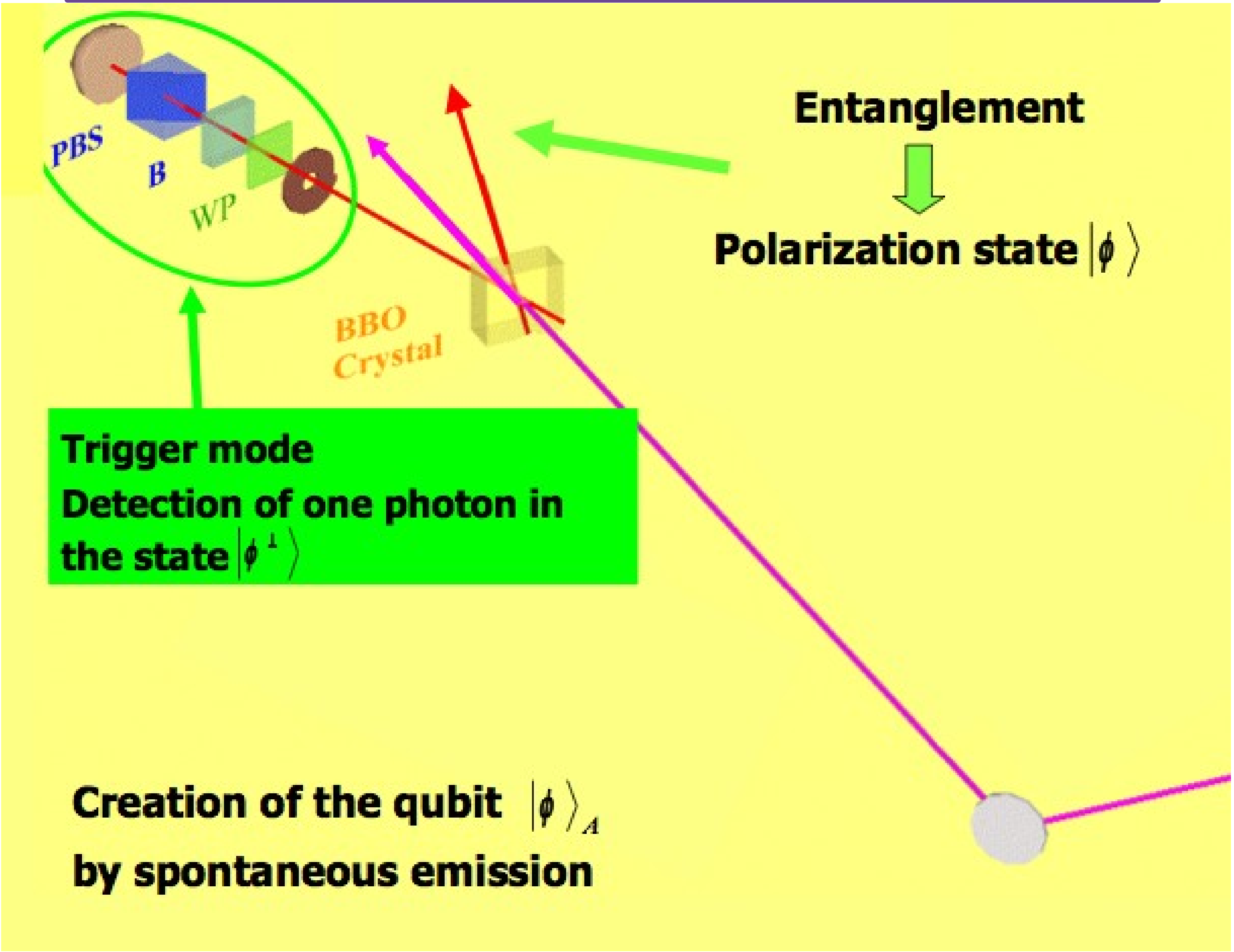
Experimental implementation



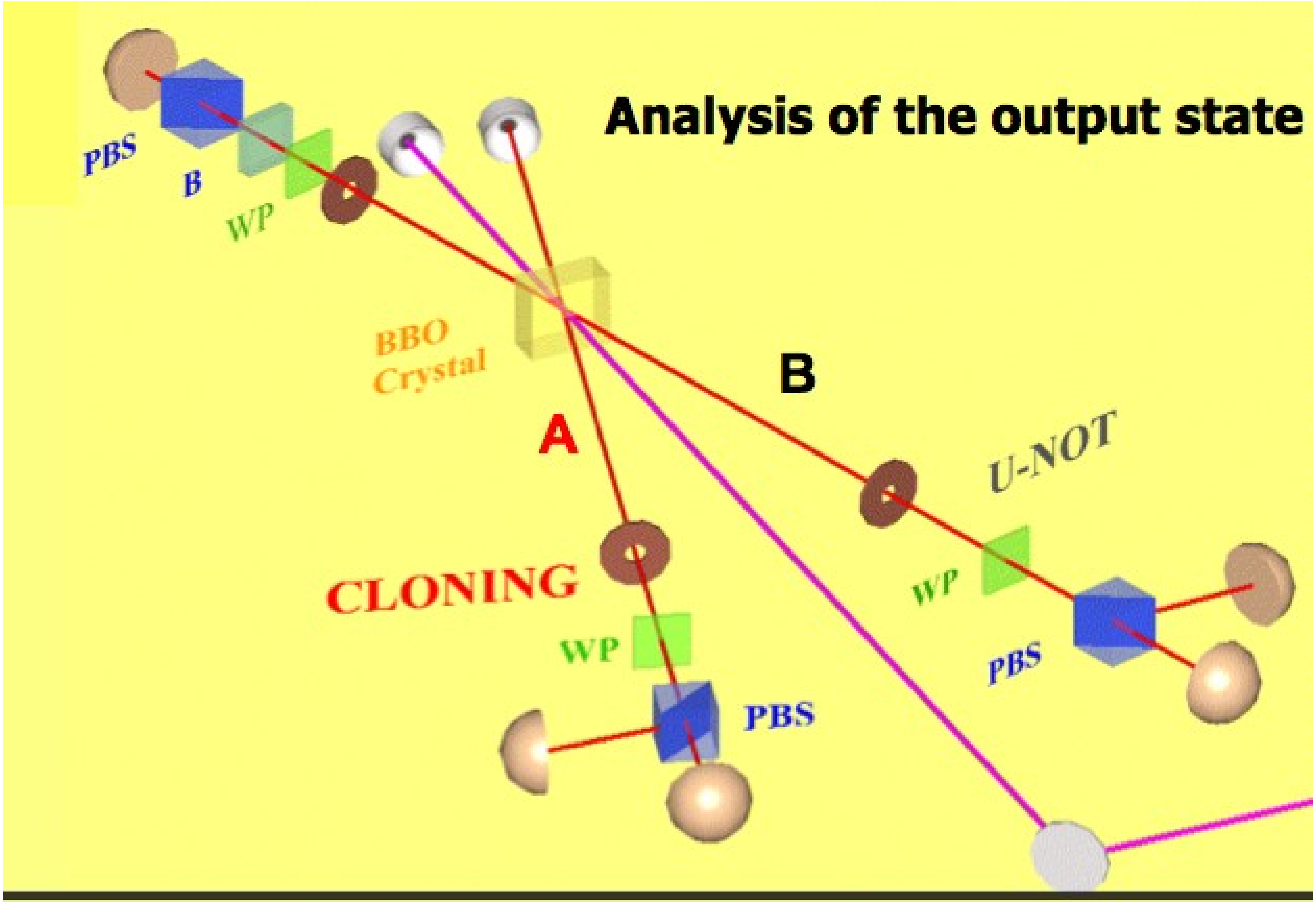
Experimental implementation



Experimental implementation



Experimental implementation

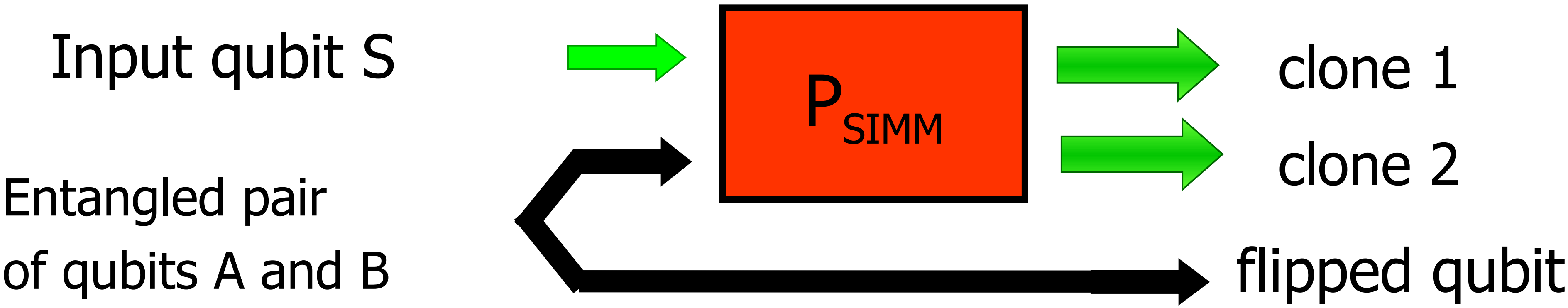


De Martini, Buzek, Sciarrino, Sias, *Nature (London)* **419**, 815 (2002)
Pelliccia, Schettini, Sciarrino, Sias, De Martini, *Physical Review A* **68,** 042306 (2003)
De Martini, Pelliccia, Sciarrino, *Physical Review Letters* **92**, 067901(2004)

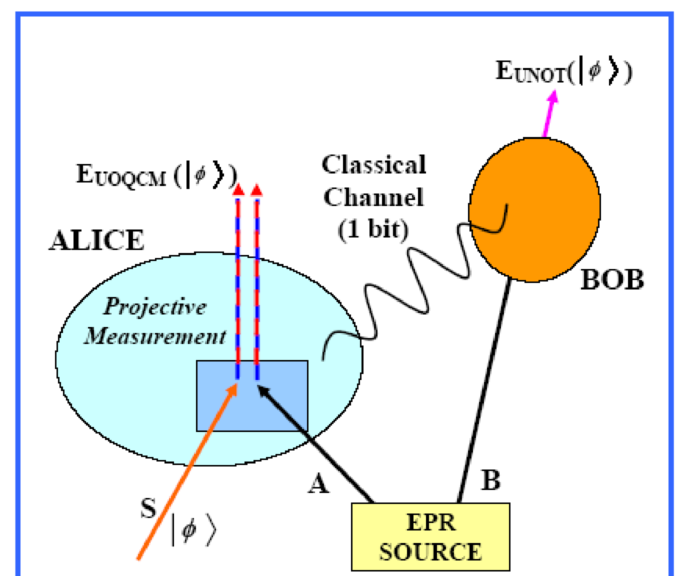
Optimal cloning by symmetrization

Projection operator of the qubits S and A into the symmetric subspace:

$$P_{SAB} = (\mathbb{I}_{SA} - |\Psi^-\rangle_{SA}\langle\Psi^-|_{SA}) \otimes \mathbb{I}_B.$$



- The process is reversible
- Contextual Realization of the Cloning and Flipping processes
- The cloning occurs with probability <1
- The Optimal Flipped qubit is teleported in a different location (no interaction between the input qubits and the "flipped" one)



Symmetrization technique

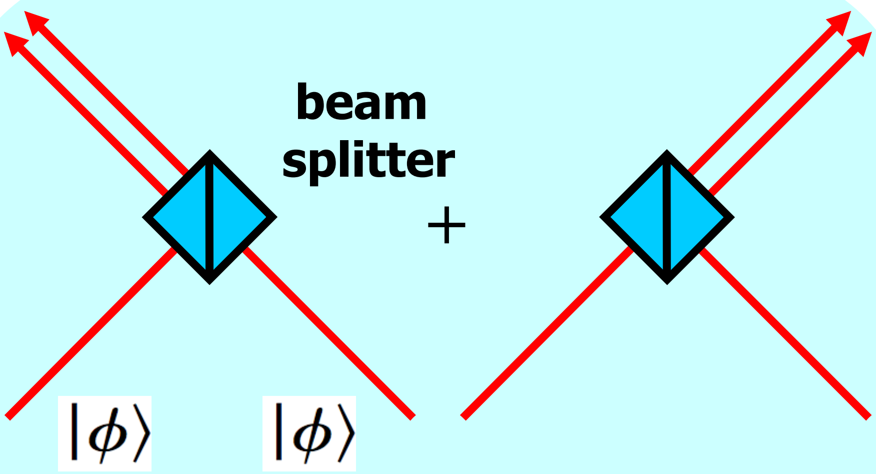
Projector over symmetric subspace P_{sym}:

Hong-Ou-Mandel Interferometer

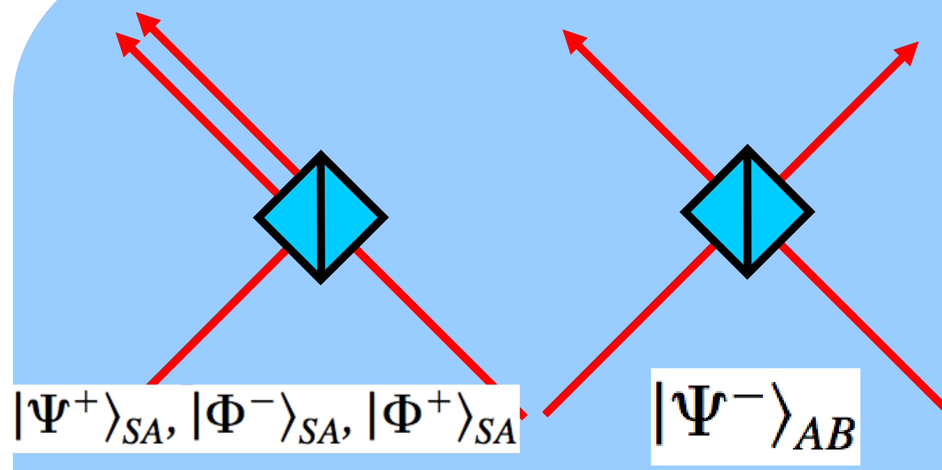
The wave packet of two single photons are superimposed on a 50: 50 beam splitter.

Indistinguishable photons (time, spectrum, wavelength and polarization)

quantum interference



Photon with same polarization emerge from the same output port. Identical photons bunch at a beam splitter, due to quantum interference



Photons in two different polarization states:

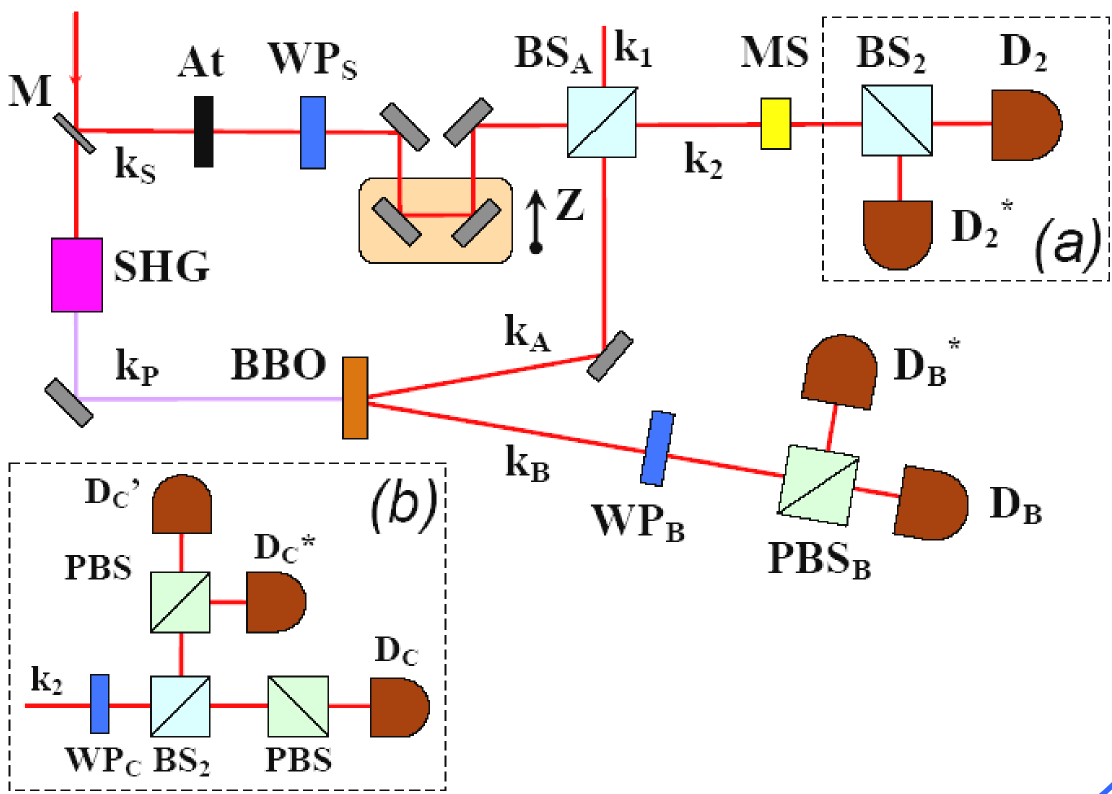
-Same mode (symmetric state)

-Different modes (antisymmetric state)

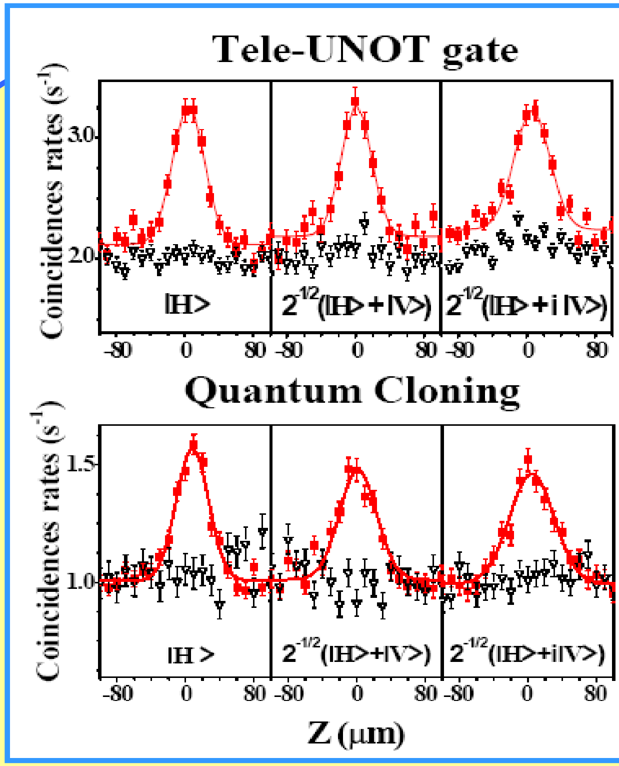
Implementation by linear optics

Polarization encoded qubit: $\alpha|0\rangle+\beta|1\rangle \longleftrightarrow \alpha|H\rangle+\beta|V\rangle$

Experimental setup



Experimental results



Fclon=0.815, $F_{flipping}$=0.630

to be compared with

F_{clon}=0.833, $F_{flipping}$=0.666

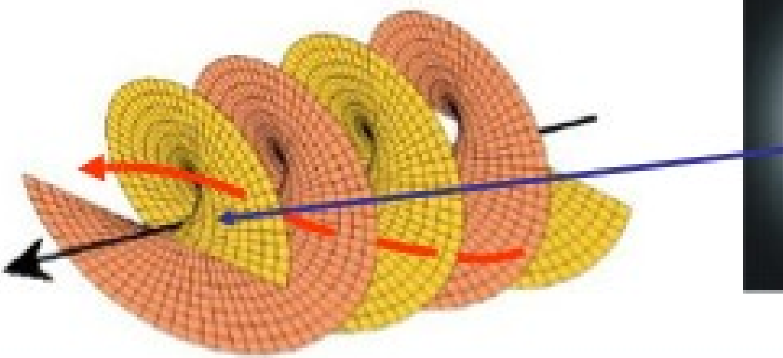
Ricci, Sciarrino, Sias, De Martini, *Physical Review Letters* **92**, 047901 (2004)

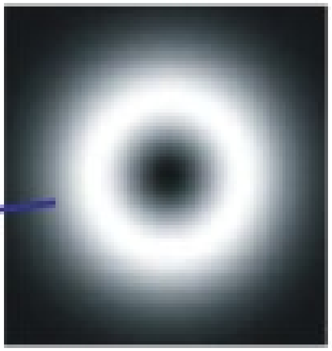
Sciarrino, Sias, Ricci, De Martini, *Physical Review A* ***70***, 052305 (2004)

Optimal cloning of "quantum images"

Degree of freedom of light associated with rotationally structured transverse spatial modes

Laguerre–Gauss modes ➡ helicoidal wavefront





nature photonics

LETTERS

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Optimal quantum cloning of orbital angular momentum photon qubits through Hong–Ou–Mandel coalescence

Eleonora Nagali[1], Linda Sansoni[1], Fabio Sciarrino[1,2]*, Francesco De Martini[1,3], Lorenzo Marrucci[4,5]*, Bruno Piccirillo[4,6], Ebrahim Karimi[4] and Enrico Santamato[4,6]

PRL 105, 073602 (2010) PHYSICAL REVIEW LETTERS week ending 13 AUGUST 2010

Experimental Optimal Cloning of Four-Dimensional Quantum States of Photons

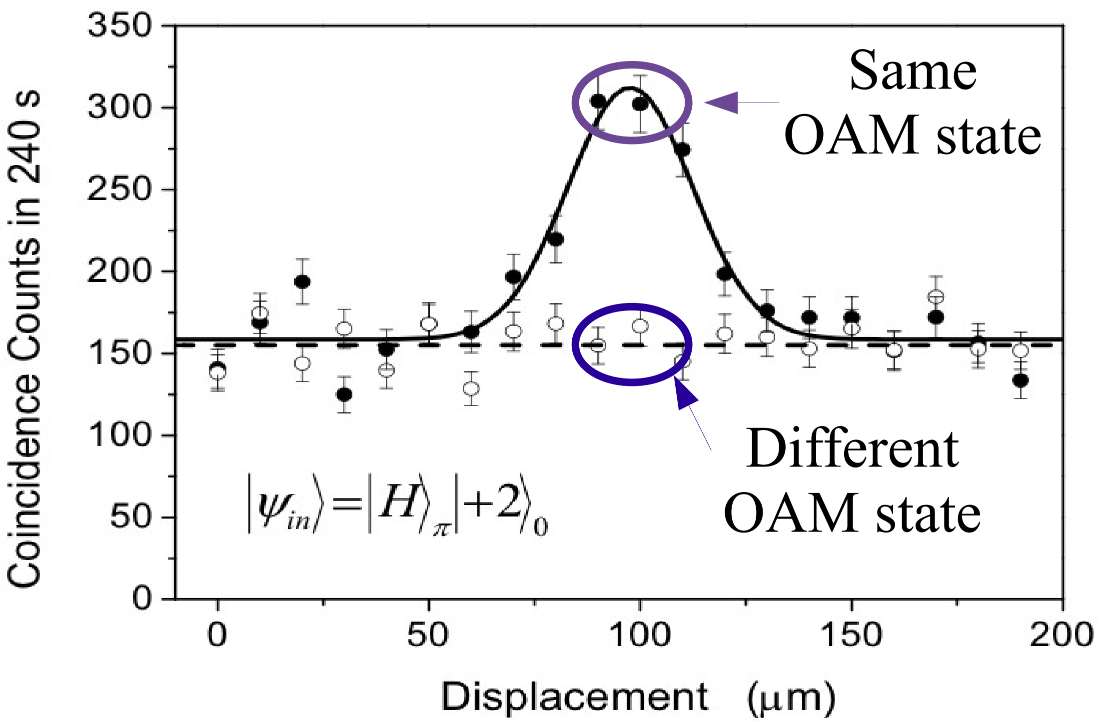
E. Nagali,[1] D. Giovannini,[1] L. Marrucci,[2,3] S. Slussarenko,[2] E. Santamato,[2] and F. Sciarrino[1,4,*]

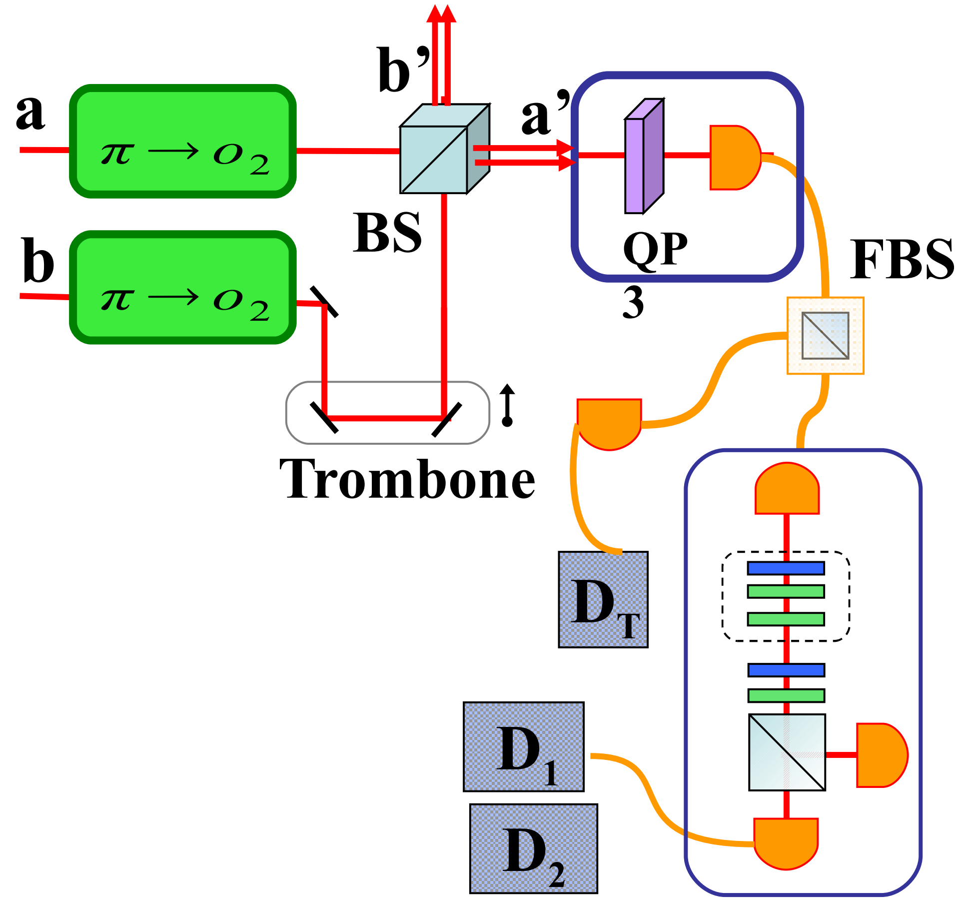
Hong-Ou-Mandel effect in the OAM

The symmetry that marks the bosonic wavefunction is revealed by a two-photon coalescence on the same spatial mode.

➡ HOM effect

$$\Psi(r,s,m) = R(r)\,\chi(s)\,\xi(m)$$



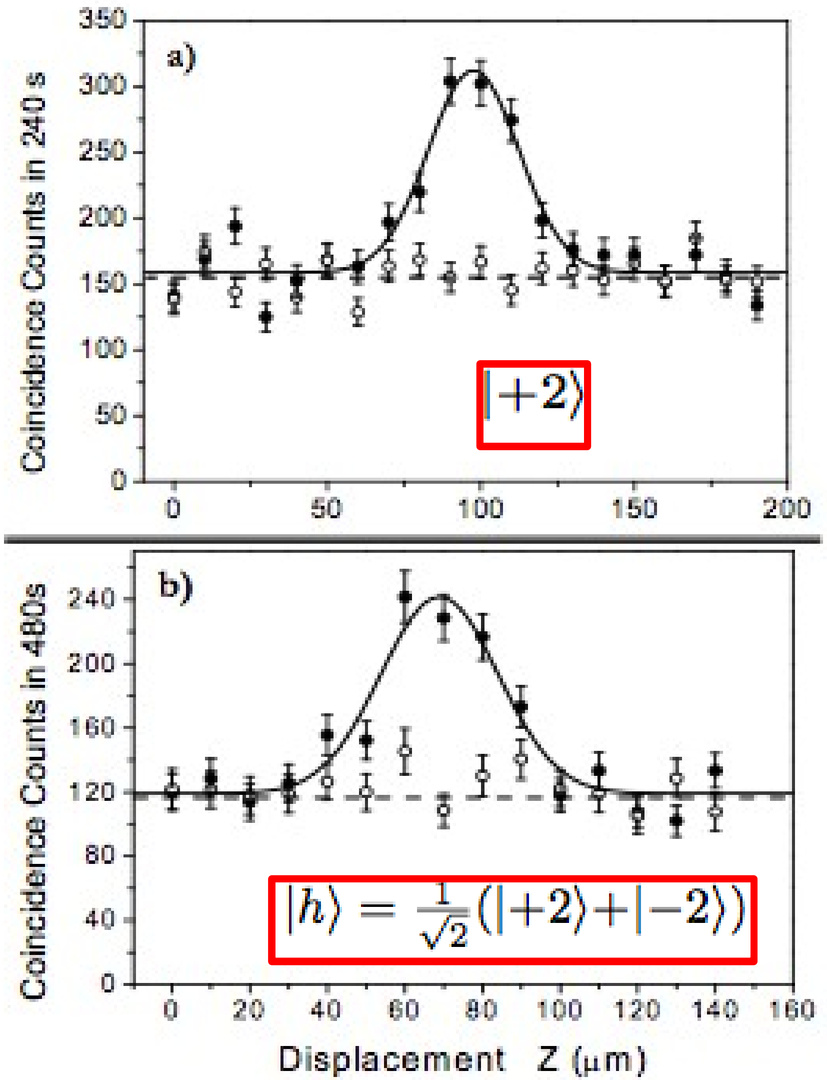


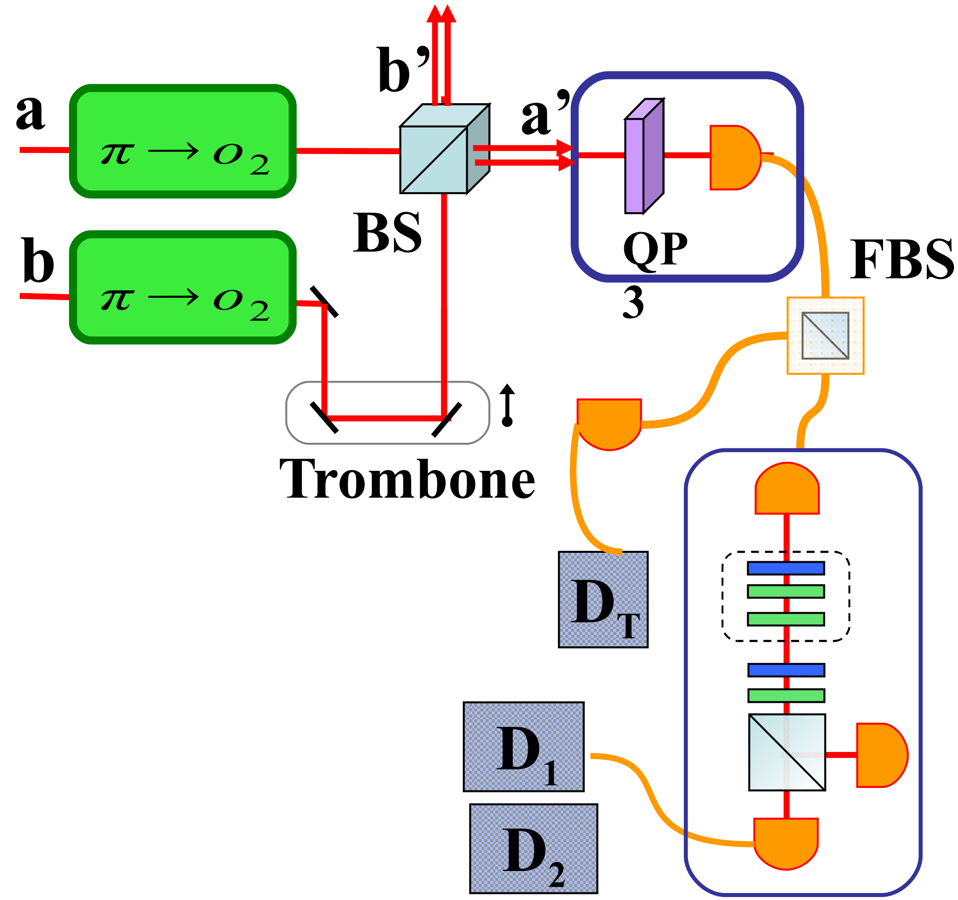
Coincidence counts enhancement R:

$$R_{th} = 2$$

$$R_{exp} = (1.98 \pm 0.05)$$

Hong-Ou-Mandel effect in the OAM





Coincidence counts enhancement R:

$$R_{th} = 2$$

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E. Nagali, *et al.*, *Nature Photonics* **3**, 720-723 (2010)

Optimal cloning of OAM qubit (1/2)

Goal: clone a generic qubit lying in the subspace OAM o_2: $|\varphi\rangle_{o_2} = \alpha|+2\rangle + \beta|-2\rangle$

No-Cloning Theorem

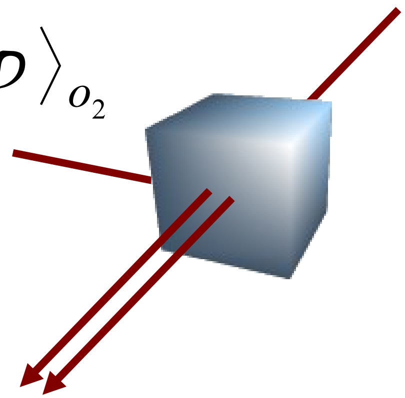
It is not possible to generate perfect copies of an unknown qubit

Optimal quantum cloning: output clones close to the input qubit as much as possible according to Quantum Mechanics working for any input state

Optimal cloner based on the symmetrization technique:
projection over the symmetric subspace
of the input cloned and a fully mixed state achieved via a
two-photon HOM coalescence effect

$\rho_A = \frac{I}{2}$

$|\varphi\rangle_{o_2}$

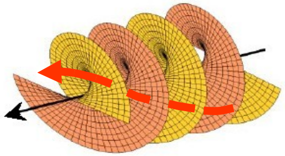


Optimal fidelity

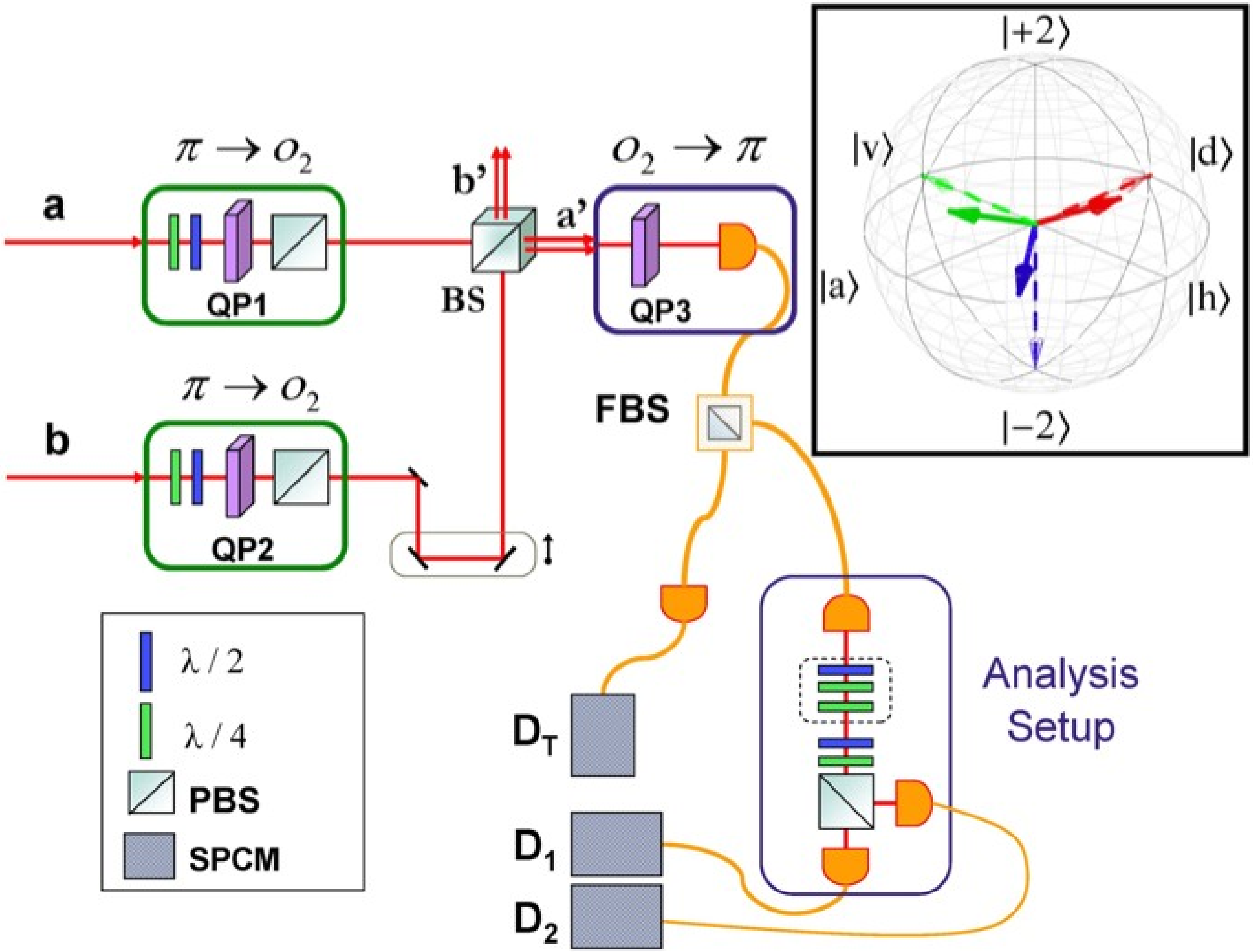
$$F_{\exp} = (0.803 \pm 0.008)$$

$$F_{th} = 0.83$$

$$\rho^{a'}_{o_2} = \frac{5}{6}|\varphi\rangle_{o_2}\langle\varphi| + \frac{1}{6}|\varphi^{\perp}\rangle_{o_2}\langle\varphi^{\perp}|$$



Optimal cloning of qubit OAM



Optimal cloning of qubit OAM

Optimal cloning

State	Fidelity
$\vert h\rangle_{o_2}$	(0.806 ± 0.023)
$\vert v\rangle_{o_2}$	(0.835 ± 0.015)
$\vert -2\rangle_o$	(0.792 ± 0.024)
$\vert +2\rangle_o$	(0.769 ± 0.022)
$\vert a\rangle_{o_2}$	(0.773 ± 0.020)
$\vert d\rangle_{o_2}$	(0.844 ± 0.019)

Stokes Parameters

The cloning process corresponds to a shrinking of the vectors lying on the Bloch's sphere

Initial State: S =1

Cloned State

$$S_{th} = 2F - 1 = 2/3$$

Experimental results

$$S_{exp} = \sqrt{S_1^2 + S_2^2 + S_3^2}$$

$$S_{exp} = (0.68 \pm 0.02)$$

E. Nagali, *et al., Nature Photonics* **3**, 720 (2009)

Optimal cloning for any qudit states

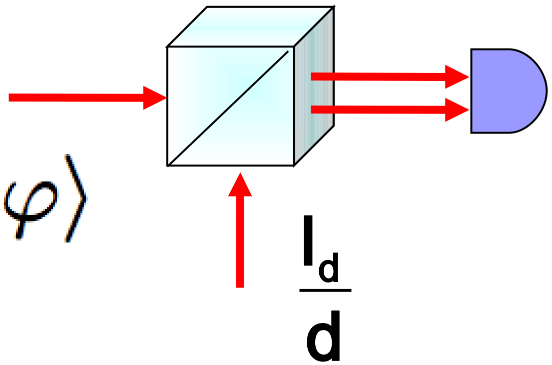
The symmetrization technique that implements the quantum cloning is **optimal** not only for qubit states, but also for **arbitrary dimension d** of the internal spaces of the quantum systems that are cloned (qudits)

Input state to be cloned $|1\rangle \equiv |\varphi\rangle$

Ancilla qudit photon

$$\rho = \frac{I_d}{d} = \frac{1}{d}\sum_n |n\rangle\langle n|$$

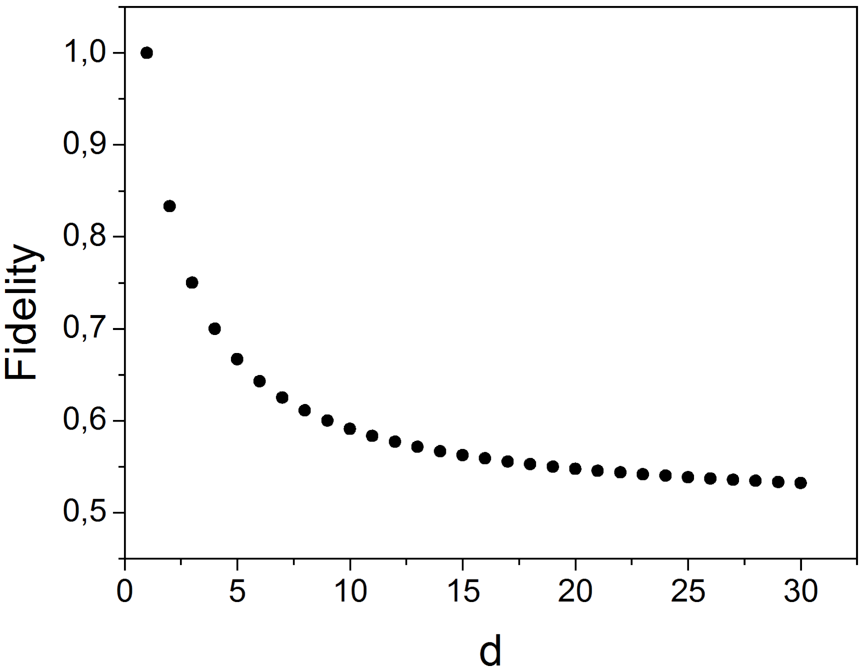
$$|1\rangle \equiv |\varphi\rangle$$



Clones $\rho_1 = \rho_2$ with fidelity

$$F = \frac{2}{d+1} \times 1 + \frac{(d-1)}{d+1} \times \frac{1}{2} =$$

$$= \frac{1}{2} + \frac{1}{d+1}$$



Optimal fidelities for any d value!

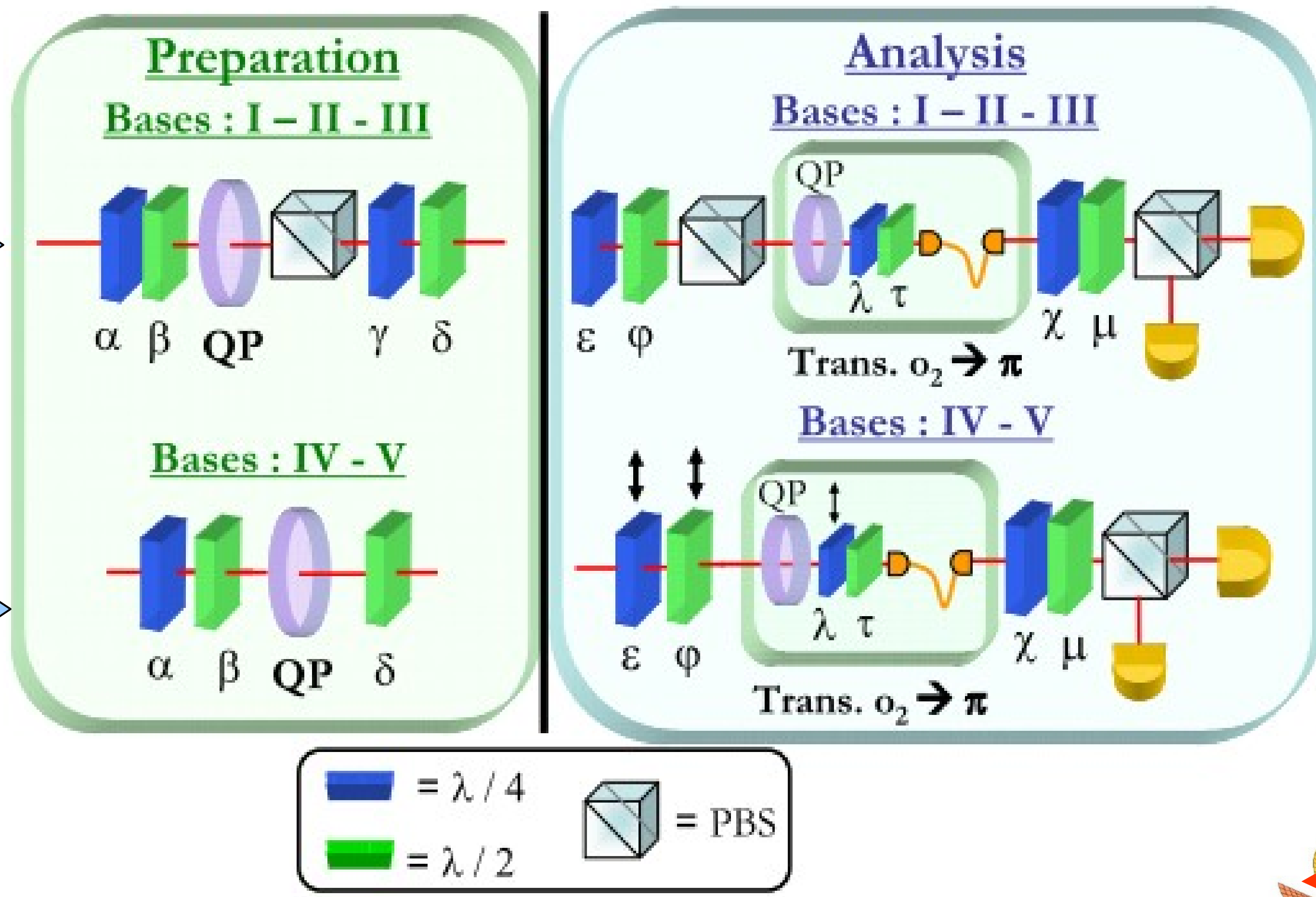
Realization of - OAM ququart

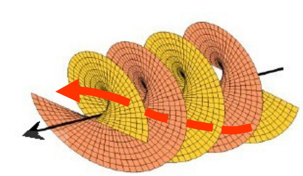
Generate ququart states (4-dimensional) encoded in a single photon by manipulating the OAM and polarization degrees of freedom

Logic ququart basis $\{|1\rangle, |2\rangle, |3\rangle, |4\rangle\} \rightarrow \{|H,+2\rangle, |H,-2\rangle, |V,+2\rangle, |V,-2\rangle\},$

Separable states of polarization and OAM

Entangled states of polarization and OAM





E. Nagali, L. Sansoni, L. Marrucci, E. Santamato, and F. Sciarrino, *Phys. Rev. A* **81**, 052317 (2010).

Optimal cloning of π-OAM ququart

PRL 105, 073602 (2010) PHYSICAL REVIEW LETTERS week ending 13 AUGUST 2010

Experimental Optimal Cloning of Four-Dimensional Quantum States of Photons

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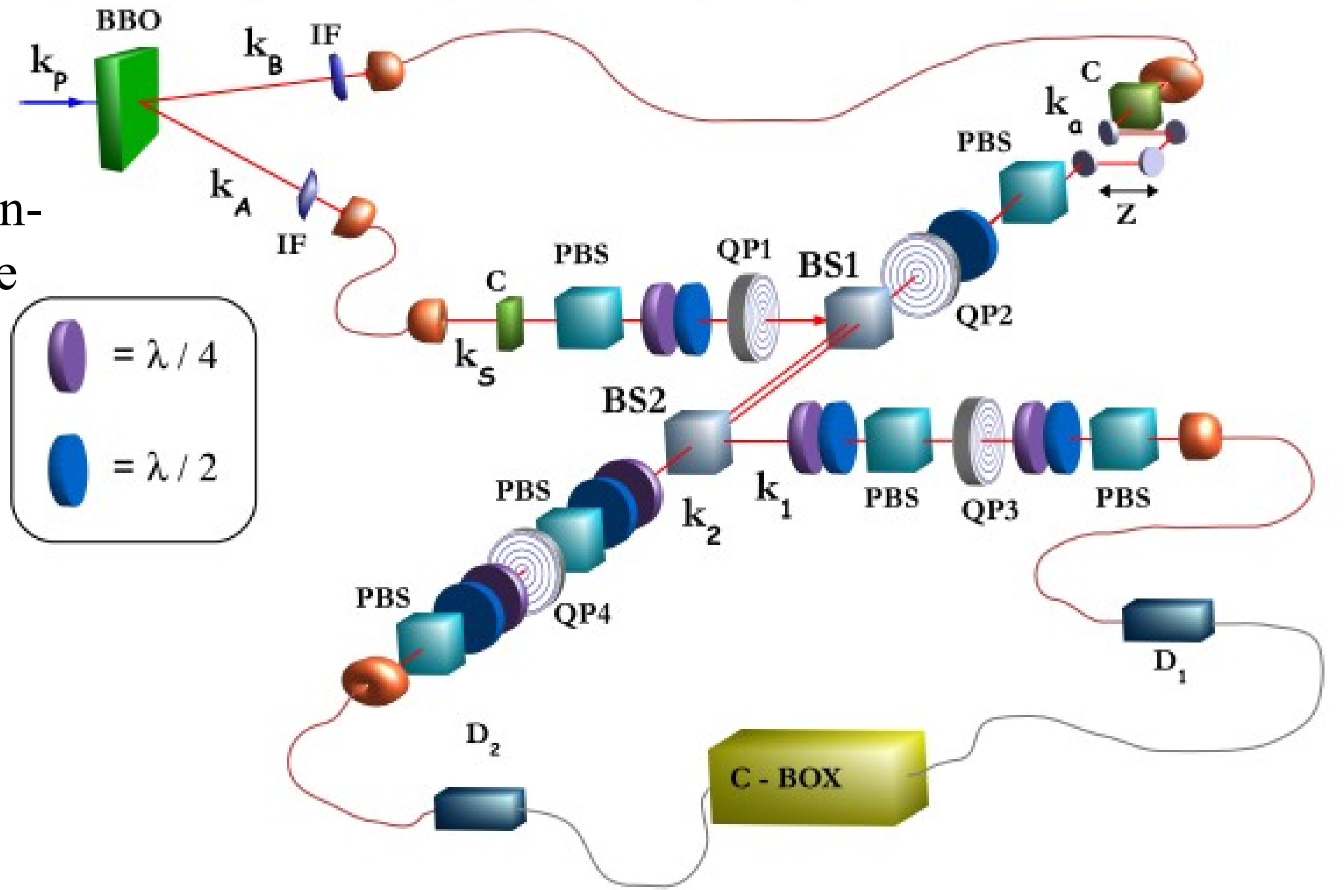
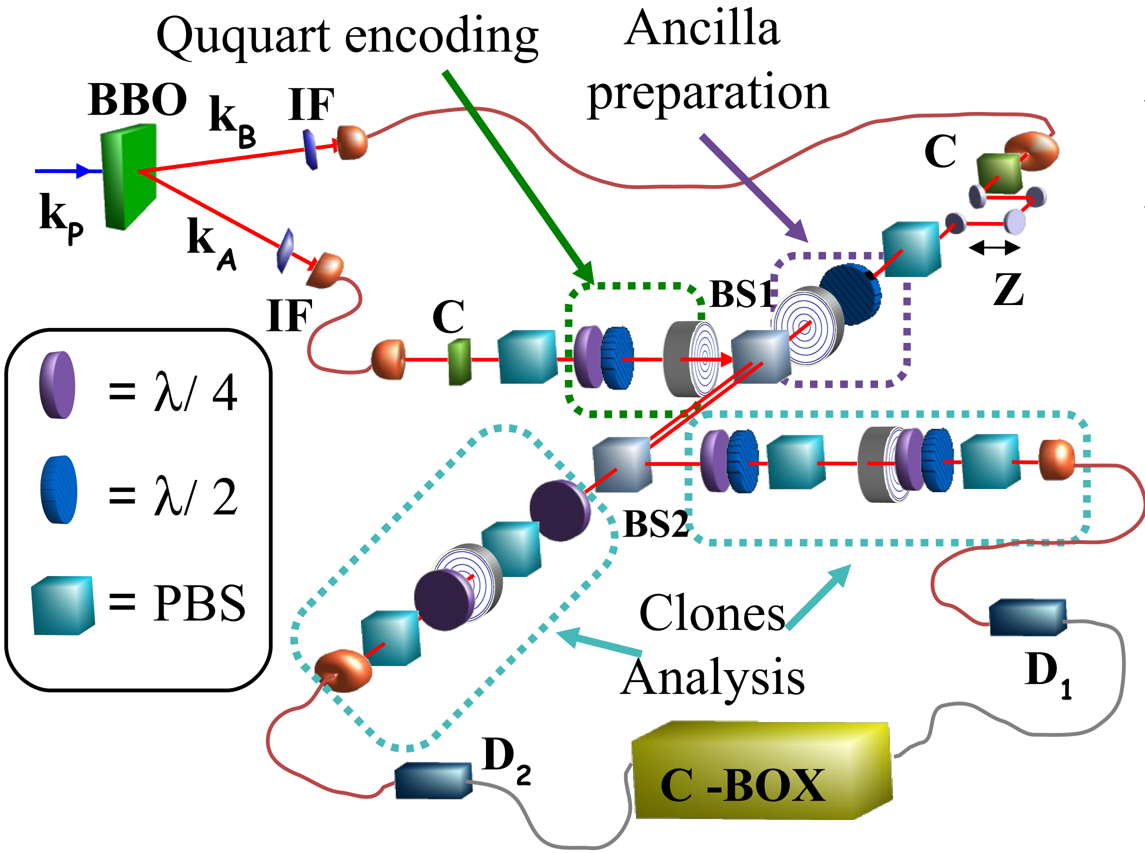
- ququart encoded in polarization-orbital angular momentum space of a single photon

- ancilla photon on mixed state of π-OAM

- optimal quantul cloning realized via Hong-Ou-Mandel coalesence over the same mode



Optimal cloning of ququart states



According to

the optimal cloning 1→2 fidelity of ququart states is expected to be $F_{d=4}=0.7$

- State to be cloned: $|\psi\rangle_s = |\varphi\rangle_\pi |\chi\rangle_{o_2}$
- Completely mixed state (*ancilla*):

$$\rho_A = \left(\frac{|H\rangle\langle H| + |V\rangle\langle V|}{2}\right)\left(\frac{|+2\rangle\langle +2| + |-2\rangle\langle -2|}{2}\right)$$

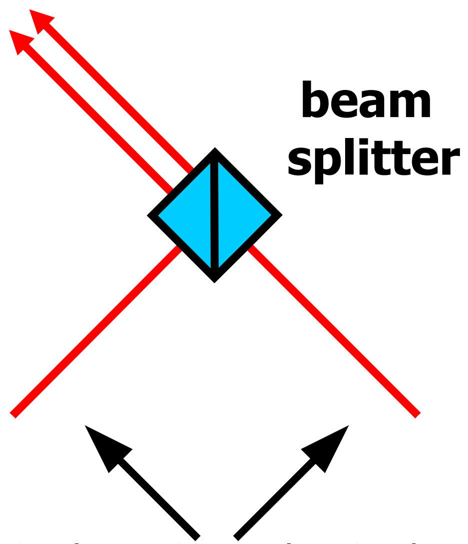
clones

$$\rho_1 = \rho_2 = \frac{1}{10}(7|1\rangle\langle 1| + |2\rangle\langle 2| + |3\rangle\langle 3| + |4\rangle\langle 4|)$$

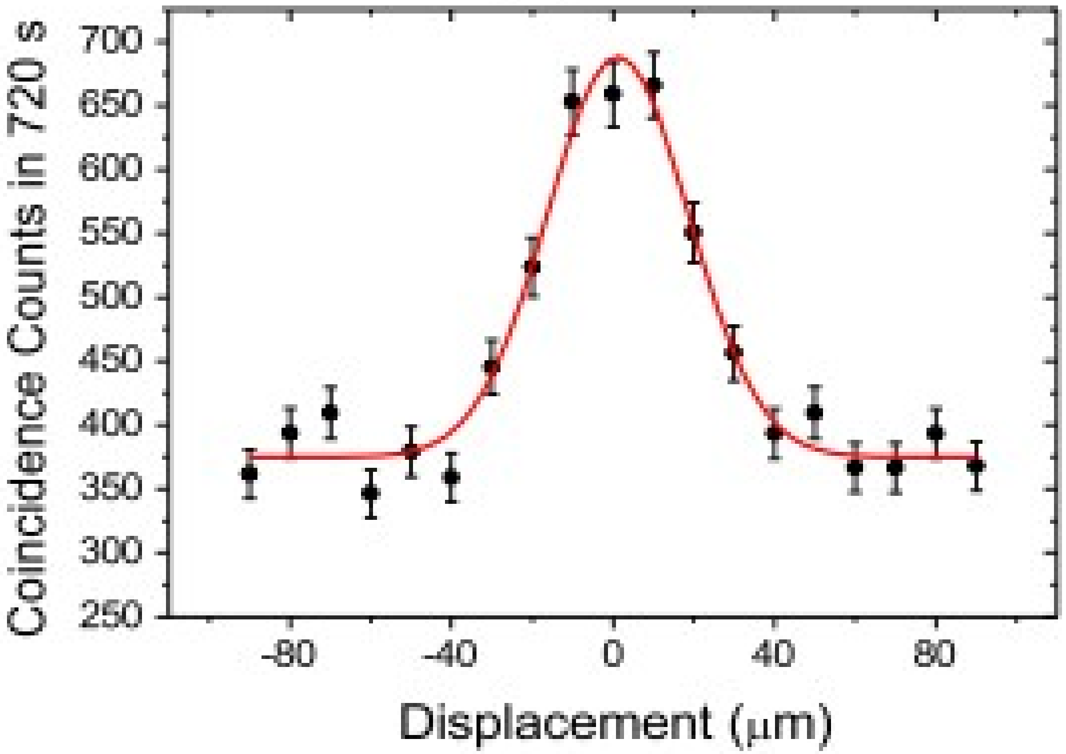
State	Fidelity
$\|1\rangle$	(0.740 ± 0.016)
$\|2\rangle$	(0.677 ± 0.016)
$\|3\rangle$	(0.707 ± 0.016)
$\|4\rangle$	(0.708 ± 0.016)

E. Nagali, et al., *Physiscal Review Letters* **105,** 073602 (2010)

Bosonic coalescence of single photon entangled state



$$\frac{1}{\sqrt{2}}\left[|L\rangle_{\pi}|-2\rangle_{o_2}+|R\rangle_{\pi}|+2\rangle_{o_2}\right]$$



This measurement underlines how the bosonic coalescence of two particles is not tied to the indistinguishability of each individual degree of freedom but rather that of the ***<u>whole quantum state</u>***

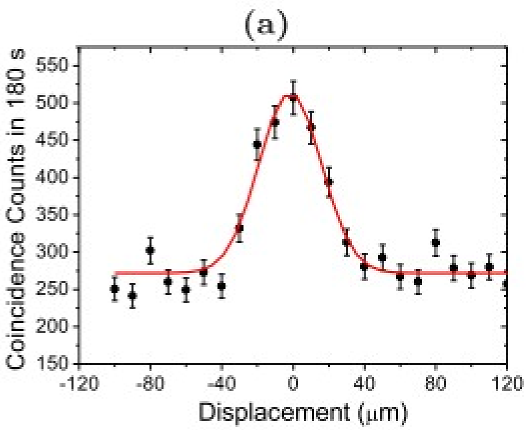
whether each of such states is entangled, as is the case in our experiment, or separable

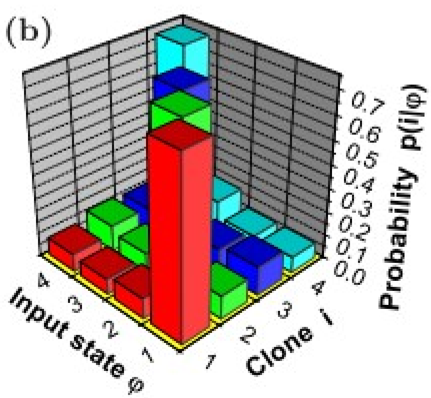
Nagali, Giovannini, Marrucci, Slussarenko, Santamato, Sciarrino, *Phys. Rev. Lett.* **105,** 073602 (2010)

Optimal cloning of π-OAM ququart

- **Expected theoretical fidelities F = 0.70**

Separable states

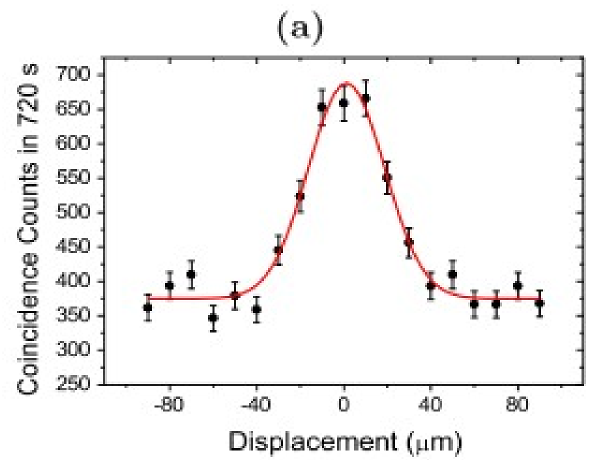


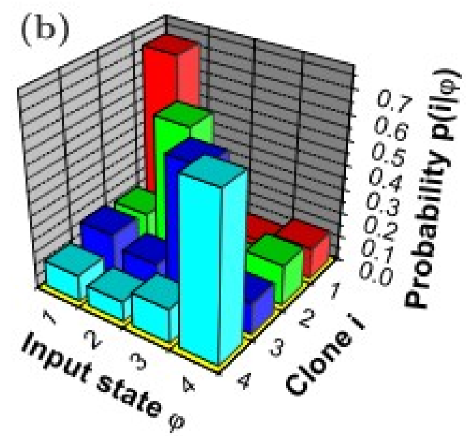


(c)

State	Fidelity
$\|1_I\rangle$	(0.740 ± 0.016)
$\|2_I\rangle$	(0.677 ± 0.012)
$\|3_I\rangle$	(0.707 ± 0.012)
$\|4_I\rangle$	(0.708 ± 0.017)

Entangled states



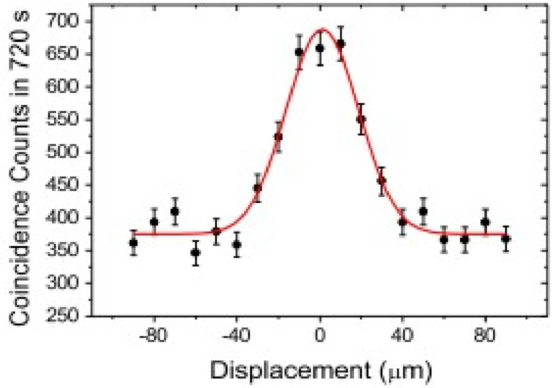


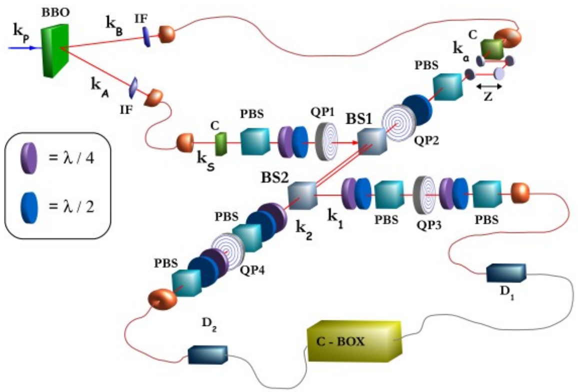
(c)

State	Fidelity
$\|1_{IV}\rangle$	(0.726 ± 0.010)
$\|2_{IV}\rangle$	(0.582 ± 0.004)
$\|3_{IV}\rangle$	(0.580 ± 0.005)
$\|4_{IV}\rangle$	(0.662 ± 0.008)

Nagali, Giovannini, Marrucci, Slussarenko, Santamato, Sciarrino, *Phys. Rev. Lett.* **105,** 073602 (2010)

Conclusions and perspectives





- Optimal quantum machines
- Optimal cloning of OAM qubit
- Bosonic coalescence of single photon entangled states
- Optimal cloning of ququart states
- NEXT STEPS: Higher dimensionality for fundamental test and protocols of quantum information

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