Optical Möbius Strips and Twisted Ribbons

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Part I will include a brief review of the canonical polarization singularities of three dimensional (3D) elliptically polarized optical fields: C lines and C points, L lines and L points. A useful summary of these singularities, that were introduced by J. F. Nye (Bristol), is given in his book: *Natural Focusing and Fine Structure of Light* (IOP Publishing, Bristol, 1999). A new, *nonconventional* class of singularities will be described that play an essential role in some, but not all, sign inversions of Nye's classical singularities. The 3D structure of the twisted ribbons and cones that surround *ordinary* (i.e. non-singular) points in a 3D field of polarization ellipses will be described, and the multitude of topological and geometrical indices that characterize these structures will be discussed.

Part II will continue with a discussion of the 3D structure of the Möbius strips and cones that surround C points, the twisted ribbons, cones, and rings, that surround L points, and the topological and geometrical indices that characterize these structures. Simple, easily applied methods will be described for generating Möbius strips with arbitrarily large odd numbers of half-twists, and twisted ribbons with arbitrarily large even numbers of half-twists. The ascending/descending multi-step quantized staircases that describe the topological and geometrical indices that characterize these multi-twist structures, the sign inversions that occur as a staircase is traversed, and the role of the nonconventional singularities in these sign inversions, will be discussed. A brief discussion will be given of experimental methods that could permit measurements to be made of the numerous unusual structures described here theoretically.

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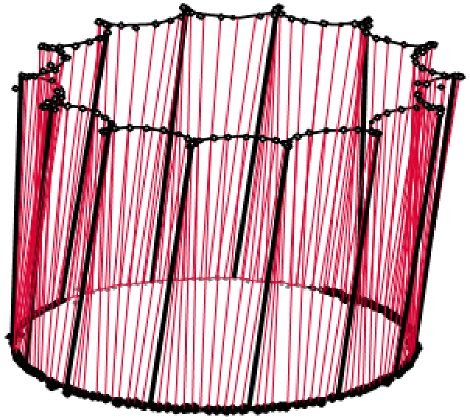
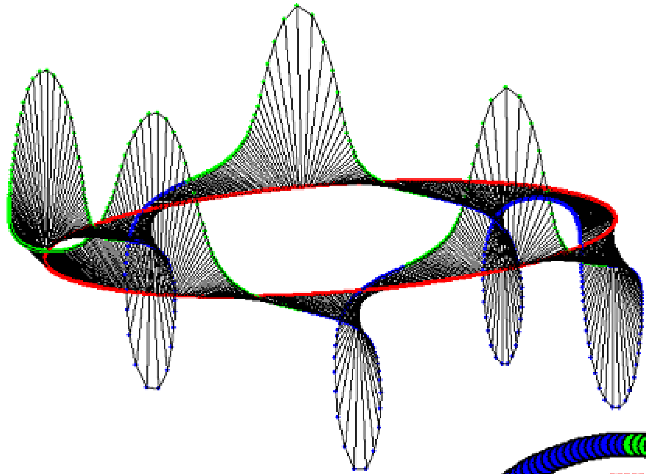
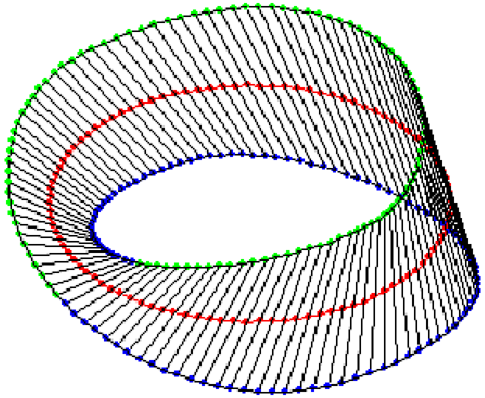
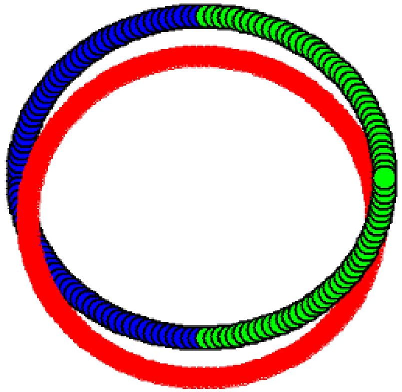
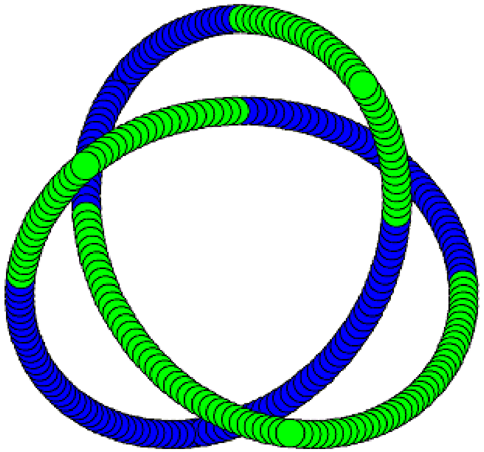


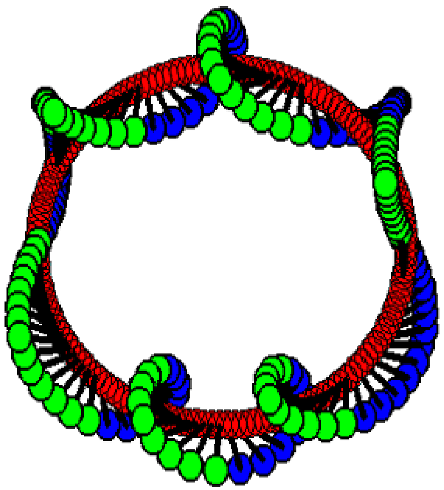


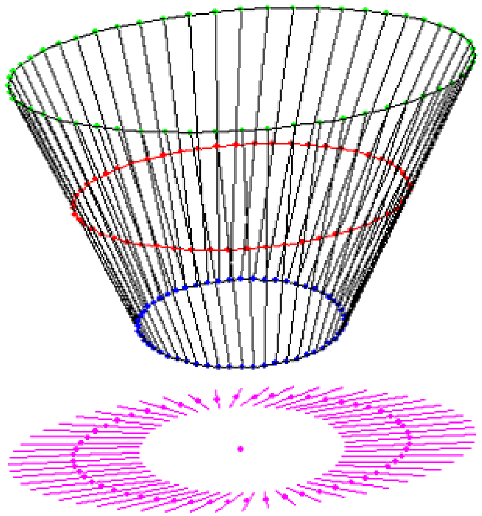


OPTICAL MÖBIUS STRIPS
AND
TWISTED RIBBONS









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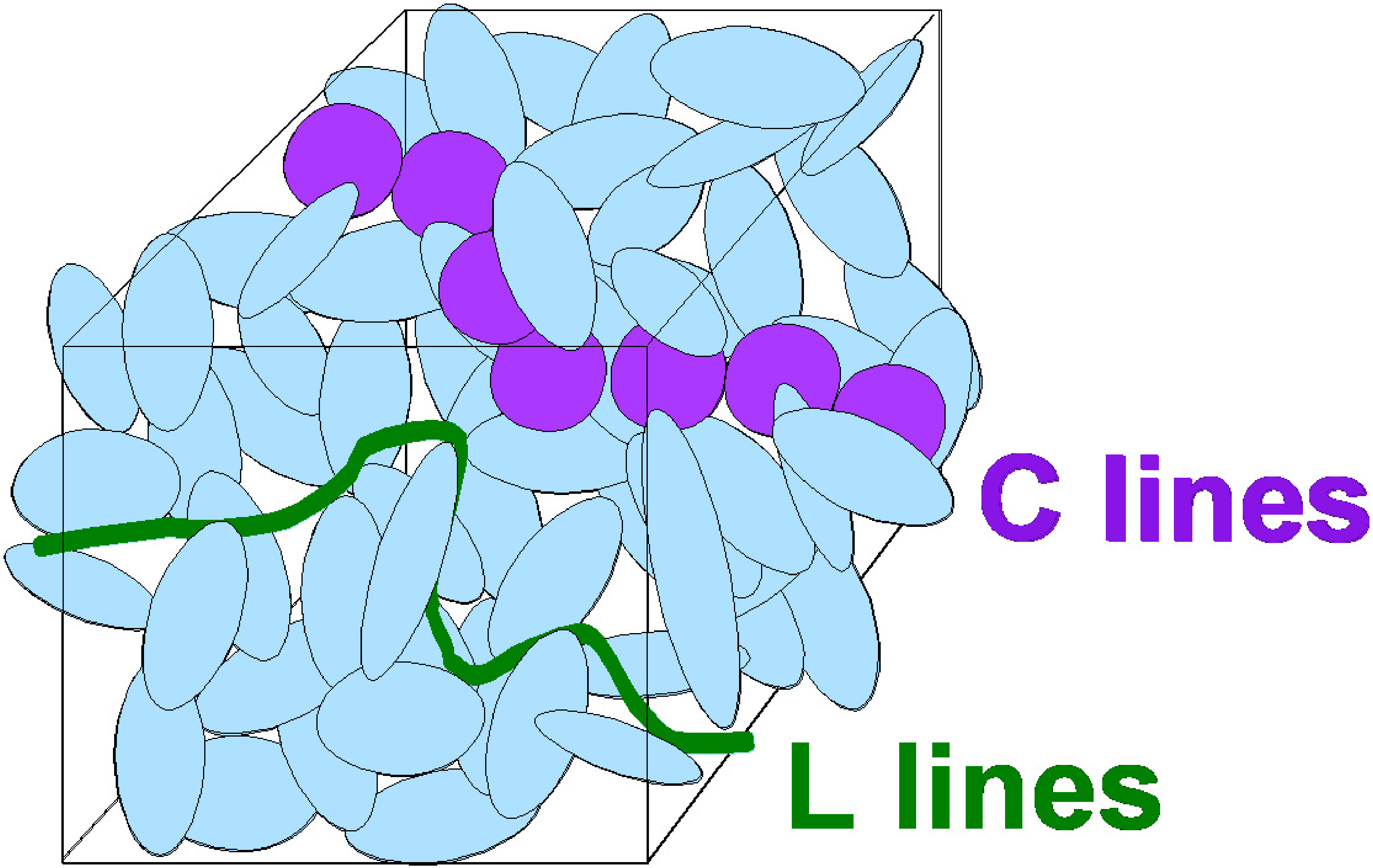
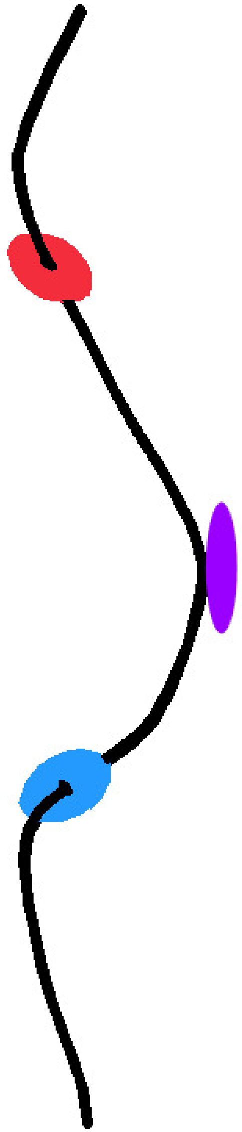
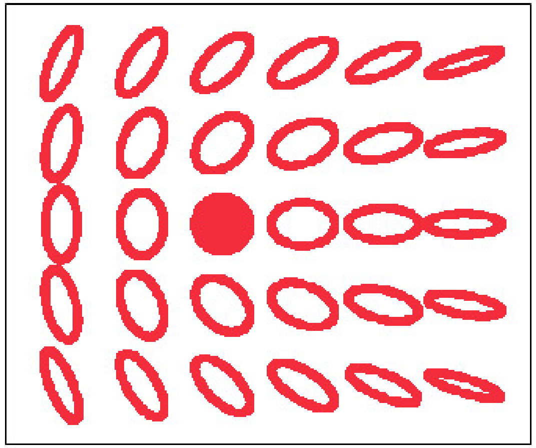
3D Ellipse Field (John Nye)

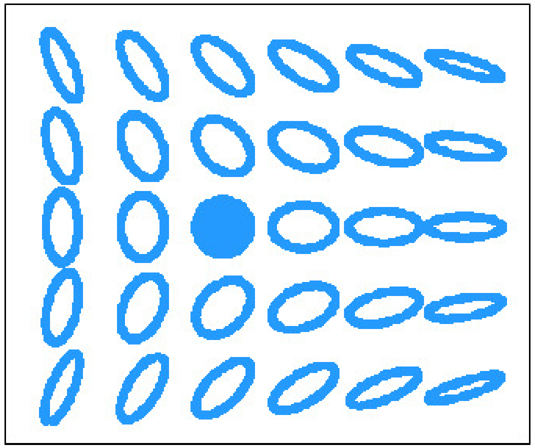


C lines & C points

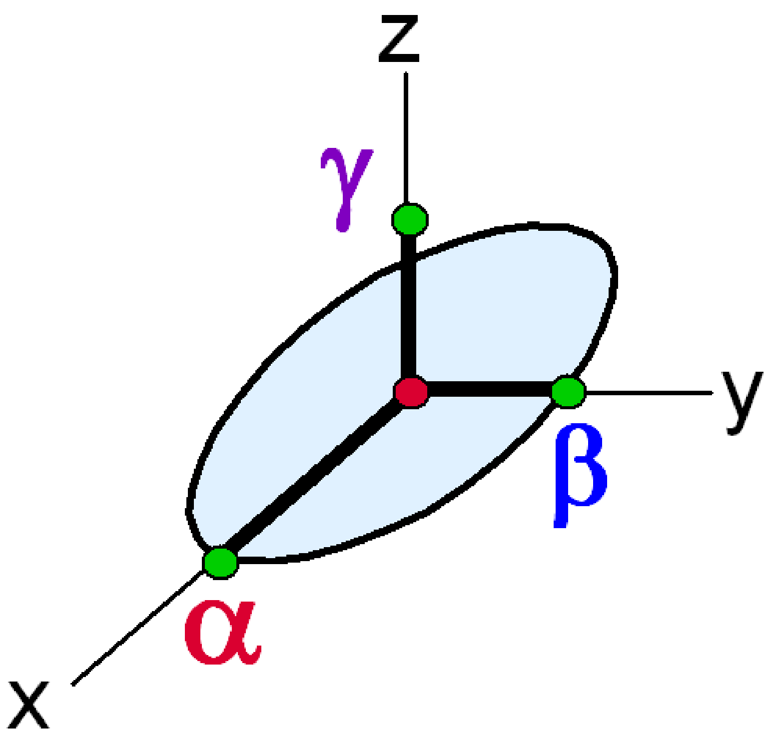




$I_C = +1/2$



$I_C = -1/2$



Berry's Eqns.

$$\alpha = Re(\mathbf{E}^* \sqrt{\mathbf{E} \bullet \mathbf{E}}\ )$$

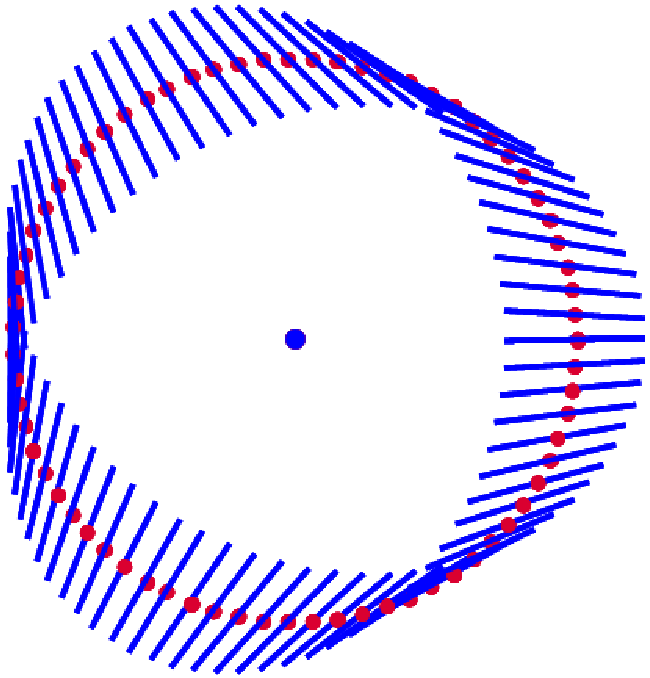
$$\beta = Im(\mathbf{E}^* \sqrt{\mathbf{E} \bullet \mathbf{E}}\ )$$

$$\gamma = Im(\mathbf{E}^* \times \mathbf{E})$$

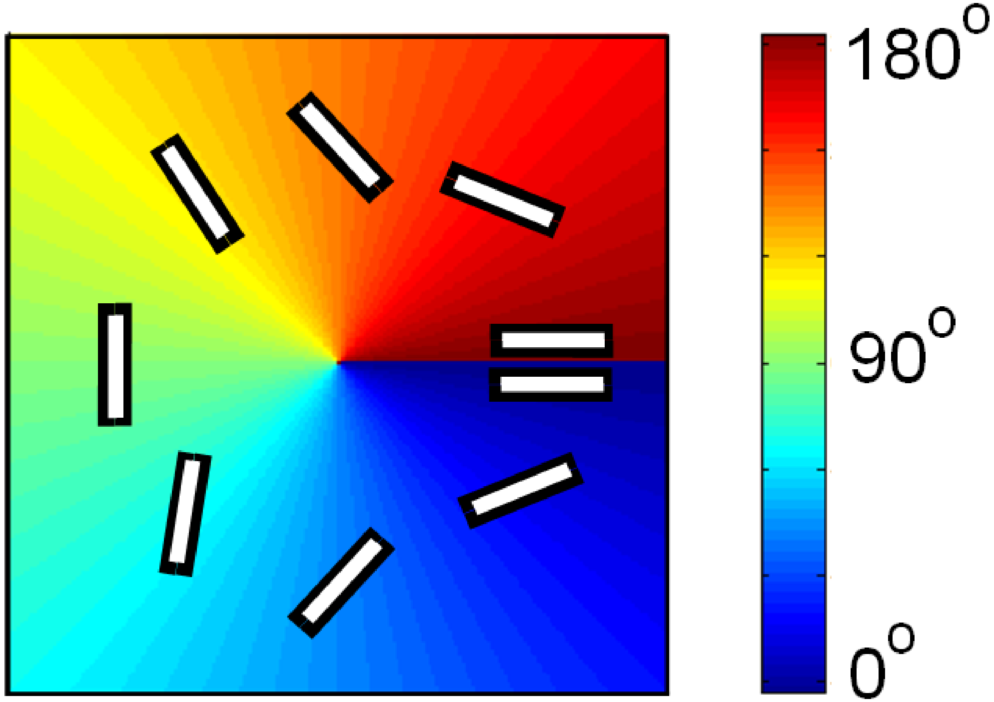
At a C point E•E = 0, so $\alpha = \beta = 0$

At an L point $\beta = \gamma = 0$

2D: Singularity Representations

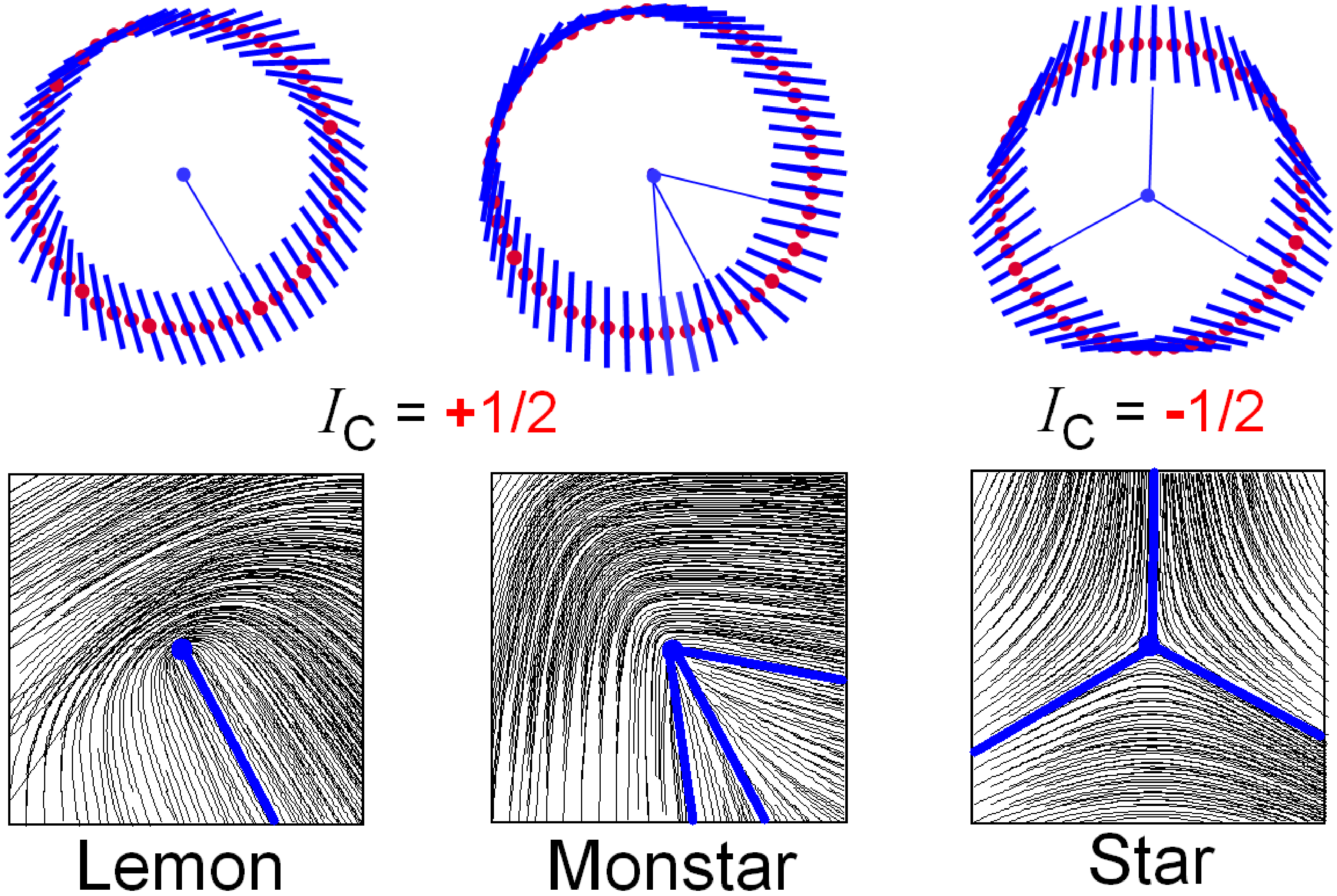


Surrounding Circle

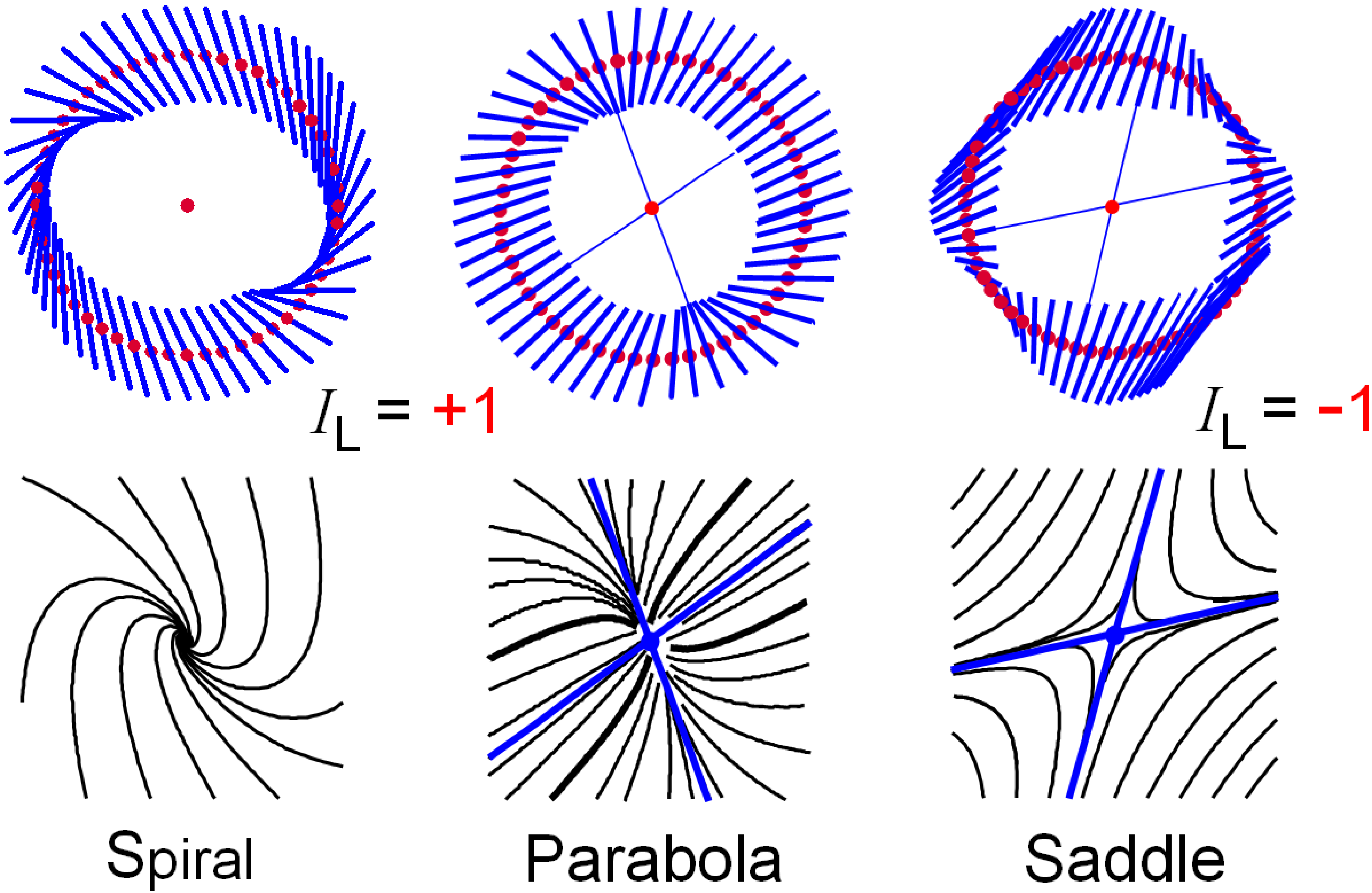


α, β, γ "vortex"

2D: C points *singular* axes α and β

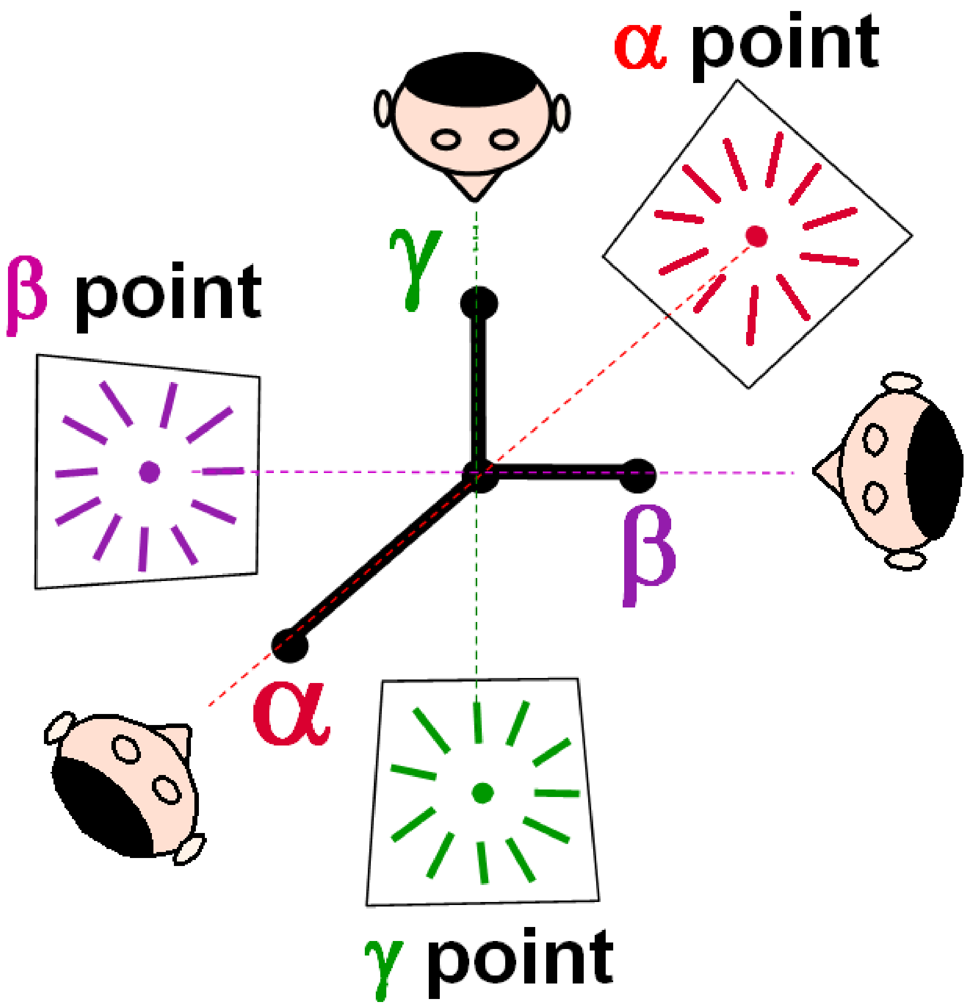


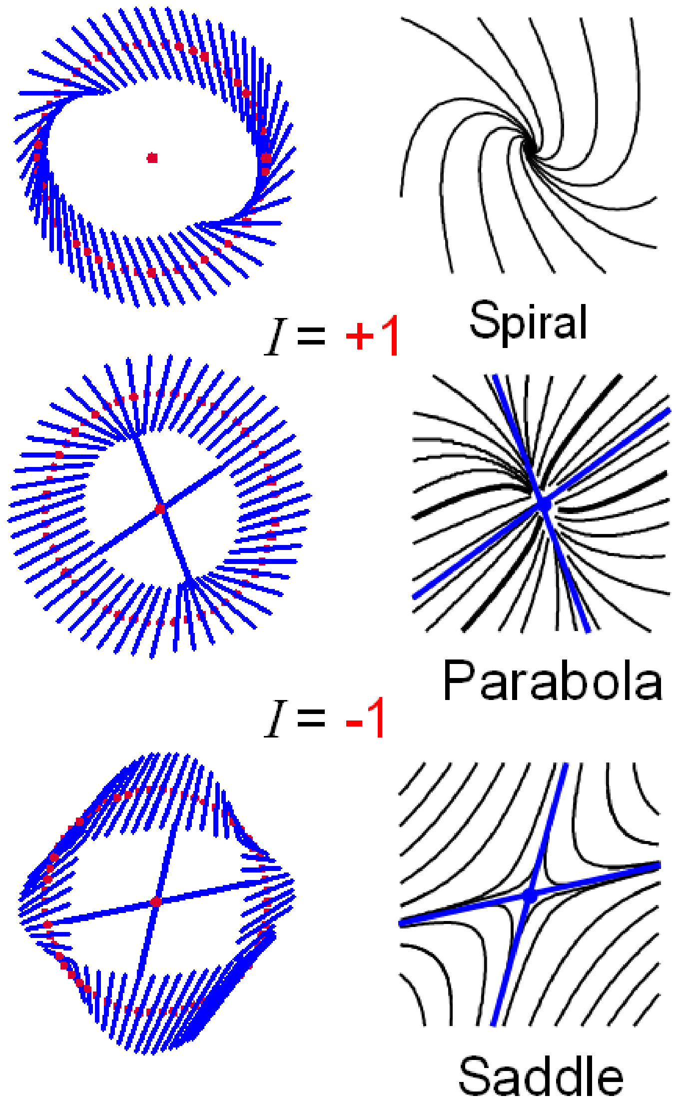
2D: L points *singular* axes β and γ



Non conventional (but essential) *2D singularities*

α, β, and γ points





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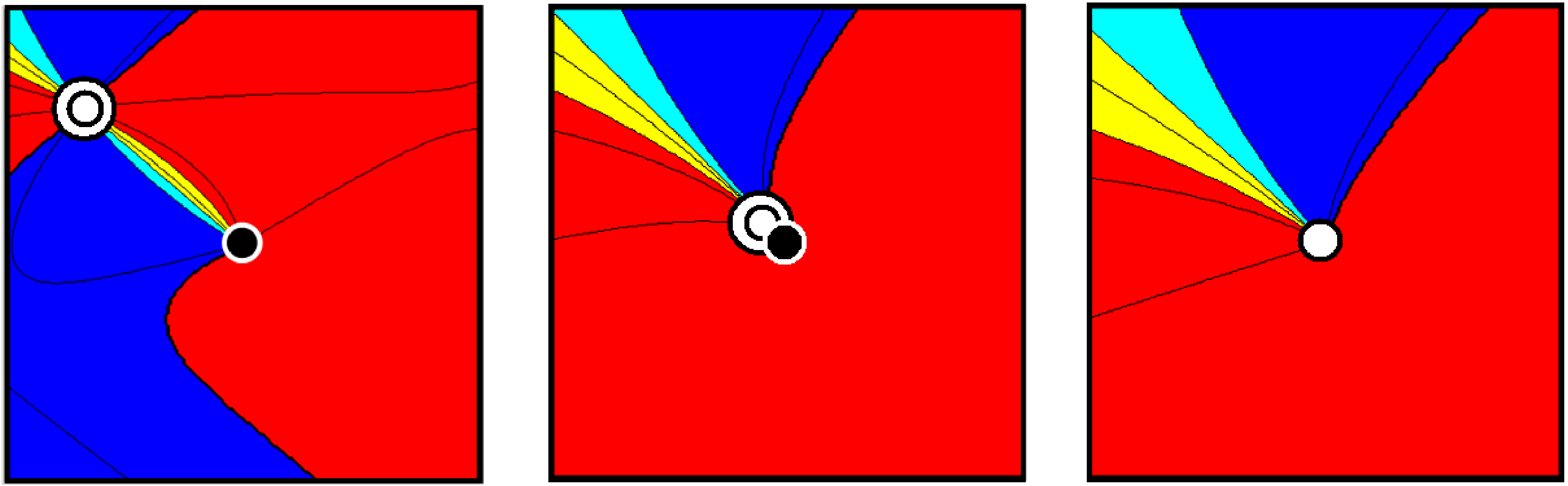
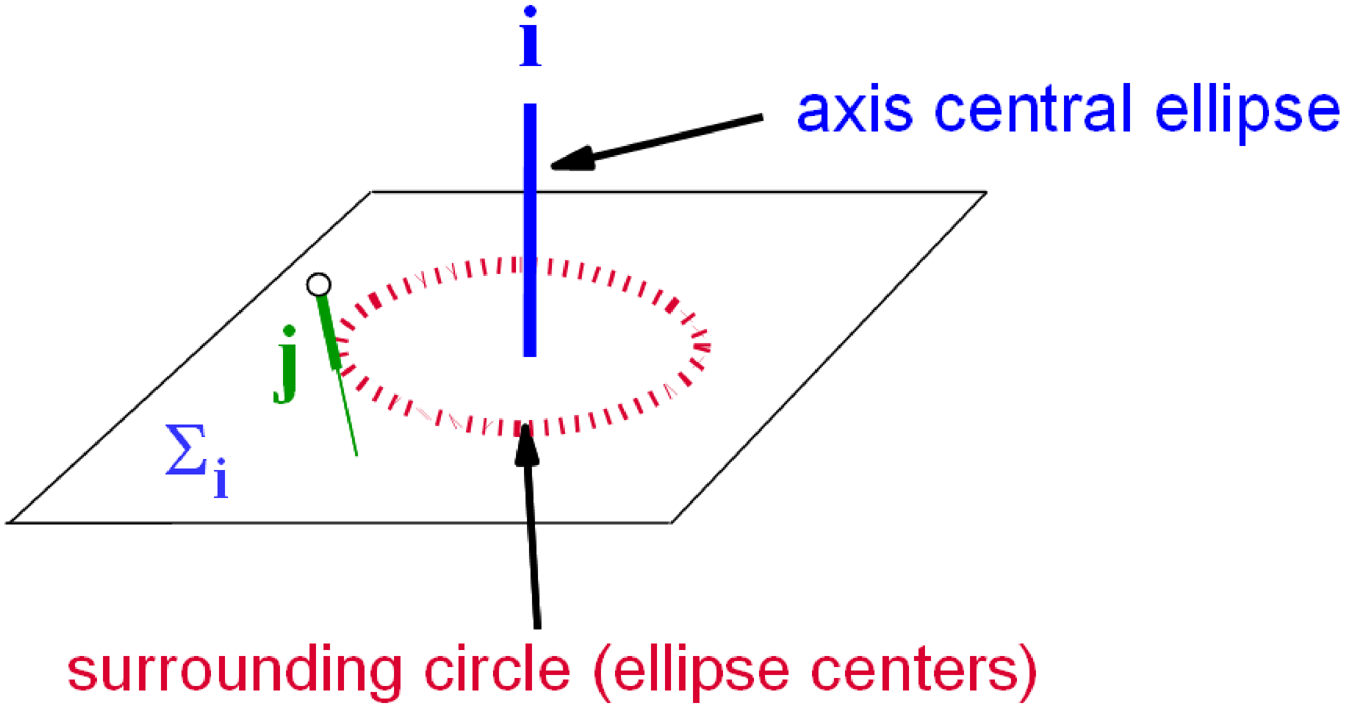
Sign inversion under rotation through the normal to the C circle: Inelastic collision of **C** & α points.



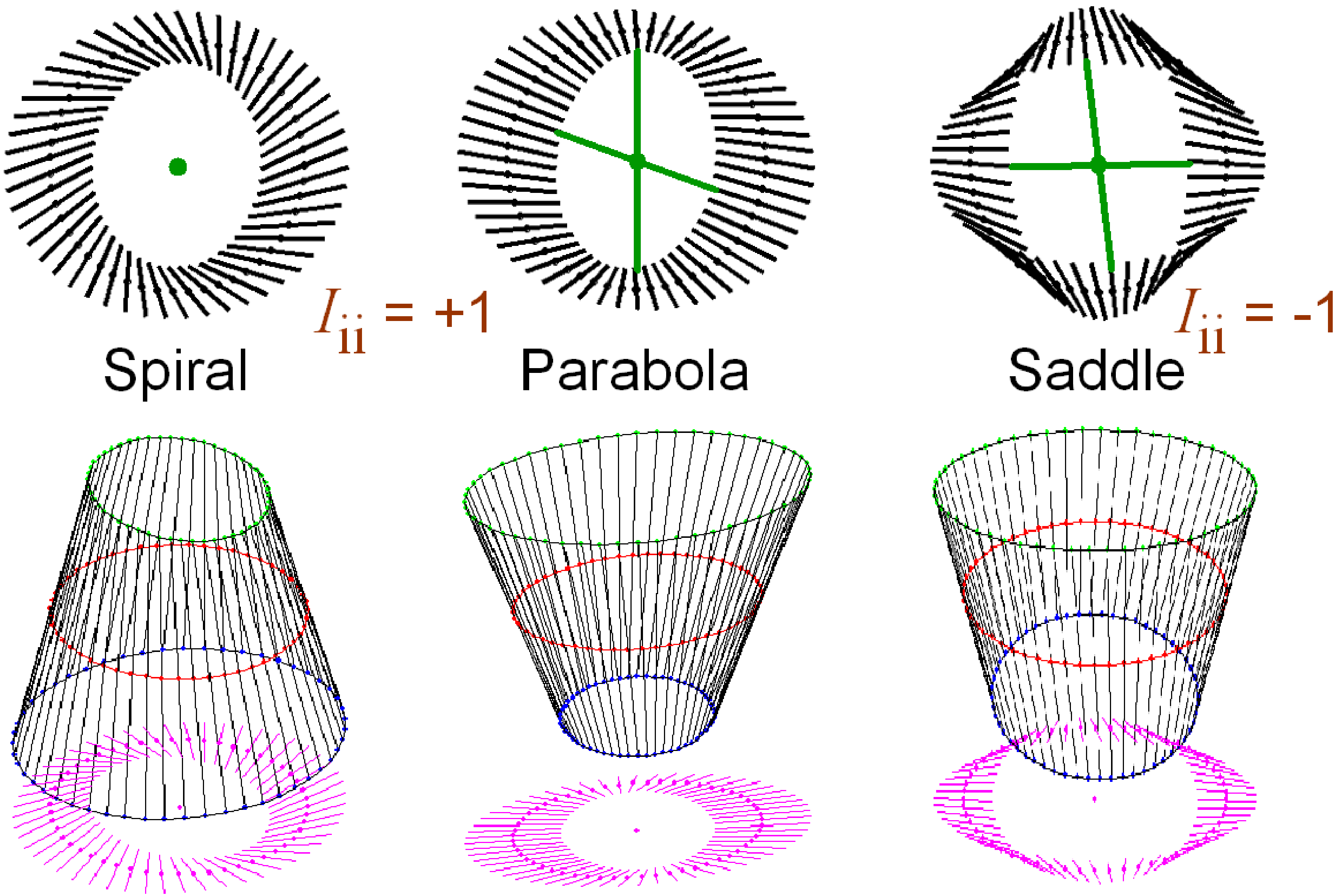
Rotate plane until normal to C circle

3D



Indices: Q_{ij}

3D: α-α, β-β, γ-γ



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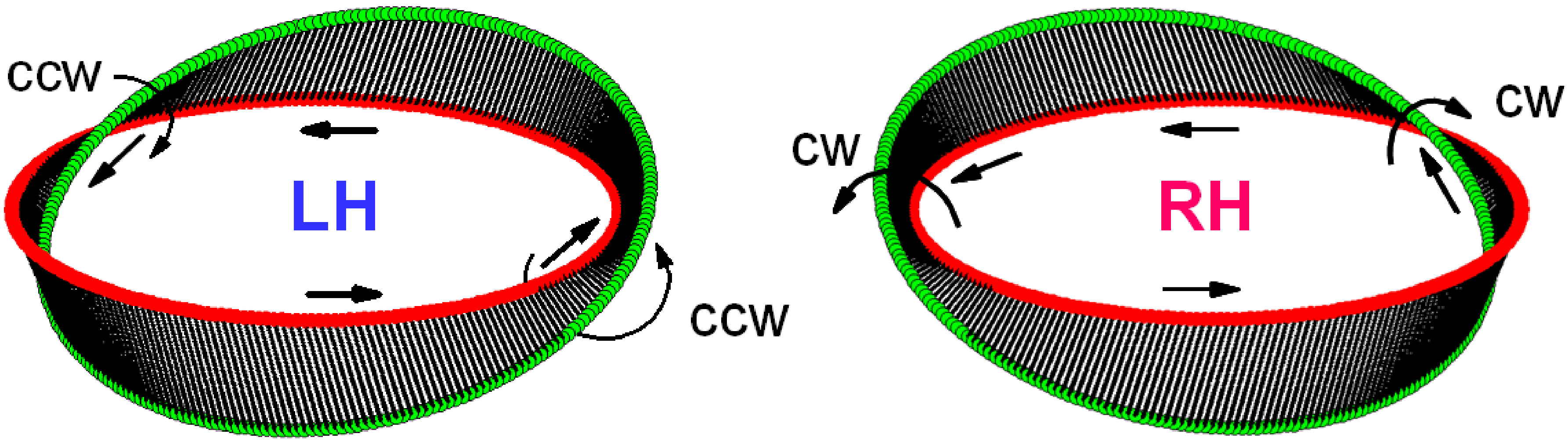
3D Off-diagonals: Ordinary Ellipses

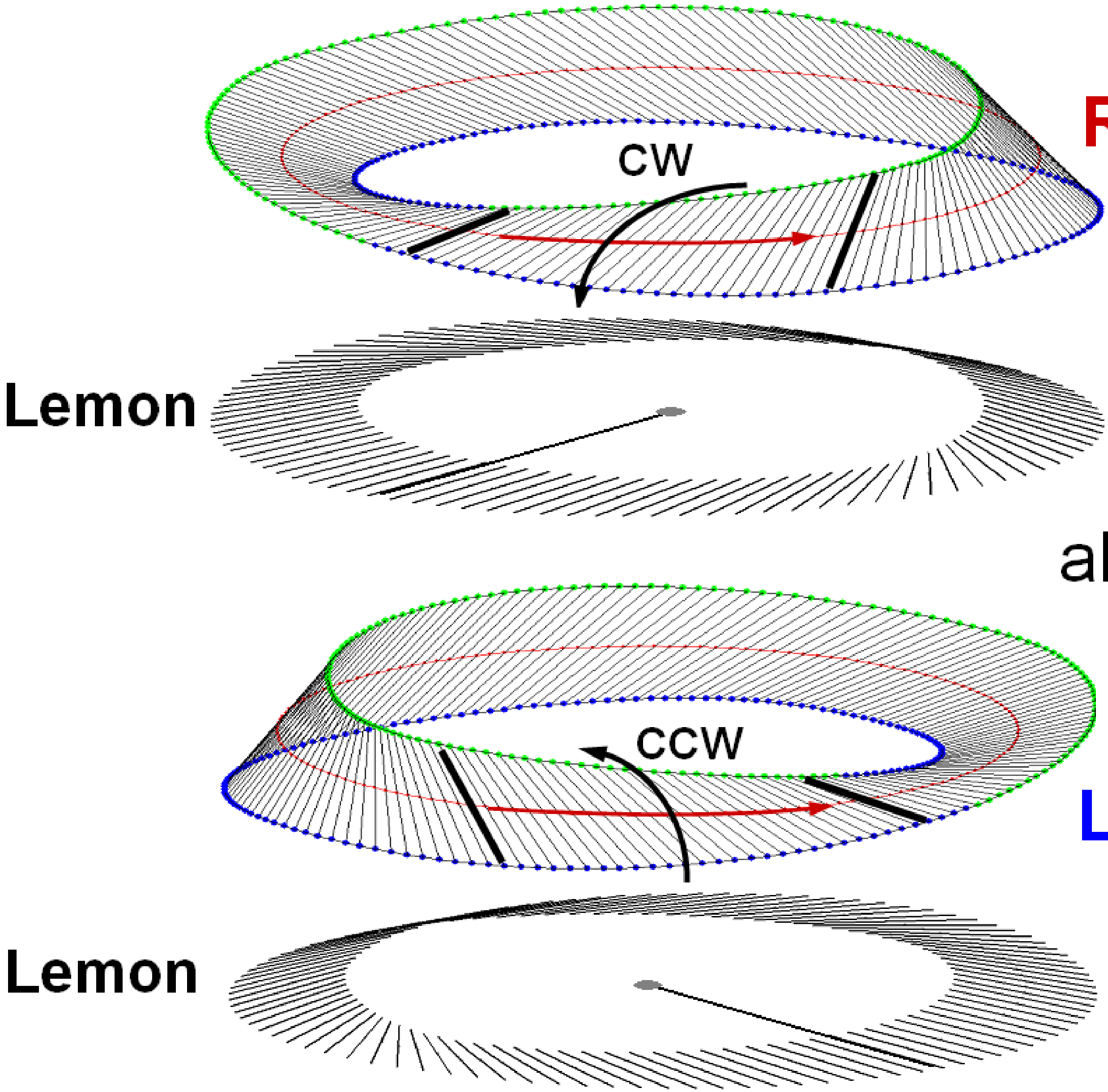
α-β, α-γ, β-α, β-γ, γ-α, γ-β



$\tau_{ij} = +1$ $\tau_{ij} = -1$

Twisted Ribbons $|\tau| = 1$, $\tau_{ij} = \tau_{ji}$

3D: C points – **Möbius Strip**

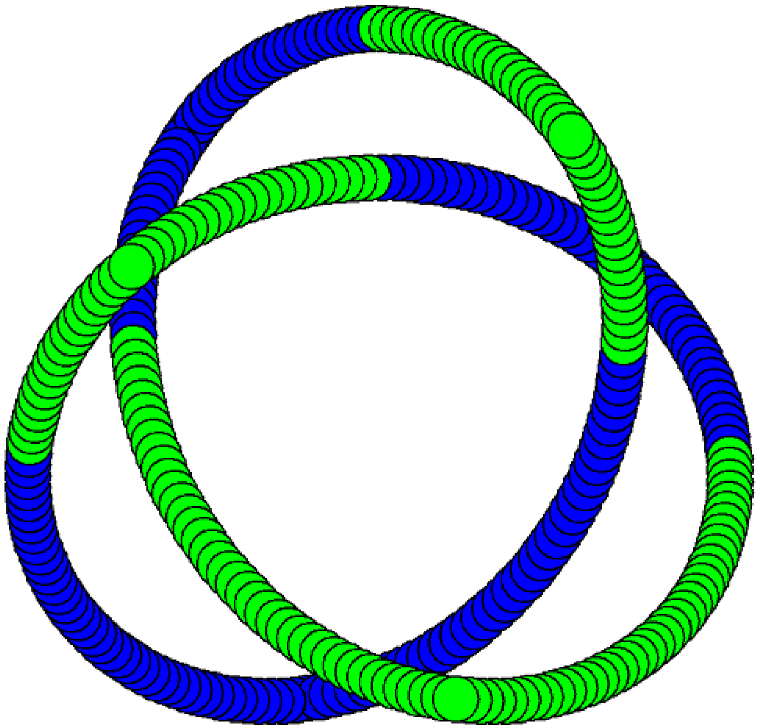


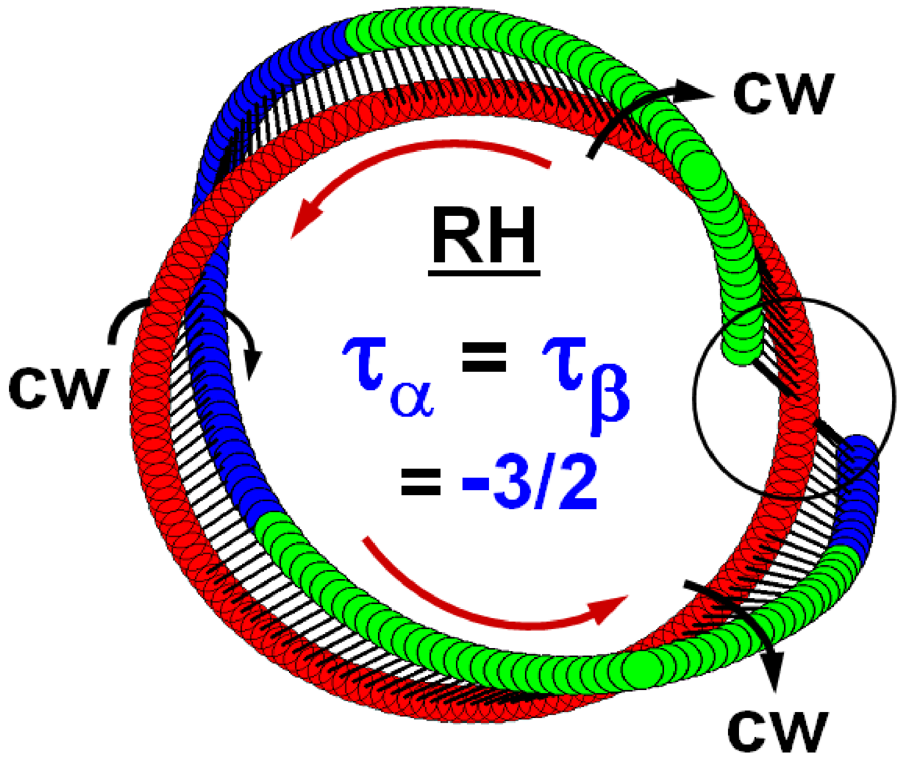
RH: $\tau_\alpha = \tau_\beta = -1/2$

also **Monstar** & **Star**

LH: $\tau_\alpha = \tau_\beta = +1/2$

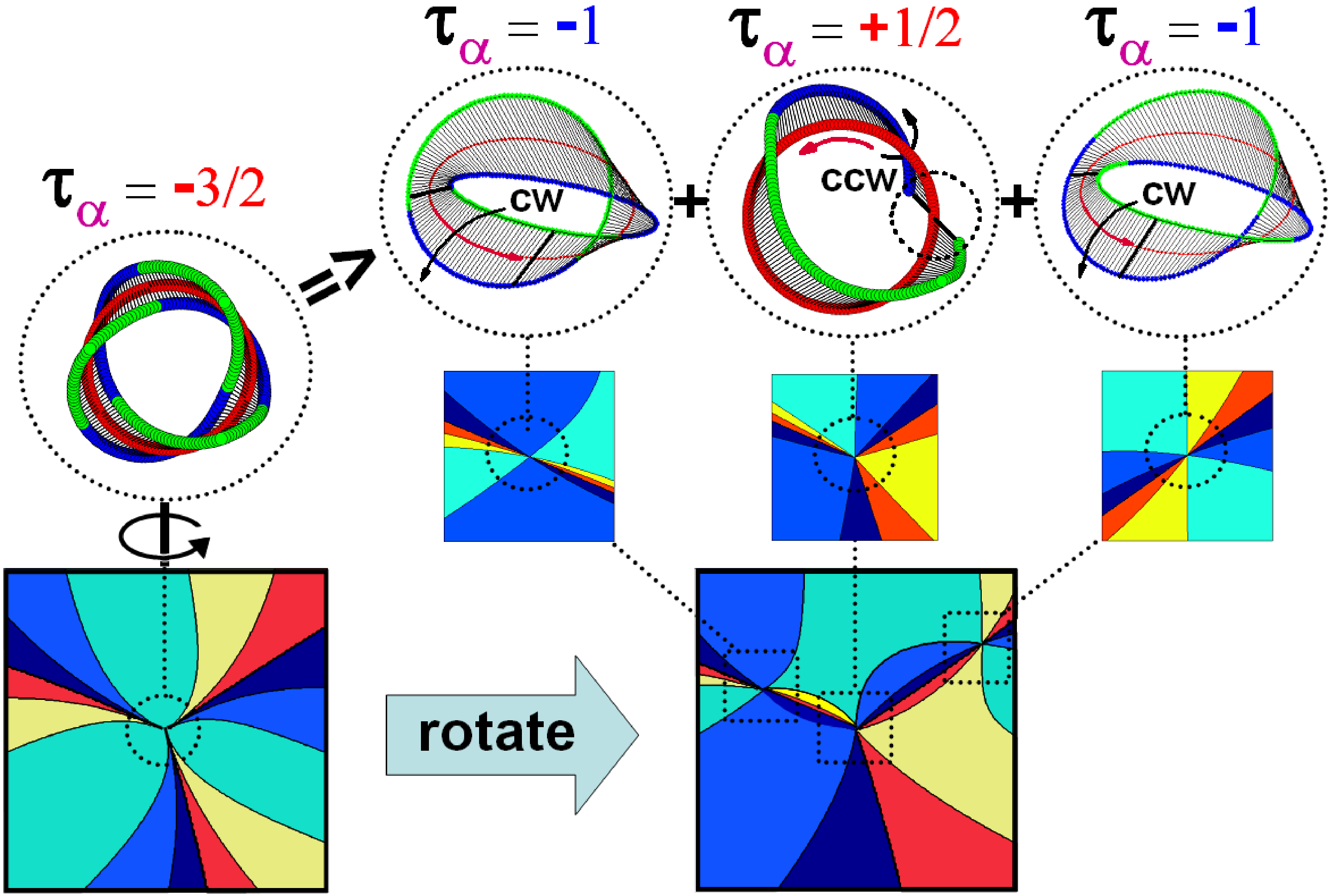
Another C Point Möbius Strip



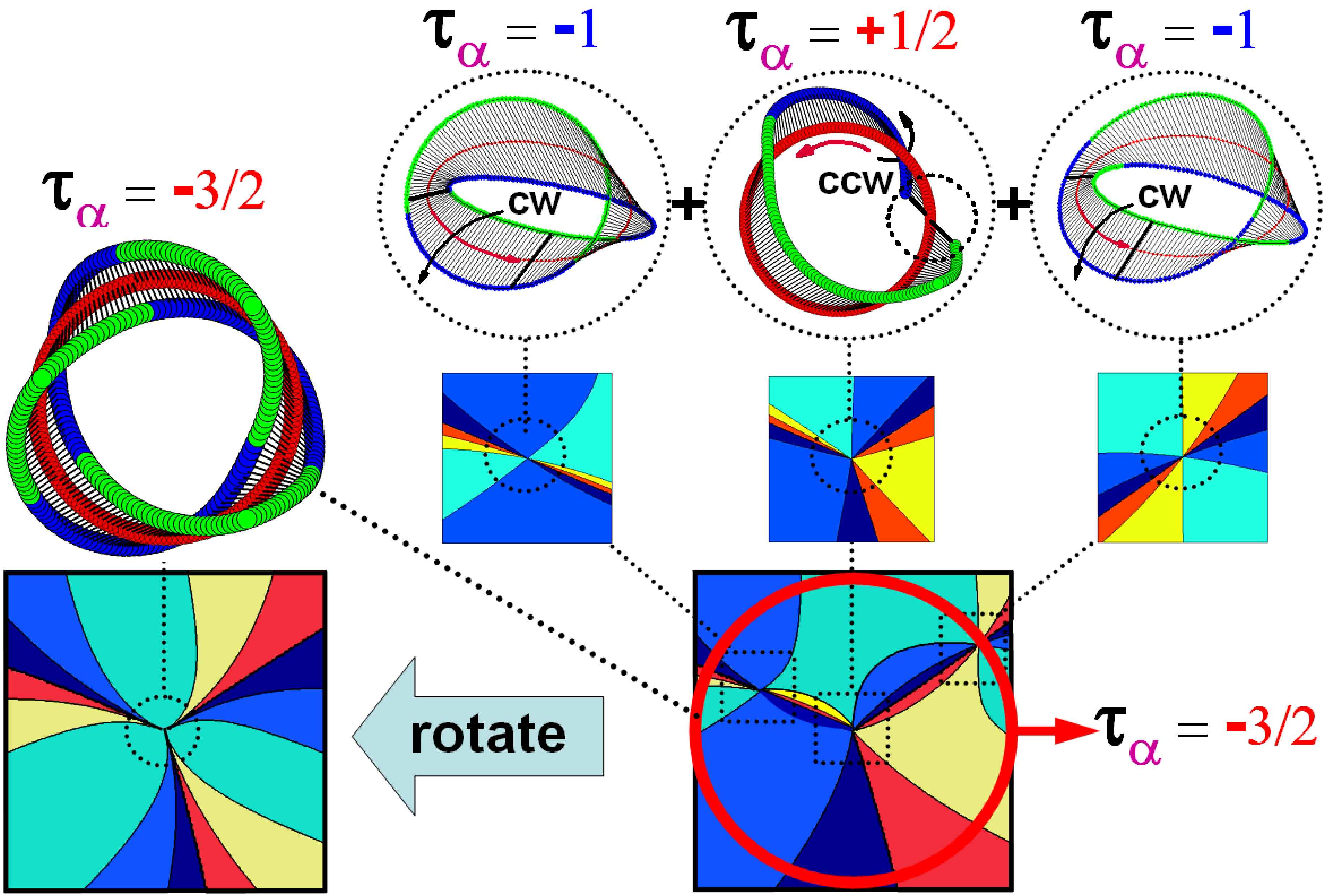


RH/LH (-3/2,+3/2), **monstar & star**

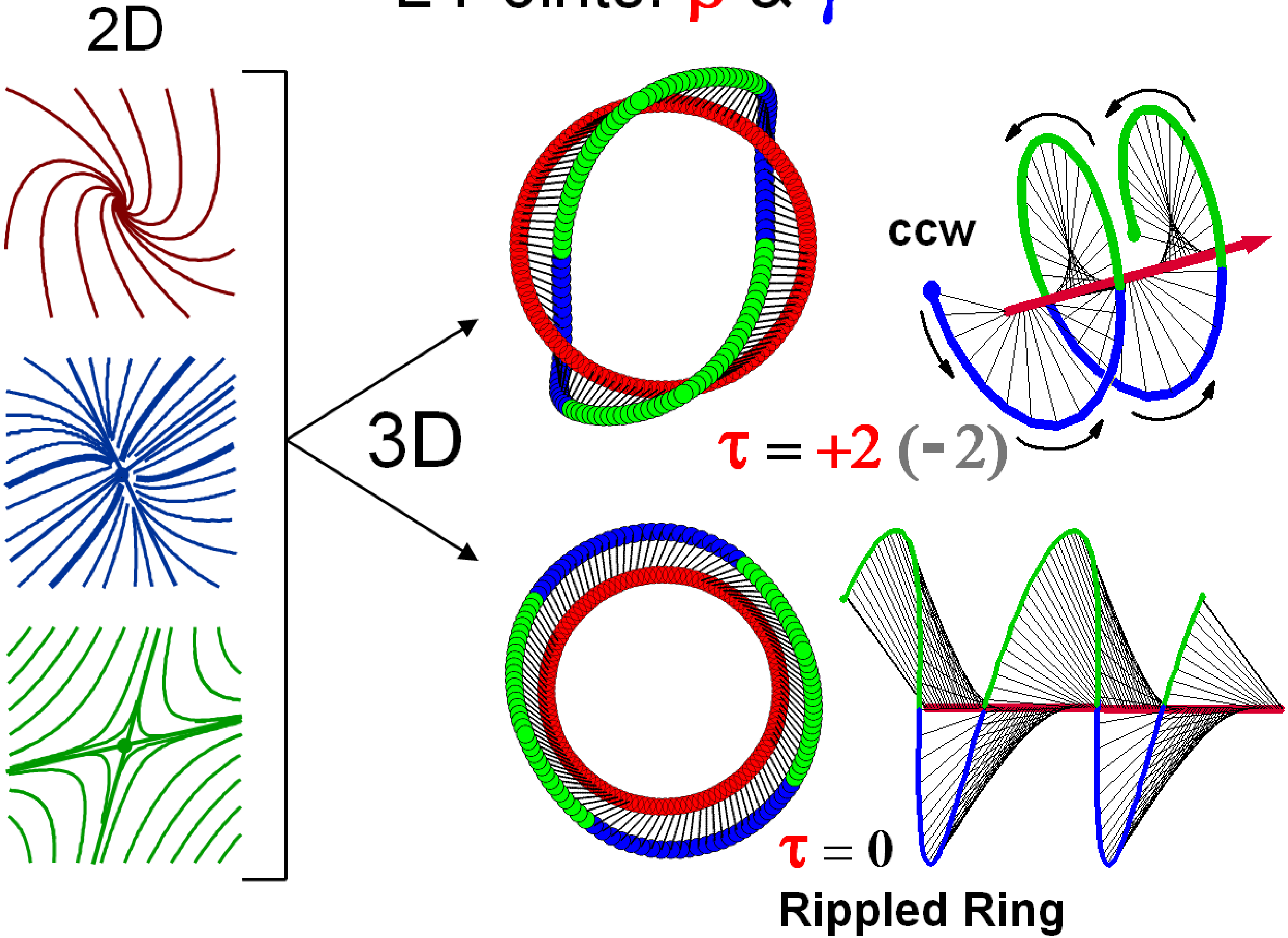
M. S. transformations -3/2 => -1 -1 +1/2



M. S. transformations -3/2 <= -1 -1 +1/2



L Points: β & γ



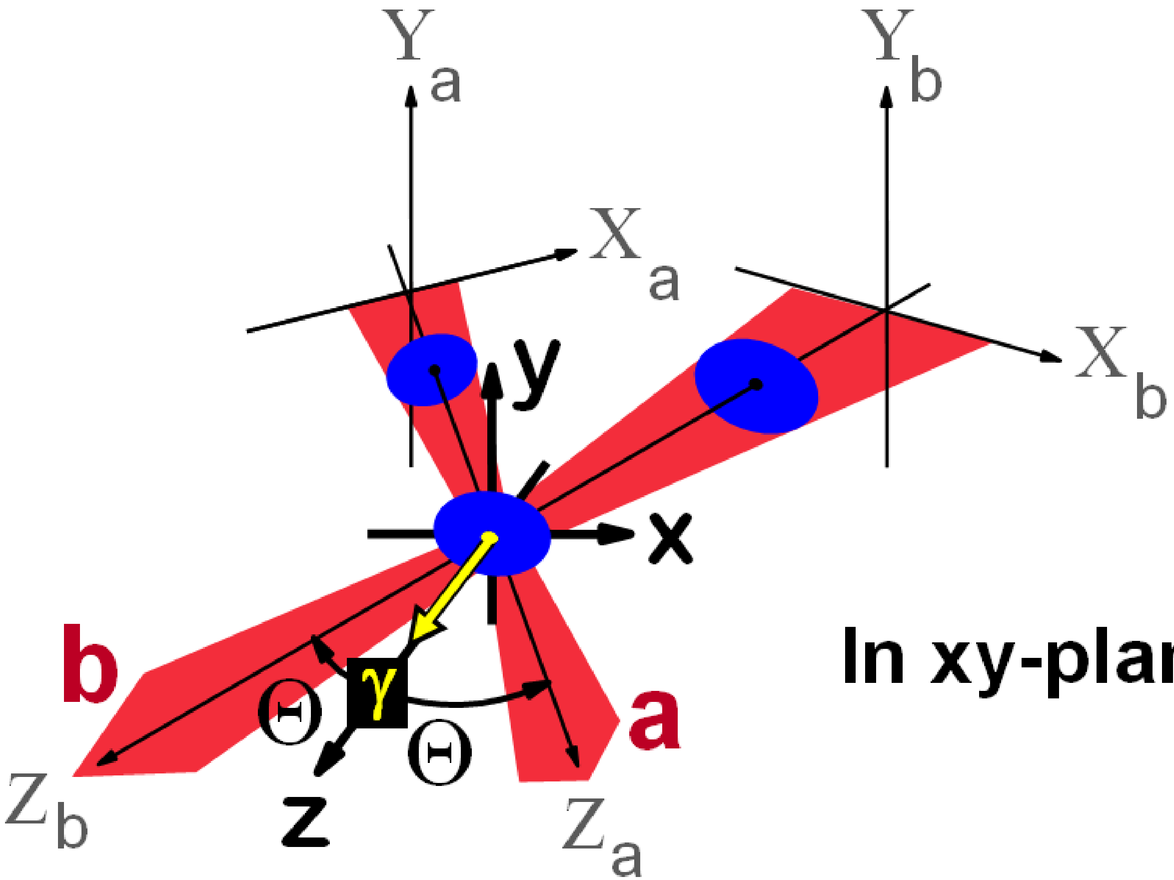
Twist (τ) Summary

$\lvert\tau\rvert$	Object	Location
0	Rippled Ring	L line
1/2	Möbius Strip	C line
1	Twisted Ribbon	Ord. Point
3/2	Möbius Strip	C line
2	Twisted Ribbon	L line

More Twists ? No Problem!

2 Intersecting Gauss-Laguerre GL_0^m Modes

[Gauss-Gouy-… $Ae^{i\chi} r^{m} e^{\pm im\phi}$, $m = 0, 1, 2, \ldots$]



Circ. Polariz.

h = +1 (LH)

h = -1 (RH)

In xy-plane we look at axes

α and β

Twist Number τ

$m_b - m_a$ even => Twisted Ribbon

$$\tau_\alpha \neq \tau_\beta$$

$m_b - m_a$ odd => Möbius Strip

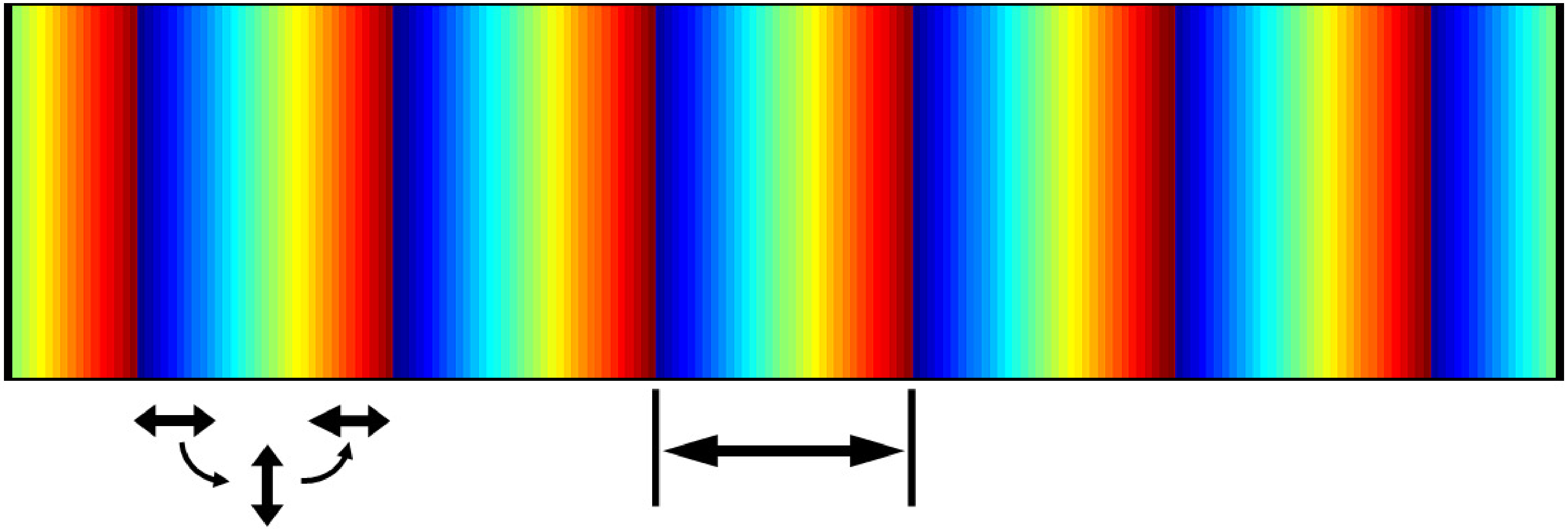
$$\tau_\alpha \equiv \tau_\beta$$

Twist Number τ is substantially *Independent* of m_a , m_b

Polarization Interference

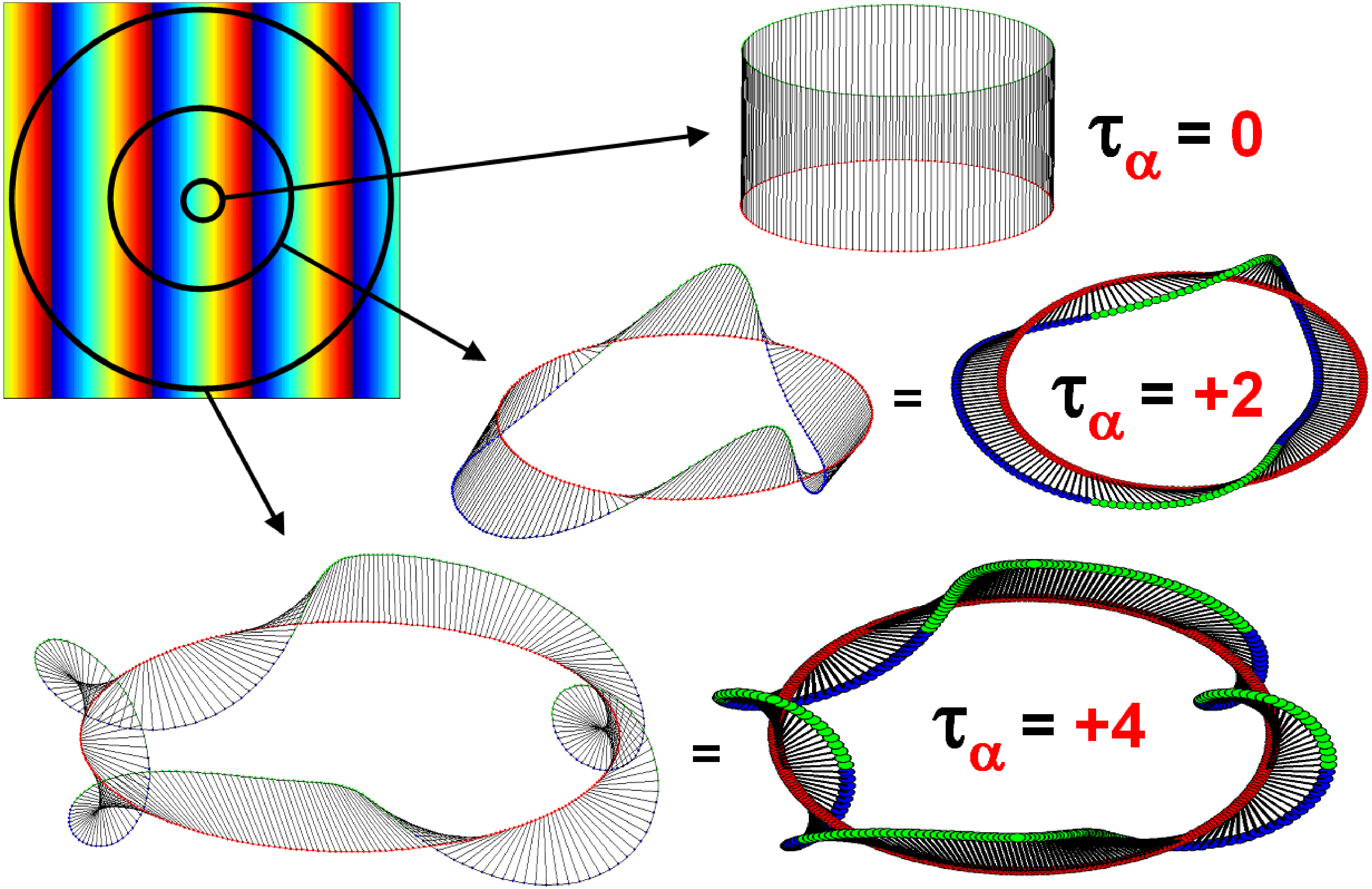
$$h_a \neq h_b,\ h_a = h_b$$

$$\alpha_{xy} = \arctan(\alpha_y, \alpha_x)$$

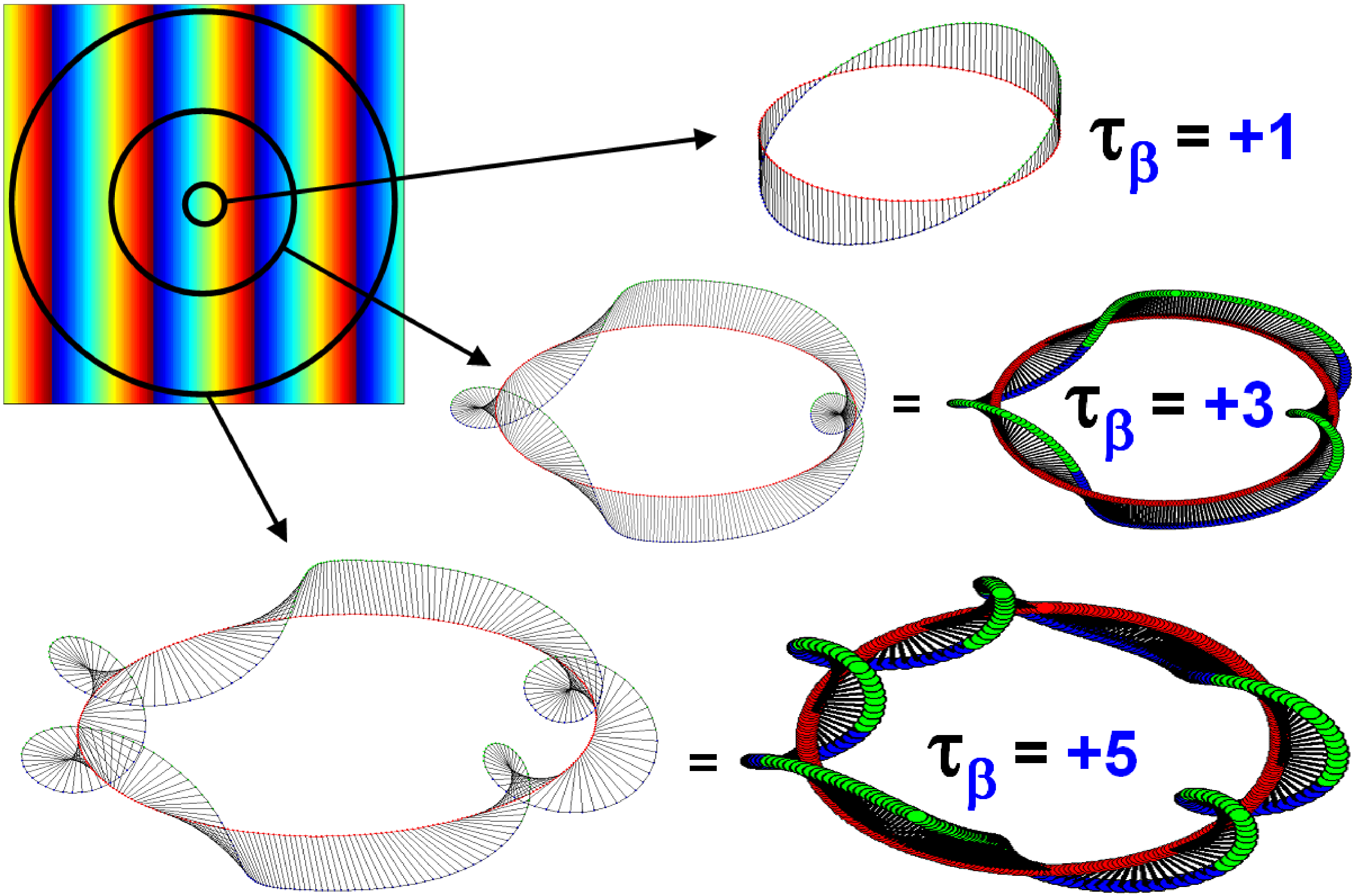


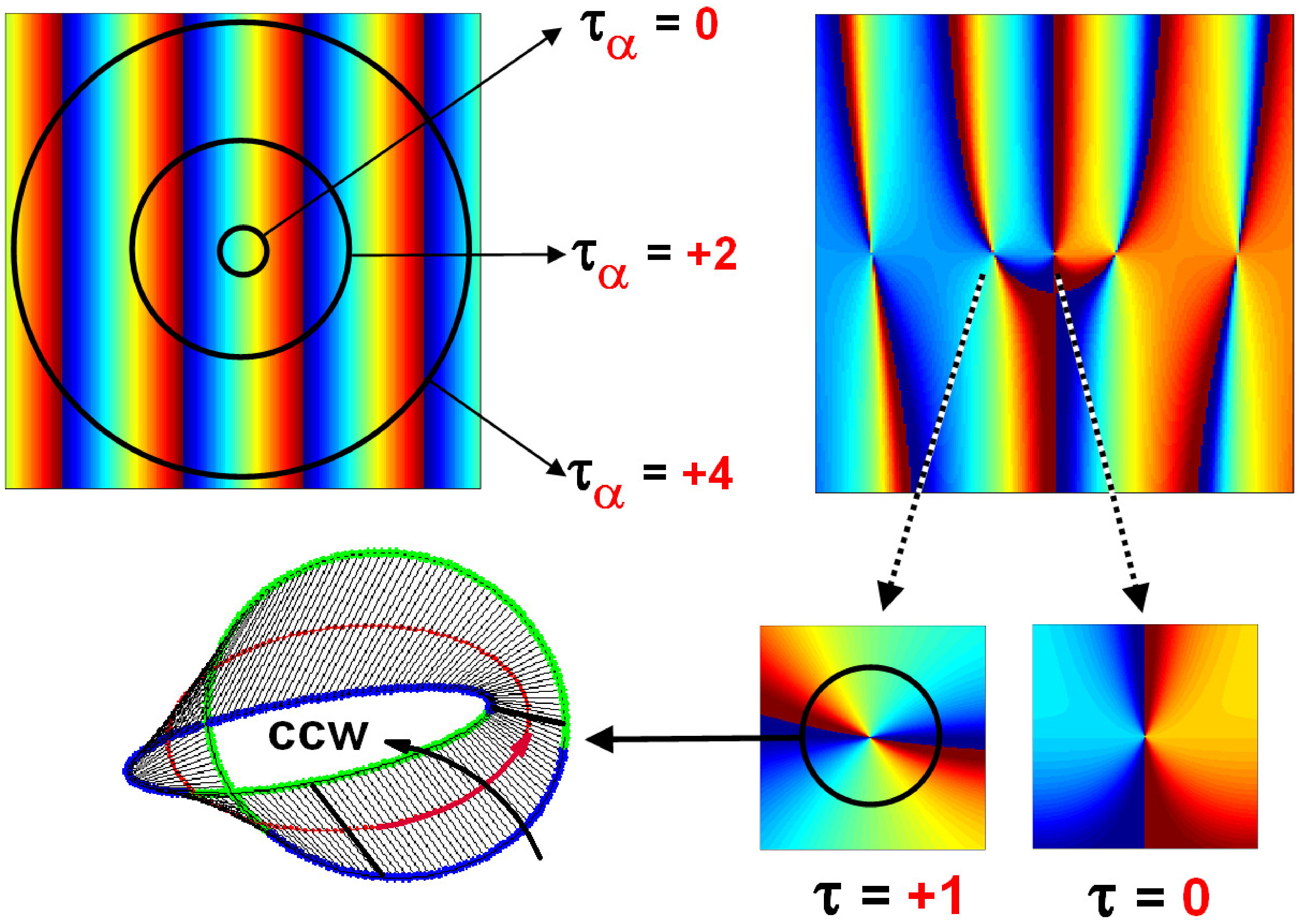
$$\mathcal{L} = \lambda/(2\sin\Theta)$$

Twisted Ribbons, axis α: $m_a = m_b = 0$, $h_a \neq h_b$

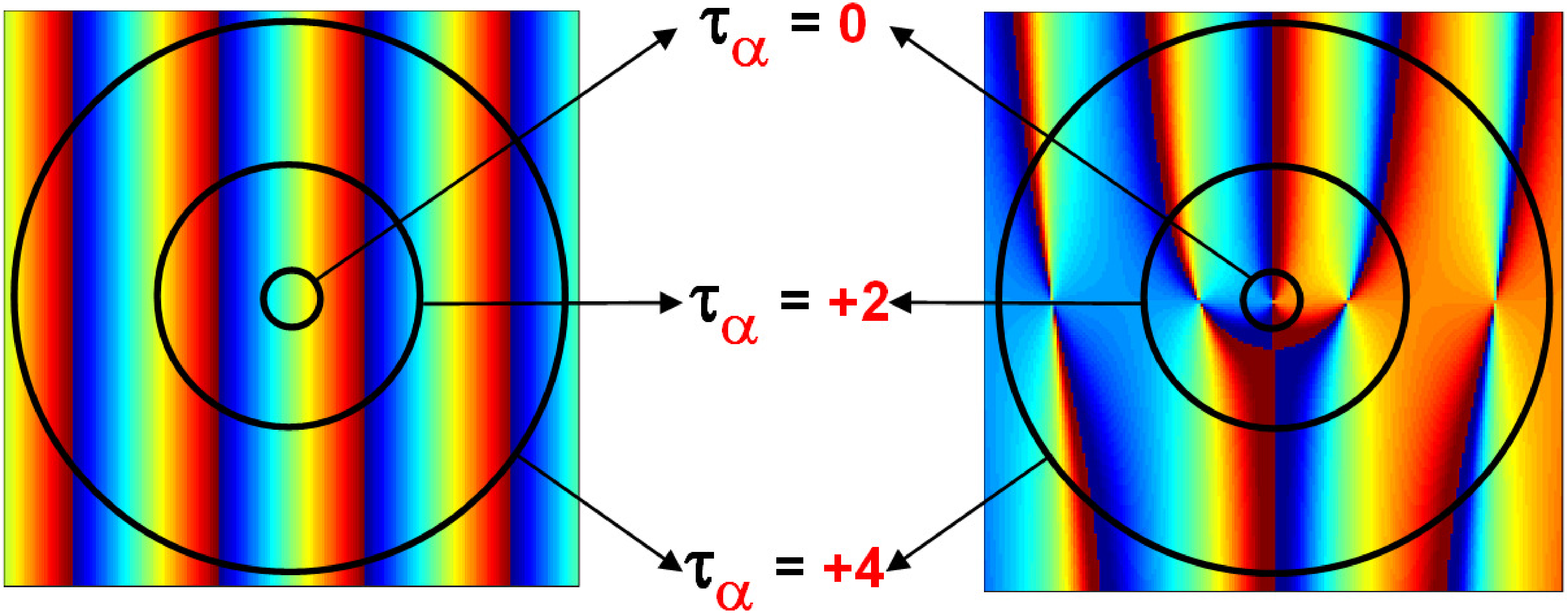


Twisted Ribbons, axis β: $m_a = m_b = 0$, $h_a \neq h_b$





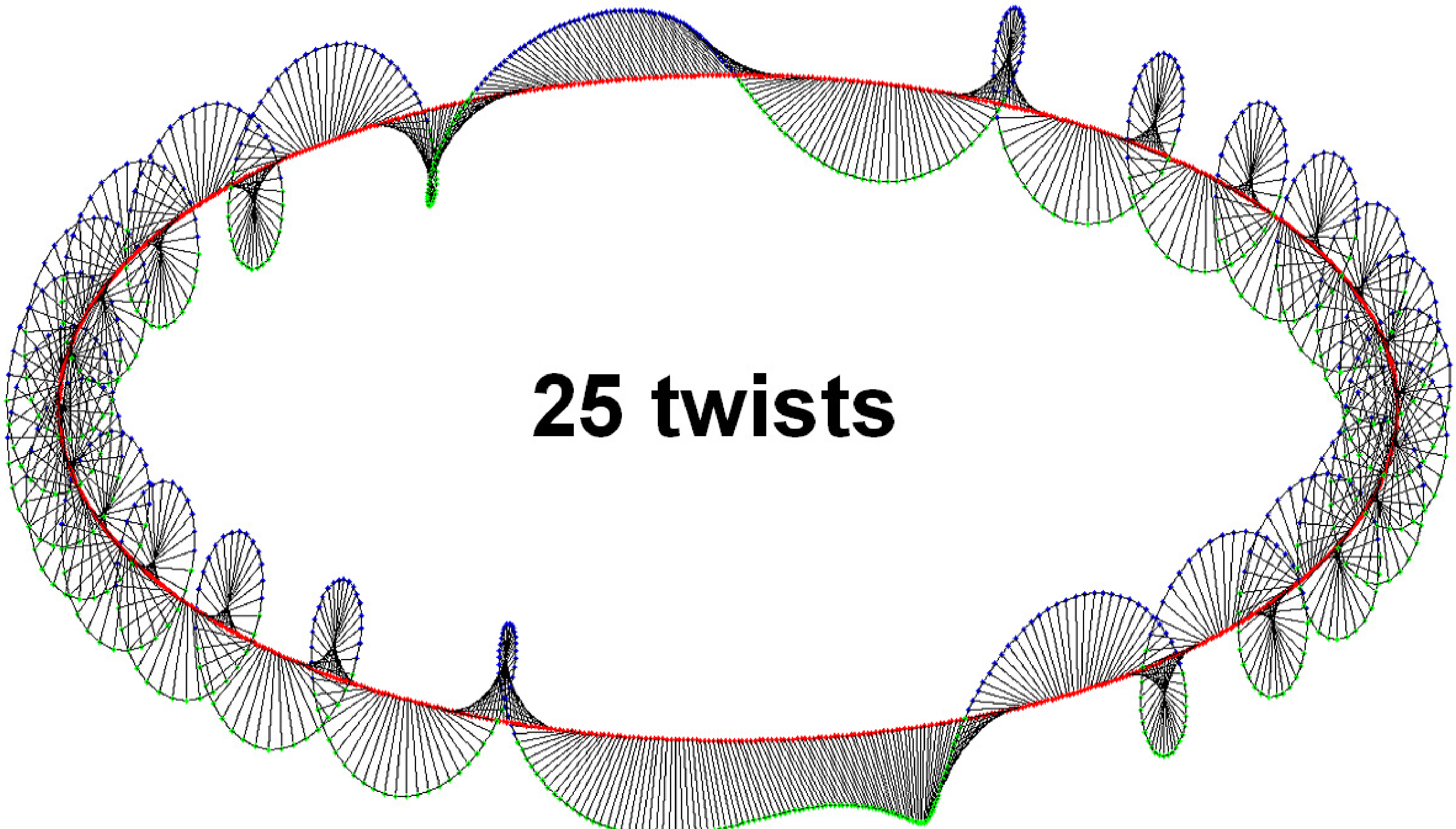
τ α = 0
τ α = +2
τ α = +4
ccw
τ = +1
τ = 0



$$\tau_\alpha = +4 = 0 + 1 + 1 + 1 + 1$$

The twist number τ on a closed path is the sum of all the twist numbers inside the path <= Index Theorem

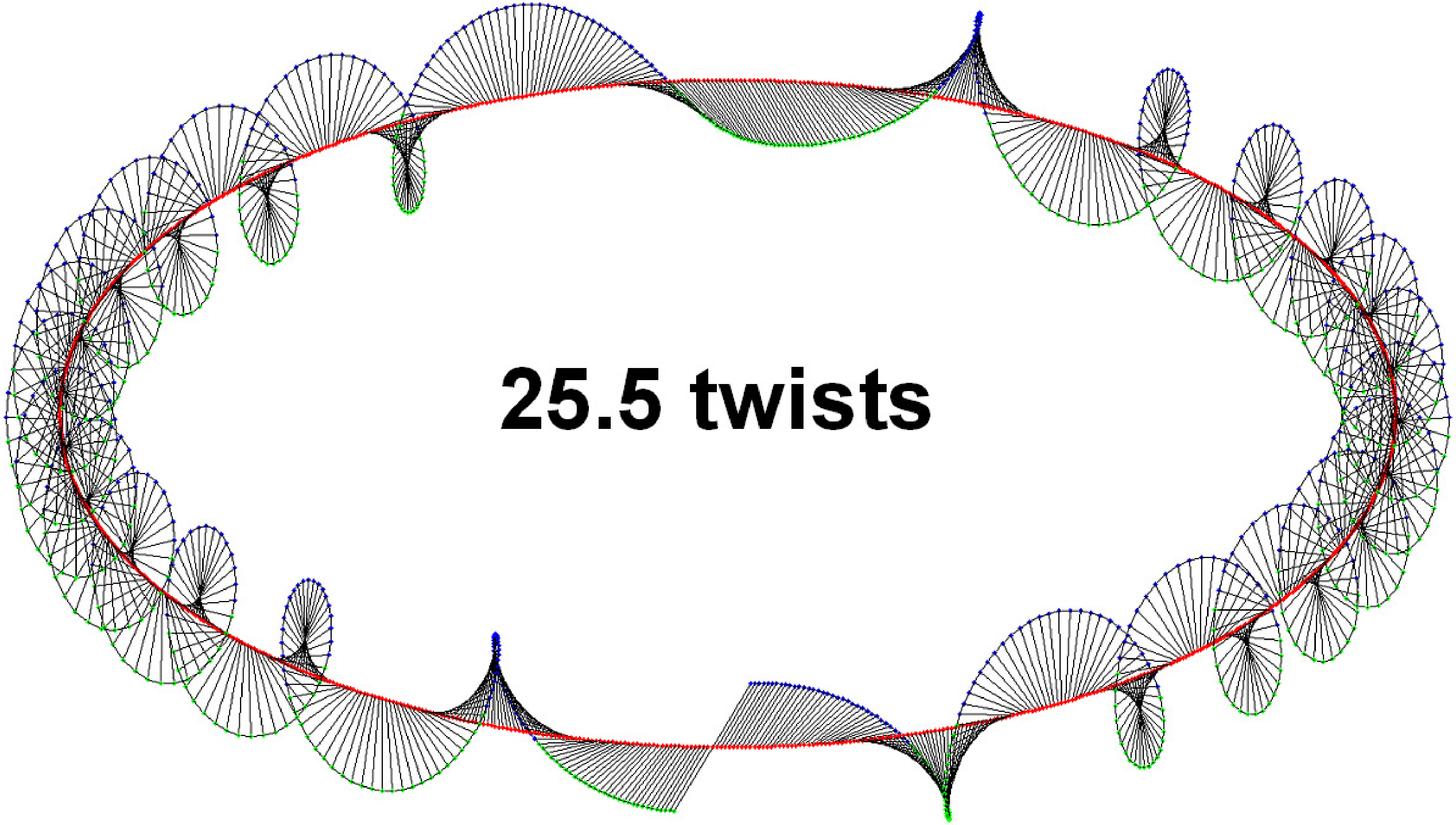
Multi-twist Ribbons and Möbius Strips



25 twists

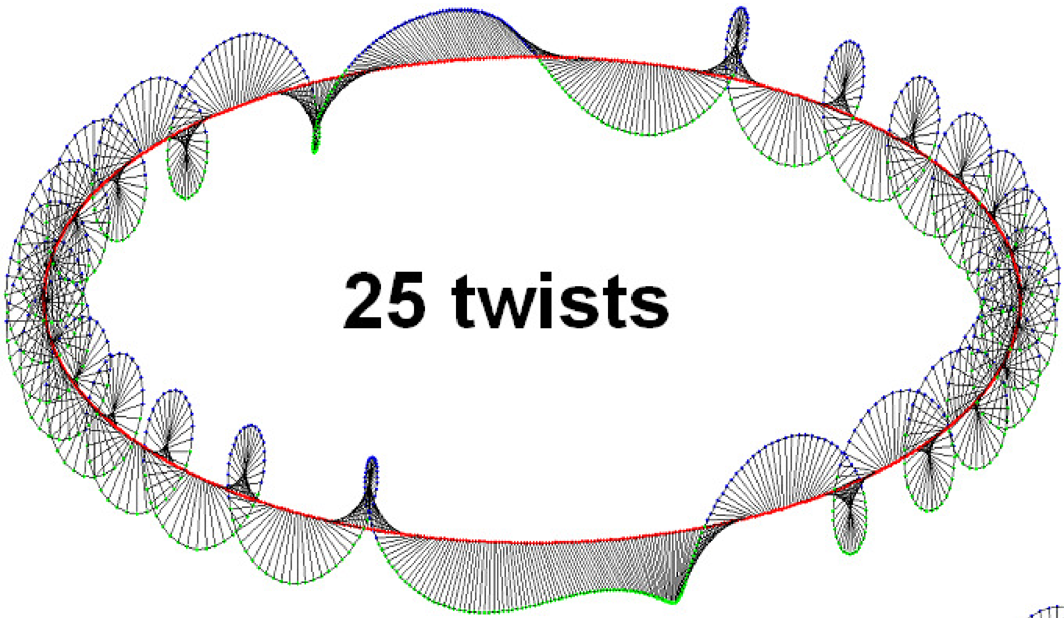
$$m_a = 0, \; m_b = 0$$

Multi-twist Ribbons and Möbius Strips

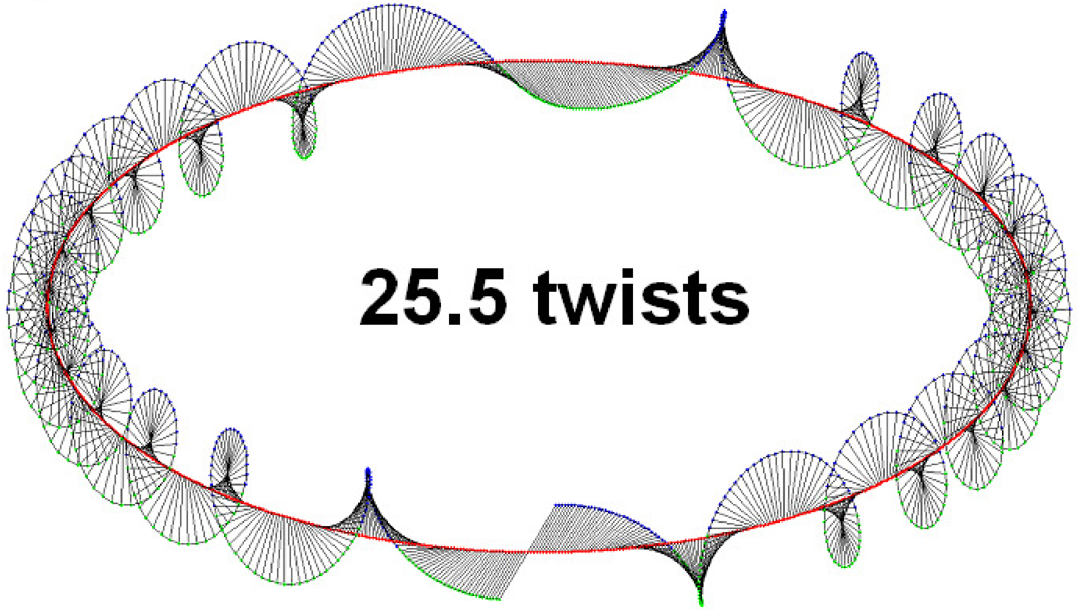


$$m_a = 0, \; m_b = 1$$

Multi-twist Ribbons and Möbius Strips



$m_a = 0, m_b = 0$



$m_a = 0, m_b = 1$